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boxcox — Box–Cox regression models

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Also see

Description

boxcox finds the maximum likelihood estimates of the parameters of the Box-Cox transform, the coefficients on the independent variables, and the standard deviation of the normally distributed errors. Any *depvar* or *indepvars* to be transformed must be strictly positive. Options can be used to control which variables remain untransformed.

Quick start

Box-Cox transform of y in a model of y as a function of x1 boxcox y x1

Same as above

boxcox y x1, model(lhsonly)

Likelihood-ratio test for each scale-variant parameter boxcox y x1, lrtest

Different transform for each side and adding covariates x2 and x3 boxcox y x1 x2 x3, model(theta)

Same transform for both sides, and include x3 as an untransformed variable transformation boxcox y x1 x2, model(lambda) notrans(x3)

Menu

Statistics > Linear models and related > Box-Cox regression

Syntax

```
boxcox depvar [indepvars] [if] [in] [weight] [, options]
```

options	Description
Model	
<u>nocons</u> tant	suppress constant term
$model(\underline{lhs}only)$	left-hand-side Box-Cox model; the default
$model(\underline{rhs}only)$	right-hand-side Box-Cox model
model(<u>lam</u> bda)	both sides Box-Cox model with same parameter
model(theta)	both sides Box-Cox model with different parameters
<pre>notrans(varlist)</pre>	do not transform specified independent variables
Reporting	
<u>l</u> evel(#)	set confidence level; default is level(95)
lrtest	perform likelihood-ratio test
Maximization	
no log	suppress all iteration logs
nologlr	suppress restricted-model lrtest iteration log
maximize_options	control the maximization process; seldom used

depvar and indepvars may contain time-series operators; see [U] 11.4.4 Time-series varlists.

 $\hbox{bootstrap, by, collect, jackknife, rolling, statsby, and xi are allowed; see [U] $11.1.10$ $Prefix commands.}$

Weights are not allowed with the bootstrap prefix; see [R] bootstrap.

fweights and iweights are allowed; see [U] 11.1.6 weight.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

Options

```
Model
```

noconstant; see [R] Estimation options.

model(lhsonly | rhsonly | lambda | theta) specifies which of the four models to fit.

model (1hsonly) applies the Box-Cox transform to depvar only. model (1hsonly) is the default.

model(rhsonly) applies the transform to the indepvars only.

model(lambda) applies the transform to both *depvar* and *indepvars*, and they are transformed by the same parameter.

model(theta) applies the transform to both *depvar* and *indepvars*, but this time, each side is transformed by a separate parameter.

notrans(varlist) specifies that the variables in varlist not be transformed when included in the model. You can specify notrans(varlist) with model(lhsonly), but the results will be the same as specifying the variables in varlist in indepvars.

Reporting

1rtest specifies that a likelihood-ratio test of significance be performed and reported for each independent variable.

Maximization

log and nolog specify whether to display the iteration log. The iteration log is displayed by default unless you used set iterlog off to suppress it; see set iterlog in [R] set iter. These options control the iteration log produced by the full model and, if option lrtest is specified, by the fitted restricted models.

nologlr suppresses the iteration log when fitting the restricted models required by the lrtest option. maximize_options: iterate(#) and from(init_specs); see [R] Maximize.

Model	Initial value specification		
lhsonly	$ extstyle{from}(heta_0, extstyle{copy})$		
rhsonly	$ extstyle{from}(\lambda_0 extstyle{,} extstyle{copy})$		
lambda	$ extstyle{from}(\lambda_0, extstyle{copy})$		
theta	from(λ_0 $ heta_0$, copy)		

Remarks and examples

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Remarks are presented under the following headings:

Introduction Theta model Lambda model Left-hand-side-only model Right-hand-side-only model

Introduction

The Box-Cox transform

$$y^{(\lambda)} = \frac{y^{\lambda} - 1}{\lambda}$$

has been widely used in applied data analysis. Box and Cox (1964) developed the transformation and argued that the transformation could make the residuals more closely normal and less heteroskedastic. Cook and Weisberg (1982) discuss the transform in this light. Because the transform embeds several popular functional forms, it has received some attention as a method for testing functional forms, in particular,

$$y^{(\lambda)} = \begin{cases} y - 1 & \text{if } \lambda = 1\\ \ln(y) & \text{if } \lambda = 0\\ 1 - 1/y & \text{if } \lambda = -1 \end{cases}$$

Davidson and MacKinnon (1993) discuss this use of the transform. Atkinson (1985) also gives a good general treatment.

