Example 28g — One-parameter logistic IRT (Rasch) model

Description Remarks and examples References Also see

Description

To demonstrate a one-parameter logistic IRT (Rasch) model, we use the following data:

```
. use https://www.stata-press.com/data/r18/gsem_cfa
(Fictional math abilities data)
. summarize
```

Variable	Obs	Mean	Std. dev.	Min	Max
school	500	10.5	5.772056	1	20
id	500	50681.71	29081.41	71	100000
q1	500	.506	.5004647	0	1
q2	500	.394	.4891242	0	1
q3	500	.534	.4993423	0	1
q4	500	.424	.4946852	0	1
q5	500	.49	.5004006	0	1
q6	500	.434	.4961212	0	1
q7	500	.52	.5001002	0	1
q8	500	.494	.5004647	0	1
att1	500	2.946	1.607561	1	5
att2	500	2.948	1.561465	1	5
att3	500	2.84	1.640666	1	5
att4	500	2.91	1.566783	1	5
att5	500	3.086	1.581013	1	5
test1	500	75.548	5.948653	55	93
test2	500	80.556	4.976786	65	94
test3	500	75.572	6.677874	50	94
test4	500	74.078	8.845587	43	96

```
. notes
_dta:
  1.  Fictional data on math ability and attitudes of 500 students from 20
      schools.
  2.  Variables q1-q8 are incorrect/correct (0/1) on individual math questions.
  3.  Variables att1-att5 are items from a Likert scale measuring each
      student's attitude toward math.
  4.  Variables test1-test4 are test scores from tests of four different
      aspects of mathematical abilities.  Range of scores: 0-100.
```

These data record results from a fictional instrument measuring mathematical ability. Variables q1 through q8 are the items from the instrument.

For discussions of Rasch models, IRT models, and their extensions, see Embretson and Reise (2000), van der Linden and Hambleton (1997), Skrondal and Rabe-Hesketh (2004), Andrich (1988), Bond and Fox (2015), and Fischer and Molenaar (1995). The standard one-parameter logistic model can be fit using the irt 1pl command; see [IRT] **irt 1pl**. This example demonstrates how to fit this model. With gsem, we can build on this model to fit many of the extensions to basic IRT models discussed in these books.

See *Item response theory (IRT) models* in [SEM] **Intro 5** for background.

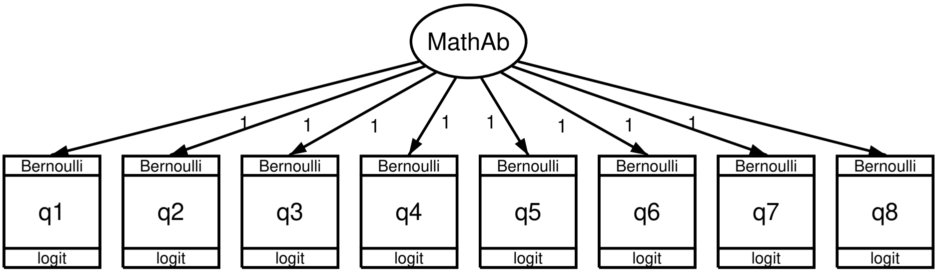
Remarks and examples

Remarks are presented under the following headings:

1-PL IRT model with unconstrained variance

Mechanically speaking, one-parameter logistic (1-PL) IRT models are similar to the probit measurement model we demonstrated in [SEM] **Example 27g**. The differences are that we will use logit rather than probit and that we will place various constraints on the logit model to obtain results that will allow us to judge the difficulty of the individual questions.

The model we wish to fit is



In the 1-PL model, we place constraints that all coefficients, the factor loadings, are equal to 1. The negative of the intercept for each question will then represent the difficulty of the question:

```
. gsem (MathAb -> (q1-q8)@1), logit
Fitting fixed-effects model:
Iteration 0:  Log likelihood = -2750.3114
Iteration 1:  Log likelihood = -2749.3709
Iteration 2:  Log likelihood = -2749.3708
Refining starting values:
Grid node 0:  Log likelihood = -2653.2353
Fitting full model:
Iteration 0:  Log likelihood = -2653.2353
Iteration 1:  Log likelihood = -2651.2171
Iteration 2:  Log likelihood = -2650.9117
Iteration 3:  Log likelihood = -2650.9116
Generalized structural equation model                      Number of obs = 500
Response: q1
Family:   Bernoulli
Link:     Logit
Response: q2
Family:   Bernoulli
Link:     Logit
Response: q3
Family:   Bernoulli
Link:     Logit
Response: q4
Family:   Bernoulli
Link:     Logit
Response: q5
Family:   Bernoulli
Link:     Logit
Response: q6
Family:   Bernoulli
Link:     Logit
Response: q7
Family:   Bernoulli
Link:     Logit
Response: q8
Family:   Bernoulli
Link:     Logit
Log likelihood = -2650.9116
 ( 1)  [q1]MathAb = 1
 ( 2)  [q2]MathAb = 1
 ( 3)  [q3]MathAb = 1
 ( 4)  [q4]MathAb = 1
 ( 5)  [q5]MathAb = 1
 ( 6)  [q6]MathAb = 1
 ( 7)  [q7]MathAb = 1
 ( 8)  [q8]MathAb = 1
```

