## MATA REFERENCE MANUAL RELEASE 18



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## Cross-referencing the documentation

When reading this manual, you will find references to other Stata manuals, for example, [U] 27 Overview of Stata estimation commands; [R] regress; and [D] reshape. The first example is a reference to chapter 27, Overview of Stata estimation commands, in the User's Guide; the second is a reference to the regress entry in the Base Reference Manual; and the third is a reference to the reshape entry in the Data Management Reference Manual.

All the manuals in the Stata Documentation have a shorthand notation:

| [GSM] | Getting Started with Stata for Mac |
| :--- | :--- |
| [GSU] | Getting Started with Stata for Unix |
| [GSW] | Getting Started with Stata for Windows |
| [U] | Stata User's Guide |
| [R] | Stata Base Reference Manual |
| [ADAPT] | Stata Adaptive Designs: Group Sequential Trials Reference Manual |
| [BAYES] | Stata Bayesian Analysis Reference Manual |
| [BMA] | Stata Bayesian Model Averaging Reference Manual |
| [CAUSAL] | Stata Causal Inference and Treatment-Effects Estimation Reference Manual |
| [CM] | Stata Choice Models Reference Manual |
| [D] | Stata Data Management Reference Manual |
| [DSGE] | Stata Dynamic Stochastic General Equilibrium Models Reference Manual |
| [ERM] | Stata Extended Regression Models Reference Manual |
| [FMM] | Stata Finite Mixture Models Reference Manual |
| [FN] | Stata Functions Reference Manual |
| [G] | Stata Graphics Reference Manual |
| [IRT] | Stata Item Response Theory Reference Manual |
| [LASSO] | Stata Lasso Reference Manual |
| [XT] | Stata Longitudinal-Data/Panel-Data Reference Manual |
| [META] | Stata Meta-Analysis Reference Manual |
| [ME] | Stata Multilevel Mixed-Effects Reference Manual |
| [MI] | Stata Multiple-Imputation Reference Manual |
| [MV] | Stata Multivariate Statistics Reference Manual |
| [PSS] | Stata Power, Precision, and Sample-Size Reference Manual |
| [P] | Stata Programming Reference Manual |
| [RPT] | Stata Reporting Reference Manual |
| [SP] | Stata Spatial Autoregressive Models Reference Manual |
| [SEM] | Stata Structural Equation Modeling Reference Manual |
| [SVY] | Stata Survey Data Reference Manual |
| [ST] | Stata Survival Analysis Reference Manual |
| [TABLES] | Stata Customizable Tables and Collected Results Reference Manual |
| [TS] | Stata Time-Series Reference Manual |
| [I] | Stata Index |
| [Pal |  |

# [M-0] Introduction to the Mata manual 

## Title

[M-0] Intro - Introduction to the Mata manual
Contents Description Remarks and examples References
Also see

## Contents

| Section | Description |
| :--- | :--- |
| $[\mathrm{M}-1]$ | Introduction and advice |
| $[\mathrm{M}-2]$ | Language definition |
| $[\mathrm{M}-3]$ | Commands for controlling Mata |
| $[\mathrm{M}-4]$ | Categorical guide to Mata functions |
| $[\mathrm{M}-5]$ | Alphabetical index to Mata functions |
| $[\mathrm{M}-6]$ | Mata glossary of common terms |

## Description

Mata is a matrix programming language that can be used by those who want to perform matrix calculations interactively and by those who want to add new features to Stata.

The Mata Reference Manual is comprehensive. If it seems overly comprehensive and too short on explanation as to why things work the way they do and how they could be used, we have a suggestion. See The Mata Book by William Gould (428 pages) or An Introduction to Stata Programming by Christopher Baum (412 pages). Baum's book assumes that you are familiar with Stata but new to programming. Gould's book assumes that you have some familiarity with programming and goes on from there. The books go well together.

## Remarks and examples

This manual is divided into six sections. Each section is organized alphabetically, but there is an introduction in front that will help you get around.

If you are new to Mata, here is a helpful reading list. Start by reading

| $[\mathrm{M}-1]$ First | Introduction and first session |
| :--- | :--- |
| $[\mathrm{M}-1]$ Interactive | Using Mata interactively |
| $[\mathrm{M}-1]$ How | How Mata works |

You may find other things in section [M-1] that interest you. For a table of contents, see
[M-1] Intro Introduction and advice

Whenever you see a term that you are unfamiliar with, see
[M-6] Glossary Mata glossary of common terms

Now that you know the basics, if you are interested, you can look deeper into Mata's programming features:
[M-2] Syntax Mata language grammar and syntax
[M-2] Syntax is pretty dense reading, but it summarizes nearly everything. The other entries in [M-2] repeat what is said there but with more explanation; see
[M-2] Intro
Language definition
because other entries in [M-2] will interest you. If you are interested in object-oriented programming, be sure to see [M-2] class.

Along the way, you will eventually be guided to sections [M-4] and [M-5]. [M-5] documents Mata's functions; the alphabetical order makes it easy to find a function if you know its name but makes learning what functions there are hopeless. That is the purpose of [M-4]-to present the functions in logical order. See
[M-4] Intro Categorical guide to Mata functions
Mathematical
[M-4] Matrix Matrix functions
[M-4] Solvers Matrix solvers and inverters
[M-4] Scalar Scalar functions
[M-4] Statistical Statistical functions
[M-4] Mathematical Other important functions
Utility and manipulation
[M-4] Standard
[M-4] Utility
[M-4] Manipulation
Stata interface
[M-4] Stata
Date and time
[M-4] Dates

Functions to create standard matrices
Matrix utility functions
Matrix manipulation functions

Stata interface functions

Date and time functions

String, I/O, and programming
[M-4] String String manipulation functions
[M-4] IO I/O functions
[M-4] Programming Programming functions

## References

Baum, C. F. 2016. An Introduction to Stata Programming. 2nd ed. College Station, TX: Stata Press.
Gould, W. W. 2018. The Mata Book: A Book for Serious Programmers and Those Who Want to Be. College Station, TX: Stata Press.

## Also see

[M-1] First - Introduction and first session
[M-6] Glossary

# [M-1] Introduction and advice 

## Title

[M-1] Intro - Introduction and advice

Contents Description Remarks and examples References Also see

## Contents

Description

Introductory material

| First | Introduction and first session |
| :--- | :--- |
| Interactive | Using Mata interactively |
| Ado | Using Mata with ado-files |
| help | Obtaining help in Stata |

How Mata works \& finding examples

| How | How Mata works |
| :--- | :--- |
| Source | Viewing the source code |

## Special topics

Returned args
Naming
Limits
Tolerance
Permutation
LAPACK

Function arguments used to return results
Advice on naming functions and variables
Limits and memory utilization
Use and specification of tolerances
An aside on permutation matrices and vectors
The LAPACK linear-algebra routines

## Description

This section provides an introduction to Mata along with reference material common to all sections.
In addition, we should mention two helpful books.
An Introduction to Stata Programming (412 pages) by Christopher Baum introduces Mata more gently than this manual. It assumes that you are familiar with Stata but new to programming.

The Mata Book (428 pages) by William Gould assumes familiarity with programming in some language, but it does not assume a lot of experience. It goes further and deeper into Mata and also covers programming, numerical accuracy, workflow, verifications, and certification.

## Remarks and examples

The most important entry in this section is [M-1] First. Also see [M-6] Glossary.
The Stata commands putmata and getmata are useful for moving data from Stata to Mata and back again; see [D] putmata.

Those looking for a textbook-like introduction to Mata may want to consider Baum (2016), particularly chapters 13 and 14.

## References

Baum, C. F. 2016. An Introduction to Stata Programming. 2nd ed. College Station, TX: Stata Press.
Gould, W. W. 2018. The Mata Book: A Book for Serious Programmers and Those Who Want to Be. College Station, TX: Stata Press.

## Also see

[M-0] Intro - Introduction to the Mata manual
[D] putmata - Put Stata variables into Mata and vice versa

## Title

[M-1] Ado - Using Mata with ado-files

## Description Remarks and examples Also see

## Description

This section provides advice to ado-file programmers on how to use Mata effectively and efficiently.

## Remarks and examples

For those interested in extending Stata by adding new commands, Mata is not a replacement for ado-files. Rather, the appropriate way to use Mata is to create subroutines used by your ado-files. Below we discuss how to do that under the following headings:

A first example<br>Where to store the Mata functions<br>Passing arguments to Mata functions<br>Returning results to ado-code<br>Advice: Use of matastrict<br>Advice: Some useful Mata functions

## A first example

We will pretend that Stata cannot produce sums and that we want to write a new command for Stata that will report the sum of one variable. Here is our first take on how we might do this:
begin varsum.ado

```
program varsum
    version 18.0 // (or version 18.5 for StataNow)
    syntax varname [if] [in]
    marksample touse
    mata: calcsum("'varlist'", "'touse'")
    display as txt " sum = " as res r(sum)
end
version 18.0 // (or version 18.5 for StataNow)
mata:
void calcsum(string scalar varname, string scalar touse)
{
    real colvector x
    st_view(x, ., varname, touse)
    st_numscalar("r(sum)", colsum(x))
}
end
```

Running this program from Stata results in the following output:

```
- varsum mpg
    sum = 1576
```

Note the following:

1. The ado-file has both ado-code and Mata code in it.
2. The ado-code handled all issues of parsing and identifying the subsample of the data to be used.
3. The ado-code called a Mata function to perform the calculation.
4. The Mata function received as arguments the names of two variables in the Stata dataset: the variable on which the calculation was to be made and the variable that identified the subsample of the data to be used.
5. The Mata function returned the result in $r()$, from where the ado-code could access it.

The outline that we showed above is a good one, although we will show you alternatives that, for some problems, are better.

## Where to store the Mata functions

Our ado-file included a Mata function. You have three choices of where to put the Mata function:

1. You can put the code for the Mata function in the ado-file, as we did. To work, your . ado file must be placed where Stata can find it.
2. You can omit code for the Mata function from the ado-file, compile the Mata function separately, and store the compiled code (the object code) in a separate file called a .mo file. You place both your . ado and .mo files where Stata can find them.
3. You can omit the code for the Mata function from the ado-file, compile the Mata function separately, and store the compiled code in a .mlib library. Here both your . ado file and your .mlib library will need to be placed where Stata can find them.

We will show you how to do each of these alternatives, but before we do, let's consider the advantages and disadvantages of each:

1. Putting your Mata source code right in the ado-file is easiest, and it sure is convenient. The disadvantage is that Mata must compile the source code into object code, and that will slow execution. The cost is small because it occurs infrequently; Mata compiles the code when the ado-file is loaded and, as long as the ado-file is not dropped from memory, Stata and Mata will use the same compiled code over and over again.
2. Saving your Mata code as .mo files saves Mata from having to compile them at all. The source code is compiled only once-at the time you create the .mo file-and thereafter, it is the already-compiled copy that Stata and Mata will use.

There is a second advantage: rather than the Mata functions (calcsum () here) being private to the ado-file, you will be able to use calcsum() in your other ado-files. calcsum() will become a utility always available to you. Perhaps calcsum() is not deserving of this honor.
3. Saving your Mata code in a .mlib library has the same advantages and disadvantages as (2); the only difference is that we save the precompiled code in a different way. The .mo/.mlib choice is of more concern to those who intend to distribute their ado-file to others. .mlib libraries allow you to place many Mata functions (subroutines for your ado-files) into one file, and so it is easier to distribute.

Let's restructure our ado-file to save calcsum() in a .mo file. First, we simply remove calcsum() from our ado-file, so it now reads


Next, we enter Mata, enter our calcsum() program, and save the object code:

```
void calcsum(string scalar varname, string scalar touse)
{
    real colvector x
    st_view(x, ., varname, touse)
    st_numscalar("r(sum)", colsum(x))
}
mata mosave calcsum(), dir(PERSONAL)
```

This step we do only once. The mata mosave command created file calcsum.mo stored in our PERSONAL sysdir directory; see [M-3] mata mosave and [P] sysdir for more details. We put the calcsum.mo file in our PERSONAL directory so that Stata and Mata would be able to find it, just as you probably did with the varsum.ado ado-file.

Typing in the calcsum() program interactively is too prone to error, so instead what we can do is create a do-file to perform the step and then run the do-file once:
begin varsum.do

```
version 18.0 // (or version 18.5 for StataNow)
mata:
void calcsum(string scalar varname, string scalar touse)
{
    real colvector x
    st_view(x, ., varname, touse)
    st_numscalar("r(sum)", colsum(x))
}
mata mosave calcsum(), dir(PERSONAL) replace
end
```

end varsum.do
We save the do-file someplace safe in case we should need to modify our code later, either to fix bugs or to add features. For the same reason, we added a replace option on the command that creates the .mo file, so we can run the do-file over and over.

To follow the third organization-putting our calcsum () subroutine in a library-is just a matter of modifying varsum. do to do mata mlib add rather than mata mosave. See [M-3] mata mlib; there are important details having to do with how you will manage all the different functions you will put in the library, but that has nothing to do with our problem here.

Where and how you store your Mata subroutines has nothing to do with what your Mata subroutines do or how you use them.

## Passing arguments to Mata functions

In designing a subroutine, you have two paramount interests: how you get data into your subroutine and how you get results back. You get data in by the values you pass as the arguments. For calcsum(), we called the subroutine by coding

```
mata: calcsum("'varlist'", "'touse'")
```

After macro expansion, that line turned into something like

```
mata: calcsum("mpg", "__0001dc")
```

The $\qquad$ 0001dc variable is a temporary variable created by the marksample command earlier in our ado-file. mpg was the variable specified by the user. After expansion, the arguments were nothing more than strings, and those strings were passed to calcsum().

Macro substitution is the most common way values are passed to Mata subroutines. If we had a Mata function $\operatorname{add}(a, b)$, which could add numbers, we might code in our ado-file

```
mata: add('x', 'y')
```

and, if macro ' $x$ ' contained 2 and macro ' $y$ ' contained 3, Mata would see

```
mata: add(2, 3)
```

and values 2 and 3 would be passed to the subroutine.
When you think about writing your Mata subroutine, the arguments your ado-file will find convenient to pass and Mata will make convenient to use are

1. numbers, which Mata calls real scalars, such as 2 and 3 (' $x$ ' and ' $y$ '), and
2. names of variables, macros, scalars, matrices, etc., which Mata calls string scalars, such as "mpg" and "__0001dc" ("'varlist'" and "'touse'").

To receive arguments of type (1), you code real scalar as the type of the argument in the function declaration and then use the real scalar variable in your Mata code.

To receive arguments of type (2), you code string scalar as the type of the argument in the function declaration, and then you use one of the Stata interface functions (in [M-4] Stata) to go from the name to the contents. If you receive a variable name, you will especially want to read about the functions st_data() and st_view() (see [M-5] st_data() and [M-5] st_view()), although there are many other utilities for dealing with variable names. If you are dealing with local or global macros, scalars, or matrices, you will want to see [M-5] st_local(), [M-5] st_global(), [M-5] st_numscalar(), and [M-5] st_matrix( ).

## Returning results to ado-code

You have many more choices on how to return results from your Mata function to the calling ado-code.

1. You can return results in $r()$-as we did in our example-or in $e()$ or in $s()$.
2. You can return results in macros, scalars, matrices, etc., whose names are passed to your Mata subroutine as arguments.
3. You can highhandedly reach back into the calling ado-file and return results in macros, scalars, matrices, etc., whose names are of your devising.

In all cases, see [M-5] st_global( ). st_global () is probably not the function you will use, but there is a wonderfully useful table in the Remarks and examples section that will tell you exactly which function to use.

Also see all other Stata interface functions in [M-4] Stata.
If you want to modify the Stata dataset in memory, see [M-5] st_store( ) and [M-5] st_view().

## Advice: Use of matastrict

The setting matastrict determines whether declarations can be omitted (see [M-2] Declarations); by default, you may. That is, matastrict is set off, but you can turn it on by typing mata set matastrict on; see [M-3] mata set. Some users do, and some users do not.

So now, consider what happens when you include Mata source code directly in the ado-file. When the ado-file is loaded, is matastrict set on, or is it set off? The answer is that it is off, because when you include the Mata source code inside an ado-file, matastrict is temporarily switched off when the ado-file is loaded even if the user running the ado-file has previously set it on.

For example, varsum. ado could read

and it will work even when run by users who have set matastrict on.
Similarly, in an ado-file, you can set matastrict on and that will not affect the setting after the ado-file is loaded, so varsum. ado could read
$\qquad$

```
program varsum
    version 18.0 // (or version 18.5 for StataNow)
    syntax varname [if] [in]
    marksample touse
    mata: calcsum("'varlist'", "'touse'")
    display as txt " sum = " as res r(sum)
end
version 18.0 // (or version 18.5 for StataNow)
mata:
mata set matastrict on
void calcsum(string scalar varname, string scalar touse)
{
    real colvector x
    st_view(x, ., varname, touse)
    st_numscalar("r(sum)", colsum(x))
}
end
```

end varsum.ado
and not only will it work, but running varsum will not change the user's matastrict setting.
This preserving and restoring of matastrict is something that is done only for ado-files when they are loaded.

## Advice: Some useful Mata functions

In the calcsum() subroutine, we used the colsum() function—see [M-5] sum()—to obtain the sum:

```
void calcsum(string scalar varname, string scalar touse)
{
    real colvector x
    st_view(x, ., varname, touse)
    st_numscalar("r(sum)", colsum(x))
}
```

We could instead have coded

```
void calcsum(string scalar varname, string scalar touse)
{
    real colvector x
    real scalar i, sum
    st_view(x, ., varname, touse)
    sum = 0
    for (i=1; i<=rows(x); i++) sum = sum + x[i]
    st_numscalar("r(sum)", sum)
}
```

The first way is preferred. Rather than construct explicit loops, it is better to call functions that calculate the desired result when such functions exist. Unlike ado-code, however, when such functions do not exist, you can code explicit loops and still obtain good performance.

Another set of functions we recommend are documented in [M-5] cross(), [M-5] crossdev(), and [M-5] quadcross( ).
cross() makes calculations of the form
$X^{\prime} X$
$X^{\prime} Z$
$X^{\prime} \operatorname{diag}(w) X$
$X^{\prime} \operatorname{diag}(w) Z$
crossdev() makes calculations of the form

$$
\begin{aligned}
& (X:-x)^{\prime}(X:-x) \\
& (X:-x)^{\prime}(Z:-z) \\
& (X:-x)^{\prime} \operatorname{diag}(w)(X:-x) \\
& (X:-x)^{\prime} \operatorname{diag}(w)(Z:-z)
\end{aligned}
$$

Both these functions could easily escape your attention because the matrix expressions themselves are so easily written in Mata. The functions, however, are quicker, use less memory, and sometimes are more accurate. Also, quad-precision versions of the functions exist; [M-5] quadcross( ).

## Also see

[M-2] version - Version control
[M-1] Intro - Introduction and advice

## Title

[M-1] First - Introduction and first session

## Description Remarks and examples Also see

## Description

Mata is a component of Stata. It is a matrix programming language that can be used interactively or as an extension for do-files and ado-files. Thus

1. Mata can be used by users who want to think in matrix terms and perform (not necessarily simple) matrix calculations interactively, and
2. Mata can be used by advanced Stata programmers who want to add features to Stata.

Mata has something for everybody.
Primary features of Mata are that it is fast and that it is C-like.

## Remarks and examples

This introduction is presented under the following headings:
Invoking Mata
Using Mata
Making mistakes: Interpreting error messages
Working with real numbers, complex numbers, and strings
Working with scalars, vectors, and matrices
Working with functions
Distinguishing real and complex values
Working with matrix and scalar functions
Performing element-by-element calculations: Colon operators
Writing programs
More functions
Mata environment commands
Exiting Mata

If you are reading the entries in the order suggested in [M-0] Intro, see [M-1] Interactive next.

## Invoking Mata

To enter Mata, type mata at Stata's dot prompt and press enter; to exit Mata, type end at Mata's colon prompt:


## Using Mata

When you type a statement into Mata, Mata compiles what you typed and, if it compiled without error, executes it:

```
:2 + 2
    4
: _
```

We typed $2+2$, a particular example from the general class of expressions. Mata responded with 4, the evaluation of the expression.

Often what you type are expressions, although you will probably choose more complicated examples. When an expression is not assigned to a variable, the result of the expression is displayed. Assignment is performed by the $=$ operator:

```
: x = 2 + 2
: x
    4
: _
```

When we type " $x=2+2$ ", the expression is evaluated and stored in the variable we just named $x$. The result is not displayed. We can look at the contents of $x$, however, simply by typing " $x$ ". From Mata's perspective, $x$ is not only a variable but also an expression, albeit a rather simple one. Just as $2+2$ says to load 2 , load another 2 , and add them, the expression x says to load x and stop there.

As an aside, Mata distinguishes uppercase and lowercase. X is not the same as x :

```
: X = 2 + 3
: x
    4
: X
    5
```


## Making mistakes: Interpreting error messages

If you make a mistake, Mata complains, and then you continue on your way. For instance,

```
: 2,,3
invalid expression
r(3000);
: _
```

2, , 3 makes no sense to Mata, so Mata complained. This is an example of what is called a compile-time error; Mata could not make sense out of what we typed.

The other kind of error is called a run-time error. For example, we have no variable called y. Let us ask Mata to show us the contents of $y$ :

```
: y
    <istmt>: 3499 y not found
r(3499);
: _
```

Here what we typed made perfect sense-show me y-but y has never been defined. This ugly message is called a run-time error message-see [M-2] Errors for a complete description—but all that's important is to understand the difference between

```
invalid expression
```

and

```
<istmt>: 3499 y not found
```

The run-time message is prefixed by an identity (<istmt> here) and a number ( 3499 here). Mata is telling us, "I was executing your istmt [that's what everything you type is called] and I got error 3499 , the details of which are that I was unable to find y."

The compile-time error message is of a simpler form: invalid expression. When you get such unprefixed error messages, that means Mata could not understand what you typed. When you get the more complicated error message, that means Mata understood what you typed, but there was a problem performing your request.

Another way to tell the difference between compile-time errors and run-time errors is to look at the return code. Compile-time errors have a return code of 3000:

```
: 2,,3
invalid expression
r(3000);
```

Run-time errors have a return code that might be in the 3000s, but is never 3000 exactly:

```
: y
    <istmt>: 3499 y not found
r(3499);
```

Whether the error is compile-time or run-time, once the error message is issued, Mata is ready to continue just as if the error never happened.

## Working with real numbers, complex numbers, and strings

As we have seen, Mata works with real numbers:

```
2+3
5
```

Mata also understands complex numbers; you write the imaginary part by suffixing a lowercase i:

```
: 1+2i + 4-1i
    5 + 1i
```

For imaginary numbers, you can omit the real part:

```
1+2i - 2i
    1
```

Whether a number is real or complex, you can use the same computer notation for the imaginary part as you would for the real part:

```
: 2.5e+3i
    2500i
:1.25e+2+2.5e+3i /* i.e., 1.25+e02 + 2.5e+03i */
125 + 2500i
```

We purposely wrote the last example in nearly unreadable form just to emphasize that Mata could interpret it.

Mata also understands strings, which you write enclosed in double quotes:

```
: "Alpha" + "Beta"
    AlphaBeta
```

Just like Stata, Mata understands simple and compound double quotes:

```
: '"Alpha"' + '"Beta"'
    AlphaBeta
```

You can add complex and reals,

```
: 1+2i + 3
    4+2i
```

but you may not add reals or complex to strings:

```
: 2 + "alpha"
type mismatch: real + string not allowed
r(3000);
```

We got a run-time error. Mata understood 2 + "alpha" all right; it just could not perform our request.

## Working with scalars, vectors, and matrices

In addition to understanding scalars-be they real, complex, or string-Mata understands vectors and matrices of real, complex, and string elements:

```
x = (1, 2)
x
    1 2
1
        1 2
```

x now contains the row vector $(1,2)$. We can add vectors:

```
x + (3, 4)
    1 2
```

1
46

The "," is the column-join operator; things like $(1,2)$ are expressions, just as $(1+2)$ is an expression:


In the above, we could have dispensed with the parentheses and typed " $y=3,4$ " followed by " $z=$ $\mathrm{x}, \mathrm{y}$ ", just as we could using the + operator, although most people find vectors more readable when enclosed in parentheses.
$\backslash$ is the row-join operator:

```
: a = (1 \ 2)
: a
    1
1 1
2
b}=(3\4
c = (a \ b)
c
```

    1
    1 1
3 3
$4 \quad 4$

Using the column-join and row-join operators, we can enter matrices:
$: A=(1,2 \backslash 3,4)$
$: A$
1

1 | 1 | 2 |
| :---: | :---: |
| 2 | 4 |

The use of these operators is not limited to scalars. Remember, $x$ is the row vector (1, 2), y is the row vector $(3,4)$, $a$ is the column vector $(1 \backslash 2)$, and $b$ is the column vector ( $3 \backslash 4$ ). Therefore,
: x \ y
$1 \quad 2$

|  | 1 | 2 |
| :--- | :--- | :--- |
| 2 | 3 | 4 |

: a, b

|  | 1 | 2 |
| :--- | :--- | :--- |
|  | 1 | 3 |
| 2 | 1 | 3 |
| 2 | 4 |  |
|  |  |  |

But if we try something nonsensical, we get an error:

$$
: \mathrm{a}, \mathrm{x}
$$

<istmt>: 3200 conformability error
We create complex vectors and matrices just as we create real ones, the only difference being that their elements are complex:

$$
\begin{aligned}
& : Z=(1+1 i, 2+3 i \backslash 3-2 i,-1-1 i) \\
& : Z \\
& \cline { 2 - 3 } \begin{array}{l}
1 \\
2
\end{array} \begin{array}{rr}
1+1 i & 2+3 i \\
2-2 i & -1-1 i
\end{array}
\end{aligned}
$$

Similarly, we can create string vectors and matrices, which are vectors and matrices with string elements:


For strings, the individual elements can be up to $2,147,483,647$ bytes long.

