sureg - Zellner's seemingly unrelated regression

Description	Quick start	Menu	Syntax
Options	Remarks and examples	Stored results	Methods and formulas
References	Also see		

Description

Title

sureg fits seemingly unrelated regression models (Zellner 1962; Zellner and Huang 1962; Zellner 1963). The acronyms SURE and SUR are often used for the estimator.

Quick start

Model of y1 and y2 as a function of x1, x2, and x3 sureg (y1 x1 x2 x3) (y2 x1 x2 x3)

Same as above

sureg (y1 y2 = x1 x2 x3)

Same as above, but obtain small-sample statistics and use small-sample adjustment sureg (y1 y2 = x1 x2 x3), small dfk

Also perform Breusch-Pagan test

sureg (y1 y2 = x1 x2 x3), small dfk corr

Model of y1 as a function of x1 and x2 and y2 as a function of x2 and lag of x2 using tsset data sureg (y1 x1 x2) (y2 x2 L.x2)

Menu

Statistics > Linear models and related > Multiple-equation models > Seemingly unrelated regression

Syntax

```
Basic syntax

sureg (depvar<sub>1</sub> varlist<sub>1</sub>) (depvar<sub>2</sub> varlist<sub>2</sub>) ... (depvar<sub>N</sub> varlist<sub>N</sub>)

[if] [in] [weight]

Full syntax

sureg ([eqname<sub>1</sub>:]depvar<sub>1a</sub> [depvar<sub>1b</sub> ...=]varlist<sub>1</sub> [, <u>noconstant</u>])

([eqname<sub>2</sub>:]depvar<sub>2a</sub> [depvar<sub>2b</sub> ...=]varlist<sub>2</sub> [, <u>noconstant</u>])

...

([eqname<sub>N</sub>:]depvar<sub>Na</sub> [depvar<sub>Nb</sub> ...=]varlist<sub>N</sub> [, <u>noconstant</u>])

[if] [in] [weight] [, options]
```

Explicit equation naming (*eqname*:) cannot be combined with multiple dependent variables in an equation specification.

options	Description
Model	
<u>i</u> sure	iterate until estimates converge
<pre><u>const</u>raints(constraints)</pre>	apply specified linear constraints
SE/Robust	
vce(<i>vcetype</i>)	vcetype may be <u>un</u> adjusted, <u>r</u> obust, or <u>cl</u> uster <i>clustvar</i>
df adj.	
<u>sm</u> all	report small-sample statistics
dfk	use small-sample adjustment
dfk2	use alternate adjustment
Reporting	
<u>l</u> evel(#)	set confidence level; default is level(95)
<u>cor</u> r	perform Breusch-Pagan test
<u>nocnsr</u> eport	do not display constraints
display_options	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
Optimization	
optimization_options	control the optimization process; seldom used
<u>noh</u> eader	suppress header table from above coefficient table
<u>not</u> able	suppress coefficient table
<u>coefl</u> egend	display legend instead of statistics

 $varlist_1, \ldots, varlist_N$ may contain factor variables; see [U] 11.4.3 Factor variables. You must have the same levels of factor variables in all equations that have factor variables.

depvars and the varlists may contain time-series operators; see [U] 11.4.4 Time-series varlists.

bootstrap, by, collect, fp, jackknife, rolling, and statsby are allowed; see [U] 11.1.10 Prefix commands. Weights are not allowed with the bootstrap prefix; see [R] bootstrap.

aweights are not allowed with the jackknife prefix; see [R] jackknife.

aweights and fweights are allowed; see [U] 11.1.6 weight.

noheader, notable, and coeflegend do not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

Options

Model

isure specifies that sureg iterate over the estimated disturbance covariance matrix and parameter estimates until the parameter estimates converge. Under seemingly unrelated regression, this iteration converges to the maximum likelihood results. If this option is not specified, sureg produces two-step estimates.

constraints(constraints); see [R] Estimation options.

SE/Robust

vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (unadjusted), that are robust to some kinds of misspecification (robust), and that allow for intragroup correlation (cluster clustvar); see [R] vce_option.

vce(unadjusted), the default, specifies that an unadjusted (nonrobust) VCE matrix be used; this results in efficient estimates when assuming homoskedasticity.

df adj.