	Coefficient	Std. err.	z	P>|z|	[95% conf. interval]	
q1						
MathAb	1	(constrained)				
_cons	.0293252	.1047674	0.28	0.780	-.1760152	.2346656
q2						
MathAb	1	(constrained)				
_cons	-.5025012	.1068768	-4.70	0.000	-.7119759	-.2930264
q3						
MathAb	1	(constrained)				
_cons	.1607425	.104967	1.53	0.126	-.044989	.3664739
q4						
MathAb	1	(constrained)				
_cons	-.3574951	.105835	-3.38	0.001	-.564928	-.1500623
q5						
MathAb	1	(constrained)				
_cons	-.0456599	.1047812	-0.44	0.663	-.2510274	.1597075
q6						
MathAb	1	(constrained)				
_cons	-.3097521	.1055691	-2.93	0.003	-.5166637	-.1028404
q7						
MathAb	1	(constrained)				
_cons	.09497	.1048315	0.91	0.365	-.1104959	.300436
q8						
MathAb	1	(constrained)				
_cons	-.0269104	.1047691	-0.26	0.797	-.232254	.1784332
var(MathAb)	.7929701	.1025406			.6154407	1.02171

Notes:

1. We had to use `gsem` and not `sem` to fit this model because the response variables were 0/1 and not continuous and because we wanted to use logit and not a continuous model.
2. To place the constraints that all coefficients are equal to 1, in the diagram we placed 1s along the path from the underlying latent factor `MathAb` to each of the questions. In the command language, we added `@1` to our command:

   ```
   gsem (MathAb -> (q1-q8)@1), logit
   ```

   Had we omitted the `@1`, we would have obtained coefficients about how well each question measured math ability.

   There are several ways we could have asked that the model above be fit. They include the following:

   ```
   gsem (MathAb -> q1@1 q2@1 q3@1 q4@1 q5@1 q6@1 q7@1 q8@1), logit
   gsem (MathAb -> (q1 q2 q3 q4 q5 q6 q7 q8)@1), logit
   gsem (MathAb -> (q1-q8)@1), logit
   ```

   Similarly, for the shorthand `logit`, we could have typed `family(bernoulli) link(logit)`.
3. The negative of the reported intercept is proportional to the difficulty of the item. The most difficult is `q2`, and the least difficult is `q3`.

1-PL IRT model with variance constrained to 1

The goal of the 1-PL model is in fact to constrain the loadings to be equal. In the previous model, that was achieved by constraining them to be 1 and letting the variance of the latent variable float. An alternative with perhaps easier-to-interpret results would constrain the variance of the latent variable to be 1—giving it a standard-normal interpretation—and constrain the loadings to be merely equal:

```
. gsem (MathAb -> (q1-q8)@b), logit var(MathAb@1) nodvheader
Fitting fixed-effects model:
Iteration 0:  Log likelihood = -2750.3114
Iteration 1:  Log likelihood = -2749.3709
Iteration 2:  Log likelihood = -2749.3708
Refining starting values:
Grid node 0:  Log likelihood = -2645.8536
Fitting full model:
Iteration 0:  Log likelihood = -2656.1973
Iteration 1:  Log likelihood = -2650.9139
Iteration 2:  Log likelihood = -2650.9116
Iteration 3:  Log likelihood = -2650.9116
Generalized structural equation model                    Number of obs = 500
Log likelihood = -2650.9116
 ( 1)  [q1]MathAb - [q8]MathAb = 0
 ( 2)  [q2]MathAb - [q8]MathAb = 0
 ( 3)  [q3]MathAb - [q8]MathAb = 0
 ( 4)  [q4]MathAb - [q8]MathAb = 0
 ( 5)  [q5]MathAb - [q8]MathAb = 0
 ( 6)  [q6]MathAb - [q8]MathAb = 0
 ( 7)  [q7]MathAb - [q8]MathAb = 0
 ( 8)  [/]var(MathAb) = 1
```