Theta model

boxcox obtains the maximum likelihood estimates of the parameters for four different models. The most general of the models, the theta model, is

$$y_j^{(\theta)} = \beta_0 + \beta_1 x_{1j}^{(\lambda)} + \beta_2 x_{2j}^{(\lambda)} + \dots + \beta_k x_{kj}^{(\lambda)} + \gamma_1 z_{1j} + \gamma_2 z_{2j} + \dots + \gamma_l z_{lj} + \epsilon_j$$

where $\epsilon \sim N(0,\sigma^2)$. Here the dependent variable, y, is subject to a Box-Cox transform with parameter θ . Each of the *indepvars*, x_1,x_2,\ldots,x_k , is transformed by a Box-Cox transform with parameter λ . The z_1,z_2,\ldots,z_l specified in the notrans() option are independent variables that are not transformed.

Box and Cox (1964) argued that this transformation would leave behind residuals that more closely follow a normal distribution than those produced by a simple linear regression model. Bear in mind that the normality of ϵ is assumed and that boxcox obtains maximum likelihood estimates of the k+l+4 parameters under this assumption. boxcox does not choose λ and θ so that the residuals are approximately normally distributed. If you are interested in this type of transformation to normality, see the official Stata commands lnskew0 and bcskew0 in [R] lnskew0. However, those commands work on a more restrictive model in which none of the independent variables is transformed.

Example 1

Below, we fit a theta model to a nonrepresentative extract of the Second National Health and Nutrition Examination Survey (NHANES II) dataset discussed in McDowell et al. (1981).

We model individual-level diastolic blood pressure (bpdiast) as a function of the transformed variables body mass index (bmi) and cholesterol level (tcresult) and of the untransformed variables age (age) and sex (sex).

```
. use https://www.stata-press.com/data/r18/nhanes2
```

. boxcox bpdiast bmi tcresult, notrans(age sex) model(theta) lrtest Fitting comparison model

```
Iteration 0: Log likelihood = -41178.61
Iteration 1: Log likelihood = -41032.51
Iteration 2: Log likelihood = -41032.488
Iteration 3: Log likelihood = -41032.488
```

Fitting full model

Iteration 0: Log likelihood = -39928.606 Iteration 1: Log likelihood = -39775.026 Iteration 2: Log likelihood = -39774.987 Iteration 3: Log likelihood = -39774.987

Fitting comparison models for LR tests

Iteration 0: Log likelihood = -39947.144 Iteration 1: Log likelihood = -39934.55 Iteration 2: Log likelihood = -39934.516 Iteration 3: Log likelihood = -39934.516 Iteration 0: Log likelihood = -39906.96 Iteration 1: Log likelihood = -39896.63 Iteration 2: Log likelihood = -39896.629 Iteration 0: Log likelihood = -40464.599 Iteration 1: Log likelihood = -40459.765 Iteration 2: Log likelihood = -40459.604 Iteration 3: Log likelihood = -40459.604Iteration 0: Log likelihood = -39829.859 Iteration 1: Log likelihood = -39815.576 Iteration 2: Log likelihood = -39815.575

> Number of obs 10,351 LR chi2(5) 2515.00 Prob > chi2 0.000

Log likelihood = -39774.987

bpdiast	Coefficient	Std. err.	z	P> z	[95% conf.	interval]
/lambda	.6383286	.1577601	4.05	0.000	.3291245	.9475327
/theta	.1988197	.0454088	4.38		.1098201	.2878193

Estimates of scale-variant parameters

	Coefficient	chi2(df)	P>chi2(df)	df of chi2
Notrans				
age	.003811	319.060	0.000	1
sex	1054887	243.284	0.000	1
_cons	5.835555			
Trans				
bmi	.0872041	1369.235	0.000	1
tcresult	.004734	81.177	0.000	1
/sigma	.3348267			