- small specifies that small-sample statistics be computed. It shifts the test statistics from χ^2 and z statistics to F statistics and t statistics. Although the standard errors from each equation are computed using the degrees of freedom for the equation, the degrees of freedom for the t statistics are all taken to be those for the first equation.
- dfk specifies the use of an alternative divisor in computing the covariance matrix for the equation residuals. As an asymptotically justified estimator, sureg by default uses the number of sample observations (n) as a divisor. When the dfk option is set, a small-sample adjustment is made and the divisor is taken to be $\sqrt{(n-k_i)(n-k_j)}$, where k_i and k_j are the number of parameters in equations i and j, respectively.
- dfk2 specifies the use of an alternative divisor in computing the covariance matrix for the equation residuals. When the dfk2 option is set, the divisor is taken to be the mean of the residual degrees of freedom from the individual equations.

Reporting

level(#); see [R] Estimation options.

corr displays the correlation matrix of the residuals between equations and performs a Breusch-Pagan test for independent equations; that is, the disturbance covariance matrix is diagonal.

nocnsreport; see [R] Estimation options.

display_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(#), fvwrapon(style), cformat(% fmt), pformat(% fmt), sformat(% fmt), and nolstretch; see [R] Estimation options.

Optimization

optimization_options control the iterative process that minimizes the sum of squared errors when isure is specified. These options are seldom used.

iterate(#) specifies the maximum number of iterations. When the number of iterations equals #, the optimizer stops and presents the current results, even if the convergence tolerance has not been reached. The default is the number set using set maxiter, which is 300 by default.

trace adds to the iteration log a display of the current parameter vector.

log and nolog specify whether to display the iteration log. The iteration log is displayed by default unless you used set iterlog off to suppress it; see set iterlog in [R] set iter.

tolerance(#) specifies the tolerance for the coefficient vector. When the relative change in the coefficient vector from one iteration to the next is less than or equal to #, the optimization process is stopped. tolerance(1e-6) is the default.

The following options are available with sureg but are not shown in the dialog box:

noheader suppresses display of the header reporting F statistics, R^2 , and root mean squared error above the coefficient table.

notable suppresses display of the coefficient table.

coeflegend; see [R] Estimation options.

Remarks and examples

stata.com

Seemingly unrelated regression models are so called because they appear to be joint estimates from several regression models, each with its own error term. The regressions are related because the (contemporaneous) errors associated with the dependent variables may be correlated. Chapter 6 of Cameron and Trivedi (2022) contains a discussion of the seemingly unrelated regression model and the feasible generalized least-squares estimator underlying it.

Example 1

When we fit models with the same set of right-hand-side variables, the seemingly unrelated regression results (in terms of coefficients and standard errors) are the same as fitting the models separately (using, say, regress). The same is true when the models are nested. Even in such cases, sureg is useful when we want to perform joint tests. For instance, let us assume that we think

$$\begin{split} \texttt{price} &= \beta_0 + \beta_1\texttt{foreign} + \beta_2\texttt{length} + u_1 \\ \texttt{weight} &= \gamma_0 + \gamma_1\texttt{foreign} + \gamma_2\texttt{length} + u_2 \end{split}$$

Because the models have the same set of explanatory variables, we could estimate the two equations separately. Yet, we might still choose to estimate them with sureg because we want to perform the joint test $\beta_1 = \gamma_1 = 0$.

We use the small and dfk options to obtain small-sample statistics comparable with regress or mvreg.

```
. use https://www.stata-press.com/data/r18/auto
(1978 automobile data)
. sureg (price foreign length) (weight foreign length), small dfk
```

Seemingly unrelated regression

Equation	Obs	Params	RMSE	"R-squar	ed"	F P>I
price	74	2	2474.593	0.31	.54 16.	35 0.000
weight	74	2	250.2515	0.89	992 316.	54 0.000
	r					
	Coefficient	Std. err.	t	P> t	[95% conf	. interval
price						
foreign	2801.143	766.117	3.66	0.000	1286.674	4315.61
length	90.21239	15.83368	5.70	0.000	58.91219	121.5120
_cons	-11621.35	3124.436	-3.72	0.000	-17797.77	-5444.93
weight						
foreign	-133.6775	77.47615	-1.73	0.087	-286.8332	19.4782
length	31.44455	1.601234	19.64	0.000	28.27921	34.60989
_cons	-2850.25	315.9691	-9.02	0.000	-3474.861	-2225.639

Note: Small-sample degrees-of-freedom adjustment applied when estimating covariance matrix of residuals.