	Coefficient	Std. err.	z	P>|z|	[95% conf.	interval]
q1						
MathAb	.8904887	.0575755	15.47	0.000	.7776429	1.003335
_cons	.0293253	.1047674	0.28	0.780	-.1760151	.2346657
q2						
MathAb	.8904887	.0575755	15.47	0.000	.7776429	1.003335
_cons	-.5025011	.1068768	-4.70	0.000	-.7119758	-.2930264
q3						
MathAb	.8904887	.0575755	15.47	0.000	.7776429	1.003335
_cons	.1607425	.104967	1.53	0.126	-.044989	.366474
q4						
MathAb	.8904887	.0575755	15.47	0.000	.7776429	1.003335
_cons	-.3574951	.105835	-3.38	0.001	-.5649279	-.1500622
q5						
MathAb	.8904887	.0575755	15.47	0.000	.7776429	1.003335
_cons	-.0456599	.1047812	-0.44	0.663	-.2510273	.1597076
q6						
MathAb	.8904887	.0575755	15.47	0.000	.7776429	1.003335
_cons	-.309752	.1055691	-2.93	0.003	-.5166637	-.1028403
q7						
MathAb	.8904887	.0575755	15.47	0.000	.7776429	1.003335
_cons	.0949701	.1048315	0.91	0.365	-.1104959	.300436

```
─────────────┬──────────────────────────────────────────────────────────────────
q8           │
      MathAb │   .8904887   .0575755    15.47   0.000     .7776429    1.003335
       _cons │  -.0269103   .1047691    -0.26   0.797    -.232254     .1784333
─────────────┼──────────────────────────────────────────────────────────────────
 var(MathAb) │          1  (constrained)
─────────────┴──────────────────────────────────────────────────────────────────
```

Notes:

1. The log-likelihood values of both models is −2650.9116. The models are equivalent.
2. Intercepts are unchanged.

Obtaining item characteristic curves

Item characteristic curves graph the conditional probability of a particular response given the latent trait. In our case, this simply amounts to graphing the probability of a correct answer against math ability. After estimation, we can obtain the predicted probabilities of a correct answer by typing

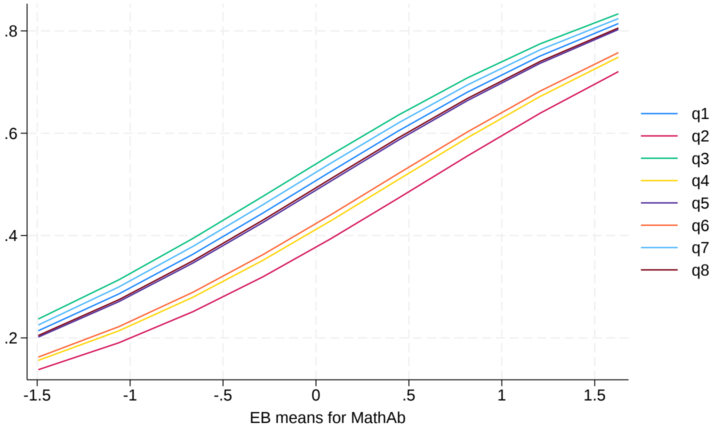
```
. predict pr*, pr
(option conditional(ebmeans) assumed)
(using 7 quadrature points)
```

We can obtain the predicted value of the latent variable by typing

```
. predict ability, latent(MathAb)
(option ebmeans assumed)
(using 7 quadrature points)
```

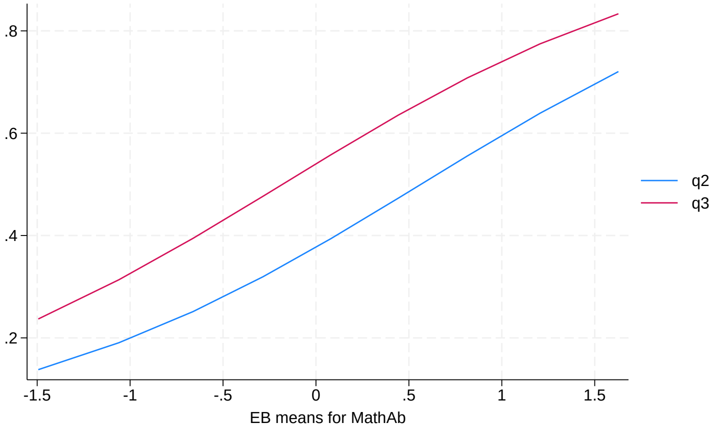
and thus we can obtain the item characteristic curves for all eight questions by typing

```
. twoway line pr1 pr2 pr3 pr4 pr5 pr6 pr7 pr8 ability, sort xlabel(-1.5(.5)1.5)
> legend(label(1 q1) label(2 q2) label(3 q3) label(4 q4) label(5 q5)
> label(6 q6) label(7 q7) label(8 q8)) xtitle(EB means for MathAb)
```



A less busy graph might show merely the most difficult and least difficult questions:

```
. twoway line pr2 pr3 ability, sort xlabel(-1.5(.5)1.5)
> legend(label(1 q2) label(2 q3)) xtitle(EB means for MathAb)
```



The slopes of each curve are identical because we have constrained them to be identical. Thus we just see the shift between difficulties with the lower items having higher levels of difficulty.