Test HO:	Restricted log likelihood	chi2	Prob > chi2
theta=lambda = -1	-40162.898	775.82	0.000
theta=lambda = 0	-39790.945	31.92	0.000
theta=lambda = 1	-39928.606	307.24	0.000

The output is composed of the iteration logs and three distinct tables. The first table contains a standard header for a maximum likelihood estimator and a standard output table for the Box–Cox transform parameters. The second table contains the estimates of the scale-variant parameters. The third table contains the output from likelihood-ratio tests on three standard functional form specifications.

The right-hand-side and the left-hand-side transformations each add to the regression fit at the 1% significance level and are both positive but less than 1. All the variables have significant impacts on diastolic blood pressure, bpdiast. As expected, the transformed variables—the body mass index, bmi, and cholesterol level, tcresult—contribute to higher blood pressure. The last output table shows that the linear, multiplicative inverse, and log specifications are strongly rejected.

□ Technical note

Spitzer (1984) showed that the Wald tests of the joint significance of the coefficients of the right-hand-side variables, either transformed or untransformed, are not invariant to changes in the scale of the transformed dependent variable. Davidson and MacKinnon (1993) also discuss this point. This problem demonstrates that Wald statistics can be manipulated in nonlinear models. Lafontaine and White (1986) analyze this problem numerically, and Phillips and Park (1988) analyze it by using Edgeworth expansions. See Drukker (2000) for a more detailed discussion of this issue. Because the parameter estimates and their Wald tests are not scale invariant, no Wald tests or confidence intervals are reported for these parameters. However, when the 1rtest option is specified, likelihood-ratio tests are performed and reported. Schlesselman (1971) showed that, if a constant is included in the model, the parameter estimates of the Box–Cox transforms are scale invariant. For this reason, we strongly recommend that you not use the noconstant option.

The lrtest option does not perform a likelihood-ratio test on the constant, so no value for this statistic is reported. Unless the data are properly scaled, the restricted model does not often converge. For this reason, no likelihood-ratio test on the constant is performed by the lrtest option. However, if you have a special interest in performing this test, you can do so by fitting the constrained model separately. If problems with convergence are encountered, rescaling the data by their means may help.

Lambda model

A less general model than the one above is called the lambda model. It specifies that the same parameter be used in both the left-hand-side and right-hand-side transformations. Specifically,

$$y_j^{(\lambda)} = \beta_0 + \beta_1 x_{1j}^{(\lambda)} + \beta_2 x_{2j}^{(\lambda)} + \dots + \beta_k x_{kj}^{(\lambda)} + \gamma_1 z_{1j} + \gamma_2 z_{2j} + \dots + \gamma_l z_{lj} + \epsilon_j$$

where $\epsilon \sim N(0, \sigma^2)$. Here the *depvar* variable, y, and each of the *indepvars*, x_1, x_2, \ldots, x_k , is transformed by a Box–Cox transform with the common parameter λ . Again, the z_1, z_2, \ldots, z_l are independent variables that are not transformed.

Left-hand-side-only model

Even more restrictive than a common transformation parameter is transforming the dependent variable only. Because the dependent variable is on the left-hand side of the equation, this model is known as the lhsonly model. Here you are estimating the parameters of the model

$$y_j^{(\theta)} = \beta_0 + \beta_1 x_{1j} + \beta_2 x_{2j} + \dots + \beta_k x_{kj} + \epsilon_j$$

where $\epsilon \sim N(0, \sigma^2)$. Here only the *depvar*, y, is transformed by a Box–Cox transform with the parameter θ .

Example 2

In this example, we model the transform of diastolic blood pressure as a linear combination of the untransformed body mass index, cholesterol level, age, and sex.