These two equations have a common set of regressors, and we could have used a shorthand syntax to specify the equations:

. sureg (price weight = foreign length), small dfk

Here the results presented by sureg are the same as if we had estimated the equations separately:

- . regress price foreign length (output omitted)
- . regress weight foreign length (output omitted)

There is, however, a difference. We have allowed u_1 and u_2 to be correlated and have estimated the full variance-covariance matrix of the coefficients. sureg has estimated the correlations, but it does not report them unless we specify the corr option. We did not remember to specify corr when we fit the model, but we can redisplay the results:

The notable and noheader options prevented sureg from redisplaying the header and coefficient tables. We find that, for the same cars, the correlation of the residuals in the price and weight equations is 0.5840 and that we can reject the hypothesis that this correlation is zero.

We can test that the coefficients on foreign are jointly zero in both equations—as we set out to do—by typing test foreign; see [R] test. When we type a variable without specifying the equation, that variable is tested for zero in all equations in which it appears:

. test foreign
(1) [price]foreign = 0
(2) [weight]foreign = 0
F(2, 142) = 17.99
Prob > F = 0.0000

4

Example 2

When the models do not have the same set of explanatory variables and are not nested, sureg may lead to more efficient estimates than running the models separately as well as allowing joint tests. This time, let us assume that we believe

price =
$$\beta_0 + \beta_1$$
foreign + β_2 mpg + β_3 displ + u_1
weight = $\gamma_0 + \gamma_1$ foreign + γ_2 length + u_2

To fit this model, we type

. sureg	(price for	eign mpg	displ)	(weight	foreign	length),	corr
Seemingly	y unrelate	d regres	sion				

Equation	Obs	Params	RMSE	"R-squa	red"	chi2	P>chi2
price	74	3	2165.321	0.4	537	49.64	0.0000
weight	74	2	245.2916	0.8	990	661.84	0.0000
	r						
	Coefficient	Std. err.	z	P> z	[95%	conf.	interval]
price							
foreign	3058.25	685.7357	4.46	0.000	1714	.233	4402.267
mpg	-104.9591	58.47209	-1.80	0.073	-219.	5623	9.644042
displacement	18.18098	4.286372	4.24	0.000	9.77	9842	26.58211
cons	3904.336	1966.521	1.99	0.047	50.	0263	7758.645
weight							
foreign	-147.3481	75.44314	-1.95	0.051	-295.3	2139	.517755
length	30.94905	1.539895	20.10	0.000	27.9	3091	33.96718
_cons	-2753.064	303.9336	-9.06	0.000	-3348	.763	-2157.365

Correlation matrix of residuals:

price weight price 1.0000 weight 0.3285 1.0000 Breusch-Pagan test of independence: chi2(1) = 7.984, Pr = 0.0047 In comparison, if we had fit the price model separately,

		F-				
Source	SS	df	MS	Number of ob	s =	74
				F(3, 70)	=	20.13
Model	294104790	3	98034929.9	Prob > F	=	0.0000
Residual	340960606	70	4870865.81	R-squared	=	0.4631
				Adj R-square	d =	0.4401
Total	635065396	73	8699525.97		=	2207
price	Coefficient	Std. err.	t	P> t [95%	conf.	interval]
foreign	3545.484	712.7763	4.97	0.000 2123.	897	4967.072
mpg	-98.88559	63.17063	-1.57	0.122 -224.8	754	27.10426
displacement	22.40416	4.634239	4.83	0.000 13.16	146	31.64686
_cons	2796.91	2137.873	1.31	0.195 -1466.	943	7060.763

. regress price foreign mpg displ

The coefficients are slightly different, but the standard errors are uniformly larger. This would still be true if we specified the dfk option to make a small-sample adjustment to the estimated covariance of the disturbances. 4

Technical note

Constraints can be applied to SURE models using Stata's standard syntax for constraints. For a general discussion of constraints, see [R] constraint; for examples similar to seemingly unrelated regression models, see [R] reg3.