Fitting the model with the Builder

Use the diagram in *1-PL IRT model with unconstrained variance* above for reference.

1. Open the dataset.

   In the Command window, type

   ```
   . use https://www.stata-press.com/data/r18/gsem_cfa
   ```

2. Open a new Builder diagram.

   Select menu item **Statistics > SEM (structural equation modeling) > Model building and estimation**.

3. Put the Builder in `gsem` mode by clicking on the button.

4. Create the measurement component for `MathAb`.

   Select the Add measurement component tool, , and then click in the diagram about one-third of the way down from the top and slightly left of the center.

   In the resulting dialog box,

   a. change the *Latent variable name* to `MathAb`;

   b. select `q1`, `q2`, `q3`, `q4`, `q5`, `q6`, `q7`, and `q8` by using the *Measurement variables* control;

   c. check *Make measurements generalized*;

   d. select `Bernoulli, Logit` in the *Family/Link* control;

   e. select `Down` in the *Measurement direction* control;

   f. click on **OK**.

   If you wish, move the component by clicking on any variable and dragging it.

5. Constrain all path coefficients to 1.

    a. Choose the Select tool, .

    b. Click on the path from `MathAb` to `q1`. In the Contextual Toolbar, type `1` in the β box and press *Enter*.

    c. Repeat this process to add the `1` constraint on the paths from `MathAb` to each of the other measurement variables.

6. Estimate.

    Click on the **Estimate** button, , in the Standard Toolbar, and then click on **OK** in the resulting *GSEM estimation options* dialog box.

7. To fit the model in *1-PL IRT model with variance constrained to 1*, change the constraints in the diagram created above.

    a. From the SEM Builder menu, select **Estimation > Clear estimates** to clear results from the previous model.

    b. Choose the Select tool, .

    c. Click on the path from `MathAb` to `q1`. In the Contextual Toolbar, type `b` in the β box and press *Enter*.

    d. Repeat this process to add the `b` constraint on the paths from `MathAb` to each of the other measurement variables.

    e. With , click on the oval for `MathAb`. In the Contextual Toolbar, type `1` in the σ^2 box and press *Enter*.

8. Estimate again.

    Click on the **Estimate** button, , in the Standard Toolbar, and then click on **OK** in the resulting *GSEM estimation options* dialog box.

You can open a completed diagram in the Builder for the first model by typing

```
. webgetsem gsem_irt1
```

You can open a completed diagram in the Builder for the second model by typing

```
. webgetsem gsem_irt2
```

References

Andrich, D. 1988. *Rasch Models for Measurement*. Newbury Park, CA: Sage.

Bond, T. G., and C. M. Fox. 2015. *Applying the Rasch Model: Fundamental Measurement in the Human Sciences*. 3rd ed. New York: Routledge.

Embretson, S. E., and S. P. Reise. 2000. *Item Response Theory for Psychologists*. Mahwah, NJ: Lawrence Erlbaum.

Fischer, G. H., and I. W. Molenaar, ed. 1995. *Rasch Models: Foundations, Recent Developments, and Applications*. New York: Springer.

Rasch, G. 1960. *Probabilistic Models for Some Intelligence and Attainment Tests*. Copenhagen: Danish Institute of Educational Research.

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Skrondal, A., and S. Rabe-Hesketh. 2004. *Generalized Latent Variable Modeling: Multilevel, Longitudinal, and Structural Equation Models*. Boca Raton, FL: Chapman and Hall/CRC.

van der Linden, W. J., and R. K. Hambleton, ed. 1997. *Handbook of Modern Item Response Theory*. New York: Springer.

Also see

[SEM] **Example 27g** — Single-factor measurement model (generalized response)

[SEM] **Example 29g** — Two-parameter logistic IRT model

[SEM] **Intro 5** — Tour of models

[SEM] **gsem** — Generalized structural equation model estimation command

[SEM] **predict after gsem** — Generalized linear predictions, etc.

[IRT] **irt 1pl** — One-parameter logistic model

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