## Working with functions

Mata's expressions also include functions:

```
: sqrt(4)
2
: sqrt(-4)
```

When we ask for the square root of -4 , Mata replies "." Further, . can be stored just like any other number:

```
: findout = sqrt(-4)
: findout
```

"." means missing, that there is no answer to our calculation. Taking the square root of a negative number is not an error; it merely produces missing. To Mata, missing is a number like any other number, and the rules for all the operators have been generalized to understand missing. For instance, the addition rule is generalized such that missing plus anything is missing:

```
: 2 + .
```

Still, it should surprise you that Mata produced missing for the sqrt ( -4 ). We said that Mata understands complex numbers, so should not the answer be $2 i$ ? The answer is that is should be if you are working on the complex plane, but otherwise, missing is probably a better answer. Mata attempts to intuit the kind of answer you want by context, and in particular, uses inheritance rules. If you ask for the square root of a real number, you get a real number back. If you ask for the square root of a complex number, you get a complex number back:

```
: sqrt(-4 + 0i)
2i
```

Here complex means multipart: $-4+0 i$ is a complex number; it merely happens to have 0 imaginary part. Thus:

```
areal = -4
acomplex = -4 + 0i
sqrt(areal)
sqrt(acomplex)
2i
```

If you ever have a real scalar, vector, or matrix, and want to make it complex, use the $C()$ function, which means "convert to complex":

```
: sqrt(C(areal))
2i
```

$C()$ is documented in [M-5] C(). C() allows one or two arguments. With one argument, it casts to complex. With two arguments, it makes a complex out of the two real arguments. Thus you could type

```
sqrt(-4 + 2i)
    .485868272 + 2.05817103i
```

or you could type

```
: sqrt(C(-4, 2))
    .485868272 + 2.05817103i
```

By the way, used with one argument, C() also allows complex, and then it does nothing:

```
sqrt(C(acomplex))
2i
```


## Distinguishing real and complex values

It is virtually impossible to tell the difference between a real value and a complex value with zero imaginary part:

```
: areal = -4
: acomplex = -4 + Oi
areal
    -4
acomplex
    -4
```

Yet, as we have seen, the difference is important: sqrt (areal) is missing, sqrt (acomplex) is $2 i$. One solution is the eltype() function:

```
: eltype(areal)
    real
: eltype(acomplex)
    complex
```

eltype() can also be used with strings,

```
: astring = "hello"
: eltype(astring)
    string
```

but this is useful mostly in programming contexts.

## Working with matrix and scalar functions

Some functions are matrix functions: they require a matrix and return a matrix. Mata's invsym ( $X$ ) is an example of such a function. It returns the matrix that is the inverse of symmetric, real matrix $X$ :

```
: X = (76, 53, 48 \ 53, 88, 46\ 48, 46, 63)
: Xi = invsym(X)
: Xi
[symmetric]
```

|  | 1 | 2 | 3 |
| :--- | ---: | ---: | ---: |
| 1 | .0298458083 |  |  |
| 2 | -.0098470272 | .0216268926 |  |
| 3 | -.0155497706 | -.0082885675 | .0337724301 |
|  |  |  |  |

```
Xi * X
```

$1 \quad 2 \quad 3$

| 1 | $-8.67362 \mathrm{e}-17$ | $-8.50015 \mathrm{e}-17$ |
| ---: | ---: | ---: |
| $-1.38778 \mathrm{e}-16$ | 1 | $-1.02349 \mathrm{e}-16$ |
| 0 | $1.11022 \mathrm{e}-16$ | 1 |

The last matrix, Xi * X, differs just a little from the identity matrix because of unavoidable computational roundoff error.

Other functions are, mathematically speaking, scalar functions. sqrt() is an example in that it makes no sense to speak of sqrt ( $X$ ). (That is, it makes no sense to speak of sqrt ( $X$ ) unless we were speaking of the Cholesky square-root decomposition. Mata has such a matrix function; see [M-5] cholesky ().)

When a function is, mathematically speaking, a scalar function, the corresponding Mata function will usually allow vector and matrix arguments and, then, the Mata function makes the calculation on each element individually:

```
:M=(1, 2\ \, 4\ 5, 6)
M
        1 2
    l|ll
S = sqrt(M)
S
```

| 1 | 1.414213562 |
| ---: | ---: |
| 1.732050808 | 2 |
| 2.236067977 | 2.449489743 |

$S[1,2] * S[1,2]$
2
$S[2,1] * S[2,1]$
3

When a function returns a result calculated in this way, it is said to return an element-by-element result.

## Performing element-by-element calculations: Colon operators

Mata's operators, such as + (addition) and $*$ (multiplication), work as you would expect. In particular, * performs matrix multiplication:

```
:A = (1, 2 \ 3, 4)
: B = (5, 6 \ 7, 8)
A*B
    1 2
    19 22
43 50
```

The first element of the result was calculated as $1 * 5+2 * 7=19$.
Sometimes, you really want the element-by-element result. When you do, place a colon in front of the operator: Mata's :* operator performs element-by-element multiplication:

```
: A:*B
    1 2
\begin{tabular}{rr}
5 & 12 \\
21 & 32
\end{tabular}
```

See [M-2] op_colon for more information.

## Writing programs

Mata is a complete programming language; it will allow you to create your own functions:

```
function add(a,b) return(a+b)
```

That single statement creates a new function, although perhaps you would prefer if we typed it as

```
: function add(a,b)
> {
> return(a+b)
> }
```

because that makes it obvious that a program can contain many lines. In either case, once defined, we can use the function:

```
add(1,2)
3
add(1+2i,4-1i)
5 + 1i
add( (1,2), (3,4) )
        1 2
1
                46
```

```
: add(x,y)
            1 2
    1
        4 6
: add(A,A)
            1 2
    1 2 4
    2 6 8
:
: Z1 = (1+1i, 1+1i \ 2, 2i)
: Z2 = (1+2i, -3+3i \ 6i, -2+2i)
add(Z1, Z2)
    1 2
2+3i -2 + 4i
2 + 6i -2 + 4i
add("Alpha","Beta")
AlphaBeta
S1 = ("one", "two" \ "three", "four")
S2 = ("abc", "def" \ "ghi", "jkl")
add(S1, S2)
                                    1 2
roneabc }r\begin{array}{rr}{\mathrm{ twodef}}\\{\mathrm{ threeghi }}&{\mathrm{ fourjkl}}
```

Of course, our little function add() does not do anything that the + operator does not already do, but we could write a program that did do something different. The following program will allow us to make $n \times n$ identity matrices:

```
real matrix id(real scalar n)
{
> real scalar i
> real matrix res
> res = J(n, n, 0)
for (i=1; i<=n; i++) {
> res[i,i] = 1
> }
return(res)
}
I3 = id(3)
I3
[symmetric]
    1 2 3
1
\begin{tabular}{|lll|}
\hline 1 & & \\
0 & 1 & \\
0 & 0 & 1 \\
\hline
\end{tabular}
```

The function $J()$ in the program line res $=J(n, n, 0)$ is a Mata built-in function that returns an $\mathrm{n} \times \mathrm{n}$ matrix containing $0 \mathrm{~s}(\mathrm{~J}(r, c, v a l)$ returns an $r \times c$ matrix, the elements of which are all equal to val); see [M-5] J().
for ( $i=1$; $i<=n ; i++$ ) says that starting with $i=1$ and so long as $i<=n$ do what is inside the braces (set res[i,i] equal to 1 ) and then (we are back to the for part again), increment i .

The final line—return(res)—says to return the matrix we have just created.
Actually, just as with add(), we do not need id() because Mata has a built-in function $I(n)$ that makes identity matrices, but it is interesting to see how the problem could be programmed.

## More functions

Mata has many functions already and much of this manual concerns documenting what those functions do; see [M-4] Intro. But right now, what is important is that many of the functions are themselves written in Mata!

One of those functions is pi(); it takes no arguments and returns the value of pi. The code for it reads

```
real scalar pi() return(3.141592653589793238462643)
```

There is no reason to type the above function because it is already included as part of Mata:

```
: pi()
    3.141592654
```

When Mata lists a result, it does not show as many digits, but we could ask to see more:

```
: printf("%17.0g", pi())
    3.14159265358979
```

Other Mata functions include the hyperbolic function $\tanh (u)$. The code for $\tanh (u)$ reads

```
numeric matrix tanh(numeric matrix u)
{
    numeric matrix eu, emu
    eu = exp(u)
    emu = exp(-u)
    return( (eu-emu):/(eu+emu) )
}
```

See for yourself: at the Stata dot prompt (not the Mata colon prompt), type

```
. viewsource tanh.mata
```

When the code for a function was written in Mata (as opposed to having been written in C), viewsource can show you the code; see [M-1] Source.

Returning to the function $\tanh ()$,

```
numeric matrix tanh(numeric matrix u)
{
    numeric matrix eu, emu
    eu = exp(u)
    emu = exp(-u)
    return( (eu-emu):/(eu+emu))
}
```

this is the first time we have seen the word numeric: it means real or complex. Built-in (previously written) function $\exp ()$ works like sqrt() in that it allows a real or complex argument and correspondingly returns a real or complex result. Said in Mata jargon: $\exp ()$ allows a numeric argument and correspondingly returns a numeric result. tanh() will also work like sqrt() and $\exp ()$.

Another characteristic tanh() shares with sqrt() and $\exp ()$ is element-by-element operation. $\tanh ()$ is element-by-element because $\exp ()$ is element-by-element and because we were careful to use the :/ (element-by-element) divide operator.

In any case, there is no need to type the above functions because they are already part of Mata. You could learn more about them by seeing their manual entry, $[\mathrm{M}-5] \sin ()$.

At the other extreme, Mata functions can become long. Here is Mata's function to solve $A X=B$ for $X$ when $A$ is lower triangular, placing the result $X$ back into $A$ :

```
real scalar _solvelower(
            numeric matrix A, numeric matrix b,
            |real scalar usertol, numeric scalar userd)
{
real scalar tol, rank, a_t, b_t, d_t
real scalar n, m, i, im1, complex_case
numeric rowvector sum
numeric scalar zero, d
d = userd
if ((n=rows(A))!=cols(A)) _error(3205)
if (n != rows(b)) _error(3200)
if (isview(b)) _error(3104)
m = cols(b)
rank = n
a_t = iscomplex(A)
b_t = iscomplex(b)
d_t = d<. ? iscomplex(d) : 0
complex_case = a_t | b_t | d_t
```

```
if (complex_case) {
    if (!a_t) A = C(A)
    if (!b_t) b = C(b)
    if (d<. & !d_t) d = C(d)
    zero = 0i
}
else zero = 0
if (n==0 | m==0) return(0)
tol = solve_tol(A, usertol)
if (abs(d) >=. ) {
    if (abs(d=A[1,1])<=tol) {
        b[1,.] = J(1, m, zero)
        --rank
        }
        else {
        b[1,.] = b[1,.] :/ d
        if (missing(d)) rank = .
        }
        for (i=2; i<=n; i++) {
            im1 = i - 1
            sum = A[|i,11,im1|] * b[|1,1\im1,m|]
                if (abs(d=A[i,i])<=tol) {
                b[i,.] = J(1, m, zero)
                --rank
            }
            else {
                        b[i,.] = (b[i,.]-sum) :/ d
                        if (missing(d)) rank = .
            }
        }
}
else {
    if (abs(d)<=tol) {
        rank = 0
        b = J(rows(b), cols(b), zero)
    }
    else {
        b[1,.] = b[1,.] :/ d
        for (i=2; i<=n; i++) {
                        im1 = i - 1
                        sum = A[|i,1l,im1|] * b[|1,1\im1,m|]
                        b[i,.] = (b[i,.]-sum) :/ d
        }
        }
}
return(rank)
}
```

If the function were not already part of Mata and you wanted to use it, you could type it into a do-file or onto the end of an ado-file (especially good if you just want to use _solvelower() as a subroutine). In those cases, do not forget to enter and exit Mata:
$\qquad$

```
program mycommand
    ado-file code appears here
end
mata:
    _solvelower() code appears here
end
```

end ado-file
Sharp-eyed readers will notice that we put a colon on the end of the Mata command. That's a detail, and why we did that is explained in [M-3] mata.

In addition to loading functions by putting their code in do- and ado-files, you can also save the compiled versions of functions in .mo files (see [M-3] mata mosave) or into .mlib Mata libraries (see [M-3] mata mlib).

For _solvelower (), it has already been saved into a library, namely, Mata's official library, so you need not do any of this.

## Mata environment commands

When you are using Mata, there is a set of commands that will tell you about and manipulate Mata's environment.

The most useful such command is mata describe; see [M-3] mata describe:

| : mata describe |  |  |
| ---: | :--- | :--- |
| \# bytes | type |  |
| 76 | transmorphic matrix | name and extent |
| 200 | real matrix | id() |
| 32 | real matrix | $\mathrm{A}[2,2]$ |
| 32 | real matrix | $\mathrm{B}[2,2]$ |
| 72 | real matrix | $\mathrm{I} 3[3,3]$ |
| 48 | real matrix | $\mathrm{M}[3,2]$ |
| 48 | real matrix | $\mathrm{S}[3,2]$ |
| 47 | string matrix | $\mathrm{S}[2,2]$ |
| 44 | string matrix | $\mathrm{S} 2[2,2]$ |
| 72 | real matrix | $\mathrm{X}[3,3]$ |
| 72 | real matrix | $\mathrm{Xi}[3,3]$ |
| 64 | complex matrix | $\mathrm{Z}[2,2]$ |
| 64 | complex matrix | $\mathrm{Z1}[2,2]$ |
| 64 | complex matrix | $\mathrm{Z2[2,2]}$ |
| 16 | real colvector | $\mathrm{a}[2]$ |
| 16 | complex scalar | acomplex |
| 8 | real scalar | areal |
| 16 | real colvector | $\mathrm{b}[2]$ |
| 32 | real colvector | $\mathrm{c}[4]$ |
| 8 | real scalar | findout |
| 16 | real rowvector | $\mathrm{x}[2]$ |
| 16 | real rowvector | $\mathrm{y}[2]$ |
| 32 | real rowvector | $\mathrm{z}[4]$ |
|  |  |  |
|  |  |  |

Another useful command is mata clear (see [M-3] mata clear), which will clear Mata without disturbing Stata:

```
: mata clear
: mata describe
    # bytes type name and extent
```

There are other useful mata commands; see [M-3] Intro. Do not confuse this command mata, which you type at Mata's colon prompt, with Stata's command mata, which you type at Stata's dot prompt and which invokes Mata.

## Exiting Mata

When you are done using Mata, type end to Mata's colon prompt:

```
: end
```

Exiting Mata does not clear it:

```
. mata
    x = 2
y=(3+2i)
function add(a,b) return(a+b)
end
```


end
Exiting Stata clears Mata, as does Stata's clear mata command; see [D] clear.

## Also see

[M-1] Intro - Introduction and advice

## Title

[M-1] help - Obtaining help in Stata

## Description Syntax Remarks and examples Also see

## Description

Help for Mata is available in Stata. This entry describes how to access it.

## Syntax

```
help m\# entryname
help mata functionname()
```

The help command may be issued at either Stata's dot prompt or Mata's colon prompt.

## Remarks and examples

To see this entry in Stata, type
. help m1 help
at Stata's dot prompt or Mata's colon prompt. You type that because this entry is [M-1] help.
To see the entry for function $\max ()$, for example, type
. help mata max()
$\max ()$ is documented in $[\mathrm{M}-5] \operatorname{minmax}()$, but that will not matter; Stata will find the appropriate entry.

To enter the Mata help system from the top, from whence you can click your way to any section or function, type
. help mata

To access Mata's PDF manual, click on the title.

## Also see

[M-3] mata help - Obtain help in Stata
[R] help - Display help in Stata
[M-1] Intro - Introduction and advice

## Title

[M-1] How — How Mata works

Description Remarks and examples Reference Also see

## Description

Below we take away some of the mystery and show you how Mata works. Everyone, we suspect, will find this entertaining, and advanced users will find the description useful for predicting what Mata will do when faced with unusual situations.

## Remarks and examples

Remarks are presented under the following headings:

```
What happens when you define a program
What happens when you work interactively
What happens when you type a mata environment command
Working with object code I: .mo files
Working with object code II: .mlib libraries
The Mata environment
```

If you are reading the entries in the order suggested in [M-0] Intro, browse [M-1] Intro next for sections that interest you, and then see [M-2] Syntax.

## What happens when you define a program

Let's say you fire up Mata and type

```
: function tryit()
> {
> real scalar i
> >
> for (i=1; i<=10; i++) i
}
```

Mata compiles the program: it reads what you type and produces binary codes that tell Mata exactly what it is to do when the time comes to execute the program. In fact, given the above program, Mata produces the binary code

```
00b4 36084000 0000 0100 0000 2000 0000
0 0 0 0 0 0 0 0 ~ f f f f ~ f f f f ~ 0 3 0 0 ~ 0 0 0 0 ~ 0 0 0 0 ~ 0 0 0 0
0100 7472 79697400 1700 0100 1f00 0700
0000 0800 0000 0200 0100 0800 2a00 0300
1e00 0300
```

which looks meaningless to you and me, but Mata knows exactly what to make of it. The compiled version of the program is called object code, and it is the object code, not the original source code, that Mata stores in memory. In fact, the original source is discarded once the object code has been stored.

It is this compilation step-the conversion of text into object code-that makes Mata able to execute programs so quickly.

Later, when the time comes to execute the program, Stata follows the instructions it has previously recorded:

```
tryit()
```

1
2
3
4
5
6
7
8
9
10

## What happens when you work interactively

Let's say you type

$$
: x=3
$$

In the jargon of Mata, that is called an istmt-an interactive statement. Obviously, Mata stores 3 in x , but how?

Mata first compiles the single statement and stores the resulting object code under the name <istmt>. The result is much as if you had typed

```
: function <istmt>()
> {
> x = 3
> }
```

except, of course, you could not define a program named <istmt> because the name is invalid. Mata has ways of getting around that.

At this point, Mata has discarded the source code $\mathrm{x}=3$ and has stored the corresponding object code. Next, Mata executes <istmt>. The result is much as if you had typed

```
: <istmt>()
```

That done, there is only one thing left to do, which is to discard the object code. The result is much as if you typed

```
: mata drop <istmt>()
```

So there you have it: you type

$$
: x=3
$$

and Mata executes

```
: function <istmt>()
> {
> x = 3
> }
: <istmt>()
: mata drop <istmt>()
```

- Technical note

The above story is not exactly true because, as told, variable x would be local to function <istmt> () so, when <istmt> () concluded execution, variable x would be discarded. To prevent that from happening, Mata makes all variables defined by <istmt> () global. Thus you can type

```
: x = 3
```

followed by

$$
: y=x+2
$$

and all works out just as you expect: y is set to 5 .

## What happens when you type a mata environment command

When you are at a colon prompt and type something that begins with the word mata, such as : mata describe
or

```
: mata clear
```

something completely different happens: Mata freezes itself and temporarily transfers control to a command interpreter like Stata's. The command interpreter accesses Mata's environment and reports on it or changes it. Once done, the interpreter returns to Mata, which comes back to life, and issues a new colon prompt:

Once something is typed at the prompt, Mata will examine it to determine if it begins with the word mata (in which case the interpretive process repeats), or if it is the beginning of a function definition (in which case the program will be compiled but not executed), or anything else (in which case Mata will try to compile and execute it as an <istmt> ()).

## Working with object code I: .mo files

Everything hinges on the object code that Mata produces, and, if you wish, you can save the object code on disk. The advantage of doing this is that, at some future date, your program can be executed without compilation, which saves time. If you send the object code to others, they can use your program without ever seeing the source code behind it.

After you type, say,

```
: function tryit()
> {
> real scalar i
>
> for (i=1; i<=10; i++) i
}
```

Mata has created the object code and discarded the source. If you now type

```
: mata mosave tryit()
```

the Mata interpreter will create file tryit.mo in the current directory; see [M-3] mata mosave. The new file will contain the object code.

At some future date, were you to type

```
: tryit()
```

without having first defined the program, Mata would look along the ado-path (see [P] sysdir and [U] 17.5 Where does Stata look for ado-files?) for a file named tryit.mo. Finding the file, Mata loads it (so Mata now has the object code and executes it in the usual way).

## Working with object code II: .mlib libraries

Object code can be saved in .mlib libraries (files) instead of .mo files. .mo files contain the object code for one program. .mlib files contain the object code for a group of files.

The first step is to choose a name (we will choose lmylib—library names are required to start with the lowercase letter $l$ ) and create an empty library of that name:

```
: mata mlib create lmylib
```

Once created, new functions can be added to the library:

```
: mata mlib add lmylib tryit()
```

New functions can be added at any time, either immediately after creation or later-even much later; see [M-3] mata mlib.

We mentioned that when Mata needs to execute a function that it does not find in memory, Mata looks for a .mo file of the same name. Before Mata does that, however, Mata thumbs through its libraries to see if it can find the function there.

## The Mata environment

Certain settings of Mata affect how it behaves. You can see those settings by typing mata query at the Mata prompt:

```
: mata query
```

```
    Mata settings
    set matastrict off
    set matalnum off
    set mataoptimize on
    set matafavor space may be space or speed
    set matacache kilobytes
    set matalibs lmatabase;lmatapss;lmatapostest;lmataerm;lmatapath;
> lmatagsem;lmataopt;lmatasem;lmataado;lmatafc;lmatasp;lmatamcmc;lmatacollect;
> lmatamixlog
    set matamofirst off
    set matasolvetol . may be . or any double-precision number
```

You can change these settings by using mata set; see [M-3] mata set. We recommend the default settings, except that we admit to being partial to mata set matastrict on.

## Reference

Gould, W. W. 2006. Mata Matters: Precision. Stata Journal 6: 550-560.

## Also see

[M-1] Intro - Introduction and advice

## Title

[M-1] Interactive - Using Mata interactively

## Description Remarks and examples Review Reference Also see

## Description

With Mata, you simply type matrix formulas to obtain the desired results. Below we provide guidelines when doing this with statistical formulas.

## Remarks and examples

You have data and statistical formulas that you wish to calculate, such as $b=\left(X^{\prime} X\right)^{-1} X^{\prime} y$. Perform the following nine steps:

1. Start in Stata. Load the data.
2. If you are doing time-series analysis, generate new variables containing any op.varname variables you need, such as l.gnp or d.r.
3. Create a constant variable (. generate cons $=1$ ). In most statistical formulas, you will find it useful.
4. Drop variables that you will not need. This saves memory and makes some things easier because you can just refer to all the variables.
5. Drop observations with missing values. Mata understands missing values, but Mata is a matrix language, not a statistical system, so Mata does not always ignore observations with missing values.
6. Put variables on roughly the same numeric scale. This is optional, but we recommend it. We explain what we mean and how to do this below.
7. Enter Mata. Do that by typing mata at the Stata command prompt. Do not type a colon after the mata. This way, when you make a mistake, you will stay in Mata.
8. Use Mata's st_view() function (see [M-5] st_view()) to create matrices based on your Stata dataset. Create all the matrices you want or find convenient. The matrices created by st_view() are in fact views onto one copy of the data.
9. Perform your matrix calculations.

If you are reading the entries in the order suggested in [M-0] Intro, see [M-1] How next.

## 1. Start in Stata; load the data

We will use the auto dataset and will fit the regression

$$
\operatorname{mpg}_{j}=b_{0}+b_{1} \text { weight }_{j}+b_{2} \text { foreign }_{j}+e_{j}
$$

by using the formulas

$$
\begin{aligned}
\mathbf{b} & =\left(\mathbf{X}^{\prime} \mathbf{X}\right)^{-1} \mathbf{X}^{\prime} \mathbf{y} \\
\mathbf{V} & =s^{2}\left(\mathbf{X}^{\prime} \mathbf{X}\right)^{-1}
\end{aligned}
$$

where

$$
\begin{aligned}
s^{2} & =\mathbf{e}^{\prime} \mathbf{e} /(n-k) \\
\mathbf{e} & =\mathbf{y}-\mathbf{X} \mathbf{b} \\
n & =\operatorname{rows}(\mathbf{X}) \\
k & =\operatorname{cols}(\mathbf{X})
\end{aligned}
$$

We begin by typing

```
. sysuse auto
(1978 automobile data)
```


## 2. Create any time-series variables

We do not have any time-series variables but, just for a minute, let's pretend we did. If our model contained lagged gnp, we would type

```
. generate lgnp = l.gnp
```

so that we would have a new variable lgnp that we would use in place of l.gnp in the subsequent steps.

## 3. Create a constant variable

```
generate cons = 1
```


## 4. Drop unnecessary variables

We will need the variables mpg, weight, foreign, and cons, so it is easier for us to type keep instead of drop:
. keep mpg weight foreign cons

## 5. Drop observations with missing values

We do not have any missing values in our data, but let's pretend we did, or let's pretend we are uncertain. Here is an easy trick for getting rid of observations with missing values:

```
. regress mpg weight foreign cons
. keep if e(sample)
```

We estimated a regression using all the variables and then kept the observations regress chose to use. It does not matter which variable you choose as the dependent variable, nor the order of the independent variables, so we just as well could have typed

```
. regress weight mpg foreign cons
. keep if e(sample)
```

or even

```
. regress cons mpg weight foreign
. keep if e(sample)
```

The output produced by regress is irrelevant, even if some variables are dropped. We are merely borrowing regress's ability to identify the subsample with no missing values.

Using regress causes Stata to make many unnecessary calculations and, if that offends you, here is a more sophisticated alternative:

```
. local O "mpg weight foreign cons"
syntax varlist
marksample touse
keep if 'touse'
drop 'touse'
```

Using regress is easier.

## 6. Put variables on roughly the same numeric scale

This step is optional, but we recommend it. You are about to use formulas that have been derived by people who assumed that the usual rules of arithmetic hold, such as $(a+b)-c=a+(b-c)$. Many of the standard rules, such as the one shown, are violated when arithmetic is performed in finite precision, and this leads to roundoff error in the final, calculated results.

You can obtain a lot of protection by making sure that your variables are on roughly the same scale, by which we mean their means and standard deviations are all roughly equal. By roughly equal, we mean equal up to a factor of 1,000 or so. So let's look at our data:
. summarize

| Variable | Obs | Mean | Std. dev. | Min | Max |
| ---: | ---: | ---: | ---: | ---: | ---: |
| mpg | 74 | 21.2973 | 5.785503 | 12 | 41 |
| weight | 74 | 3019.459 | 777.1936 | 1760 | 4840 |
| foreign | 74 | .2972973 | .4601885 | 0 | 1 |
| cons | 74 | 1 | 0 | 1 | 1 |

Nothing we see here bothers us much. Variable weight is the largest, with a mean and standard deviation that are 1,000 times larger than those of the smallest variable, foreign. We would feel comfortable, but only barely, ignoring scale differences. If weight were 10 times larger, we would begin to be concerned, and our concern would grow as weight grew.

The easiest way to address our concern is to divide weight so that, rather than measuring weight in pounds, it measures weight in thousands of pounds:

```
. replace weight = weight/1000
variable weight was int now float
(74 real changes made)
```

. summarize

| Variable | Obs | Mean | Std. dev. | Min | Max |
| ---: | ---: | ---: | ---: | ---: | ---: |
| mpg | 74 | 21.2973 | 5.785503 | 12 | 41 |
| weight | 74 | 3.019459 | .7771936 | 1.76 | 4.84 |
| foreign | 74 | .2972973 | .4601885 | 0 | 1 |
| cons | 74 | 1 | 0 | 1 | 1 |

What you are supposed to do is make the means and standard deviations of the variables roughly equal. If weight had a large mean and reasonable standard deviation, we would have subtracted, so that we would have had a variable measuring weight in excess of some number of pounds. Or we could do both, subtracting, say, 2,000 and then dividing by 100 , so we would have weight in excess of 2,000 pounds, measured in 100 -pound units.

Remember, the definition of roughly equal allows lots of leeway, so you do not have to give up easy interpretation.

## 7. Enter Mata

We type

```
. mata
```

: _

Mata uses a colon prompt, whereas Stata uses a period.