. boxcox bpdiast bmi tcresult age sex, model(lhsonly) lrtest nolog nologlr Fitting comparison model $\,$

Fitting full model

Fitting comparison models for LR tests

Number of obs = 10,351 LR chi2(4) = 2509.56 Log likelihood = -39777.709 Prob > chi2 = 0.000

bpdiast	Coefficient	Std. err.	z	P> z	[95% conf.	interval]
/theta	.2073268	.0452895	4.58	0.000	.1185611	.2960926

Estimates of scale-variant parameters

	Coefficient	chi2(df)	P>chi2(df)	df of chi2
Notrans				
bmi	.0272628	1375.841	0.000	1
tcresult	.0006929	82.380	0.000	1
age	.0040141	334.117	0.000	1
sex	1122274	263.219	0.000	1
_cons	6.302855			
/sigma	.3476615			

Test HO:	Restricted log likelihood	LR statistic chi2	Prob > chi2
theta = -1	-40146.678	737.94	0.000
theta = 0	-39788.241	21.06	0.000
theta = 1	-39928.606	301.79	0.000

The maximum likelihood estimate of the transformation parameter for this model is positive and significant. Once again, all the scale-variant parameters are significant, and we find a positive impact of body mass index (bmi) and cholesterol levels (tcresult) on the transformed diastolic blood pressure (bpdiast). This model rejects the linear, multiplicative inverse, and log specifications.

4

Right-hand-side-only model

The fourth model leaves the *depvar* alone and transforms a subset of the *indepvars* using the parameter λ . This is the rhsonly model. In this model, the *depvar*, y, is given by

$$y_j = \beta_0 + \beta_1 x_{1j}^{(\lambda)} + \beta_2 x_{2j}^{(\lambda)} + \dots + \beta_k x_{kj}^{(\lambda)} + \gamma_1 z_{1j} + \gamma_2 z_{2j} + \dots + \gamma_l z_{lj} + \epsilon_j$$

where $\epsilon \sim N(0, \sigma^2)$. Here each of the *indepvars*, x_1, x_2, \ldots, x_k , is transformed by a Box-Cox transform with the parameter λ . Again, the z_1, z_2, \ldots, z_l are independent variables that are not transformed.

Example 3

Now, we consider a rhsonly model in which the regressors sex and age are not transformed.

Number of obs

10.351

. boxcox bpdiast bmi tcresult, notrans(sex age) model(rhsonly) lrtest nolog

> nologlr

Fitting full model

Fitting comparison models for LR tests

LR chi2(5) 2500.79 Log likelihood = -39928.212Prob > chi2 0.000 **bpdiast** Coefficient Std. err. P>|z| [95% conf. interval] z /lambda .8658841 .1522387 5.69 0.000 .5675018 1.164266

Estimates of scale-variant parameters

	Coefficient	chi2(df)	P>chi2(df)	df of chi2
Notrans				
sex	-3.544042	235.020	0.000	1
age	.128809	311.754	0.000	1
_cons	50.01498			
Trans				
bmi	1.418215	1396.709	0.000	1
tcresult	.0462964	78.500	0.000	1
/sigma	11.4557			

Test	Restricted	LR statistic	Prob > chi2
HO:	log likelihood	chi2	
lambda = -1 $lambda = 0$ $lambda = 1$	-39989.331	122.24	0.000
	-39942.945	29.47	0.000
	-39928.606	0.79	0.375

The maximum likelihood estimate of the transformation parameter in this model is positive and significant at the 1% level. The transformed bmi coefficient behaves as expected, and the remaining scale-variant parameters are significant at the 1% level. This model rejects the multiplicative inverse and log specifications strongly. However, we cannot reject the hypothesis that the model is linear.

Stored results

Scalars

e(N)

boxcox stores the following in e():