Stored results

sureg stores the following in e():

Scalars

e(N)	number of observations
e(k)	number of parameters
e(k_eq)	number of equations in e(b)
e(mss_#)	model sum of squares for equation #
e(df_m#)	model degrees of freedom for equation #
e(rss_#)	residual sum of squares for equation #
e(df_r)	residual degrees of freedom
e(r2_#)	R^2 for equation #
e(F_#)	F statistic for equation # (small only)
e(rmse_#)	root mean squared error for equation #
e(dfk2_adj)	divisor used with VCE when dfk2 specified
e(11)	log likelihood
e(N_clust)	number of clusters
e(chi2_#)	χ^2 for equation #
e(p_#)	<i>p</i> -value for equation #
e(cons_#)	1 if equation # has a constant, 0 otherwise
e(chi2_bp)	Breusch–Pagan χ^2
e(df_bp)	degrees of freedom for Breusch–Pagan χ^2 test
e(rank)	rank of e(V)
e(ic)	number of iterations

Macros	
e(cmd)	sureg
e(cmdline)	command as typed
e(method)	sure or isure
e(depvar)	names of dependent variables
e(exog)	names of exogenous variables
e(eqnames)	names of equations
e(wtype)	weight type
e(wexp)	weight expression
e(corr)	correlation structure
e(small)	small, if specified
e(dfk)	dfk or dfk2, if specified
e(clustvar)	name of cluster variable
e(vce)	vcetype specified in vce()
e(vcetype)	title used to label Std. err.
e(properties)	b V
e(predict)	program used to implement predict
e(marginsok)	predictions allowed by margins
e(marginsnotok)	predictions disallowed by margins
e(marginsdefault)	default predict() specification for margins
e(asbalanced)	factor variables fvset as asbalanced
e(asobserved)	factor variables fvset as asobserved
Matrices	
e(b)	coefficient vector
e(Cns)	constraints matrix
e(Sigma)	$\widehat{\Sigma}$, covariance matrix of residuals
e(V)	variance–covariance matrix of the estimators
	variance covariance matrix of the estimators
Functions	1
e(sample)	marks estimation sample

In addition to the above, the following is stored in r():

Matrices

r(table) matrix containing the coefficients with their standard errors, test statistics, *p*-values, and confidence intervals

Note that results stored in r() are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.

Methods and formulas

sureg uses the asymptotically efficient, feasible, generalized least-squares algorithm described in Greene (2018, 328–339). The computing formulas are given on page 328–335.

The R^2 reported is the percent of variance explained by the predictors. It may be used for descriptive purposes, but R^2 is not a well-defined concept when GLS is used.

sureg will refuse to compute the estimators if the same equation is named more than once or the covariance matrix of the residuals is singular.

The Breusch and Pagan (1980) χ^2 statistic—a Lagrange multiplier statistic—is given by

$$\lambda = T \sum_{m=2}^{M} \sum_{n=1}^{m-1} r_{mn}^2$$

where r_{mn} is the estimated correlation between the residuals of the M equations and T is the number of observations. It is distributed as χ^2 with M(M-1)/2 degrees of freedom.

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using vce(robust) and vce(cluster *clustvar*), respectively. See [P] <u>robust</u>, particularly *Introduction* and *Methods and formulas*.

Arnold Zellner (1927–2010) was born in New York. He studied physics at Harvard and economics at Berkeley, and then he taught economics at the Universities of Washington and Wisconsin before settling in Chicago in 1966. Among his many major contributions to econometrics and statistics are his work on seemingly unrelated regression, three-stage least squares, and Bayesian econometrics.

References

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Also see

- [R] sureg postestimation Postestimation tools for sureg
- [R] **nlsur** Estimation of nonlinear systems of equations
- [R] reg3 Three-stage estimation for systems of simultaneous equations
- [R] regress Linear regression
- [MV] **mvreg** Multivariate regression
- [SEM] Example 12 Seemingly unrelated regression
- [SEM] Intro 5 Tour of models
- [TS] dfactor Dynamic-factor models

[U] 20 Estimation and postestimation commands

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