## 8. Use Mata's st_view( ) function to access your data

Our matrix formulas are

$$
\begin{aligned}
\mathbf{b} & =\left(\mathbf{X}^{\prime} \mathbf{X}\right)^{-1} \mathbf{X}^{\prime} \mathbf{y} \\
\mathbf{V} & =s^{2}\left(\mathbf{X}^{\prime} \mathbf{X}\right)^{-1}
\end{aligned}
$$

where

$$
\begin{aligned}
s^{2} & =\mathbf{e}^{\prime} \mathbf{e} /(n-k) \\
\mathbf{e} & =\mathbf{y}-\mathbf{X b} \\
n & =\operatorname{rows}(\mathbf{X}) \\
k & =\operatorname{cols}(\mathbf{X})
\end{aligned}
$$

so we are going to need $\mathbf{y}$ and $\mathbf{X}$. $\mathbf{y}$ is an $n \times 1$ column vector of dependent-variable values, and $\mathbf{X}$ is an $n \times k$ matrix of the $k$ independent variables, including the constant. Rows are observations, columns are variables.

We make the vector and matrix as follows:

```
: st_view(y=., ., "mpg")
: st_view(X=., ., ("weight", "foreign", "cons"))
```

Let us explain. We wish we could type

```
: y = st_view(., "mpg")
: X = st_view(., ("weight", "foreign", "cons"))
```

because that is what the functions are really doing. We cannot because st_view () (unlike all other Mata functions), returns a special kind of matrix called a view. A view acts like a regular matrix in nearly every respect, but views do not consume nearly as much memory, because they are in fact views onto the underlying Stata dataset!

We could instead create y and $X$ with Mata's st_data() function (see [M-5] st_data()), and then we could type the creation of y and X the natural way,

```
: y = st_data(., "mpg")
: X = st_data(., ("weight", "foreign", "cons"))
```

st_data() returns a real matrix, which is a copy of the data Stata has stored in memory.
We could use st_data() and be done with the problem. For our automobile-data example, that would be a fine solution. But were the automobile data larger, you might run short of memory, and views can save lots of memory. You can create views willy-nilly-lots and lots of them-and never consume much memory! Views are wonderfully convenient and it is worth mastering the little bit of syntax to use them.
st_view() requires three arguments: the name of the view matrix to be created, the observations (rows) the matrix is to contain, and the variables (columns). If we wanted to create a view matrix Z containing all the observations and all the variables, we could type

```
: st_view(Z, ., .)
```

st_view() understands missing value in the second and third positions to mean all the observations and all the variables. Let's try it:

```
: st_view(Z, ., .)
    <istmt>: 3499 Z not found
r(3499);
: _
```

That did not work because Mata requires Z to be predefined. The reasons are technical, but it should not surprise you that function arguments need to be defined before a function can be used. Mata just does not understand that st_view() really does not need $Z$ defined. The way around Mata's confusion is to define Z and then let st_view() redefine it:

```
: Z = .
: st_view(Z, ., .)
```

You can, if you wish, combine all that into one statement

```
: st_view(Z=., ., .)
```

and that is what we did when we defined y and X :

```
: st_view(y=., ., "mpg")
: st_view(X=., ., ("weight", "foreign", "cons"))
```

The second argument (.) specified that we wanted all the observations, and the third argument specified the variables we wanted. Be careful not to omit the "extra" parentheses when typing the variables. Were you to type

```
: st_view(X=., ., "weight", "foreign", "cons")
```

you would be told you typed an invalid expression. st_view() expects three arguments, and the third argument is a row vector specifying the variables to be selected: ("weight", "foreign", "cons").

At this point, we suggest you type

```
: y
    (output omitted)
X
    (output omitted)
```

to see that y and X really do contain our data. In case you have lost track of what we have typed, here is our complete session so far:
. sysuse auto

- generate cons = 1
- keep mpg weight foreign cons
. regress mpg weight foreign cons
. keep if e(sample)
. replace weight $=$ weight/1000
. mata
: st_view (y=., ., "mpg")
: st_view(X=., ., ("weight", "foreign", "cons"))


## 9. Perform your matrix calculations

To remind you: our matrix calculations are

$$
\begin{aligned}
\mathbf{b} & =\left(\mathbf{X}^{\prime} \mathbf{X}\right)^{-1} \mathbf{X}^{\prime} \mathbf{y} \\
\mathbf{V} & =s^{2}\left(\mathbf{X}^{\prime} \mathbf{X}\right)^{-1}
\end{aligned}
$$

where

$$
\begin{aligned}
s^{2} & =\mathbf{e}^{\prime} \mathbf{e} /(n-k) \\
\mathbf{e} & =\mathbf{y}-\mathbf{X b} \\
n & =\operatorname{rows}(\mathbf{X}) \\
k & =\operatorname{cols}(\mathbf{X})
\end{aligned}
$$

Let's get our regression coefficients,

```
: b = invsym(X'X)*X'y
    : b
```

    1
    1
-6.587886358
-1.650029004
41.67970227
and let's form the residuals, define $n$ and $k$, and obtain $s^{2}$,

```
: e = y - X*b
: n = rows(X)
: k = cols(X)
: s2 = (e'e)/(n-k)
```

so we are able to calculate the variance matrix:

```
: V = s2*invsym(X'X)
: V
[symmetric]
```

1
2
3


We are done.

We can present the results in more readable fashion by pulling the diagonal of $V$ and calculating the square root of each element:

```
: se = sqrt(diagonal(V))
: (b, se)
```

    \(1 \quad 2\)
    $-6.587886358 \quad .6371129122$
$-1.6500290041 .075994086$
$41.67970227 \quad 2.165547114$

You know that if we were to type

```
2+3
5
```

Mata evaluates the expression and shows us the result, and that is exactly what happened when we typed
: (b, se)
( $b$, se) is an expression, and Mata evaluated it and displayed the result. The expression means to form the matrix whose first column is $b$ and second column is se. We could obtain a listing of the coefficient, standard error, and its $t$ statistic by asking Mata to display (b, se, b:/se),

```
: (b, se, b:/se)
```

$$
\begin{array}{ll}
1 & 2
\end{array}
$$

3

| 1 | -6.587886358 | .6371129122 | -10.34021793 |
| :--- | ---: | ---: | ---: |
| 2 | -1.650029004 | 1.075994086 | -1.533492633 |
| 3 | 41.67970227 | 2.165547114 | 19.24673077 |
|  |  |  |  |

In the expression above, $\mathrm{b}: /$ se means to divide the elements of b by the elements of se. :/ is called a colon operator and you can learn more about it by seeing [M-2] op_colon.

We could add the significance level by typing

|  | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| 1 | -6.587886358 | . 6371129122 | -10.34021793 | 8.28286e-16 |
| 2 | -1.650029004 | 1.075994086 | -1.533492633 | . 1295987129 |
| 3 | 41.67970227 | 2.165547114 | 19.24673077 | $6.89556 \mathrm{e}-30$ |

Those are the same results reported by regress; type
. sysuse auto
. replace weight = weight/1000
. regress mpg weight foreign
and compare results.

## Review

Our complete session was

```
. sysuse auto
generate cons = 1
. keep mpg weight foreign cons
regress mpg weight foreign cons
keep if e(sample)
replace weight = weight/1000
mata
st_view(y=., ., "mpg")
st_view(X=., ., ("weight", "foreign", "cons"))
b = invsym(X'X)*X'y
b
e = y - X*b
n = rows(X)
k = cols(X)
s2= (e'e)/(n-k)
V = s2*invsym(X'X)
V
se = sqrt(diagonal(V))
(b, se)
(b, se, b:/se)
(b, se, b:/se, 2*ttail(n-k, abs(b:/se)))
end
```


## Reference

Gould, W. W. 2006. Mata Matters: Interactive use. Stata Journal 6: 387-396.

## Also see

[M-1] Intro - Introduction and advice

## Title

[M-1] LAPACK — Linear algebra package (LAPACK) routines

| Description | Syntax | Option for set lapack_mkl |
| :--- | :--- | :--- |
| Remarks and examples | Acknowledgments | References |

Also see

## Description

LAPACK stands for Linear Algebra PACKage and is a freely available set of Fortran 90 routines for solving systems of simultaneous equations, eigenvalue problems, and singular value problems. Many of the LAPACK routines are based on older EISPACK and LINPACK routines, and the more modern LAPACK does much of its computation by using Basic Linear Algebra Subprograms (BLAS).

Stata contains two sets of LAPACK and BLAS libraries; one is from Netlib, and the other is from the Intel Math Kernel Library (MKL).
set lapack_mkl sets which LAPACK library will be used.
set lapack_mkl_cnr sets the conditional numerical reproducibility mode for the Intel MKL LAPACK routines.

## Syntax

Set whether Intel MKL LAPACK routines will be used
set lapack_mkl $\{$ on $\mid$ off $\}[$, permanently]
Set the conditional numerical reproducibility mode for the Intel MKL LAPACK routines
set lapack_mkl_cnr $\{$ default|auto|compatible|off $\}$

## Option for set lapack_mkl

permanently specifies that, in addition to making the change right now, the setting be remembered and become the default setting when you invoke Stata in the future.

## Remarks and examples

Remarks are presented under the following headings:

```
LAPACK in Mata
set lapack_mkl
Intel MKL conditional numerical reproducibility
set lapack_mkl_cnr
```


## LAPACK in Mata

The LAPACK and BLAS routines form the basis for many of Mata's linear algebra capabilities. Individual functions of Mata that use LAPACK routines always make note of that fact.

Stata on all platforms, except ARM-based Mac (Apple Silicon), contains two sets of LAPACK and BLAS libraries. One is based on the source code from Netlib's LAPACK and has been used in Stata since Stata 9. Stata, since Stata 17, also contains LAPACK and BLAS libraries from the Intel MKL. Because the Intel MKL does not support ARM-based Mac, ARM-based Mac supports only Netlib's LAPACK library.

For up-to-date information on LAPACK, see http://www.netlib.org/lapack/.
For up-to-date information on Intel MKL, see https://software.intel.com/mkl.
Advanced programmers can directly access the LAPACK functions; see [M-5] lapack().

## set lapack_mkI

For platforms that support both LAPACK libraries, the default is set lapack_mkl on, meaning the Intel MKL LAPACK routines are used. To instead use Netlib's LAPACK library, you can set lapack_mkl off. To determine which library is being used, check the contents of c (lapack_mkl). On ARM-based Mac, lapack_mkl is not settable and is always off.

Note that set lapack_mkl should be specified in Stata, not in Mata.

## Intel MKL conditional numerical reproducibility

Regardless of which LAPACK routine you use, you may encounter slight numeric differences when you run the same code in the same version and platform of Stata on a different OS. This is normal and can be caused by several reasons. For example, Stata is compiled with different compilers on different operating systems, and different compilers could produce slightly different numeric results.

You might also get different results on the same operating system if you use the Intel MKL routines. Rosenquist (2011) explains why: "To get the best performance wherever a program is run, Intel MKL will check on the processor type at run time and can dispatch processor-specific code accordingly. If a particular instruction set or cache of a certain size is available a specialized code path may exploit it. These code paths are different enough that they will again cause a different order of operations and will cause slightly different results on different processors." This means that there could be slight numeric differences when running the same code on two machines with the same flavor of Stata, the same operating system, but with different CPUs. That said, you can obtain different levels of numerical reproducibility with set lapack_mkl_cnr, discussed below.

## set lapack_mkl_cnr

Intel offers a couple of different conditional numerical reproducibility (CNR) functions for obtaining different levels of reproducibility, and the type of reproducibility desired can be specified with the lapack_mkl_cnr setting. The lapack_mkl_cnr setting should be specified in Stata, not Mata, and it will take effect the next time you launch Stata. The possible values are default, auto, compatible, or off.
set lapack_mkl_cnr default sets the reproducibility level to SSE4_2 on Intel CPUs. On non-Intel CPUs, set lapack_mkl_cnr default is the same as set lapack_mkl_cnr auto.
set lapack_mkl_cnr auto allows the Intel MKL to automatically determine the code path based on the different features the CPU supports. The setting takes advantage of the specific features of the CPU, hence the improved performance. The cost is the lower level of numerical reproducibility; the same LAPACK function may return different results on different CPUs, assuming you are using the function on the same operating system and flavor of Stata.

Note that the automatically determined code path will not necessarily be the most efficient. For example, we noticed that the AVX instruction set caused excessively high CPU usage on Intel CPUs when Stata was idle.
set lapack_mkl_cnr compatible provides a higher level of numerical reproducibility compared with set lapack_mkl_cnr auto. In this setting, the LAPACK functions try to produce the same results by using a set of the most common features supported by different CPUs. This setting is also the slowest in terms of performance.
set lapack_mkl_cnr off turns off the code that tries to maintain conditional numerical reproducibility, which results in greater performance. With this setting, you may get different results across different runs of the same LAPACK function on the same copy of Stata and on the same machine.

For a detailed introduction to the Intel MKL conditional numerical reproducibility, see Rosenquist (2011).

## Acknowledgments

We thank the authors of LAPACK for their excellent work:
E. Anderson, Z. Bai, C. Bischof, S. Blackford, J. Demmel, J. Dongarra, J. Du Croz, A. Greenbaum, S. Hammarling, A. McKenney, and D. Sorensen.

## References

Anderson, E., Z. Bai, C. Bischof, S. Blackford, J. Demmel, J. J. Dongarra, J. Du Croz, A. Greenbaum, S. Hammarling, A. McKenney, and D. Sorensen. 1999. LAPACK Users' Guide. 3rd ed. Philadelphia: Society for Industrial and Applied Mathematics.

Rosenquist, T. 2011. Getting reproducible results with Intel MKL.
https://software.intel.com/content/www/us/en/develop/articles/getting-reproducible-results-with-intel-mkl.html.

[^0]
## Title

[M-1] Limits - Limits and memory utilization

## Description Summary Remarks and examples Also see

## Description

Mata imposes limits, but those limits are of little importance compared with the memory requirements.
Mata stores matrices in memory and requests the memory for them from the operating system.

## Summary

Limits:

|  | Minimum | Maximum |
| :--- | :---: | :---: |
| Scalars, vectors, matrices |  |  |
| rows | 0 | $2^{48}-1$ |
| columns | 0 | $2^{48}-1$ |
| String elements, length | 0 | $2,147,483,647$ |

Size approximations:
Memory requirements

| real matrices | $o h+r * c * 8$ |
| :--- | :--- |
| complex matrices | $o h+r * c * 16$ |
| pointer matrices | $o h+r * c * 8$ |
| string matrices | $o h+r * c * 8+$ total_length_of_strings |

where $r$ and $c$ represent the number of rows and columns and where $o h$ is overhead and is approximately 64 bytes

## Remarks and examples

Mata requests (and returns) memory from the operating system as it needs it, and if the operating system cannot provide it, Mata issues the following error:

```
: x = foo(A, B)
    <istmt>: 3499 foo() not found
r(3499);
```

Stata's set min_memory and set max_memory values (see [D] memory) play no role in Mata or, at least, they play no direct role.

## Also see

[M-3] mata memory - Report on Mata's memory usage
[M-5] mindouble() - Minimum and maximum nonmissing value
[M-1] Intro - Introduction and advice

## Title

[M-1] Naming - Advice on naming functions and variables
Description Syntax Remarks and examples Also see

## Description

Advice is offered on how to name variables and functions.

## Syntax

A name is $1-32$ characters long, the first character of which must be

$$
\mathrm{A}-\mathrm{Z} \quad \mathrm{a}-\mathrm{Z} \quad-
$$

and the remaining characters of which may be

$$
A-Z \quad a-z \quad-\quad 0-9
$$

except that names may not be a word reserved by Mata (see [M-2] reswords for a list).
Examples of names include

| x | x2 | alpha |
| :--- | :--- | :--- |
| logarithm_of_x | Log0fX |  |

Case matters: alpha, Alpha, and ALPHA are different names.
Variables and functions have separate name spaces, which means a variable and a function can have the same name, such as value and value(), and Mata will not confuse them.

## Remarks and examples

Remarks are presented under the following headings:

```
Interactive use
Naming variables
Naming functions
What happens when functions have the same names
How to determine if a function name has been taken
```


## Interactive use

Use whatever names you find convenient: Mata will tell you if there is a problem.
The following sections are for programmers who want to write code that will require the minimum amount of maintenance in the future.

## Naming variables

Mostly, you can name variables however you please. Variables are local to the program in which they appear, so one function can have a variable or argument named $x$ and another function can have a variable or argument of the same name, and there is no confusion.

If you are writing a large system of programs that has global variables, on the other hand, we recommend that you give the global variables long names, preferably with a common prefix identifying your system. For instance,

```
multeq_no_of_equations
multeq_eq
multeq_inuse
```

This way, your variables will not become confused with variables from other systems.

## Naming functions

Our first recommendation is that, for the most part, you give functions all-lowercase names: foo() rather than FOO () or FOO (). If you have a function with one or more capital letters in the name, and if you want to save the function's object code, you must do so in .mlib libraries; .mo files are not sufficient. .mo files require that filename be the same as the function name, which means Foo.mo for Foo(). Not all operating systems respect case in filenames. Even if your operating system does respect case, you will not be able to share your .mo files with others whose operating systems do not.

We have no strong recommendation against mixed case; we merely remind you to use .mlib library files if you use it.

Of greater importance is the name you choose. Mata provides many functions and more will be added over time. You will find it more convenient if you choose names that StataCorp and other users do not choose.

That means to avoid words that appear in the English-language dictionary and to avoid short names, say, those four characters or fewer. You might have guessed that $\operatorname{svd}($ ) would be taken, but who would have guessed lud()? Or qrd()? Or e()?

Your best defense against new official functions, and other community-contributed functions, is to choose long function names.

## What happens when functions have the same names

There are two kinds of official functions: built-in functions and library functions. Communitycontributed functions are invariably library functions (here we draw no distinction between functions supplied in .mo files and those supplied in .mlib files).

Mata will issue an error message if you attempt to define a function with the same name as a built-in function.

Mata will let you define a new function with the same name as a library function if the library function is not currently in memory. If you store your function in a .mo file or a .mlib library, however, in the future the official Mata library function will take precedence over your function: your function will never be loaded. This feature works nicely for interactive users, but for long-term programming, you will want to avoid naming your functions after Mata functions.

A similar result is obtained if you name your function after a community-contributed function that is installed on your computer. You can do so if the community-contributed function is not currently in memory. In the future, however, one or the other function will take precedence and, no matter which, something will break.

## How to determine if a function name has been taken

Use mata which (see [M-3] mata which):

```
: mata which det_of_triangular()
function det_of_triangular() not found
r(111);
: mata which det()
        det(): lmatabase
```


## Also see

[M-2] reswords - Reserved words
[M-1] Intro - Introduction and advice

## Title

[M-1] Permutation - An aside on permutation matrices and vectors

Description Syntax Remarks and examples Also see

## Description

Permutation matrices are a special kind of orthogonal matrix that, via multiplication, reorder the rows or columns of another matrix. Permutation matrices cast the reordering in terms of multiplication.

Permutation vectors also reorder the rows or columns of another matrix, but they do it via subscripting. This alternative method of achieving the same end is, computerwise, more efficient, in that it uses less memory and less computer time.

The relationship between the two is shown below.

## Syntax

|  | Permutation matrix <br> notation | Permutation vector <br> notation |
| :--- | :--- | :--- |
| permute rows | $B=P * A$ | $B=A[p,]$. |
| permute columns | $B=A * P$ | $B=A[., p]$ |
| unpermute rows | $B=P^{\prime} A$ | $B=A ; B[p,]=A$. |
|  |  | or |
| unpermute columns | $B=A * P^{\prime}$ | $B=A[\operatorname{invorder}(p),]$. |
|  | $B=A ; B[, . p]=A$ |  |
|  |  | or |
|  | $B=A[., \operatorname{invorder}(p)]$ |  |

A permutation matrix is an $n \times n$ matrix that is a row (or column) permutation of the identity matrix.
A permutation vector is a $1 \times n$ or $n \times 1$ vector of the integers 1 through $n$.
The following permutation matrix and permutation vector are equivalent:

$$
P=\left[\begin{array}{lll}
0 & 1 & 0 \\
0 & 0 & 1 \\
1 & 0 & 0
\end{array}\right] \Longleftrightarrow p=\left[\begin{array}{l}
2 \\
3 \\
1
\end{array}\right]
$$

Either can be used to permute the rows of

$$
A=\left[\begin{array}{llll}
a & b & c & d \\
e & f & g & h \\
i & j & k & l
\end{array}\right] \text { to produce }\left[\begin{array}{llll}
e & f & g & h \\
i & j & k & l \\
a & b & c & d
\end{array}\right]
$$

and to permute the columns of

$$
P=\left[\begin{array}{ccc}
m & n & o \\
p & q & r \\
s & t & u \\
v & w & x
\end{array}\right] \text { to produce }\left[\begin{array}{ccc}
n & o & m \\
q & r & p \\
t & u & s \\
w & x & v
\end{array}\right]
$$

## Remarks and examples

Remarks are presented under the following headings:

Permutation matrices<br>How permutation matrices arise<br>Permutation vectors

## Permutation matrices

A permutation matrix is a square matrix whose rows are a permutation of the identity matrix. The following are the full set of all $2 \times 2$ permutation matrices:

$$
\begin{align*}
& {\left[\begin{array}{ll}
1 & 0 \\
0 & 1
\end{array}\right]}  \tag{1}\\
& {\left[\begin{array}{ll}
0 & 1 \\
1 & 0
\end{array}\right]} \tag{2}
\end{align*}
$$

Let $P$ be an $n \times n$ permutation matrix. If $n \times m$ matrix $A$ is premultiplied by $P$, the result is to reorder its rows. For example,

$$
\left.\begin{array}{rl}
P & \left.* \begin{array}{cc}
A &
\end{array}{ }_{\left[\begin{array}{ll}
0 & P A \\
0 & 1
\end{array} 0\right.}^{1} \begin{array}{lll}
0 & 1 \\
1 & 0 & 0
\end{array}\right]
\end{array} *^{*} \begin{array}{lll}
1 & 2 & 3  \tag{3}\\
4 & 5 & 6 \\
7 & 8 & 9
\end{array}\right] \quad=\left[\begin{array}{lll}
4 & 5 & 6 \\
7 & 8 & 9 \\
1 & 2 & 3
\end{array}\right]
$$

Above, we illustrated the reordering using square matrix $A$, but $A$ did not have to be square.
If $m \times n$ matrix $B$ is postmultiplied by $P$, the result is to reorder its columns. We illustrate using square matrix $A$ again:

$$
\begin{array}{rl}
A & * \begin{array}{ll}
P & \\
{\left[\begin{array}{lll}
1 & 2 & 3 \\
4 & 5 & 6 \\
7 & 8 & 9
\end{array}\right]}
\end{array} *\left[\begin{array}{lll}
0 & 1 & 0 \\
0 & 0 & 1 \\
1 & 0 & 0
\end{array}\right]
\end{array}=\begin{array}{lll}
A P \tag{4}
\end{array}
$$

Say that we reorder the rows of $A$ by forming $P A$. Obviously, we can unreorder the rows by forming $P^{-1} P A$. Because permutation matrices are orthogonal, their inverses are equal to their transpose. Thus the inverse of the permutation matrix ( $0,1,0 \backslash 0,0,1 \backslash 1,0,0$ ) we have been using is $(0,0,1 \backslash 1,0,0 \backslash 0,1,0)$. For instance, taking our results from (3)

Allow us to summarize:

1. A permutation matrix $P$ is a square matrix whose rows are a permutation of the identity matrix.
2. $P A=$ a row permutation of $A$.
3. $A P=$ a column permutation of $A$.
4. The inverse permutation is given by $P^{\prime}$.
5. $P^{\prime} P A=A$.
6. $A P P^{\prime}=A$.

## How permutation matrices arise

Some of Mata's matrix functions implicitly permute the rows (or columns) of a matrix. For instance, the LU decomposition of matrix $A$ is defined as

$$
A=L U
$$

where $L$ is lower triangular and $U$ is upper triangular. For any matrix $A$, one can solve for $L$ and $U$, and Mata has a function that will do that (see [M-5] lud()). However, Mata's function does not solve the problem as stated. Instead, it solves

$$
P^{\prime} A=L U
$$

where $P^{\prime}$ is a permutation matrix that Mata makes up! Just to be clear; Mata's function solves for $L$ and $U$, but for a row permutation of $A$, not $A$ itself, although the function does tell you what permutation it chose (the function returns $L, U$, and $P$ ). The function permutes the rows because, that way, it can produce a more accurate answer.

You will sometimes read that a function engages in pivoting. What that means is that, rather than solving the problem for the matrix as given, it solves the problem for a permutation of the original matrix, and the function chooses the permutation in a way to minimize numerical roundoff error. Functions that do this invariably return the permutation matrix along with the other results, because you are going to need it.

For instance, one use of LU decomposition is to calculate inverses. If $A=L U$ then $A^{-1}=U^{-1} L^{-1}$. Calculating the inverses of triangular matrices is an easy problem, so one recipe for calculating inverses is

1. decompose $A$ into $L$ and $U$,
2. calculate $U^{-1}$,
3. calculate $L^{-1}$, and
4. multiply the results together.

That would be the solution except that the LU decomposition function does not decompose $A$ into $L$ and $U$; it decomposes $P^{\prime} A$, although the function does tell us $P$. Thus we can write,

$$
\begin{array}{rlr}
P^{\prime} A & =L U & \\
A & =P L U & \\
A^{-1} & =U^{-1} L^{-1} P^{\prime} & \text { remember } \left.P^{\prime-1}=P\right)
\end{array}
$$

Thus the solution to our problem is to use the $U$ and $L$ just as we planned-calculate $U^{-1} L^{-1}$ —and then make a column permutation of that, which we can do by postmultiplying by $P^{\prime}$.

There is, however, a detail that we have yet to reveal to you: Mata's LU decomposition function does not return $P$, the permutation matrix. It instead returns $p$, a permutation vector equivalent to $P$, and so the last step-forming the column permutation by postmultiplying by $P^{\prime}$-is done differently. That is the subject of the next section.

Permutation vectors are more efficient than permutation matrices, but you are going to discover that they are not as mathematically transparent. Thus when working with a function that returns a permutation vector-when working with a function that permutes the rows or columns-think in terms of permutation matrices and then translate back into permutation vectors.

## Permutation vectors

Permutation vectors are used with Mata's subscripting operator, so before explaining permutation vectors, let's understand subscripting.