number of observations

```
e(11)
                             log likelihood
    e(chi2)
                            LR statistic of full vs. comparison
    e(df_m)
                             full model degrees of freedom
    e(110)
                            log likelihood of the restricted model
    e(df_r)
                            restricted model degrees of freedom
                            log likelihood of model \lambda = \theta = 1
    e(ll_t1)
                            LR of \lambda = \theta = 1 vs. full model
    e(chi2_t1)
                             p-value of \lambda = \theta = 1 vs. full model
    e(p_t1)
    e(ll_tm1)
                            log likelihood of model \lambda = \theta = -1
    e(chi2_tm1)
                            LR of \lambda = \theta = -1 vs. full model
                             p-value of \lambda = \theta = -1 vs. full model
    e(p_tm1)
    e(11_t0)
                             log likelihood of model \lambda = \theta = 0
    e(chi2_t0)
                             LR of \lambda = \theta = 0 vs. full model
                            p-value of \lambda = \theta = 0 vs. full model
    e(p_t0)
                            rank of e(V)
    e(rank)
                            number of iterations
    e(ic)
    e(rc)
                            return code
Macros
    e(cmd)
                            boxcox
    e(cmdline)
                            command as typed
    e(depvar)
                            name of dependent variable
    e(model)
                             lhsonly, rhsonly, lambda, or theta
    e(wtype)
                             weight type
    e(wexp)
                             weight expression
    e(ntrans)
                             yes if untransformed indepvars
    e(chi2type)
                            LR; type of model \chi^2 test
    e(lrtest)
                             1rtest, if requested
    e(properties)
    e(predict)
                            program used to implement predict
    e(marginsnotok)
                            predictions disallowed by margins
Matrices
    e(b)
                             coefficient vector
    e(V)
                             variance-covariance matrix of the estimators (see note below)
                            p-values for LR tests on indepvars
    e(pm)
                            degrees of freedom of LR tests on indepvars
    e(df)
    e(chi2m)
                            LR statistics for tests on indepvars
Functions
    e(sample)
                            marks estimation sample
e(V) contains all zeros, except for the elements that correspond to the parameters of the Box-Cox
```

e(V) contains all zeros, except for the elements that correspond to the parameters of the Box–Cox transform.

Methods and formulas

In the internal computations,

$$y^{(\lambda)} = \begin{cases} \frac{y^{\lambda} - 1}{\lambda} & \text{if } |\lambda| > 10^{-10} \\ \ln(y) & \text{otherwise} \end{cases}$$

The unconcentrated log likelihood for the theta model is

$$\ln\!L = \left(\frac{-N}{2}\right) \left\{ \, \ln(2\pi) + \, \ln(\sigma^2) \right\} + (\theta-1) \sum_{i=1}^N \, \ln(y_i) - \left(\frac{1}{2\sigma^2}\right) \mathrm{SSR}$$

where

$$SSR = \sum_{i=1}^{N} (y_i^{(\theta)} - \beta_0 + \beta_1 x_{i1}^{(\lambda)} + \beta_2 x_{i2}^{(\lambda)} + \dots + \beta_k x_{ik}^{(\lambda)} + \gamma_1 z_{i1} + \gamma_2 z_{i2} + \dots + \gamma_l z_{il})^2$$

Writing the SSR in matrix form,

$$SSR = (\mathbf{y}^{(\theta)} - \mathbf{X}^{(\lambda)}\mathbf{b}' - \mathbf{Z}\mathbf{g}')'(\mathbf{y}^{(\theta)} - \mathbf{X}^{(\lambda)}\mathbf{b}' - \mathbf{Z}\mathbf{g}')$$

where $\mathbf{y}^{(\theta)}$ is an $N \times 1$ vector of elementwise transformed data, $\mathbf{X}^{(\lambda)}$ is an $N \times k$ matrix of elementwise transformed data, \mathbf{Z} is an $N \times l$ matrix of untransformed data, \mathbf{b} is a $1 \times k$ vector of coefficients, and \mathbf{g} is a $1 \times l$ vector of coefficients. Letting

$$\mathbf{W}_{\lambda} = \left(\mathbf{X}^{(\lambda)} \; \mathbf{Z}\right)$$

be the horizontal concatenation of $\mathbf{X}^{(\lambda)}$ and \mathbf{Z} and

$$\mathbf{d}' = \begin{pmatrix} \mathbf{b}' \\ \mathbf{g}' \end{pmatrix}$$

be the vertical concatenation of the coefficients yields

$$SSR = (\mathbf{y}^{(\theta)} - \mathbf{W}_{\lambda} \mathbf{d}')' (\mathbf{y}^{(\theta)} - \mathbf{W}_{\lambda} \mathbf{d}')$$