Not surprisingly, Mata allows subscripting. Given matrix

$$
A=\left[\begin{array}{lll}
1 & 2 & 3 \\
4 & 5 & 6 \\
7 & 8 & 9
\end{array}\right]
$$

Mata understands that

$$
A[2,3]=6
$$

Mata also understands that if one or the other subscript is specified as . (missing value), the entire column or row is to be selected:

$$
\begin{gathered}
A[., 3]=\left[\begin{array}{l}
3 \\
6 \\
9
\end{array}\right] \\
A[2, .]=\left[\begin{array}{lll}
4 & 5 & 6
\end{array}\right]
\end{gathered}
$$

Mata also understands that if a vector is specified for either subscript

$$
A\left[\left[\begin{array}{l}
2 \\
3 \\
2
\end{array}\right], .\right]=\left[\begin{array}{lll}
4 & 5 & 6 \\
7 & 8 & 9 \\
4 & 5 & 6
\end{array}\right]
$$

In Mata, we would actually write the above as $A[(2 \backslash 3 \backslash 2),$.$] , and Mata would understand that we$ want the matrix made up of rows 2,3 , and 2 of $A$ and all columns. Similarly, we can request all rows and columns 2,3 , and 2 :

$$
A[.,(2,3,2)]=\left[\begin{array}{lll}
2 & 3 & 2 \\
5 & 6 & 5 \\
8 & 9 & 8
\end{array}\right]
$$

In the above, we wrote $(2,3,2)$ as a row vector because it seems more logical that way, but we could just as well have written $A[.,(2 \backslash 3 \backslash 2)]$. In subscripting, Mata does not care whether the vectors are rows or columns.

In any case, we can use a vector of indices inside Mata's subscripts to select rows and columns of a matrix, and that means we can permute them. All that is required is that the vector we specify contain a permutation of the integers 1 through $n$ because, otherwise, we would repeat some rows or columns and omit others.

A permutation vector is an $n \times 1$ or $1 \times n$ vector containing a permutation of the integers 1 through $n$. For example, the permutation vector equivalent to the permutation matrix

$$
P=\left[\begin{array}{lll}
0 & 1 & 0 \\
0 & 0 & 1 \\
1 & 0 & 0
\end{array}\right]
$$

is

$$
p=\left[\begin{array}{l}
2 \\
3 \\
1
\end{array}\right]
$$

$p$ can be used with subscripting to permute the rows of $A$

$$
A[p, .]=\left[\begin{array}{lll}
4 & 5 & 6 \\
7 & 8 & 9 \\
1 & 2 & 3
\end{array}\right]
$$

and similarly, $A[., p]$ would permute the columns.
Also subscripting can be used on the left-hand side of the equal-sign assignment operator. So far, we have assumed that the subscripts are on the right-hand side of the assignment operator, such as

$$
B=A[p, .]
$$

We have learned that if $p=(2 \backslash 3 \backslash 1$ ) (or $p=(2,3,1)$ ), the result is to copy the second row of $A$ to the first row of $B$, the third row of $A$ to the second row of $B$, and the first row of $A$ to the third row of $B$. Coding

$$
\mathrm{B}[\mathrm{p}, .]=\mathrm{A}
$$

does the inverse: it copies the first row of $A$ to the second row of $B$, the second row of $A$ to the third row of $B$, and the third row of $A$ to the first row of $B . B[p,]=$.$A really is the inverse of C=$ $A[p,$.$] in that, if we code$

$$
\begin{aligned}
& C=A[p, .] \\
& B[p, .]=C
\end{aligned}
$$

$B$ will be equal to $A$, and if we code

$$
\begin{aligned}
& C[p, .]=A \\
& B=C[p, .]
\end{aligned}
$$

$B$ will also be equal to $A$.
There is, however, one pitfall that you must watch for when using subscripts on the left-hand side: the matrix on the left-hand side must already exist and it must be of the appropriate (here same) dimension. Thus when performing the inverse copy, it is common to code

$$
\begin{aligned}
& B=C \\
& B[p, .]=C
\end{aligned}
$$

The first line is not unnecessary; it is what ensures that $B$ exists and is of the proper dimension, although we could just as well code

$$
\begin{aligned}
& B=J(\operatorname{rows}(C), \operatorname{cols}(C), .) \\
& B[p, .]=C
\end{aligned}
$$

The first construction is preferred because it ensures that $B$ is of the same type as $C$. If you really like the second form, you should code

$$
\begin{aligned}
& B=J(\operatorname{rows}(C), \text { cols }(C), \text { missingof }(C)) \\
& B[p, .]=C
\end{aligned}
$$

Going back to the preferred code

$$
\begin{aligned}
& B=C \\
& B[p, .]=C
\end{aligned}
$$

some programmers combine it into one statement:

$$
(B=C)[p, .]=C
$$

Also Mata provides an invorder ( $p$ ) (see [M-5] invorder()) that will return an inverted $p$ appropriate for use on the right-hand side, so you can also code

$$
B=C[\text { invorder }(p), .]
$$

## Also see

[M-5] invorder( ) - Permutation vector manipulation
[M-1] Intro - Introduction and advice

## Title

[M-1] Returned args - Function arguments used to return results

Description Syntax Remarks and examples Also see

## Description

Most Mata functions leave their arguments unchanged and return a result:

$$
: y=f(x, \ldots)
$$

Some Mata functions, however, return nothing and instead return results in one or more arguments:

$$
: g(x, \ldots, y)
$$

If you use such functions interactively and the arguments that are to receive results are not already defined ( $y$ in the above example), you will get a variable-not-found error. The solution is to define the arguments to contain something-anything-before calling the function:

```
: y = .
:g(x, ..., y)
```

You can combine this into one statement:

$$
: g(x, \ldots, y=.)
$$

## Syntax

$$
\begin{array}{ll}
y=f(x, \ldots) & \text { (function returns result the usual way) } \\
g(x, \ldots, y) & \text { (function returns result in argument } y \text { ) }
\end{array}
$$

## Remarks and examples

$\operatorname{sqrt}(a)$-see $[\mathrm{M}-5] \operatorname{sqrt}()$-calculates the (element-by-element) square root of $a$ and returns the result:

```
    : x = 4
    : y = sqrt(x)
    : y // y now contains 2
    2
    x // x is unchanged
    4
```

Most functions work like sqrt(), although many take more than one argument.
On the other hand, polydiv $\left(c_{a}, c_{b}, c_{q}, c_{r}\right)$ —see [M-5] polyeval()—takes the polynomial stored in $c_{a}$ and the polynomial stored in $c_{b}$ and divides them. It returns the quotient in the third argument $\left(c_{q}\right)$ and the remainder in the fourth $\left(c_{r}\right) . c_{a}$ and $c_{b}$ are left unchanged. The function itself returns nothing:

```
: A = (1,2,3)
: B = (0,1)
: polydiv(A, B, Q, R)
```

```
: Q
// Q has been redefined
            1 2
1
                2 3
R
// as has R
    1
A // while A and B are unchanged
1
1 2 3
B
            1 2
1
    0
```

As another example, st_view ( $V, i, j$ )—see [M-5] st_view( )—creates a view onto the Stata dataset. Views are like matrices but consume less memory. Arguments $i$ and $j$ specify the observations and variables to be selected. Rather than returning the matrix, however, the result is returned in the first argument ( $V$ ).

```
: st_view(V, (1\5), ("mpg", "weight"))
: V
            1 2
\begin{array} { | l l } { 2 2 } & { 2 9 3 0 } \\ { 1 5 } & { 4 0 8 0 } \\ { \hline } \end{array}
```

If you try to use these functions interactively, you will probably get an error:

```
: polydiv(A, B, Q, R)
        <istmt>: 3499 Q not found
r(3499);
: st_view(V, (1\5), ("mpg", "weight"))
    <istmt>: 3499 V not found
r(3499);
```

Arguments must be defined before they are used, even if their only purpose is to receive a newly calculated result. In such cases, it does not matter how the argument is defined because its contents will be replaced. Easiest is to fill in a missing value:

```
: Q = .
: R = .
: polydiv(A, B, Q, R)
: V = .
st_view(V, (1\5), ("mpg", "weight"))
```

You can also define the argument inside the function:

```
: polydiv(A, B, Q=., R=.)
: st_view(V=., (1\5), ("mpg", "weight"))
```

When you use functions like these inside a program, however, you need not worry about defining the arguments, because they are defined by virtue of appearing in your program:

```
function foo()
{
    polydiv(A, B, Q, R)
    st_view(V, (1\5), ("mpg", "weight"))
}
```

When Mata compiles your program, however, you may see warning messages:

```
: function foo()
> {
> ...
> polydiv(A, B, Q, R)
> st_view(V, (1\5), ("mpg", "weight"))
> ...
> }
note: variable Q may be used before set
note: variable R may be used before set
note: variable V may be used before set
```

If the warning messages bother you, either define the variables before they are used just as you would interactively or use pragma to suppress the warning messages; see [M-2] pragma.

## Also see

[M-1] Intro - Introduction and advice

## Title

[M-1] Source - Viewing the source code

Description Syntax Remarks and examples Also see

## Description

Many Mata functions are written in Mata. viewsource will allow you to examine their source code.

## Syntax

viewsource functionname . mata

## Remarks and examples

Some Mata functions are implemented in C (they are part of Mata itself), and others are written in Mata.
viewsource allows you to look at the official source code written in Mata. Reviewing this code is a great way to learn Mata.

The official source code is stored in .mata files. To see the source code for diag() (see [M-5] diag( )), for instance, type

```
. viewsource diag.mata
```

You type this at Stata's dot prompt, not at Mata's colon prompt.
If a function is built in, such as abs () (see [M-5] abs()), here is what will happen when you attempt to view the source code:

```
. viewsource abs.mata
file "abs.mata" not found
r(601);
```

You can verify that abs () is built in by using the mata which (see [M-3] mata which) command:

```
. mata: mata which abs()
abs(): built-in
```

viewsource can be also used to look at source code of community-contributed functions if the distribution included the source code (it might not).

## Also see

[P] viewsource - View source code
[M-1] Intro - Introduction and advice

## Title

[M-1] Tolerance - Use and specification of tolerances
Description Syntax Remarks and examples Also see

## Description

The results provided by many of the numerical routines in Mata depend on tolerances. Mata provides default tolerances, but those can be overridden.

## Syntax

somefunction (..., real scalar tol, ... )
where, concerning argument tol,
optional Argument tol is usually optional; not specifying tol is equivalent to specifying tol $=1$.
tol $>0 \quad$ Specifying tol $>0$ specifies the amount by which the usual tolerance is to be multiplied: tol $=2$ means twice the usual tolerance; tol $=0.5$ means half the usual tolerance.
tol $<0 \quad$ Specifying tol $<0$ specifies the negative of the value to be used for the tolerance: tol $=-1 \mathrm{e}-14$ means $1 \mathrm{e}-14$ is to be used.
tol $=0 \quad$ Specifying $t o l=0$ means all numbers are to be taken at face value, no matter how close to 0 they are. The single exception is when tol is applied to values that, mathematically, must be greater than or equal to zero. Then negative values (which arise from roundoff error) are treated as if they were zero.

The default tolerance is given by formula, such as

$$
\text { eta }=1 \mathrm{e}-14
$$

or

$$
e t a=\operatorname{epsilon}(1) \quad(\text { see }[M-5] \text { epsilon }())
$$

or

$$
\text { eta }=1000 * \operatorname{epsilon}(\operatorname{trace}(\operatorname{abs}(A)) / \text { rows }(A))
$$

Specifying tol $>0$ specifies a value to be used to multiply eta. Specifying tol $<0$ specifies that - tol be used in place of eta. Specifying tol $=0$ specifies that eta be set to 0 .

The formula for eta and how eta is used are found under Remarks and examples. For instance, the Remarks and examples might say that $A$ is declared to be singular if any diagonal element of $U$ of its LU decomposition is less than or equal to eta.

## Remarks and examples

Remarks are presented under the following headings:

## The problem

Absolute versus relative tolerances
Specifying tolerances

## The problem

In many formulas, zero is a special number in that, when the number arises, sometimes the result cannot be calculated or, other times, something special needs to be done.

The problem is that zero- 0.00000000000 -seldom arises in numerical calculation. Because of roundoff error, what would be zero were the calculation performed in infinite precision in fact is $1.03948 \mathrm{e}-15$, or $-4.4376 \mathrm{e}-16$, etc.

If one behaves as if these small numbers are exactly what they seem to be (1.03948e-15 is taken to mean $1.03948 \mathrm{e}-15$ and not zero), some formulas produce wildly inaccurate results; see [M-5] lusolve() for an example.

Thus routines use tolerances-preset numbers-to determine when a number is small enough to be considered to be zero.

The problem with tolerances is determining what they ought to be.

## Absolute versus relative tolerances

Tolerances come in two varieties: absolute and relative.
An absolute tolerance is a fixed number that is used to make direct comparisons. If the tolerance for a particular routine were $1 \mathrm{e}-14$, then $8.99 \mathrm{e}-15$ in some calculation would be considered to be close enough to zero to act as if it were, in fact, zero, and $1.000001 \mathrm{e}-14$ would be considered a valid, nonzero number.

But is $1 \mathrm{e}-14$ small? The number may look small to you, but whether $1 \mathrm{e}-14$ is small depends on what is being measured and the units in which it is measured. If all the numbers in a certain problem were around $1 \mathrm{e}-12$, you might suspect that $1 \mathrm{e}-14$ is a reasonable number.

That leads to relative measures of tolerance. Rather than treating, say, a predetermined quantity as being so small as to be zero, one specifies a value (for example, 1e-14) multiplied by something and uses that as the definition of small.

Consider the following matrix:

$$
\left[\begin{array}{ll}
5.5 \mathrm{e}-15 & 1.2 \mathrm{e}-16 \\
1.3 \mathrm{e}-16 & 6.4 \mathrm{e}-15
\end{array}\right]
$$

What is the rank of the matrix? One way to answer that question would be to take the LU decomposition of the matrix and then count the number of diagonal elements of $U$ that are greater than zero. Here, however, we will just look at the matrix.

The absolutist view is that the matrix is full of roundoff error and that the matrix is really indistinguishable from the matrix

$$
\left[\begin{array}{ll}
0 & 0 \\
0 & 0
\end{array}\right]
$$

The matrix has rank 0 . The relativist view is that the matrix has rank 2 because, other than a scale factor of $1 \mathrm{e}-16$, the matrix is indistinguishable from

$$
\left[\begin{array}{cc}
55.0 & 1.2 \\
1.3 & 64.0
\end{array}\right]
$$

There is no way this question can be answered until someone tells you how the matrix arose and the units in which it is measured.

Nevertheless, most Mata routines would (by default) adopt the relativist view: the matrix is of full rank. That is because most Mata routines are implemented using relative measures of tolerance, chosen because Mata routines are mostly used by people performing statistics, who tend to make calculations such as $X^{\prime} X$ and $X^{\prime} Z$ on data matrices, and those resulting matrices can contain very large numbers. Such a matrix might contain

$$
\left[\begin{array}{ll}
5.5 \mathrm{e}+14 & 1.2 \mathrm{e}+12 \\
1.3 \mathrm{e}+13 & 2.4 \mathrm{e}+13
\end{array}\right]
$$

Given a matrix with such large elements, one is tempted to change one's view as to what is small. Calculate the rank of the following matrix:

$$
\left[\begin{array}{lll}
5.5 \mathrm{e}+14 & 1.2 \mathrm{e}+12 & 1.5 \mathrm{e}-04 \\
1.3 \mathrm{e}+13 & 2.4 \mathrm{e}+13 & 2.8 \mathrm{e}-05 \\
1.3 \mathrm{e}-04 & 2.4 \mathrm{e}-05 & 8.7 \mathrm{e}-05
\end{array}\right]
$$

This time, we will do the problem correctly: we will take the LU decomposition and count the number of nonzero entries along the diagonal of $U$. For the above matrix, the diagonal of $U$ turns out to be $(5.5 \mathrm{e}+14,2.4 \mathrm{e}+13,0.000087)$.

An absolutist would tell you that the matrix is of full rank; the smallest number along the diagonal of $U$ is 0.000087 (8.7e-5), and that is still a respectable number, at least when compared with computer precision, which is about $2.22 \mathrm{e}-16$ (see [M-5] epsilon()).

Most Mata routines would tell you that the matrix has rank 2. Numbers such as 0.000087 may seem respectable when compared with machine precision, but 0.000087 is, relatively speaking, a very small number, being about $4.6 \mathrm{e}-19$ relative to the average value of the diagonal elements.

## Specifying tolerances

Most Mata routines use relative tolerances, but there is no rule. You must read the documentation for the function you are using.

When the tolerance entry for a function directs you here, [M-1] Tolerance, then the tolerance works as summarized under Syntax above. Specify a positive number, and that number multiplies the default; specify a negative number, and the corresponding positive number is used in place of the default.

[^1]
## [M-2] Language definition

## Title

[M-2] Intro - Language definition
Contents Description Remarks and examples Also see

## Contents

[M-2] Entry

Description

|  |  |
| :--- | :--- |
|  | Syntax |
| Syntax |  |
| Subscripts | Grammar and syntax |
| reswords | Use of subscripts |
| Comments | Reserved words |
|  | Comments |


|  | Expressions \& operators |
| :--- | :--- |
| exp | Expressions |
| op_assignment | Assignment operator |
| op_arith | Arithmetic operators |
| op_increment | Increment and decrement operators |
| op_logical | Logical operators |
| op_conditional | Conditional operator |
| op_colon | Colon operators |
| op_join | Row- and column-join operators |
| op_range | Range operators |
| op_transpose | Conjugate transpose operator |
| op_kronecker | Kronecker direct-product operator |

Declarations \& arguments

Declarations
optargs
struct
class
pragma
version

Declarations and types
Optional arguments
Structures
Object-oriented programming (classes)
Suppressing warning messages
Version control

Flow of control

```
if if (exp)...else...
for for (exp1; exp2; exp3) stmt
while while (exp) stmt
do do ... while (exp)
break Break out of for, while, or do loop
continue Continue with next iteration of for, while, or do loop
goto goto label
return return and return(exp)
```


## Special topics

| Semicolons | Use of semicolons |
| :--- | :--- |
| void | Void matrices |
| pointers | Pointers |
| ftof | Passing functions to functions |

Error codes
Errors
Error codes

## Description

This section defines the Mata programming language.

## Remarks and examples

[M-2] Syntax provides an overview, dense and brief, and the other sections expand on it.
Also see [M-1] Intro for an introduction to Mata.

Augusta Ada King, Lady Lovelace (1815-1852), is popularly believed to have written the first computer program. She was born Augusta Ada Byron in London, England. She was the daughter of Lord Byron, a well-known Romantic poet and infamous libertine. Because of her marriage to William King, Count of Lovelace, most people know her informal name, Ada Lovelace.

Shortly after Lovelace's birth, Lady Byron divorced Lovelace's father. Attempting to discourage Lovelace from Lord Byron's poetry, Lady Byron hired private tutors in mathematics and science. One of these tutors introduced Lovelace to Charles Babbage in 1833. Lovelace later translated Menabrea's article on Babbage's Analytical Engine. At Babbage's request, she added her own explanation about the engine's usefulness. At the time, few scientists recognized that the engine could be programmed to solve specific problems. Lovelace also noted the potential for the engine to use symbols in its computations, anticipating the functionality of modern computers.

Her notes on Menabrea's work also included algorithms that could be used for computation. Although the first several algorithms are recognized as Babbage's work, the algorithm to compute Bernoulli numbers is attributed to Lovelace. In honor of this work, the U.S. Department of Defense named the computer language it developed in 1979 "Ada". The British Computer Society awards a medal and sponsors an annual lecture in her name.

## Also see

[M-0] Intro - Introduction to the Mata manual

## Title

[M-2] break - Break out of for, while, or do loop
Description Syntax Remarks and examples Also see

## Description

break exits the innermost for, while, or do loop. Execution continues with the statement immediately following the close of the loop, just as if the loop had terminated normally.
break nearly always occurs following an if.

## Syntax

```
for, while, or do {
    if (...) {
        break
    }
}
stmt }\leftarrow\mathrm{ break jumps here
```


## Remarks and examples

In the following code,

```
for (i=1; i<=rows(A); i++) {
    for (j=1; j<=cols(A); j++) {
        if (A[i,j]==0) break
    }
    printf("j = %g\n", j)
}
```

the break statement will be executed if any element of $\mathrm{A}[\mathrm{i}, \mathrm{j}]$ is zero. Assume that the statement is executed for $i=2$ and $j=3$. Execution will continue with the printf() statement, which is to say, the $j$ loop will be canceled but the i loop will continue. The value of $j$ upon exiting the loop will be 3 ; when you break out of the loop, the $\mathrm{j}^{++}$is not executed.

## Also see

[M-2] continue - Continue with next iteration of for, while, or do loop
[M-2] do - do ... while (exp)
[M-2] for - for $(\exp 1 ; \exp 2 ; \exp 3)$ stmt
[M-2] while - while (exp) stmt
[M-2] Intro - Language definition

## Title

[M-2] class - Object-oriented programming (classes)

Description Syntax Remarks and examples Reference<br>Also see

## Description

class provides object-oriented programming, also known as class programming, to Mata.
A class is a set of variables or related functions (methods) (or both) tied together under one name. Classes may be derived from other classes according to inheritance.

For a more detailed description of classes, see Gould (2018).
Let's look at the details of the example from the first page of this entry (under the heading Example):

1. First, we created a class called coord. When we coded
```
class coord {
    real scalar x, y
    real scalar length(), angle()
}
```

we specified that each element of a coord stores two real values, which we called x and y . coord also contains two functions, which we called length() and angle(). length() and angle() are functions because of the open and close parentheses at the end of the names, and $x$ and $y$ are variables because of the absence of parentheses. In the jargon, $x$ and y are called member variables, and length() and angle() are called member functions.

The above, called the class's definition, defines a blueprint for the type coord.
A variable that is of type coord is called an instance of a coord. Say variables b and c are instances of coord, although we have not yet explained how you might arrange that. Then $\mathrm{b} . \mathrm{x}$ and $\mathrm{b} . \mathrm{y}$ would be b's values of x and y , and $\mathrm{c} . \mathrm{x}$ and $\mathrm{c} . \mathrm{y}$ would be c 's values. We could run the functions on the values in b by coding b . length() and b. angle(), or on the values in c by coding c.length() and c.angle().
2. Next we defined coord's length() and angle() functions. The definitions were

```
real scalar coord::length()
{
    return(sqrt(x^2 + y^2))
}
real scalar coord::angle()
{
    return(atan2(y, x)*360/(2*pi()))
}
```

These functions are similar to regular Mata functions. These functions do not happen to take any arguments, but that is not a requirement. It often happens that member functions do not require arguments because the functions are defined in terms of the class and its concepts. That is exactly what is occurring here. The first function says that when someone has an instance of a coord and they ask for its length, return the square root of the sum of the x and $y$ values, individually squared. The second function says that when someone asks for the angle, return the arctangent of y and x , multiplied by $360 /(2 * \mathrm{pi}())$ to convert the result from radians to degrees.
3. Next we defined another new concept, a rotated_coord, by extending (inheriting from) class coord:

```
class rotated_coord extends coord {
    real scalar theta
    real scalar angle()
    void new()
}
```

So think of a coord and read the above as the additions. In addition to x and y of a regular coord, a rotated_coord has new variable theta. We also declared that we were adding two functions, angle() and new(). But wait, angle() already existed! What we are actually saying by explicitly mentioning angle() is that we are going to change the definition of angle(). Function new () is indeed new.

Notice that we did not mention previously existing function length(). Our silence indicates that the concept of length() remains unchanged.

So what is a rotated_coord? It is a coord with the addition of theta, a redefinition of how angle() is calculated, and the addition of a function called new ().

This is an example of inheritance. class rotated_coord is an extension of class coord. In object-oriented programming, we would say this as "class rotated_coord inherits from class coord". A class that inherits from another class is known as a "subclass".
4. Next we defined our replacement and new functions. The definitions are

```
real scalar rotated_coord::angle()
{
    return(super.angle() - theta)
}
void rotated_coord::new()
{
    theta = 0
}
```

Concerning angle(), we stated that it is calculated by taking the result of super.angle() and subtracting theta from it. super .angle() is how one refers to the parent's definition of a function. If you are unfamiliar with object-oriented programming, parent may seem like an odd word in this context. We are inheriting concepts from coord to define rotated_coord, and in that sense, coord is the parent concept. Anyway, the new definition of an angle is the old definition, minus theta, the angle of rotation.
new () is a special function and is given a special name in the sense that the name new () is reserved. The new () function, if it exists, is a function that is called automatically whenever
a new instance of the class is created. Our new () function says that when a new instance of a rotated_coord is created, initialize the instance's theta to 0 .

Well, that seems like a good idea. But we did not have a new () function when we defined our previous class, coord. Did we forget something? Maybe. When you do not specify a new () function-when you do not specify how variables are to be initialized-they are initialized in the usual Mata way: missing values. $x$ and $y$ will be initialized to contain missing. Given our new () function for rotated_coord, however, theta will be initialized to 0 .
new () is called a "constructor", because it is used to construct, or initialize, the class when a new instance of the class is created.

And that completes the definition of our two, related classes.
There are two ways to create instances of a coord or a rotated_coord. One is mostly for interactive use and the other for programming use.

If you interactively type $\mathrm{a}=$ coord() (note the parentheses), you will create a coord and store it in
 in $b$. In the first example, typing $b=r o t a t e d \_c o o r d() ~ i s ~ e x a c t l y ~ w h a t ~ w e ~ c h o s e ~ t o ~ d o: ~$

```
: b = rotated_coord()
```

Recall that a rotated_coord contains an x , y , and theta. At this stage, x and y equal missing, and theta is 0 . In the example, we set b's x and y values to 1 , and then asked for the resulting angle():

```
: b.x = b.y = 1
: b.angle()
    4 5
```

b-dot-x is how one refers to b's value of x . One can use $\mathrm{b} . \mathrm{x}$ (and $\mathrm{b} . \mathrm{y}$ ) just as one would use any real scalar variable in Mata.

If we reset b's theta to be 30 degrees, then b's angle ought to change to being 15 degrees. That is exactly what happens:

```
: b.theta = 30
: b.angle()
    15
```

b-dot-angle() is how one specifies that member function angle() is to be run on instance b. Now you know why member functions so seldom have additional arguments: they are, in effect, passed an instance of a class and so have access to all the values of member variables of that class. We repeat, however, that a member function could take additional arguments. Had we coded

```
real scalar rotated_coord::angle(real scalar base)
{
    return(super.angle() - theta - base)
}
```

then angle() would have taken an argument and returned the result measured from base.
The difference between using rotated_coord interactively and using it inside a Mata program is that if we declare a variable (say, b) to be a class rotated_coord scalar, with the emphasis on
scalar, then we do not need to bother coding $\mathrm{b}=$ rotated_coord() to fill in b initially. Coding class rotated_coord scalar b implies that b needs to be initialized because it is a scalar, and so that happens automatically. It would not hurt if we also coded $b=r o t a t e d \_c o o r d(), ~ b u t ~ i t ~ w o u l d ~$ be a waste of our time and of the computer's once it got around to executing our program.

Now let's show you something we did not show in the first example. Remember when we defined length() for a coord? Remember how we did not define length() for a rotated_coord? Remember how we did not even mention length()? Even so, length() is a concept of a rotated_coord, because part of the definition of rotated_coord was inherited from coord, and that happened because when we declared rotated_coord, we said

```
class rotated_coord extends coord
```

The inheritance happened because we said extends. Let's test that length() works with our rotated_coord class instance, b:

$$
\begin{aligned}
: & \text { b.length () } \\
& 1.414213562
\end{aligned}
$$

In the above, inheritance is what saved us from having to write additional, repeated code.
Let's review. First, we defined a coord. From that, we defined a rotated_coord. You might now define translated_and_rotated_coord using rotated_coord as a starting point. It would not be difficult.

Classes have lots of properties, features, and details, but it is the property of inheritance that is at the heart of object-oriented programming.

## Syntax

```
class classname [extends classname] {
    declaration(s)
}
```

Syntax is presented under the following headings:
Introduction
Example
Declaration of member variables
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Description and Remarks and examples follow that.

## Introduction

Stata's two programming languages, ado and Mata, each support object-oriented programming. This manual entry explains object-oriented programming in Mata. Most users interested in object-oriented programming will wish to program in Mata. See [P] class to learn about object-oriented programming in ado.

## Example

The following example is explained in detail in Description.

```
class coord {
    real scalar x, y
    real scalar length(), angle()
}
real scalar coord::length()
{
    return(sqrt(x^2 + y^2))
}
real scalar coord::angle()
{
    return(atan2(y, x)*360/(2*pi()))
}
class rotated_coord extends coord {
    real scalar theta
    real scalar angle()
    void new()
}
real scalar rotated_coord::angle()
{
    return(super.angle() - theta)
}
void rotated_coord::new()
{
    theta = 0
}
```

One could use the class interactively:

```
: b = rotated_coord()
: b.x = b.y = 1
: b.angle() // displayed will be 45
: b.theta = 30
: b.angle() // displayed will be 15
```

Note that executing the class as if it were a function creates an instance of the class. When using the class inside other functions, it is not necessary to create the instance explicitly as long as you declare the member instance variable to be a scalar:

```
void myfunc()
{
    class rotated_coord scalar b
    b.x = b.y = 1
    b.angle() // displayed will be 45
    b.theta = 30
    b.angle() // displayed will be 15
}
```


## Declaration of member variables

Declarations are of the form

$$
\begin{aligned}
& {[\text { exposure }][\text { static }]} \\
& {[\text { final }] \text { matatype name }[, \text { name }[, \ldots]]}
\end{aligned}
$$

where

```
exposure := {public|protected|private}
matatype := { eltype orgtype |eltype |orgtype }
    eltype := transmorphic orgtype := matrix
        numeric vector
        real rowvector
        complex colvector
        string scalar
        pointer
        class classname
        struct structname
```

For example,

```
class S {
    real matrix M
    private real scalar type
    static real scalar count
    class coord scalar c
}
```


## Declaration and definition of methods (member functions)

Declarations are of the form

$$
\begin{aligned}
& \text { [exposure] [static] } \\
& \text { [final] [virtual] } \\
& \text { matatype name() }[\text {, name() }[\ldots]]
\end{aligned}
$$

For example,

```
class T {
    real matrix inverse()
    protected real scalar type()
    class coord scalar c()
}
```

Note that function arguments, even if allowed, are not declared.

Member functions (methods) and member variables may share the same names and no special meaning is attached to the fact. type and c below are variables, and type() and c() are functions:

```
class U {
    real matrix
    private real scalar type
    static real scalar count
    class coord scalar c
    real matrix inverse()
    protected real scalar type()
    class coord scalar c()
}
```

Member functions are defined separately, after the class is defined. For example,

```
class V {
    real matrix
    private real scalar type
    static real scalar count
    class coord scalar c
    real matrix inverse()
    protected real scalar type()
    class coord scalar c()
}
real matrix V::inverse(...)
{
}
real scalar V::type(...)
{
}
class coord scalar V::c(...)
{
}
```

When you define member functions, they must be of the same matatype as they were previously declared to be, but you omit exposure (as well as static, final, and virtual).

## Default exposure in declarations

Variables and functions are public unless explicitly declared otherwise. (They are also not static, not final, and not virtual, but that is not part of exposure and so has nothing to do with this subsection.)

You may use any of the exposure modifiers public, protected, and private, followed by a colon, to create blocks with a different default:

```
class V {
    public:
        real matrix M
        static real scalar count
        class coord scalar c
        real matrix inverse()
        class coord scalar c()
    private:
        real scalar type
    protected:
        real scalar type()
}
```


## Remarks and examples

Remarks are presented under the following headings:

```
Notation and jargon
Declaring and defining a class
Saving classes in files
Workflow recommendation
When you need to recompile
Obtaining instances of a class
Constructors and destructors
Setting member variable and member function exposure
Making a member final
Making a member static
Virtual functions
Referring to the current class using this
Using super to access the parent's concept
Casting back to a parent
Accessing external functions from member functions
Pointers to classes
```


## Notation and jargon

$\therefore$ (double colon)
The double-colon notation is used as a shorthand in documentation to indicate that a variable or function is a member of a class and, in two cases, the double-colon notation is also syntax that is understood by Mata.
$\mathrm{S}:: \mathrm{s}$ indicates that the variable s is a member of class $\mathrm{S} . \mathrm{S}:: \mathrm{s}$ is documentation shorthand.
$\mathrm{S}:: \mathrm{f}()$ indicates that function f() is a member of class $\mathrm{S} . \mathrm{S}:: \mathrm{f}()$ is documentation shorthand. $\mathrm{S}:: \mathrm{f}()$ is something Mata itself understands in two cases:

1. Notation $S:: f()$ is used when defining member functions.
2. Notation $S:: f()$ can be used as a way of calling static member functions.
be an instance of
When we write "let $s$ be an instance of $S$ " we are saying that $s$ is an instance of class $S$, or equivalently, that in some Mata code, $s$ is declared as a class S scalar.
class definition, class declaration
A class definition or declaration is the definition of a class, such as the following:
```
class S {
    private real scalar a, b
    real scalar f(),g()
}
```

Note well that the declaration of a Mata variable to be of type class S , such as the line class S scalar s in

```
void myfunction()
{
        class S scalar s
}
```

is not a class definition. It is a declaration of an instance of a class.
class instance, instance, class A instance
A class instance is a variable defined according to a class definition. In the code

```
void myfunction()
{
        class S scalar s
        real scalar b
}
```

$s$ is a class instance, or, more formally, an instance of class $S$. The term "instance" can be used with all element types, not just with classes. Thus b is an instance of a real. The term "instance" is more often used with classes and structures, however, because one needs to distinguish definitions of variables containing classes or structures from the definitions of the classes and structures themselves.
inheritance, extends, subclass, parent, child
Inheritance is the property of one class definition using the variables and functions of another just as if they were its own. When a class does this, it is said to extend the other class. T extending S means the same thing as T inheriting from S . T is also said to be a subclass of S .

Consider the following definitions:

```
class S {
    real scalar a, b
    real scalar f()
}
class T extends S {
    real scalar c
    real scalar g()
}
```

Let $s$ be an instance of $S$ and $t$ be an instance of T. It is hardly surprising that s.a, s.b, s.f(), $t . c$, and $\mathrm{t} . \mathrm{g}()$ exist, because each is explicitly declared. It is because of inheritance that $\mathrm{t} . \mathrm{a}$, $\mathrm{t} . \mathrm{b}$, and $\mathrm{t} . \mathrm{f}()$ also exist, and they exist with no additional code being written.

If $U$ extends $T$ extends $S$, then $U$ is said to be a child of $T$ and to be a child of $S$, although formally $U$ is the grandchild of $S$. Similarly, both $S$ and $T$ are said to be parents of $U$, although formally $S$ is the grandparent of $U$. It is usually sufficient to label the relationship parent/child without going into details.

## external functions

An external function is a regular function, such as sqrt(), $\sin ()$, and $m y f \operatorname{cn}()$, defined outside of the class and, as a matter of fact, outside of all classes. The function could be a function provided by Mata (such as sqrt() or $\sin ())$ or it could be a function you have written, such as myfen().

An issue arises when calling external functions from inside the code of a member function. When coding a member function, references such as sqrt(), $\sin ()$, and $m y f \operatorname{cn}()$ are assumed to be references to the class's member functions if a member function of the name exists. If one wants to ensure that the external-function interpretation is made, one codes ::sqrt(), ::sin(), and ::myfcn(). See Accessing external functions from member functions below.
member, member variable, member function, method
A variable or function declared within a class is said to be a member variable, or member function, of the class. Member functions are also known as methods. In what follows, let ex1 be an instance of $S$, and assume class $S$ contains member function $f()$ and member variable $v$.

When member variables and functions are used inside member functions, they can simply be referred to by their names, $v$ and $f()$. Thus, if we were writing the code for $S:: g()$, we could code

```
real scalar S::g()
{
        return(f()*v)
}
```

When member variables and functions are used outside of member functions, which is to say, are used in regular functions, the references must be prefixed with a class instance and a period. Thus one codes

```
real scalar myg(class S scalar ex1)
{
    return(ex1.f()*ex1.v)
}
```

variable and method overriding
Generically, a second variable or function overrides a first variable or function when they share the same name and the second causes the first to be hidden, which causes the second variable to be accessed or the second function to be executed in preference to the first. This arises in two ways in Mata's implementation of classes.

In the first way, a variable or function in a parent class is said to be overridden if a child defines a variable or function of the same name. For instance, let $U$ extend $T$ extend $S$, and assume $S:: f()$ and $T:: f()$ are defined. By the rules of inheritance, instances of $U$ and $T$ that call $f()$ will cause $T:: f()$ to execute. Instances of $S$ will cause $S:: f()$ to be executed. Here $S . f()$ is said to be overridden by T.f(). Because T::f() will usually be implemented in terms of $\mathrm{S}:: \mathrm{f}(), \mathrm{T}:: \mathrm{f}$ () will find it necessary to call $\mathrm{S}:: \mathrm{f}()$. This is done by using the super prefix; see Using super to access the parent's concept.

The second way has to do with stack variables having precedence over member variables in member functions. For example,

```
class S {
        real scalar a, b
        void f()
}
void S::f()
{
    real scalar a
    a = 0
    b = 0
}
```

Let $s$ be an instance of $S$. Then execution of $s . f()$ sets $s . b$ to be 0 ; it does not change $s . a$, although it was probably the programmer's intent to set s . a to zero, too. Because the programmer declared a variable named a within the program, however, the program's variable took precedence over the member variable a . One solution to this problem would be to change the $\mathrm{a}=0$ line to read this.a $=0$; see Referring to the current class using this.

## Declaring and defining a class

Let's say we declared a class by executing the code below:

```
class coord {
    real scalar x, y
    real scalar length(), angle()
}
```

At this point, class coord is said to be declared but not yet fully defined because the code for its member functions length() and angle() has not yet been entered. Even so, the class is partially functional. In particular,

1. The class will work. Obviously, if an attempt to execute length() or angle() is made, an error message will be issued.
2. The class definition can be saved in an .mo or .mlib file for use later, with the same proviso as in 1).

Member functions of the class are defined in the same way regular Mata functions are defined but the name of the function is specified as classname: :functionname().

```
real scalar coord::length()
{
    return(sqrt(x^2 + y^2))
}
real scalar coord::angle()
{
    return(atan2(y, x)*360/(2*pi()))
}
```

The other difference is that member functions have direct access to the class's member variables and functions. x and y in the above are the x and y values from an instance of class coord.

Class coord is now fully defined.

## Saving classes in files

The notation coord () —classname-open parenthesis-close parenthesis-is used to refer to the entire class definition by Mata's interactive commands such as mata describe, mata mosave, and mata mlib.
mata describe coord() would show

| : mata describe coord() <br> \# bytes type | name and extent |  |
| :---: | :--- | :---: |
| 344 | classdef scalar | coord() |
| 176 | real scalar | $::$ angle() |
| 136 | real scalar | $:: l e n g t h()$ |

The entire class definition-the compiled coord() and all its compiled member functions-can be stored in a .mo file by typing

```
: mata mosave coord()
```

The entire class definition could be stored in an already existing .mlib library, lpersonal, by typing

```
: mata mlib add lpersonal coord()
```

When saving a class definition, both commands allow the additional option complete. The option specifies that the class is to be saved only if the class definition is complete and that, otherwise, an error message is to be issued.

```
: mata mlib add lpersonal coord(), complete
```


## Workflow recommendation

Our recommendation is that the source code for classes be kept in separate files, and that the files include the complete definition. At StataCorp, we would save coord in file coord.mata:

```
*! version 1.0.0 class coord
version 18.0 // (or version 18.5 for StataNow)
mata:
class coord {
        real scalar x, y
    real scalar length(), angle()
}
real scalar coord::length()
{
    return(sqrt(x^2 + y^2))
}
real scalar coord::angle()
{
        return(atan2(y, x)*360/(2*pi()))
}
end
```

end coord.mata
Note that the file does not clear Mata, nor does it save the class in a .mo or .mlib file; the file merely records the source code. With this file, to save coord in a .mo file, we need only to type

```
. clear mata
. do coord.mata
. mata mata mosave coord(), replace
```

(Note that although file coord.mata does not have the extension .do, we can execute it just like we would any other do-file by using the do command.) Actually, we would put those three lines in yet another do-file called, perhaps, cr_coord.do.

We similarly use files for creating libraries, such as

| version 18.0 | begin cr_lourlib.do |
| :--- | :--- |
| clear mata |  |
| do coord.mata |  |
| do anotherfile.mata |  |
| do yetanotherfile.mata |  |
| /* etc. $* /$ |  |
| mata: |  |
| mata mlib create lourlib, replace |  |
| mata mlib add lourlib*() |  |
| end |  |

With the above, it is easy to rebuild libraries after updating code.

## When you need to recompile

If you change a class declaration, you need to recompile all programs that use the class. That includes other class declarations that inherit from the class. For instance, in the opening example, if you change anything in the coord declaration,

```
class coord {
    real scalar x, y
    real scalar length(), angle()
}
```

even if the change is so minor as to reverse the order of $x$ and $y$, or to add a new variable or function, you must recompile all of coord's member functions, recompile all functions that use coord, and recompile rotated_coord because rotated_coord is inherited from coord.

You must do this because Mata seriously compiles your code. The Mata compiler deconstructs all uses of classes and substitutes low-level, execution-time-efficient constructs that record the address of every member variable and member function. The advantage is that you do not have to sacrifice run-time efficiency to have more readable and easily maintainable code. The disadvantage is that you must recompile when you make changes in the definition.

You do not need to recompile outside functions that use a class if you only change the code of member functions. You do need to recompile if you add or remove member variables or member functions.

To minimize the need for recompiling, especially if you are distributing your code to others physically removed from you, you may want to adopt the pass-through approach.

Assume you have written large, wonderful systems in Mata that all hinge on object-oriented programming. One class inherits from another and that one from another, but the one class you need to tell your users about is class wonderful. Rather than doing that, however, merely tell your users that they should declare a transmorphic named wonderfulhandle-and then just have them pass wonderfulhandle around. They begin using your system by making an initialization call:

```
wonderfulhandle = wonderful_init()
```

After that, they use regular functions you provide that require wonderful as an argument. From their perspective, wonderfulhandle is a mystery. From your perspective, wonderful_init() returned an instance of a class wonderful, and the other functions you provided receive an instance of a wonderful. Sadly, this means that you cannot reveal to your users the beauty of your underlying class system, but it also means that they will not have to recompile their programs when you distribute an update. If the Mata compiler can compile their code knowing only that wonderfulhandle is a transmorphic, then it is certain their code does not depend on how wonderfulhandle is constructed.

## Obtaining instances of a class

Declaring a class, such as

```
class coord {
    real scalar x, y
    real scalar length(), angle()
}
```

in fact creates a function, coord(). Function coord () will create and return instances of the class:

- coord()—coord() without arguments—returns a scalar instance of the class.
- coord (3) —coord () with one argument, here 3—returns a $1 \times 3$ vector, each element of which is a separate instance of the class.
- coord (2,3) - coord () with two arguments, here 2 and 3 —returns a $2 \times 3$ matrix, each element of which is a separate instance of the class.

Function coord() is useful when using Mata interactively. It is the only way to create instances of a class interactively.

In functions, you can create scalar instances by coding class coord scalar name, but in all other cases, you will also need to use function coord(). In the following example, the programmer wants a vector of coords:

```
void myfunction()
{
        real scalar i
        class coord vector v
        v = coord(3)
        for (i=1; i<=3; i++) v[i].x = v[i].y = 1
}
```

This program would have generated a compile-time error had the programmer omitted the line class coord vector v. Variable declarations are usually optional in Mata, but variable declarations of type class are not optional.

This program would have generated a run-time error had the programmer omitted the line $\mathrm{v}=$ coord(3). Because v was declared to be vector, v began life as $1 \times 0$. The first thing the programmer needed to do was to expand v to be $1 \times 3$.

In practice, one seldom needs class vectors or matrices. A more typical program would read

```
void myfunction()
{
    real scalar i
    class coord scalar v
    v.x = v.y = 1
}
```

Note particularly the line class coord scalar v. The most common error programmers make is to omit the word scalar from the declaration. If you do that, then matrix is assumed, and then, rather than v being $1 \times 1$, it will be $0 \times 0$. If you did omit the word scalar, you have two alternatives. Either go back and put the word back in, or add the line $\mathrm{v}=\operatorname{coord}()$ before initializing $\mathrm{v} . \mathrm{x}$ and $\mathrm{v} . \mathrm{y}$. It does not matter which you do. In fact, when you specify scalar, the compiler merely inserts the line $\mathrm{v}=\operatorname{coord}()$ for you.

## Constructors and destructors

You can specify how variables are initialized, and more, each time a new instance of a class is created, but before we get to that, let's understand the default initialization. In

```
class coord {
    real scalar x, y
    real scalar length(), angle()
}
```

there are two variables. When an instance of a coord is created, say, by coding $b=\operatorname{coord}()$, the values are filled in the usual Mata way. Here, because $x$ and $y$ are scalars, $b . x$ and $b . y$ will be filled in with missing. Had they been vectors, row vectors, column vectors, or matrices, they would have been dimensioned $1 \times 0$ (vectors and row vectors), $0 \times 1$ (column vectors), and $0 \times 0$ (matrices).

If you want to control initialization, you may declare a constructor function named new() in your class:

```
class coord {
    real scalar x, y
    real scalar length(), angle()
    void new()
}
```

Function new () must be declared to be void and must take no arguments. You never bother to call new () yourself-in fact, you are not allowed to do that. Instead, function new () is called automatically each time a new instance of the class is created.

If you wanted every new instance of a coord to begin life with x and y equal to 0 , you could code

```
void coord::new()
{
    x = y = 0
}
```

Let's assume we do that and we then inherit from coord when creating the new class rotated_coord, just as shown in the first example.

```
class rotated_coord extends coord {
    real scalar theta
    real scalar angle()
    void new()
}
void rotated_coord::new()
{
    theta = 0
}
```

When we create an instance of a rotated_coord, all the variables were initialized to 0 . That is, function rotated_coord() will know to call both coord::new() and rotated_coord::new(), and it will call them in that order.

In your new () function, you are not limited to initializing values. new() is a function and you can code whatever you want, so if you want to set up a link to a radio telescope and check for any incoming messages, you can do that. Closer to home, if you were implementing a file system, you might use a static variable to count the number of open files. (We will explain static member variables later.)

On the other side of instance creation-instance destruction-you can declare and create void destroy(), which also takes no arguments, to be called each time an instance is destroyed.
destroy() is known as a destructor. destroy() works like new() in the sense that you are not allowed to call the function directly. Mata calls destroy () for you whenever Mata is releasing the memory associated with an instance. destroy () does not serve much use in Mata because, in Mata, memory management is automatic and the freeing of memory is not your responsibility. In some other object-oriented programming languages, such as C++, you are responsible for memory management, and the destructor is where you free the memory associated with the instance.

Still, there is an occasional use for destroy () in Mata. Let's consider a system that needs to count the number of instances of itself that exists, perhaps so it can release a resource when the instance count goes to 0 . Such a system might, in part, read

```
class bigsystem {
    static real scalar counter
    void new(), destroy()
}
void bigsystem::new()
{
    counter = (counter==. ? 1 : counter+1)
}
void bigsystem::destroy()
{
    counter--
}
```

Note that bigsystem: : new () must deal with two initializations, first and subsequent. The first time it is called, counter is . and new() must set counter to be 1 . After that, new () must increment counter.

If this system needed to obtain a resource, such as access to a radio telescope, on first initialization and release it on last destruction, and be ready to repeat the process, the code could read

```
void bigsystem::new()
{
    if (counter==.) {
        get_resource()
        counter = 1
    }
    else {
        ++counter
    }
}
void bigsystem::destroy()
{
    if (--counter == 0) {
        release_resource()
    counter = .
    }
}
```

Note that destroy()s are run whenever an instance is destroyed, even if that destruction is due to an abort-with-error in the user's program or even in the member functions. The radio telescope will
be released when the last instance of bigsystem is destroyed, except in one case. The exception is when there is an error in destroy() or any of the subroutines destroy() calls.

For inheritance, child destroy()s are run before parent destroy()s.

## Setting member variable and member function exposure

Exposure specifies who may access the member variables and member functions of your class. There are three levels of exposure: from least restrictive to most restrictive, public, protected, and private.

- Public variables and functions may be accessed by anyone, including callers from outside a class.
- Protected variables and functions may be accessed only by member functions of a class and its children.
- Private variables and functions may be accessed only by member functions of a class (excluding children).

When you do not specify otherwise, variables and functions are public. For code readability, you may declare member variables and functions to be public with public, but this is not necessary.

In programming large systems, it is usually considered good style to make member variables private and to provide public functions to set and to access any variables that users might need. This way, the internal design of the class can be subsequently modified and other classes that inherit from the class will not need to be modified, they will just need to be recompiled.

You make member variables and functions protected or private by preceding their declaration with protected or private:

```
class myclass {
    private real matrix X
    private real vector y
    void setup()
    real matrix invert()
    protected real matrix invert_subroutine()
}
```

Alternatively, you can create blocks with different defaults:

```
class myclass {
        public:
            void setup()
            real matrix invert()
        protected:
            real matrix invert_subroutine()
        private:
            real matrix X
            real vector y
}
```

You may combine public, private, and protected, with or without colons, freely.

## Making a member final

A member variable or function is said to be final if no children define a variable or function of the same name. Ensuring that a definition is the final definition can be enforced by including final in the declaration of the member, such as

```
class myclass {
    public:
        void setup()
        real matrix invert()
    protected:
        final real matrix
    private:
        real matrix X
        real vector y
}
```

In the above, no class that inherits from myclass can redefine invert_subroutine().

## Making a member static

Being static is an optional property of a class member variable function. In what follows, let ex1 and ex2 both be instances of $S$, assume $c$ is a static variable of the class, and assume $f()$ is a static function of the class.

A static variable is a variable whose value is shared across all instances of the class. Thus, ex1.c and ex2.c will always be equal. If ex1.c is changed, say, by ex1.c=5, then ex2.c will also be 5 .

A static function is a function that does not use the values of nonstatic member variables of the class. Thus ex1.f(3) and ex2.f(3) will be equal regardless of how ex1 and ex2 differ.

Outside of member functions, static functions may also be invoked by coding the class name, two colons, and the function name, and thus used even if you do not have a class instance. For instance, equivalent to coding ex1.f(3) or ex2.f(3) is $S:: f(3)$.

## Virtual functions

When a function is declared to be virtual, the rules of inheritance act as though they were reversed. Usually, when one class inherits from another, if the child class does not define a function, then it inherits the parent's function. For virtual functions, however, the parent has access to the child's definition of the function, which is why we say that it is, in some sense, reverse inheritance.

The canonical motivational example of this deals with animals. Without loss of generality, we will use barnyard animals to illustrate. A class, animal, is defined. Classes cow and sheep are defined that extend (inherit from) animal. One of the functions defined in animal needs to make the sound of the specific animal, and it calls virtual function sound () to do that. At the animal level, function animal::sound() is defined to display "Squeak!". Because function sound() is virtual, however, each of the specific-animal classes is supposed to define their own sound. Class cow defines cow::sound () that displays "Moo!" and class sheep defines sheep::sound() that displays "Baa!" The result is that when a routine in animal calls sound (), it behaves as if the inheritance is reversed, the appropriate sound () routine is called, and the user sees "Moo!" or "Ba!!" If a new specific animal is added, and if the programmer forgets to define sound(), then animal::sound() will run, and the user sees "Squeak!". In this case, that is supposed to be the sound of the system in trouble, but it is really just the default action.

Let's code this example. First, we will code the usual case:

```
class animal {
    void sound()
    void poke()
}
void animal::sound() { "Squeak!" }
void animal::poke()
{
    sound()
}
class cow extends animal {
}
class sheep extends animal {
}
```

In the above example, when an animal, cow, or sheep is poked, among other things, it emits a sound. Poking is defined at the animal level because poking is mostly generic across animals, except for the sound they emit. If $c$ is an instance of cow, then one can poke that particular cow by coding c. poke(). poke(), however, is inherited from animal, and the generic action is taken, along with the cow emitting a generic squeak.

Now we make sound() virtual:

```
class animal {
    virtual void sound()
    void poke()
}
void animal::sound() { "Squeak!" }
void animal::poke()
{
    sound()
}
class cow extends animal {
    virtual void sound()
}
```

```
void cow::sound() { "Moo!" }
class sheep extends animal {
        virtual void sound()
}
void sheep::sound() { "Baa!" }
```

Now let's trace through what happens when we poke a particular cow by coding c.poke(). c, to remind you, is a cow. There is no cow: :poke() function; however, c.poke() executes animal::poke(). animal::poke() calls sound(), which, were sound() not a virtual function, would be animal::sound(), which would emit a squeak. Poke a cow, get a "Squeak!" Because sound () is virtual, animal : : poke() called with a cow calls cow : : sound (), and we hear a "Moo!"

Focusing on syntax, it is important that both cow and sheep repeated virtual void sound() in their declarations. If you define function $S:: f()$, then $f()$ must be declared in $S$. Consider a case, however, where we have two breeds of cows, Angus and Holstein, and they emit slightly different sounds. The Holstein, being of Dutch origin, gets a bit of a $U$ sound into the moo in a way no native English speaker can duplicate. Thus we would declare two new classes, angus extends cow and holstein extends cow, and we would define angus:: sound() and holstein::sound(). Perhaps we would not bother to define angus: : sound(); then an Angus would get the generic cow sound.

But let's pretend that, instead, we defined angus: :sound() and removed the function for cow: :sound(). Then it does not matter whether we include the line virtual void sound() in cow's declaration. Formally, it should be included, because the line of reverse declaration should not be broken, but Mata does not care one way or the other.

A common use of virtual functions is to allow you to process a list of objects without any knowledge of the specific type of object, as long as all the objects are subclasses of the same base class:

```
class animal animals
animals = animal(3,1)
animals[1] = cow()
animals[2] = sheep()
animals[3] = holstein()
for(i=1; i<=length(animals); i++) {
    animals[i].sound()
}
```

Note the use of animals = animal $(3,1)$ to initialize the vector of animals. This is an example of how to create a nonscalar class instance, as discussed in Obtaining instances of a class.

When the code above is executed, the appropriate sound for each animal will be displayed even though animals is a vector declared to be of type class animal. Because sound() is virtual, the appropriate sound from each specific animal child class is called.