For given values of λ and θ , the solutions for \mathbf{d}' and σ^2 are

$$\widehat{\mathbf{d}}' = (W_{\lambda}' W_{\lambda})^{-1} W_{\lambda}' y^{(\theta)}$$

and

$$\widehat{\sigma}^{2} = \frac{1}{N} \left(\mathbf{y}^{(\theta)} - W_{\lambda} \widehat{d}' \right)' \left(y^{(\theta)} - W_{\lambda} \widehat{d}' \right)$$

Substituting these solutions into the log-likelihood function yields the concentrated log-likelihood function

$$\ln L_c = \left(-\frac{N}{2} \right) \left\{ \ln(2\pi) + 1 + \ln(\widehat{\sigma}^2) \right\} + (\theta - 1) \sum_{i=1}^{N} \ln(y_i)$$

Similar calculations yield the concentrated log-likelihood function for the lambda model,

$$\ln L_c = \left(-\frac{N}{2} \right) \left\{ \ln(2\pi) + 1 + \ln(\widehat{\sigma}^{\,2}) \right\} + (\lambda - 1) \sum_{i=1}^{N} \ln(y_i)$$

the lhsonly model,

$$\ln L_c = \left(-\frac{N}{2}\right) \left\{ \ln(2\pi) + 1 + \ln(\widehat{\sigma}^2) \right\} + (\theta - 1) \sum_{i=1}^{N} \ln(y_i)$$

and the rhsonly model,

$$\ln L_c = \left(-\frac{N}{2}\right) \left\{ \ln(2\pi) + 1 + \ln(\widehat{\sigma}^2) \right\}$$

where $\hat{\sigma}^2$ is specific to each model and is defined analogously to that in the theta model.

References

Atkinson, A. C. 1985. Plots, Transformations, and Regression: An Introduction to Graphical Methods of Diagnostic Regression Analysis. Oxford: Oxford University Press.

Box, G. E. P., and D. R. Cox. 1964. An analysis of transformations. *Journal of the Royal Statistical Society, Series B* 26: 211–252.

Carroll, R. J., and D. Ruppert. 1988. Transformation and Weighting in Regression. New York: Chapman and Hall.

Cook, R. D., and S. Weisberg. 1982. Residuals and Influence in Regression. New York: Chapman and Hall/CRC.

Davidson, R., and J. G. MacKinnon. 1993. Estimation and Inference in Econometrics. New York: Oxford University Press.

Drukker, D. M. 2000. sg131: On the manipulability of Wald tests in Box–Cox regression models. Stata Technical Bulletin 54: 36–42. Reprinted in Stata Technical Bulletin Reprints, vol. 9, pp. 319–327. College Station, TX: Stata Press.

Lafontaine, F., and K. J. White. 1986. Obtaining any Wald statistic you want. *Economics Letters* 21: 35–40. https://doi.org/10.1016/0165-1765(86)90117-5.

Lindsey, C., and S. J. Sheather. 2010a. Power transformation via multivariate Box-Cox. Stata Journal 10: 69-81.

—. 2010b. Optimal power transformation via inverse response plots. Stata Journal 10: 200–214.

McDowell, A., A. Engel, J. T. Massey, and K. Maurer. 1981. Plan and operation of the Second National Health and Nutrition Examination Survey, 1976–1980. Vital and Health Statistics 1(15): 1–144.

Phillips, P. C. B., and J. Y. Park. 1988. On the formulation of Wald tests of nonlinear restrictions. *Econometrica* 56: 1065–1083. https://doi.org/10.2307/1911359.

Schlesselman, J. J. 1971. Power families: A note on the Box and Cox transformation. *Journal of the Royal Statistical Society, Series B* 33: 307–311. https://doi.org/10.1111/j.2517-6161.1971.tb00882.x.

Spitzer, J. J. 1984. Variance estimates in models with the Box–Cox transformation: Implications for estimation and hypothesis testing. Review of Economics and Statistics 66: 645–652. https://doi.org/10.2307/1935988.

Also see

- [R] **boxcox postestimation** Postestimation tools for boxcox
- [R] **lnskew0** Find zero-skewness log or Box-Cox transform

- [R] regress Linear regression
- [U] 20 Estimation and postestimation commands

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