## Referring to the current class using this

this is used within member functions to refer to the class as a whole. For instance, consider the following:

```
class S {
    real scalar n
    real matrix M
    void make_M()
}
void S::make_M(real scalar n)
{
    real scalar i, j
    this.n = n
    M = J (n, n, 0)
    for (i=1; i<=n; i++) {
        for (j=1; j<=i; j++) M[i,j] = 1
    }
}
```

In the above program, references to M are understood to be the class instance definition of M . References to $n$, however, refer to the function's definition of $n$ because the program's $n$ has precedence over the class's definition of it. this.n is how one refers to the class's variable in such cases. M also could have been referred to as this.M, but that was not necessary.

If, in function $S:: f()$, it was necessary to call a function outside of the class, and if that function required that we pass the class instance as a whole as an argument, we could code this for the class instance. The line might read outside_function(this).

## Using super to access the parent's concept

The super modifier is a way of dealing with variables and functions that have been overridden in subclasses or, said differently, is a way of accessing the parent's concept of a variable or function.

Let $T$ extend $S$ and let $t$ be an instance of Then $t . f()$ refers to $T: f()$ if the function exists, and otherwise to $S:: f()$. t.super.f() always refers to $S:: f()$.

More generally, in a series of inheritances, z.super.f() refers to the parent's concept of $f()$ the $f()$ the parent would call if no super were specified-z.super.super.f() refers to the grandparent's concept of $f()$, and so on. For example, let $W$ extend $V$ extend $U$ extend $T$ extend $S$. Furthermore, assume

| $S:: f()$ | exists |  |
| :--- | :--- | :--- |
| $T:: f()$ |  | does not exist |
| $U:: f()$ | exists |  |
| $V:: f()$ |  | does not exist |
| $W:: f()$ |  | does not exist |

Finally, let $s$ be an instance of $S$, $t$ be an instance of $T$, and so on. Then calls of the form $w . f()$, w.super.f(), ..., w.super.super.super.super.f(), v.f(), v.super.f(), and so on, result in execution of

|  | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| s. | S::f() |  |  |  |  |
| t. | S::f() | S::f() |  |  |  |
| u. | $U:: f()$ | $S:: f()$ | $S:: f()$ |  |  |
| v. | $U:: f()$ | $U:: f()$ | $S:: f()$ | $S:: f()$ |  |
| w. | $U:: f()$ | $U:: f()$ | $U:: f()$ | $S:: f()$ | $S:: f()$ |

## Casting back to a parent

A class instance may be treated as a class instance of a parent by casting. Assume U extends T extends $S$, and let $u$ be an instance of $U$. Then

$$
(c l a s s ~ T) u
$$

is treated as an instance of $T$, and
(class S) u
is treated as an instance of $S$. ( (class $T$ ) u) could be used anywhere an instance of $T$ is required and ( (class S) u) could be used anywhere an instance of $S$ is required.

For instance, assume $S:: f()$ is overridden by $T:: f()$. If an instance of $U$ found it necessary to call $S:: f()$, one way it could do that would be $u$.super.super.f(). Another would be

$$
((c l a s s ~ S) u) . f()
$$

## Accessing external functions from member functions

In the opening example, class coord contained a member function named length(). Had we been coding a regular (nonclass) function, we could not have created that function with the name length (). The name length() is reserved by Mata for its own function that returns the length (number of elements) of a vector; see [M-5] rows( ). We are allowed to create a function named length() inside classes. Outside of member functions, c.length() is an unambiguous reference to c's definition of length(), and length() by itself is an unambiguous reference to length()'s usual definition.

There is, however, possible confusion when coding member functions. Let's add another member function to coord: ortho(), the code for which reads

```
class coord scalar coord::ortho()
{
    class coord scalar newcoord
    real scalar r, t
    r = length()
    t = angle()
    newcoord.x = r*cos(t)
    newcoord.y = r*sin(t)
    return(newcoord)
}
```

Note that in the above code, we call length(). Because we are writing a member function, and because the class defines a member function of that name, length() is interpreted as being a call to coord's definition of length(). If coord did not define such a member function, then length() would have been interpreted as a call to Mata's length().

So what do we do if the class provides a function, externally there is a function of the same name, and we want the external function? We prefix the function's name with double colons. If we wanted length() to be interpreted as Mata's function and not the class's, we would have coded
r = : :length()

There is nothing special about the name length() here. If the class provided member function $m y f c n()$, and there was an external definition as well, and we wanted the externally defined function, we would code $:: m y f c n()$. In fact, it is not even necessary that the class provide a definition. If we cannot remember whether class coord includes myfen(), but we know we want the external myfen() no matter what, we can code : :myfcn().

Placing double-colons in front of external function names used inside the definitions of member functions is considered good style. This way, if you ever go back and add another member function to the class, you do not have to worry that some already written member function is assuming that the name goes straight through to an externally defined function.

## Pointers to classes

Just as you can obtain a pointer to any other Mata variable, you can obtain a pointer to an instance of a class. Recall our example in Virtual functions of animals that make various sounds when poked.

For the sake of example, we will obtain a pointer to an instance of a cow () and access the poke() function through the pointer:

```
void pokeacow() {
    class cow scalar c
    pointer(class cow scalar) scalar p
    p = &c
    p->poke()
}
```

You access the member variables and functions through a pointer to a class with the operator $->$ just like you would for structures. See Pointers to structures in [M-2] struct for more information.

## Reference

Gould, W. W. 2018. The Mata Book: A Book for Serious Programmers and Those Who Want to Be. College Station, TX: Stata Press.

[^2]
## Title

[M-2] Comments - Comments

Description Syntax Remarks and examples Also see

## Description

/* and */ and // are how you place comments in Mata programs.

## Syntax

/* enclosed comment */
// rest-of-line comment

Notes:

1. Comments may appear in do-files and ado-files; they are not allowed interactively.
2. Stata's beginning-of-the-line asterisk comment is not allowed in Mata:
. * valid in Stata but not in Mata

## Remarks and examples

There are two comment styles: /* and $* /$ and //. You may use one, the other, or both.
Remarks are presented under the following headings:
The /* */ enclosed comment
The // rest-of-line comment

## The /* */ enclosed comment

Enclosed comments may appear on a line:

```
/* What follows uses an approximation formula: */
```

Enclosed comments may appear within a line and even in the middle of a Mata expression:

$$
\mathrm{x}=\mathrm{x}+/ * \text { left-single quote*/ char(96) }
$$

Enclosed comments may themselves contain multiple lines:

## /*

We use the approximation based on $\sin (x)$ approximately equaling $x$ for small $x$; $x$ measure in radians
*/

Enclosed comments may be nested, which is useful for commenting out code that itself contains comments:

```
/*
for (i=1; i<=rows(x); i++) { /* normalization */
    x[i] = x[i] :/ value[i]
}
*/
```


## The // rest-of-line comment

The rest-of-line comment may appear by itself on a line
// What follows uses an approximation formula:
or it may appear at the end of a line:

$$
\mathrm{x}=\mathrm{x}+\operatorname{char}(96) \quad / / \text { append single quote }
$$

In either case, the comment concludes when the line ends.

## Also see

[M-2] Intro - Language definition

## Title

[M-2] continue - Continue with next iteration of for, while, or do loop
Description Syntax Remarks and examples Also see

## Description

continue restarts the innermost for, while, or do loop. Execution continues just as if the loop had reached its logical end.
continue nearly always occurs following an if.

## Syntax

```
for, while, or do {
    if (...) {
        continue
    }
}
```


## Remarks and examples

The following two code fragments are equivalent:

```
for (i=1; i<=rows(A); i++) {
    for (j=1; j<=cols(A); j++) {
        if (i==j) continue
        ... action to be performed on A[i,j] ...
    }
}
```

and

```
for (i=1; i<=rows(A); i++) {
    for (j=1; j<=cols(A); j++) {
        if (i!=j) {
            ... action to be performed on A[i,j] ...
        }
    }
}
```

continue operates on the innermost for or while loop, and even when the continue action is taken, standard end-of-loop processing takes place (which is $\mathrm{j}^{++}$here).

## Also see

[M-2] break - Break out of for, while, or do loop
[M-2] do — do . . . while (exp)
[M-2] for - for $(\exp 1 ; \exp 2 ; \exp 3)$ stmt
[M-2] while - while (exp) stmt
[M-2] Intro - Language definition

## Title

[M-2] Declarations - Declarations and types

## Description Syntax Remarks and examples Also see

## Description

The type and the use of declarations are explained. Also discussed is the calling convention (functions are called by address, not by value, and so may change the caller's arguments), and the use of external globals.

Mata also has structures-the eltype is struct name—but these are not discussed here. For a discussion of structures, see [M-2] struct.

Mata also has classes-the eltype is class name-but these are not discussed here. For a discussion of classes, see [M-2] class.

Declarations are optional but, for careful work, their use is recommended.

## Syntax

```
declaration}1 fcnname(declaration 2)
{
            declaration3
}
```

such as

```
real matrix myfunction(real matrix X, real scalar i)
{
        real scalar j, k
        real vector v
    }
```

declaration $_{1}$ is one of
function
type [function]
void [function]
declaration $_{2}$ is
$[$ type $]$ argname $[,[$ type $]$ argname $[, \ldots]]$
where argname is the name you wish to assign to the argument.
declaration $_{3}$ are lines of the form of either of
type varname $[$, varname $[, \ldots]]$
external $[$ type $]$ varname $[$, varname $[, \ldots]]$
type is defined as one of

| eltype orgtype | such as real vector |
| :--- | :--- |
| eltype | such as real |
| orgtype | such as vector |

eltype and orgtype are each one of

| eltype | orgtype |
| :--- | :--- |
| transmorphic | matrix |
| numeric | vector |
| real | rowvector |
| complex | colvector |
| string | scalar |
| pointer |  |

If eltype is not specified, transmorphic is assumed. If orgtype is not specified, matrix is assumed.

## Remarks and examples

Remarks are presented under the following headings:
The purpose of declarations
Types, element types, and organizational types
Implicit declarations
Element types
Organizational types
Function declarations
Argument declarations
The by-address calling convention
Variable declarations
Linking to external globals

## The purpose of declarations

Declarations occur in three places: in front of function definitions, inside the parentheses defining the function's arguments, and at the top of the body of the function, defining private variables the function will use. For instance, consider the function

```
real matrix swaprows(real matrix A, real scalar i1, real scalar i2)
{
    real matrix B
    real rowvector v
    B = A
        v = B[i1, .]
        B[i1, .] = B[i2, .]
        B[i2, .] = v
        return(B)
}
```

This function returns a copy of matrix A with rows i1 and i2 swapped.
There are three sets of declarations in the above function. First, there is a declaration in front of the function name:

```
real matrix swaprows(...)
{
}
```

That declaration states that this function will return a real matrix.
The second set of declarations occur inside the parentheses:

```
... swaprows(real matrix A, real scalar i1, real scalar i2)
{
}
```

Those declarations state that this function expects to receive three arguments, which we chose to call A, i1, and i2, and which we expect to be a real matrix, a real scalar, and a real scalar, respectively.

The third set of declarations occur at the top of the body of the function:

```
... swaprows(...)
```

\{
real matrix $B$
real rowvector $v$
\}

Those declarations state that we will use variables $B$ and $v$ inside our function and that, as a matter of fact, B will be a real matrix and v a real row vector.

We could have omitted all those declarations. Our function could have read

```
function swaprows(A, i1, i2)
{
    B = A
    v = B[i1, .]
    B[i1, .] = B[i2, .]
    B[i2, .] = v
    return(B)
}
```

and it would have worked just fine. So why include the declarations?

1. By including the outside declaration, we announced to other programs what to expect. They can depend on swaprows() returning a real matrix because, when swaprows() is done, Mata will verify that the function really is returning a real matrix and, if it is not, abort execution.

Without the outside declaration, anything goes. Our function could return a real scalar in one case, a complex row vector in another, and nothing at all in yet another case.

Including the outside declaration makes debugging easier.
2. By including the argument declaration, we announced to other programmers what they are expected to pass to our function. We have made it easier to understand our function.

We have also told Mata what to expect and, if some other program attempts to use our function incorrectly, Mata will stop execution.

Just as in (1), we have made debugging easier.
3. By including the inside declaration, we have told Mata what variables we will need and how we will be using them. Mata can do two things with that information: first, it can make sure that we are using the variables correctly (making debugging easier again), and second, Mata can produce more efficient code (making our function run faster).

Interactively, we admit that we sometimes define functions without declarations. For more careful work, however, we include them.

## Types, element types, and organizational types

When you use Mata interactively, you just willy-nilly create new variables:

```
: n = 2
:A=(1,2\ \ 3,4)
: z = (sqrt(-4+0i), sqrt(4))
```

When you create a variable, you may not think about the type, but Mata does. n above is, to Mata, a real scalar. A is a real matrix. $z$ is a complex row vector.

Mata thinks of the type of a variable as having two parts:

1. the type of the elements the variable contains (such as real or complex) and
2. how those elements are organized (such as a row vector or a matrix).

We call those two types the eltype-element type-and orgtype-organizational type. The eltypes and orgtypes are

| eltype | orgtype |
| :--- | :--- |
| transmorphic | matrix |
| numeric | vector |
| real | rowvector |
| complex | colvector |
| string | scalar |
| pointer |  |

You may choose one of each and so describe all the types Mata understands.

## Implicit declarations

When you do not declare an object, Mata behaves as if you declared it to be transmorphic matrix:

1. transmorphic means that the matrix can be real, complex, string, or pointer.
2. matrix means that the organization is to be $r \times c, r \geq 0$ and $c \geq 0$.

At one point in your function, a transmorphic matrix might be a real scalar (real being a special case of transmorphic and scalar being a special case of a matrix when $r=c=1$ ), and at another point, it might be a string colvector (string being a special case of transmorphic, and colvector being a special case of a matrix when $c=1$ ).

Consider our swaprows () function without declarations,

```
function swaprows(A, i1, i2)
{
        B = A
        v = B[i1, .]
        B[i1, .] = B[i2, .]
        B[i2, .] = v
        return(B)
}
```

The result of compiling this function is just as if the function read

```
transmorphic matrix swaprows(transmorphic matrix A,
                                    transmorphic matrix i1,
                                    transmorphic matrix i2)
```

\{
transmorphic matrix $B$
transmorphic matrix $v$
$\mathrm{B}=\mathrm{A}$
$\mathrm{v}=\mathrm{B}[\mathrm{i} 1$, .]
$B[i 1,]=.B[i 2,$.
$\mathrm{B}[\mathrm{i} 2,]=$.
return (B)
\}

When we declare a variable, we put restrictions on it.

## Element types

There are six eltypes, or element types:

1. transmorphic, which means real, complex, string, or pointer.
2. numeric, which means real or complex.
3. real, which means that the elements are real numbers, such as $1,3,-50$, and 3.14159 .
4. complex, which means that each element is a pair of numbers, which are given the interpretation $a+b i$. complex is a storage type; the number stored in a complex might be real, such as $2+0 i$.
5. string, which means the elements are strings of text. Each element may contain up to $2,147,483,647$ bytes and strings may (need not) contain binary 0 ; that is, strings may be binary strings or text strings. Mata strings are similar to the strL type in Stata in that they can be very long and may contain binary 0 . Mata strings, like all other strings in Stata, can contain Unicode characters and are stored in UTF-8 encoding.
6. pointer means the elements are pointers to (addresses of) other Mata matrices, vectors, scalars, or even functions; see [M-2] pointers.

## Organizational types

There are five orgtypes, or organizational types:

1. matrix, which means $r \times c, r \geq 0$ and $c \geq 0$.
2. vector, which means $1 \times n$ or $n \times 1, n \geq 0$.
3. rowvector, which means $1 \times n, n \geq 0$.
4. colvector, which means $n \times 1, n \geq 0$.
5. scalar, which means $1 \times 1$.

Sharp-eyed readers will note that vectors and matrices can have zero rows or columns! See [M-2] void for more information.

## Function declarations

Function declarations are the declarations that appear in front of the function name, such as

```
real matrix swaprows(...)
{
}
```

The syntax for what may appear there is

```
function
type [function]
void [function]
```

Something must appear in front of the name, and if you do not want to declare the type (which makes the type transmorphic matrix), you just put the word function:

```
function swaprows(...)
{
}
```

You may also declare the type and include the word function if you wish,

```
real matrix function swaprows(...)
{
}
```

but most programmers omit the word function; it makes no difference.

In addition to all the usual types, void is a type allowed only with functions-it states that the function returns nothing:

```
void _swaprows(real matrix A, real scalar i1, real scalar i2)
{
    real rowvector v
    v = A[i1, .]
    A[i1, .] = A[i2, .]
    A[i2, .] = v
}
```

The function above returns nothing; it instead modifies the matrix it is passed. That might be useful to save memory, especially if every use of the original swaprows () was going to be

$$
\mathrm{A}=\operatorname{swaprows(A,i1,~i2)}
$$

In any case, we named this new function _swaprows() (note the underscore), to flag the user that there is something odd and deserving caution concerning the use of this function.
void, that is to say, returning nothing, is also considered a special case of a transmorphic matrix because Mata secretly returns a $0 \times 0$ real matrix, which the caller just discards.

## Argument declarations

Argument declarations are the declarations that appear inside the parentheses, such as

```
... swaprows(real matrix A, real scalar i1, real scalar i2)
{
}
```

The syntax for what may appear there is

$$
[\text { type }] \text { argname }[,[\text { type }] \text { argname }[, \ldots]]
$$

The names are required-they specify how we will refer to the argument-and the types are optional. Omit the type and transmorphic matrix is assumed. Specify the type, and it will be checked when your function is called. If the caller attempts to use your function incorrectly, Mata will stop the execution and complain.

## The by-address calling convention

Arguments are passed to functions by address, not by value. If you change the value of an argument, you will change the caller's argument. That is what made _swaprows() (above) work. The caller passed us A and we changed it. And that is why in the original version of swaprows (), the first line read

$$
\mathrm{B}=\mathrm{A}
$$

we did our work on $B$, and returned $B$. We did not want to modify the caller's original matrix.

You do not ordinarily have to make copies of the caller's arguments, but you do have to be careful if you do not want to change the argument. That is why in all the official functions (with the single exception of st_view () -see [M-5] st_view()), if a function changes the caller's argument, the function's name starts with an underscore. The reverse logic does not hold: some functions start with an underscore and do not change the caller's argument. The underscore signifies caution, and you need to read the function's documentation to find out what it is you need to be cautious about.

## Variable declarations

The variable declarations are the declarations that appear at the top of the body of a function:

```
... swaprows(...)
{
    real matrix B
    real rowvector v
}
```

These declarations are optional. If you omit them, Mata will observe that you are using B and vin your code, and then Mata will compile your code just as if you had declared the variables to be transmorphic matrix, meaning that the resulting compiled code might be a little more inefficient than it could be, but that is all.

The variable declarations are optional as long as you have not mata set matastrict on; see [M-3] mata set. Some programmers believe so strongly that variables really ought to be declared that Mata provides a provision to issue an error when they forget.

In any case, these declarations-explicit or implicit-define the variables we will use. The variables we use in our function are private-it does not matter if there are other variables named B and v floating around somewhere. Private variables are created when a function is invoked and destroyed when the function ends. The variables are private but, as explained above, if we pass our variables to another function, that function may change their values. Most functions do not.

The syntax for declaring variables is

real matrix $B$ and real rowvector $v$ match the first syntax.

## Linking to external globals

The second syntax has to do with linking to global variables. When you use Mata interactively and type

$$
: n=2
$$

you create a variable named $n$. That variable is global. When you code inside a function

```
... myfunction(...)
{
```

    external n
    \}

The n variable your function will use is the global variable named n . If your function were to examine the value of n right now, it would discover that it contained 2 .

If the variable does not already exist, the statement external $n$ will create it. Pretend that we had not previously defined $n$. If myfunction() were to examine the contents of $n$, it would discover that n is a $0 \times 0$ matrix. That is because we coded

```
external n
```

and Mata behaved as if we had coded

```
external transmorphic matrix n
```

Let's modify myfunction() to read:

```
... myfunction(...)
{
    external real scalar n
}
```

Let's consider the possibilities:

1. n does not exist. Here external real scalar n will create n -as a real scalar, of course-and set its value to missing.

If n had been declared a rowvector, a $1 \times 0$ vector would have been created.
If n had been declared a colvector, a $0 \times 1$ vector would have been created.
If $n$ had been declared a vector, a $0 \times 1$ vector would have been created. Mata could just as well have created a $1 \times 0$ vector, but it creates a $0 \times 1$.

If n had been declared a matrix, a $0 \times 0$ matrix would have been created.
2. n exists, and it is a real scalar. Our function executes, using the global n .
3. n exists, and it is a real $1 \times 1$ rowvector, colvector, or matrix. The important thing is that it is $1 \times 1$; our function executes, using the global n .
4. n exists, but it is complex or string or pointer, or it is real but not $1 \times 1$. Mata issues an error message and aborts execution of our function.

Complicated systems of programs sometimes find it convenient to communicate via globals. Because globals are globals, we recommend that you give your globals long names. A good approach is to put the name of your system as a prefix:

```
... myfunction(...)
{
        external real scalar mysystem_n
}
```

For another approach to globals, see [M-5] findexternal() and [M-5] valofexternal( ).

## Also see

[M-2] Intro - Language definition

## Title

[M-2] do - do $\ldots$ while (exp)

## Description Syntax Remarks and examples Also see

## Description

do executes stmt or stmts one or more times, until exp is zero.

## Syntax

```
do stmt while (exp)
do {
    stmts
} while (exp)
```

where $\exp$ must evaluate to a real scalar.

## Remarks and examples

One common use of do is to loop until convergence:

```
do {
    lasta = a
    a = get_new_a(lasta)
} while (mreldif(a, lasta)>1e-10)
```

The loop is executed at least once, and the conditioning expression is not executed until the loop's body has been executed.

## Also see

[M-2] break - Break out of for, while, or do loop
[M-2] continue - Continue with next iteration of for, while, or do loop
[M-2] for - for $(\exp 1 ; \exp 2 ; \exp 3)$ stmt
[M-2] while - while (exp) stmt
[M-2] Intro - Language definition

## Title

[M-2] Errors - Error codes

## Description Remarks and examples Also see

## Description

When an error occurs, Mata presents a number as well as text describing the problem. The codes are presented below.

Also the error codes can be used as an argument with _error (), see [M-5] error().
Mata's error codes are a special case of Stata's return codes. In particular, they are the return codes in the range 3000-3999. In addition to the 3000-level codes, it is possible for Mata functions to generate any Stata error message and return code.

## Remarks and examples

Error messages in Mata break into two classes: errors that occur when the function is compiled (code 3000) and errors that occur when the function is executed (codes 3001-3999).

Compile-time error messages look like this:

```
: 2,,3
invalid expression
r(3000);
: "this" + "that
mismatched quotes
r(3000);
```

The text of the message varies according to the error made, but the error code is always 3000 .
Run-time errors look like this:

```
: myfunction(2,3)
    solve(): 3200 conformability error
    mysub(): - function returned error
    myfunction(): - function returned error
    <istmt>: - function returned error
r(3200);
```

The output is called a traceback log. Read from bottom to top, it says that what we typed (the <istmt>) called myfunction(), which called mysub(), which called solve() and, at that point, things went wrong. The error is possibly in solve(), but because solve() is a library function, it is more likely that the error is in how mysub() called solve(). Moreover, the error is seldom in the program listed at the top of the traceback log because the log lists the identity of the program that detected the error. Say solve() did have an error. Then the traceback $\log$ would probably have read something like

```
                            *: 3200 conformability error
                            solve(): - function returned error
                            mysub(): - function returned error
myfunction(): - function returned error
    <istmt>: - function returned error
```

The above log says the problem was detected by * (the multiplication operator), and at that point, solve() would be suspect, because one must ask, why did solve() use the multiplication operator incorrectly?

In any case, let's assume that the problem is not with solve(). Then you would guess the problem lies with mysub (). If you have used mysub() in many previous programs without problems, however, you might now shift your suspicion to myfunction(). If myfunction() is always trustworthy, perhaps you should not have typed myfunction $(2,3)$. That is, perhaps you are using myfunction() incorrectly.

## The error codes

3000. (message varies)

There is an error in what you have typed. Mata cannot interpret what you mean.
3001. incorrect number of arguments

The function expected, say, three arguments and received two, or five. Or the function allows between three and five arguments, but you supplied too many or too few. Fix the calling program.
3002. identical arguments not allowed

You have called a function specifying the same variable more than once. Usually this would not be a problem, but here, it is, usually because the supplied arguments are matrices that the function wishes to overwrite with different results. For instance, say function $f(A, B$, $C$ ) examines matrix $A$ and returns a calculation based on $A$ in $B$ and $C$. The function might well complain that you specified the same matrix for $B$ and $C$.
3010. attempt to dereference NULL pointer

The program made reference to $* s$, and $s$ contains NULL; see [M-2] pointers.
3011. invalid lval

In an assignment, what appears on the left-hand side of the equals is not something to which a value can be assigned; see [M-2] op_assignment.
3012. undefined operation on pointer

You have, for instance, attempted to add two pointers; see [M-2] pointers.
3020. class child/parent compiled at different times

One class is being used to extend another class, and the parent has been updated since the class was compiled.
3021. class compiled at different times

A function using a predefined class has not been recompiled since the class has last been changed.
3022. function not supported on this platform

You have tried to use a function that is defined for some operating system or edition supported by Stata but not for the one you are currently using. For example, you may have tried to use a Mac-only function in Stata for Windows.
3101. matrix found where function required

A particular argument to a function is required to be a function, and a matrix was found instead.

## 3102. function found where matrix required

A particular argument to a function is required to be a matrix, vector, or scalar, and a function was found instead.
3103. view found where array required

In general, view matrices can be used wherever a matrix is required, but there are a few exceptions, both in low-level routines and in routines that wish to write results back to the argument. Here a view is not acceptable. If $V$ is the view variable, simply code $X=V$ and then pass $X$ in its stead. See [M-5] st_view().
3104. array found where view required

A function argument was specified with a matrix that was not a view, and a view was required. See [M-5] st_view().
3200. conformability error

A matrix, vector, or scalar has the wrong number of rows and/or columns for what is required. Adding a $2 \times 3$ matrix to a $1 \times 4$ would result in this error.
3201. vector required

An argument is required to be $r \times 1$ or $1 \times c$, and a matrix was found instead.
3202. rowvector required

An argument is required to be $1 \times c$ and it is not.
3203. colvector required

An argument is required to be $r \times 1$ and it is not.
3204. matrix found where scalar required

An argument is required to be $1 \times 1$ and it is not.
3205. square matrix required

An argument is required to be $n \times n$ and it is not.
3206. invalid use of view containing op.vars

Factor variables have been used in a view, and the view is now being used in a context that does not support the use of factor variables.
3208. more than 2 billion rows or columns (LAPACK)

A call was made to a LAPACK () function with a matrix containing more than $2^{31}-1$ rows or columns. The LAPACK functions used by Mata cannot accept matrices larger than this.
3209. more than 281 terarows or teracolumns

A call was made to a function with a matrix containing more than $2^{48}-1$ rows or columns. This is not allowed.
3211. view frame not found

The referenced view is linked to a frame that no longer exists.
3212. view vars \& obs out of range

An attempt was made to access a view using both a variable number and an observation number that are out of range.
3213. view vars out of range

An attempt was made to access a view using a variable number out of range.
3214. view obs out of range

An attempt was made to access a view using an observation number out of range.
3250. type mismatch

The eltype of an argument does not match what is required. For instance, perhaps a real was expected and a string was received. See eltype in [M-6] Glossary.
3251. nonnumeric found where numeric required

An argument was expected to be real or complex and it is not.
3252. noncomplex found where complex required

An argument was expected to be complex and it is not.
3253. nonreal found where real required

An argument was expected to be real and it is not.
3254. nonstring found where string required

An argument was expected to be string and it is not.
3255. real or string required

An argument was expected to be real or string and it is not.
3256. numeric or string required

An argument was expected to be real, complex, or string and it is not.
3257. nonpointer found where pointer required

An argument was expected to be a pointer and it is not.
3258. nonvoid found where void required

An argument was expected to be void and it is not.
3259. nonstruct found where struct required

A variable that is not a structure was passed to a function that expected the variable to be a structure.
3260. nonclass found where class required

A variable that is not a class was passed to a function that expected the variable to be a class.
3261. non class/struct found where class/struct required

A variable that is not a class or a structure was passed to a function that expected the variable to be a class or a structure.
3300. argument out of range

The eltype and orgtype of the argument are correct, but the argument contains an invalid value, such as if you had asked for the 20th row of a $4 \times 5$ matrix. See eltype and orgtype in [M-6] Glossary.
3301. subscript invalid

The subscript is out of range (refers to a row or column that does not exist) or contains the wrong number of elements. See [M-2] Subscripts.
3302.
invalid \%fmt
The \%fmt for formatting data is invalid. See [M-5] printf() and see [U] 12.5 Formats: Controlling how data are displayed.

## 3303. invalid permutation vector

The vector specified does not meet the requirements of a permutation vector, namely, that an $n$-element vector contain a permutation of the integers 1 through $n$. See [M-1] Permutation.
3304. struct nested too deeply

Structures may contain structures that contain structures, and so on, but only to a depth of 500.
3305. class nested too deeply

Classes may contain classes that contain classes, and so on, but only to a depth of 500 .
3351. argument has missing values

In general, Mata is tolerant of missing values, but there are exceptions. This function does not allow the matrix, vector, or scalar to have missing values.
3352. singular matrix

The matrix is singular and the requested result cannot be carried out. If singular matrices are a possibility, then you are probably using the wrong function.
3353. matrix not positive definite

The matrix is non-positive definite and the requested results cannot be computed. If non-positive-definite matrices are a possibility, then you are probably using the wrong function.
3360. failure to converge

The function that issued this message used an algorithm that the function expected would converge but did not, probably because the input matrix was extreme in some way.
3492. resulting string too long

A string that the function was attempting to produce became too long. Because the maximum length of strings in Mata is $2,147,483,647$ characters, it is unlikely that Mata imposed the limit. Review the documentation on the function for the source of the limit that was imposed (for example, perhaps a string was being produced for use by Stata). In any case, this error does not arise because of an out-of-memory situation. It arises because some limit was imposed.
3498. (message varies)

An error specific to this function arose. The text of the message should describe the problem.
3499. $\qquad$ not found
The specified variable or function could not be found. For a function, it was not already loaded, it is not in the libraries, and there is no .mo file with its name.
3500. invalid Stata variable name

A variable name-which name is contained in a Mata string variable-is not appropriate for use with Stata.
3598. Stata returned error

You are using a Stata interface function and have asked Stata to perform a task. Stata could not or refused.
3601. invalid file handle

The number specified does not correspond to an open file handle; see [M-5] fopen( ).
3602. invalid filename

The filename specified is invalid.
3603. invalid file mode

The file mode (whether read, write, read-write, etc.) specified is invalid; see [M-5] fopen().
3610. file from more recent version of Stata

An attempt was made to read a file created by a newer version of Stata than you are currently using. The file is in a format that your version of Stata does not understand.
3611. too many open files

The maximum number of files that may be open simultaneously is 50 , although your operating system may not allow that many.
3621. attempt to write read-only file

The file was opened read-only and an attempt was made to write into it.
3622. attempt to read write-only file

The file was opened write-only and an attempt was made to read it.
3623. attempt to seek append-only file

The file was opened append-only and then an attempt was made to seek into the file; see [M-5] fopen().
3698. file seek error

An attempt was made to seek to an invalid part of the file, or the seek failed for other reasons; see [M-5] fopen().
3900. out of memory

Mata is out of memory; the operating system refused to supply what Mata requested. There is no Mata or Stata setting that affects this, and so nothing in Mata or Stata to reset in order to get more memory. You must take up the problem with your operating system.
3901. macro memory in use

This error message should not occur; please notify StataCorp if it does.
3930. error in LAPACK routine

Linear algebra package (LAPACK) routines-see [M-1] LAPACK-generated an error that Mata did not expect. Please notify StataCorp if you should receive this error.
3995. unallocated function

This error message should not occur; please notify StataCorp if it does.
3996. built-in unallocated

This error message should not occur; please notify StataCorp if it does.
3997. unimplemented opcode

This error message should not occur; please notify StataCorp if it does.
3998. stack overflow

Your program nested too deeply. For instance, imagine calculating the factorial of $n$ by recursively calling yourself and then requesting the factorial of $1 \mathrm{e}+100$. Functions that call themselves in an infinite loop inevitably cause this error.
3999. system assertion false

Something unexpected happened in the internal Mata code. Please contact Technical Services and report the problem. You do not need to exit Stata or Mata. Mata stopped doing what was leading to a problem.

## Also see

[M-5] error() - Issue error message
[M-2] Intro - Language definition

## Title

[M-2] exp - Expressions

Description Syntax Remarks and examples Reference Also see

## Description

exp is used in syntax diagrams to mean "any valid expression may appear here". Expressions can range from being simple constants

```
2
"this"
3+2i
```

to being names of variables

```
A
beta
varwithverylongname
```

to being a full-fledged scalar, string, or matrix expression:

```
sqrt(2)/2
substr(userinput, 15, strlen(otherstr))
conj(X)'X
```


## Syntax

$\exp$

## Remarks and examples

Remarks are presented under the following headings:

```
What's an expression
Assignment suppresses display, as does (void)
The pieces of an expression
Numeric literals
String literals
Variable names
Operators
Functions
```


## What's an expression

Everybody knows what an expression is: expressions are things like $2+3$ and invsym ( $\mathrm{X}^{\prime} \mathrm{X}$ ) $* \mathrm{X}^{\prime} \mathrm{y}$. Simpler things are also expressions, such as numeric constants
and string literals
"hi there" is an expression
and function calls:

```
sqrt(2) is an expression
```

Even when functions do not return anything (the function is void), the code that causes the function to run is an expression. For instance, the function $\operatorname{swap}()$ (see [M-5] swap()) interchanges the contents of its arguments and returns nothing. Even so,

```
swap (A, B) is an expression
```


## Assignment suppresses display, as does (void)

The equal sign assigns the result of an expression to a variable. For instance,

$$
a=2+3
$$

assigns 5 to a. When the result of an expression is not assigned to a variable, the result is displayed at the terminal. This is true of expressions entered interactively and of expressions coded in programs. For instance, given the program

```
function example(a, b)
{
    "the answer is"
    a+b
}
```

executing example() produces

```
example(2, 3)
    the answer is
    5
```

The fact that 5 appeared is easy enough to understand; we coded the expression $\mathrm{a}+\mathrm{b}$ without assigning it to another variable. The fact that "the answer is" also appeared may surprise you. Nevertheless, we coded "the answer is" in our program, and that is an example of an expression, and because we did not assign the expression to a variable, it was displayed.

In programming situations, there will be times when you want to execute a function-call it setup()but do not care what the function returns, even though the function itself is not void (that is, it returns something). If you code

```
function example(...)
{
    setup(...)
}
```

the result will be to display what setup() returns. You have two alternatives. You could assign the result of setup to a variable even though you will subsequently not use the variable

```
function example(...)
{
    result = setup(...)
}
```

or you could cast the result of the function to be void:

```
function example(...)
{
    (void) setup(...)
}
```

Placing (void) in front of an expression prevents the result from being displayed.

## The pieces of an expression

Expressions comprise
numeric literals
string literals
variable names
operators
functions

## Numeric literals

Numeric literals are just numbers

```
2
3.14159
-7.2
5i
1.213e+32
1.213E+32
1.921fb54442d18X+001
1.921fb54442d18x+001
.a
.b
```

but you can suffix an i onto the end to mean imaginary, such as $5 i$ above. To create complex numbers, you combine real and imaginary numbers using the + operator, as in $2+5 i$. In any case, you can put the $i$ on the end of any literal, so $1.213 \mathrm{e}+32 \mathrm{i}$ is valid, as is $1.921 \mathrm{fb} 54442 \mathrm{~d} 18 \mathrm{X}+001 \mathrm{i}$.
$1.921 \mathrm{fb} 54442 \mathrm{~d} 18 \mathrm{X}+001 \mathrm{i}$ is a formidable-looking beast, with or without the i .
$1.921 \mathrm{fb} 54442 \mathrm{~d} 18 \mathrm{X}+001$ is a way of writing floating-point numbers in binary; it is described in [U] 12.5 Formats: Controlling how data are displayed. Most people never use it.

Also, numeric literals include Stata's missing values, ., .a, .b, ..., .z.

Complex variables may contain missing just as real variables may, but they get only one: . $\mathrm{a}+. \mathrm{bi}$ is not allowed. A complex variable contains a valid complex value, or it contains ., .a, .b,..., .z.

## String literals

String literals are enclosed in double quotes or in compound double quotes:

```
"the answer is"
"a string"
'"also a string"'
""The "factor" of a matrix"'
""
6 | | )
```

Strings in Mata contain between 0 and $2,147,483,647$ bytes. "" or '""' is how one writes the 0 -length string.

Any plain ASCII or UTF-8 character may appear in the string, but no provision is provided for typing unprintable characters into the string literal. Instead, you use the char () function; see [M-5] ascii( ). For instance, char (13) is carriage return, so the expression

```
"my string" + char(13)
```

produces "my string" followed by a carriage return.
No character is given a special interpretation. In particular, backslash ( $\backslash$ ) is given no special meaning by Mata. The string literal "my string $\backslash n$ " is just that: the characters "my string" followed by a backslash followed by an " $n$ ". Some functions, such as printf() (see [M-5] printf()), give a special meaning to the two-character sequence $\backslash \mathrm{n}$, but that special interpretation is a property of the function, not Mata, and is noted in the function's documentation.

Strings are not zero (null) terminated in Mata. Mata knows that the string "hello" is of length 5, but it does not achieve that knowledge by padding a binary 0 as the string's fifth character. Thus strings may be used to hold binary information.

Although Mata gives no special interpretation to binary 0, some Mata functions do. For instance, strmatch ( $s$, pattern) returns 1 if $s$ matches pattern and 0 otherwise; see [M-5] strmatch(). For this function, both strings are considered to end at the point they contain a binary 0 , if they contain a binary 0 . Most strings do not, and then the function considers the entire string. In any case, if there is special treatment of binary 0 , that is on a function-by-function basis, and a note of that is made in the function's documentation.

Some string functions in Mata have variants that are designed specifically to deal with Unicode. For examples, usubstr () is the Unicode-aware version of substr (). See [U] 12.4.2 Handling Unicode strings for more details on working with Unicode strings.

## Variable names

Variable names are just that. Names are case sensitive and no abbreviations are allowed:

```
X
x
MyVar
VeryLongVariableNameForUseInMata
MyVariable
```

The maximum length of a variable name is 32 characters.

## Operators

## Operators, listed by precedence, low to high

| Operator | Operator name | Documentation |
| :---: | :---: | :---: |
| $a=b$ | assignment | [M-2] op_assignment |
| $a \geq b: c$ | conditional | [M-2] op_conditional |
| $a \backslash b$ | column join | [M-2] op_join |
| $a:: b$ | column to | [M-2] op_range |
| $a, b$ | row join | [M-2] op_join |
| $a \ldots b$ | row to | [M-2] op_range |
| $a: \mid \quad b$ | e.w. or | [M-2] op_colon |
| $a \mid b$ | or | [M-2] op_logical |
| $a: \& b$ | e.w. and | [M-2] op_colon |
| $a \& b$ | and | [M-2] op_logical |
| $a:==b$ | e.w. equal | [M-2] op_colon |
| $a==b$ | equal | [M-2] op_logical |
| $a:>=b$ | e.w. greater than or equal | [M-2] op_colon |
| $a>=b$ | greater than or equal | [M-2] op_logical |
| $a:<=b$ | e.w. less than or equal | [M-2] op_colon |
| $a<=b$ | less than or equal | [M-2] op_logical |
| $a:<b$ | e.w. less than | [M-2] op_colon |
| $a<b$ | less than | [M-2] op_logical |
| $a \quad:>b$ | e.w. greater than | [M-2] op_colon |
| $a>b$ | greater than | [M-2] op_logical |
| $a:!=b$ | e.w. not equal | [M-2] op_colon |
| $a \quad!=b$ | not equal | [M-2] op_logical |
| $a:+b$ | e.w. addition | [M-2] op_colon |
| $a+b$ | addition | [M-2] op_arith |
| $a:-b$ | e.w. subtraction | [M-2] op_colon |
| $a-b$ | subtraction | [M-2] op_arith |
| $a: * b$ | e.w. multiplication | [M-2] op_colon |
| $a * b$ | multiplication | [M-2] op_arith |
| $a$ \# b | Kronecker | [M-2] op_kronecker |
| $a: / b$ | e.w. division | [M-2] op_colon |
| $a / b$ | division | [M-2] op_arith |
| -a | negation | [M-2] op_arith |
| $a:^{\wedge} b$ | e.w. power | [M-2] op_colon |
| $a \sim b$ | power | [M-2] op_arith |
| $a^{\prime}$ | transposition | [M-2] op_transpose |
| *a | contents of | [M-2] pointers |
| \& $a$ | address of | [M-2] pointers |
| $!a$ | not | [M-2] op_logical |
| $a[\exp ]$ | subscript | [M-2] Subscripts |
| $a[\|\exp \|]$ | range subscript | [M-2] Subscripts |
| $a++$ | increment | [M-2] op_increment |
| $a--$ | decrement | [M-2] op_increment |
| ++a | increment | [M-2] op_increment |
| --a | decrement | [M-2] op_increment |

(e.w. = elementwise)

## Functions

Functions supplied with Mata are documented in [M-5]. An index to the functions can be found in [M-4] Intro.

## Reference

Gould, W. W. 2006. Mata Matters: Precision. Stata Journal 6: 550-560.

## Also see

[M-2] Intro - Language definition

## Title

[M-2] for - for (exp1; exp2; exp3) stmt
Description Syntax Remarks and examples Also see

## Description

for is equivalent to

```
exp1
while ( exp 2) {
    stmt(s)
    exp3
}
```

$\operatorname{stmt}(s)$ is executed zero or more times. The loop continues as long as exp2 is not equal to zero.

## Syntax

```
for \(\left(\exp _{1} ; \exp _{2} ; \exp _{3}\right)\) stmt
for \(\left(\exp _{1} ; \exp _{2} ; \exp _{3}\right)\) \{
        stmts
\}
```

where $\exp _{1}$ and $\exp _{3}$ are optional, and $\exp _{2}$ must evaluate to a real scalar.

## Remarks and examples

To understand for, enter the following program

```
function example(n)
{
        for (i=1; i<=n; i++) {
        printf("i=%g\n", i)
    }
    printf("done\n")
}
```

and run example(3), example(2), example(1), example(0), and example(-1).
Common uses of for include

```
for (i=1; i<=rows(A); i++) {
    for (j=1; j<=cols(A); j++) {
    }
}
```


## Also see

[M-2] break - Break out of for, while, or do loop
[M-2] continue - Continue with next iteration of for, while, or do loop
[M-2] do - do ... while (exp)
[M-2] Semicolons - Use of semicolons
[M-2] while - while (exp) stmt
[M-2] Intro - Language definition

## Title

[M-2] ftof - Passing functions to functions

Description Syntax Remarks and examples Also see

## Description

Functions can receive other functions as arguments.
Below is described (1) how to call a function that receives a function as an argument and (2) how to write a function that receives a function as an argument.

## Syntax

example(..., \&somefunction(), ...)
where example() is coded

```
function example(..., f, ...)
{
    (*f)(...)
}
```


## Remarks and examples

Remarks are presented under the following headings:
Passing functions to functions
Writing functions that receive functions, the simplified convention
Passing built-in functions

## Passing functions to functions

Someone has written a program that receives a function as an argument. We will imagine that function is

```
real scalar fderiv(function(), x)
```

and that feriv() numerically evaluates the derivative of function() at $x$. The documentation for fderiv() tells you to write a function that takes one argument and returns the evaluation of the function at that argument, such as

```
real scalar expratio(real scalar x)
{
    return(exp(x)/exp(-x))
}
```

To call fderiv() and have it evaluate the derivative of expratio() at 3, you code

```
fderiv(&expratio(), 3)
```

To pass a function to a function, you code \& in front of the function's name and () after. Coding \&expratio() passes the address of the function expratio() to fderiv().

## Writing functions that receive functions, the simplified convention

To receive a function, you include a variable among the program arguments to receive the function-we will use $f$-and you then code $(* f)(\ldots)$ to call the passed function. The code for fderiv() might read

```
function fderiv(f, x)
{
    return( ((*f) (x+1e-6) - (*f)(x)) / 1e-6 )
}
```

or, if you prefer to be explicit about your declarations,

```
real scalar fderiv(pointer scalar f, real scalar x)
{
    return( ((*f)(x+1e-6) - (*f)(x)) / 1e-6 )
}
```

or, if you prefer to be even more explicit:

```
real scalar fderiv(pointer(real scalar function) scalar f,
    real scalar x)
{
    return( ((*f)(x+1e-6) - (*f)(x)) / 1e-6 )
}
```

In any case, using pointers, you type ( $* \mathrm{f}$ ) (...) to execute the function passed. See [M-2] pointers for more information.

Aside: the function fderiv() would work but, because of the formula it uses, would return very inaccurate results.

## Passing built-in functions

You cannot pass built-in functions to other functions. For instance, $[\mathrm{M}-5] \exp ()$ is built in, which is revealed by [M-3] mata which:

```
: mata which exp()
    exp(): built-in
```

Not all official functions are built in. Many are implemented in Mata as library functions, but exp() is built in and coding $\& \exp ()$ will result in an error. If you wanted to pass $\exp ()$ to a function, create your own version of it

```
: function myexp(x) return(exp(x))
```

and then pass \&myexp().

## Also see

[M-2] Intro - Language definition

## Title

[M-2] goto - goto label
Description Syntax Remarks and examples Reference Also see

## Description

goto label causes control to pass to the statement following label: label may be any name up to eight characters long.

## Syntax

```
label: ...
    goto label
```

where label: may occur before or after the goto and must be a valid Stata name.

## Remarks and examples

These days, good style is to avoid using goto.
goto is useful when translating a FORTRAN program, such as

$$
\begin{aligned}
& \mathrm{A}=4.0 \mathrm{e} 0 / 3 \cdot 0 \mathrm{e} 0 \\
& 10 \mathrm{~B}=\mathrm{A}-1.0 \mathrm{e} 0 \\
& \mathrm{C}=\mathrm{B}+\mathrm{B}+\mathrm{B} \\
& \mathrm{EPS}=\mathrm{DABS}(\mathrm{C}-1.0 \mathrm{e} 0) \\
& \text { if (EPS.EQ.0.0e0) GOTO } 10
\end{aligned}
$$

The Mata translation is

$$
\begin{array}{ll}
\text { s10: } \quad & \mathrm{b}=4 / 3 \\
& \mathrm{c}=\mathrm{a}-1 \\
& \text { eps }=\mathrm{abs}(c-1) \\
& \text { if (eps==0) goto s10 }
\end{array}
$$

although

$$
\begin{aligned}
& a=4 / 3 \\
& \text { do \{ } \\
& \qquad \begin{array}{l}
b=a-1 \\
c=b+b+b \\
e p s=a b s(c-1)
\end{array} \\
& \} \text { while (eps==0)}
\end{aligned}
$$

is more readable.

## Reference

Gould, W. W. 2005. Mata Matters: Translating Fortran. Stata Journal 5: 421-441.

## Also see

[M-2] break - Break out of for, while, or do loop
[M-2] continue - Continue with next iteration of for, while, or do loop
[M-2] do - do ... while (exp)
[M-2] for - for $(\exp 1 ; \exp 2 ; \exp 3)$ stmt
[M-2] while - while (exp) stmt
[M-2] Intro - Language definition

## Title

[M-2] if — if (exp) ... else ...
Description Syntax Remarks and examples Also see

## Description

if evaluates the expression, and if it is true (evaluates to a nonzero number), if executes the statement or statement block that immediately follows it; otherwise, if skips the statement or block.
if ... else evaluates the expression, and if it is true (evaluates to a nonzero number), if executes the statement or statement block that immediately follows it and skips the statement or statement block following the else; otherwise, it skips the statement or statement block immediately following it and executes the statement or statement block following the else.

## Syntax

```
if (exp) stmtl
if (exp) stmtl
else stmt2
if (exp) {
        stmts1
}
else {
        stmts2
}
if (exp1)...
else if (exp )...
else if (exp )...
else ...
```

where $\exp , \exp _{1}, \exp _{2}, \exp _{3}, \ldots$ must evaluate to real scalars.

## Remarks and examples

if followed by multiple elses is interpreted as being nested, that is,

```
if (exp ) ...
else if (exp 2)...
else if (exp )...
else ...
```

is equivalent to

```
if (exp1) ...
else {
    if (exp2)...
    else {
        if (exp3) ...
        else {
        }
    }
}
```


## Also see

[M-2] Intro - Language definition

## Title

[M-2] op_arith - Arithmetic operators

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

The above operators perform basic arithmetic.

## Syntax

| $a+b$ | addition |
| :--- | :--- |
| $a-b$ | subtraction |
| $a * b$ | multiplication |
| $a / b$ | division |
| $a^{\wedge} b$ | power |
| $-a$ | negation |

where $a$ and $b$ may be numeric scalars, vectors, or matrices.

## Remarks and examples

Also see [M-2] op_colon for the :+, :-, :*, and :/ operators. Colon operators have relaxed conformability restrictions.

The $*$ and :* multiplication operators can also perform string duplication-3*"a" = "aaa"-see [M-5] strdup ().

## Conformability

$$
\begin{array}{rlr}
a+b, a-b: & & \\
a: & & r \times c \\
b: & r \times c \\
\text { result }: & r \times c
\end{array}
$$

$a * b:$

| $a:$ | $k \times n$ | $k \times n$ | $1 \times 1$ |
| ---: | :--- | :--- | :--- |
| $b:$ | $n \times m$ | $1 \times 1$ | $n \times m$ |
| result: | $k \times m$ | $k \times n$ | $n \times m$ |

$a / b:$

| $a:$ |  | $r \times c$ |
| ---: | :--- | ---: |
| $b:$ |  | $1 \times 1$ |
| result $:$ |  | $r \times c$ |

$a^{\wedge} b:$

$$
\begin{aligned}
& a: \\
& b: \\
& 1 \times 1 \times 1 \\
& \text { result: } \\
& 1 \times 1
\end{aligned}
$$

$-a$ :

$$
\begin{aligned}
a: & & r \times c \\
\text { result: } & & r \times c
\end{aligned}
$$

## Diagnostics

All operators return missing when arguments are missing.
$a * b$ with $a: k \times 0$ and $b: 0 \times m$ returns a $k \times m$ matrix of zeros.
$a / b$ returns missing when $b==0$ or when $a / b$ would result in overflow.
$a^{\wedge} b$ returns a real when both $a$ and $b$ are real; thus, ( -4$)^{\wedge} .5$ evaluates to missing, whereas $(-4+0 i)^{\wedge} .5$ evaluates to $2 i$.
$a^{\wedge} b$ returns missing on overflow.

## Also see

[M-2] exp — Expressions
[M-2] Intro - Language definition

## Title

[M-2] op_assignment - Assignment operator

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

$=$ assigns the evaluation of exp to lval.
Do not confuse the $=$ assignment operator with the $==$ equality operator. Coding

$$
x=y
$$

assigns the value of y to x . Coding

```
if (x==y)... (note doubled equal signs)
```

performs the action if the value of $x$ is equal to the value of $y$. See [M-2] op_logical for a description of the $==$ equality operator.

If the result of an expression is not assigned to a variable, then the result is displayed at the terminal; see [M-2] exp.

## Syntax

$$
l v a l=\exp
$$

where exp is any valid expression and where lval is

```
name
name [exp]
name[exp, exp]
name[|exp|]
```

In pointer use (advanced), name may be

```
*lval
* (lval)
*(lval[exp])
*(lval[exp, exp])
* (lval[|exp|])
```

in addition to being a variable name.

## Remarks and examples

Remarks are presented under the following headings:

[^3]
## Assignment suppresses display

When you interactively enter an expression or code an expression in a program without the equalassignment operator, the result of the expression is displayed at the terminal:

```
:2+3
5
```

When you assign the expression to a variable, the result is not displayed:

```
: x = 2 + 3
```


## The equal-assignment operator

Equals is an operator, so in addition to coding

$$
a=2+3
$$

you can code

$$
\mathrm{a}=\mathrm{b}=2+3
$$

or

```
y = x / (denominator = sqrt (a+b))
```

or even

$$
\mathrm{y} 1=\mathrm{y} 2=\mathrm{x} /(\text { denominator }=\operatorname{sqrt}(\text { sum=a+b }))
$$

This last is equivalent to

```
sum = a + b
denominator = sqrt(sum)
y2 = x / denominator
y1 = y2
```

Equals binds weakly, so

$$
\mathrm{a}=\mathrm{b}=2+3
$$

is interpreted as

$$
a=b=(2+3)
$$

and not

$$
a=(b=2)+3
$$

## Ivals, what appears on the left-hand side

What appears to the left of the equals is called an lval, short for left-hand-side value. It would make no sense, for instance, to code

$$
\operatorname{sqrt}(4)=3
$$

and, as a matter of fact, you are not allowed to code that because sqrt (4) is not an lval:

```
: sqrt(4) = 3
invalid lval
r(3000);
```

An lval is anything that can hold values. A scalar can hold values

$$
\begin{aligned}
& \mathrm{a}=3 \\
& \mathrm{x}=\operatorname{sqrt}(4)
\end{aligned}
$$

a matrix can hold values

$$
\begin{aligned}
& A=(1,2 \backslash 3,4) \\
& B=\operatorname{invsym}(C)
\end{aligned}
$$

a matrix row can hold values

$$
\mathrm{A}[1, .]=(7,8)
$$

a matrix column can hold values

$$
\mathrm{A}[., 2]=(9 \backslash 10)
$$

and finally, a matrix element can hold a value

$$
\mathrm{A}[1,2]=7
$$

lvals are usually one of the above forms. The other forms have to do with pointer variables, which most programmers never use; they are discussed under Pointer lvals below.

## Row, column, and element Ivals

When you assign to a row, column, or element of a matrix,

$$
\begin{aligned}
& \mathrm{A}[1, .]=(7,8) \\
& \mathrm{A}[., 2]=(9 \backslash 10) \\
& \mathrm{A}[1,2]=7
\end{aligned}
$$

the row, column, or element must already exist:

```
:A = (1, 2 \ 3,4)
: A[3,4] = 4
    <istmt>: 3301 subscript invalid
r(3301);
```

This is usually not an issue because, by the time you are assigning to a row, column, or element, the matrix has already been created, but in the event you need to create it first, use the J() function; see $[\mathrm{M}-5] \mathbf{J}()$. The following code fragment creates a $3 \times 4$ matrix containing the sum of its indices:

```
\(\mathrm{A}=\mathrm{J}(3,4,\).
for (i=1; i<=3; i++) \{
        for ( \(j=1 ; j<=4 ; j++\) ) \(A[i, j]=i+j\)
\}
```


## Pointer Ivals

In addition to the standard lvals

$$
\begin{aligned}
& \mathrm{A}=(1,2 \backslash 3,4) \\
& \mathrm{A}[1, .]=(7,8) \\
& \mathrm{A}[., 2]=(9 \backslash 10) \\
& \mathrm{A}[1,2]=7
\end{aligned}
$$

pointer lvals are allowed. For instance,

$$
* p=3
$$

stores 3 in the address pointed to by pointer scalar p .

$$
(* q)[1,2]=4
$$

stores 4 in the $(1,2)$ element of the address pointed to by pointer scalar q , whereas

$$
* Q[1,2]=4
$$

stores 4 in the address pointed to by the $(1,2)$ element of pointer matrix Q .

```
*Q[2,1][1,3] = 5
```

is equivalent to

$$
*(Q[2,1])[1,3]=5
$$

and stores 5 in the $(1,3)$ element of the address pointed to by the $(2,1)$ element of pointer matrix Q . Pointers to pointers, pointers to pointers to pointers, etc., are also allowed. For instance,

$$
* * r=3
$$

stores 3 in the address pointed to by the address pointed to by pointer scalar $r$, whereas

$$
*((*(Q[1,2]))[2,1])[3,4]=7
$$

stores 7 in the $(3,4)$ address pointed to by the $(2,1)$ address pointed to by the $(1,2)$ address of pointer matrix Q.

## Conformability

$a=b:$
input:

```
            b: r
```

output:

$$
a: \quad r \times c
$$

## Diagnostics

$a=b$ aborts with error if there is insufficient memory to store a copy of $b$ in $a$.

## Also see

[M-5] swap () - Interchange contents of variables
[M-2] exp - Expressions
[M-2] Intro - Language definition

## Title

[M-2] op_colon - Colon operators

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

Colon operators perform element-by-element operations.

## Syntax

| $a:+\quad b$ | addition |
| :---: | :---: |
| $a:-\quad b$ | subtraction |
| $a: * \quad b$ | multiplication |
| $a: / b$ | division |
| $a:^{\wedge} \quad b$ | power |
| $a:==b$ | equality |
| $a:!=b$ | inequality |
| $a:>b$ | greater than |
| $a:>=b$ | greater than or equal to |
| $a:<b$ | less than |
| $a:<=b$ | less than or equal to |
| $a$ : \& $b$ | and |
| $a: \mid \quad b$ | or |

## Remarks and examples

Remarks are presented under the following headings:
C-conformability: element by element Usefulness of colon logical operators Use parentheses

## C-conformability: element by element

The colon operators perform the indicated operation on each pair of elements of $a$ and $b$. For instance,

$$
\left[\begin{array}{cc}
c & d \\
f & g \\
h & i
\end{array}\right]: *\left[\begin{array}{cc}
j & k \\
l & m \\
n & o
\end{array}\right]=\left[\begin{array}{cc}
c * j & d * k \\
f * l & g * m \\
h * n & i * o
\end{array}\right]
$$

Also colon operators have a relaxed definition of conformability:

$$
\begin{aligned}
{\left[\begin{array}{l}
c \\
f \\
g
\end{array}\right]: *\left[\begin{array}{ll}
j & k \\
l & m \\
n & o
\end{array}\right] } & =\left[\begin{array}{cc}
c * j & c * k \\
f * l & f * m \\
g * n & g * o
\end{array}\right] \\
{\left[\begin{array}{ll}
c & d \\
f & g \\
h & i
\end{array}\right]: *\left[\begin{array}{l}
j \\
l \\
n
\end{array}\right] } & =\left[\begin{array}{ll}
c * j & d * j \\
f * l & g * l \\
h * n & i * n
\end{array}\right] \\
{\left[\begin{array}{ll}
c & d
\end{array}\right]: *\left[\begin{array}{ll}
j & k \\
l & m \\
n & o
\end{array}\right] } & =\left[\begin{array}{ll}
c * j & d * k \\
c * l & d * m \\
c * n & d * o
\end{array}\right] \\
{\left[\begin{array}{ll}
c & d \\
f & g \\
h & i
\end{array}\right]: *\left[\begin{array}{ll}
l & m
\end{array}\right] } & =\left[\begin{array}{ll}
c * l & d * m \\
f * l & g * m \\
h * l & i * m
\end{array}\right] \\
{\left[\begin{array}{ll}
c & d \\
f & g \\
h & i
\end{array}\right]: * } & =\left[\begin{array}{ll}
c * j & c * k \\
c * l & c * m \\
c * n & c * o
\end{array}\right] \\
l & =\left[\begin{array}{ll}
c * j & d * j \\
f * j & g * j \\
h * j & i * j
\end{array}\right]
\end{aligned}
$$

The matrices above are said to be c-conformable; the $c$ stands for colon. The matrices have the same number of rows and columns, or one or the other is a vector with the same number of rows or columns as the matrix, or one or the other is a scalar.

C-conformability is relaxed, but not everything is allowed. The following is an error:

$$
\left(\begin{array}{lll}
c & d & e
\end{array}\right): *\left[\begin{array}{l}
f \\
g \\
h
\end{array}\right]
$$

## Usefulness of colon logical operators

It is worth paying particular attention to the colon logical operators because they can produce pattern vectors and matrices. Consider the matrix

```
:x = (5, 0\ \ 0, 2\ \ 3, 8)
x
    1 2
    1 
```

Which elements of x contain 0 ?

| $\mathrm{x}:==0$ |  |  |
| :---: | :---: | :---: |
|  | 1 | 2 |
| 1 | 0 | 1 |
| 2 | 1 | 0 |
| 3 | 0 | 0 |

How many zeros are there in $x$ ?

```
: sum(x:==0)
    2
```


## Use parentheses

Because of their relaxed conformability requirements, colon operators are not associative even when the underlying operator is. For instance, you expect $(a+b)+c==a+(b+c)$, at least ignoring numerical roundoff error. Nevertheless, $(a:+b):+c==a:+(b:+c)$ does not necessarily hold. Consider what happens when

| $a:$ |  |
| :--- | :--- |
| $b:$ |  |
| $c$ | $5 \times 4$ |
| $c:$ | $5 \times 4$ |

Then $(a:+b):+c$ is an error because $a:+b$ is not c-conformable.
Nevertheless, $a:+(b:+c)$ is not an error and in fact produces a $5 \times 4$ matrix because $b:+c$ is $5 \times 4$, which is c-conformable with $a$.

For nonassociative operations, parentheses are useful when using colon operators for even the most basic computations. For example, consider the column vectors

```
: x = (4 \ 5 \ 6)
: y = (1 \ 2 \ 3)
```

Below, we attempt to compute $4-x-y$ with two different statements. The actual computations that are performed are listed as comments:
: 4 :- x :- y /* (4-x)-y */
1

| 1 | -1 |
| :--- | :--- |
| 2 | -3 |
| 3 |  |

4 :- $\mathrm{x}-\mathrm{y} \quad / * 4-(\mathrm{x}-\mathrm{y})$ */
1
1
1
1

As stated in [M-2] Syntax, an operator preceded by a colon (that is, a colon operator) has lower precedence than the operator itself. This is why Mata first subtracts y from x in the second statement above. But, if you plan to use a combination of operators and colon operators, you can still set the precedence with parentheses:
: (4 :- x) - y /* (4-x)-y */
1
1
-1
$-3$
$-5$

This produces the desired result and the same output as the first statement above.

## Conformability

$a: o p b:$

$$
\begin{aligned}
a: & r_{1} \times c_{1} \\
b: & r_{2} \times c_{2}, \quad a \text { and } b \text { c-conformable } \\
\text { result: } & \max \left(r_{1}, r_{2}\right) \times \max \left(c_{1}, c_{2}\right)
\end{aligned}
$$

## Diagnostics

The colon operators return missing and abort with error under the same conditions that the underlying operator returns missing and aborts with error.

## Also see

[M-2] exp — Expressions
[M-2] Intro - Language definition

## Title

[M-2] op_conditional - Conditional operator

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

The conditional operator returns $b$ if $a$ is true ( $a$ is not equal to 0 ) and $c$ otherwise.

## Syntax

$$
a ? b: c
$$

where $a$ must evaluate to a real scalar, and $b$ and $c$ may be of any type whatsoever.

## Remarks and examples

Conditional operators

$$
\text { dof }=(\mathrm{k}==0 \text { ? } \mathrm{n}-1 \quad: \mathrm{n}-\mathrm{k})
$$

are more compact than the if-else alternative

$$
\begin{array}{ll}
\text { if }(k==0) & \text { dof }=n-1 \\
\text { else } & \text { dof }=n-k
\end{array}
$$

and they can be used as parts of expressions:

```
mse = ess/(k==0 ? n-1 : n-k)
```


## Conformability

$$
a ? b: c
$$

| $a:$ | $1 \times 1$ |
| ---: | :--- |
| $b:$ | $r_{1} \times c_{1}$ |
| $c:$ | $r_{2} \times c_{2}$ |
| result: | $r_{1} \times c_{1} \quad$ or $\quad r_{2} \times c_{2}$ |

## Diagnostics

In $a$ ? $b: c$, only the necessary parts are evaluated: $a$ and $b$ if $a$ is true, or $a$ and $c$ if $a$ is false. However, the ++ and -- operators are always evaluated:

$$
(k==0 \text { ? } \quad i++: j++)
$$

increments both $i$ and $j$, regardless of the value of $k$.

## Also see

[M-2] exp - Expressions
[M-2] Intro - Language definition

## Title

[M-2] op_increment - Increment and decrement operators

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

$++i$ and $i++$ increment $i$; they perform the operation $i=i+1 .++i$ performs the operation before the evaluation of the expression in which it appears, whereas $i++$ performs the operation afterward.
$--i$ and $i--$ decrement $i$; they perform the operation $i=i-1 .--i$ performs the operation before the evaluation of the expression in which is appears, whereas $i^{--}$performs the operation afterward.

## Syntax

$++i \quad$ increment before
$--i$ decrement before
$i++\quad$ increment after
$i$-- decrement after
where $i$ must be a real scalar.

## Remarks and examples

These operators are used in code, such as

```
x[i++] = 2
x[--i] = 3
for (i=0; i<100; i++) {
}
if (++n > 10) {
}
```

Where these expressions appear, results are as if the current value of i were substituted, and in addition, $i$ is incremented, either before or after the expression is evaluated. For instance,

$$
x[i++]=2
$$

is equivalent to

$$
x[i]=2 ; i=i+1
$$

and

$$
x[++i]=3
$$

is equivalent to

$$
i=i+1 ; x[i]=3
$$

Coding

```
for (i=0; i<100; i++) {
}
```

or

```
for (i=0; i<100; ++i) {
}
```

is equivalent to

```
for (i=0; i<100; i=i+1) {
}
```

because it does not matter whether the incrementation is performed before or after the otherwise null expression.

```
if (++n > 10) {
}
```

is equivalent to

```
n = n + 1
if (n > 10) {
}
```

whereas

```
if (n++ > 10) {
}
```

is equivalent to

```
if (n > 10) {
    n = n + 1
}
else n = n + 1
```

The ++ and -- operators may be used only with real scalars and are usually associated with indexing or counting. They result in fast and readable code.

## Conformability

$++i,--i, i++$, and $i--$ :

| $i:$ | $1 \times 1$ |
| ---: | :--- |
| result: | $1 \times 1$ |

## Diagnostics

++ and -- are allowed with real scalars only. That is, ++i or $\mathrm{i}++$ is valid, assuming i is a real scalar, but $x[i, j]++$ is not valid.
++ and -- abort with error if applied to a variable that is not a real scalar.
$++i$, $i++$, --i, and $i--$ should be the only reference to $i$ in the expression. Do not code, for instance,

$$
\begin{aligned}
& x[i++]=y[i] \\
& x[++i]=y[i] \\
& x[i]=y[i++] \\
& x[i]=y[++i]
\end{aligned}
$$

The value of $i$ in the above expressions is formally undefined; whatever is its value, you cannot depend on that value being obtained by earlier or later versions of the compiler. Instead code
i++ ; x[i] = y[i]
or code

$$
\mathrm{x}[\mathrm{i}]=\mathrm{y}[\mathrm{i}] \text {; i++ }
$$

according to the desired outcome.
It is, however, perfectly reasonable to code

$$
x[i++]=y[j++]
$$

That is, multiple ++ and -- operators may occur in the same expression; it is multiple references to the target of the ++ and -- that must be avoided.

## Also see

[M-2] exp - Expressions
[M-2] Intro - Language definition

## Title

[M-2] op_join — Row- and column-join operators

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

, and $\backslash$ are Mata's row-join and column-join operators.

## Syntax

$$
\begin{aligned}
& a, b \\
& a \backslash b
\end{aligned}
$$

## Remarks and examples

Remarks are presented under the following headings:

```
Comma and backslash are operators
Comma as a separator
Warning about the misuse of comma and backslash operators
```


## Comma and backslash are operators

That , and $\backslash$ are operators cannot be emphasized enough. When one types

$$
:(1,2 \backslash 3,4)
$$

1 $\begin{array}{ll}1 & 2 \\ 3 & 4\end{array}$
one is tempted to think, "Ah, comma and backslash are how you separate elements when you enter a matrix." If you think like that, you will not appreciate the power of , and $\backslash$.
, and $\backslash$ are operators in the same way that $*$ and + are operators.
, is the operator that takes a $r \times c_{1}$ matrix and a $r \times c_{2}$ matrix, and returns a $r \times\left(c_{1}+c_{2}\right)$ matrix.
$\backslash$ is the operator that takes a $r_{1} \times c$ matrix and a $r_{2} \times c$ matrix, and returns a $\left(r_{1}+r_{2}\right) \times c$ matrix. , and $\backslash$ may be used with scalars, vectors, or matrices:

$$
\begin{aligned}
& : a=(1 \backslash 2) \\
& : b=(3 \backslash 4)
\end{aligned}
$$

|  |  | 2 |
| :---: | :---: | :---: |
| 1 | 1 | 3 |
| 2 | 2 | 4 |
| c | (1, | ) |
| d | (3, | ) |
| c | d |  |
|  | 1 | 2 |
| 1 | 1 | 2 |
| 2 | 3 | 4 |

binds more tightly than $\backslash$, meaning that $e, f \backslash g, h$ is interpreted as $(e, f) \backslash(g, h)$. In this, , and $\backslash$ are no different from $*$ and + operators: $*$ binds more tightly than + and $e * f+g * h$ is interpreted as $(e * f)+(g * h)$.

Just as it sometimes makes sense to type $e^{*}(f+g) * h$, it can make sense to type $e,(f \backslash g), h$ :

```
:e = 1 \ 2
: f = 5 \ 6
:g = 3
: h = 4
: e,(g\h),f
    1 2 3
\begin{tabular}{|lll|}
\hline 1 & 3 & 5 \\
2 & 4 & 6 \\
\hline
\end{tabular}
```


## Comma as a separator

, has a second meaning in Mata: it is the argument separator for functions. When you type

```
: myfunc(a, b)
```

the comma that appears inside the parentheses is not the comma row-join operator; it is the comma argument separator. If you wanted to call myfunc() with second argument equal to row vector $(1,2)$, you must type

```
: myfunc(a, (1,2))
```

and not

```
: myfunc(a, 1, 2)
```

because otherwise Mata will think you are trying to pass three arguments to myfunc(). When you open another set of parentheses inside a function's argument list, comma reverts to its usual row-join meaning.

## Warning about the misuse of comma and backslash operators

Misuse or mere overuse of , and $\backslash$ can substantially reduce the speed with which your code executes. Consider the actions Mata must take when you code, say,

$$
a \backslash b
$$

First, Mata must allocate a matrix or vector containing rows $(a)+$ rows $(b)$ rows, then it must copy $a$ into the new matrix or vector, and then it must copy $b$. Nothing inefficient has happened yet, but now consider

$$
(a \backslash b) \backslash c
$$

Picking up where we left off, Mata must allocate a matrix or vector containing rows (a)+rows $(b)+$ rows ( $c$ ) rows, then it must copy ( $a \backslash b$ ) into the new matrix or vector, and then it must copy $c$. Something inefficient just happened: $a$ was copied twice!

Coding

$$
r e s=(a \backslash b) \backslash c
$$

is convenient, but execution would be quicker if we coded

```
res \(=\mathrm{J}(\) rows \((a)+\) rows \((b)+\) rows \((c), \operatorname{cols}(a),\).
\(\operatorname{res}[1,]=\).
\(\operatorname{res}[2,]=\).
\(\operatorname{res}[3,]=\).
```

We do not want to cause you concern where none is due. In general, you would not be able to measure the difference between the more efficient code and coding res $=(a \backslash b) \backslash c$. But as the number of row or column operators stack up, the combined result becomes more and more inefficient. Even that is not much of a concern. If the inefficient construction itself is buried in a loop, however, and that loop is executed thousands of times, the inefficiency can become important.

With a little thought, you can always substitute predeclaration using J () (see [M-5] J()) and assignment via subscripting.

## Conformability

$a, b:$

$$
\begin{aligned}
& a: r \times c_{1} \\
& b: \\
& \text { result: } \\
& r \times c_{2} \\
&\left.r \times c_{1}+c_{2}\right)
\end{aligned}
$$

$a \backslash b:$

| $a:$ | $r_{1} \times c$ |
| ---: | :--- |
| $b:$ | $r_{2} \times c$ |
| result: | $\left(r_{1}+r_{2}\right) \times c$ |

## Diagnostics

, and $\backslash$ abort with error if $a$ and $b$ are not of the same broad type.

## Also see

[M-2] exp - Expressions
[M-2] Intro - Language definition

## Title

[M-2] op_kronecker — Kronecker direct-product operator
Description Syntax Remarks and examples Conformability

Diagnostics
Reference
Also see

## Description

$A \# B$ returns the Kronecker direct product.
\# binds tightly: $X * A \# B * Y$ is interpreted as $X *(A \# B) * Y$.

## Syntax

A\#B
where $A$ and $B$ may be real or complex.

## Remarks and examples

The Kronecker direct product is also known as the Kronecker product, the direct product, the tensor product, and the outer product.

The Kronecker product $A \# B$ is the matrix $\left\|a_{i j} * B\right\|$.

## Conformability

$A \# B:$

| $A:$ | $r_{1} \times c_{1}$ |
| ---: | :--- |
| $B:$ | $r_{2} \times c_{2}$ |
| result: | $r_{1} * r_{2} \times c_{1} * c_{2}$ |

## Diagnostics

None.

Leopold Kronecker (1823-1891) was born in Liegnitz, Prussia (now Legnica, Poland), to a well-off family. He attended the universities of Berlin, Bonn, and Breslau before completing a doctorate on the complex roots of unity. For several years, Kronecker devoted himself to business interests while working on mathematics in his spare time, publishing particularly in number theory, elliptic functions, and the theory of equations. He later started giving lectures at the university in Berlin, as was his right as a member of the Academy of Science. In 1883, he was appointed as the chair. Kronecker came to believe that mathematical arguments should involve only finite numbers and a finite number of operations, which led to increasing mathematical and personal disagreements with those who worked on irrational numbers or nonconstructive existence proofs.

## Reference

James, I. M. 2002. Remarkable Mathematicians: From Euler to von Neumann. Cambridge: Cambridge University Press.

## Also see

[M-2] exp — Expressions
[M-2] Intro - Language definition

## Title

[M-2] op_logical - Logical operators

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

The operators above perform logical comparisons, and operator ! performs logical negation. All operators evaluate to 1 or 0 , meaning true or false.

## Syntax

| $a==b$ true if $a$ equals $b$ <br> $a!=b$ true if $a$ not equal to $b$ |  |
| :--- | :--- |
| $a>b$ | true if $a$ greater than $b$ |
| $a>=b$ | true if $a$ greater than or equal to $b$ |
| $a<b$ | true if $a$ less than $b$ |
| $a<=b$ | true if $a$ less than or equal to $b$ |
| $l a$ | logical negation; true if $a==0$ and false otherwise |
| $a \& b$ | true if $a!=0$ and $b!=0$ <br> true if $a!=0$ or $b!=0$ |
| $a \mid b$ | synonym for $a \& b$ <br> $a \& \& b$ <br> $a\|\mid b$ |

## Remarks and examples

Remarks are presented under the following headings:

Introduction<br>Use of logical operators with pointers

## Introduction

The operators above work as you would expect when used with scalars, and the comparison operators and the not operator have been generalized for use with matrices.
$a==b$ evaluates to true if $a$ and $b$ are p-conformable, of the same type, and the corresponding elements are equal. Of the same type means $a$ and $b$ are both numeric, both strings, or both pointers. Thus it is not an error to ask if a $2 \times 2$ matrix is equal to a $4 \times 1$ vector or if a string variable is equal to a real variable; they are not. Also $a==b$ is declared to be true if $a$ or $b$ are p-conformable and the number of rows or columns is zero.
$a!=b$ is equivalent to $!(a==b) . a!=b$ evaluates to true when $a==b$ would evaluate to false and evaluates to true otherwise.

The remaining comparison operators $>,>=,<$, and $<=$ work differently from $==$ and $!=$ in that they require $a$ and $b$ be p-conformable; if they are not, they abort with error. They return true if the corresponding elements have the stated relationship, and return false otherwise. If $a$ or $b$ is complex, the comparison is made in terms of the length of the complex vector; for instance, $a>b$ is equivalent to $\mathrm{abs}(a)>\mathrm{abs}(b)$, and so $-3>2+0 \mathrm{i}$ is true.
$!a$, when $a$ is a scalar, evaluates to 0 if $a$ is not equal to zero and 1 otherwise. Applied to a vector or matrix, the same operation is carried out, element by element: $!(-1,0,1,2,$.$) evaluates$ to ( $0,1,0,0,0$ ).
\& and \| (and and or) may be used with scalars only. Because so many people are familiar with programming in the C language, Mata provides \&\& as a synonym for \& and \| as a synonym for I.

## Use of logical operators with pointers

In a pointer expression, NULL is treated as false and all other pointer values (address values) are treated as true. Thus the following code is equivalent

```
pointer x pointer x
if (x) { if (x!=NULL) {
} }
```

The logical operators $a==b, a!=b, a \& b$, and $a \mid b$ may be used with pointers.

## Conformability

$a==b, a!=b:$

| $a:$ | $r_{1} \times c_{1}$ |
| ---: | :--- |
| $b:$ | $r_{2} \times c_{2}$ |
| result $:$ | $1 \times 1$ |

$a>b, a>=b, a<b, a<=b$ :

| $a:$ | $r \times c$ |
| ---: | :--- |
| $b:$ | $r \times c$ |
| result: | $1 \times 1$ |

!a:

$$
\begin{aligned}
a: & & r \times c \\
\text { result: } & & r \times c
\end{aligned}
$$

$a \& b, a \mid b$ :

| $a:$ | $1 \times 1$ |
| ---: | :--- |
| $b:$ | $1 \times 1$ |
| result: |  |
|  | $1 \times 1$ |

## Diagnostics

$a==b$ and $a!=b$ cannot fail.
$a>b, a>=b, a<b, a<=b$ abort with error if $a$ and $b$ are not p-conformable, if $a$ and $b$ are not of the same general type (numeric and numeric or string and string), or if $a$ or $b$ are pointers.
$!a$ aborts with error if $a$ is not real.
$a \& b$ and $a \mid b$ abort with error if $a$ and $b$ are not both real or not both pointers. If $a$ and $b$ are pointers, pointer value NULL is treated as false and all other pointer values are treated as true. In all cases, a real equal to 0 or 1 is returned.

## Also see

[M-2] exp - Expressions
[M-2] Intro - Language definition

## Title

[M-2] op_range - Range operators

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

The range operators create vectors that count from $a$ to $b$.
$a . . b$ returns a row vector.
$a:: b$ returns a column vector.

## Syntax

$$
\begin{array}{ll}
a . . b & \text { row range } \\
a:: b & \text { column range }
\end{array}
$$

## Remarks and examples

$a . . b$ and $a:: b$ count from $a$ up to but not exceeding $b$, incrementing by 1 if $b>=a$ and by -1 if $b<a$.
1.. 4 creates row vector $(1,2,3,4)$.
$1:: 4$ creates column vector $(1 \backslash 2 \backslash 3 \backslash 4)$.
$-1 . .-4$ creates row vector $(-1,-2,-3,-4)$.
$-1::-4$ creates column vector $(-1 \backslash-2 \backslash-3 \backslash-4)$.
1.5..4.5 creates row vector $(1.5,2.5,3.5,4.5)$.
$1.5:: 4.5$ creates column vector ( $1.5 \backslash 2.5 \backslash 3.5 \backslash 4.5$ ).
1.5..4.4 creates row vector ( $1.5,2.5,3.5$ ).
$1.5:: 4.4$ creates column vector ( $1.5 \backslash 2.5 \backslash 3.5$ ).
$-1.5 . .-4.4$ creates row vector $(-1.5,-2.5,-3.5)$.
$-1.5::-4.4$ creates column vector $(-1.5 \backslash-2.5 \backslash-3.5)$.
1..1 and 1::1 both return (1).

## Conformability

$a . . b$

$$
\begin{aligned}
a: & 1 \times 1 \\
b: & 1 \times 1 \\
\text { result: } & 1 \times \operatorname{trunc}(\operatorname{abs}(b-a))+1
\end{aligned}
$$

$a:=b$

$$
\begin{aligned}
a: & 1 \times 1 \\
b: & 1 \times 1 \\
\text { result: } & \operatorname{trunc}(\operatorname{abs}(b-a))+1 \times 1
\end{aligned}
$$

## Diagnostics

$a . . b$ and $a:: b$ return missing if $a>=$. or $b>=.$.

## Also see

[M-2] exp - Expressions
[M-2] Intro - Language definition

## Title

[M-2] op_transpose - Conjugate transpose operator

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

$A^{\prime}$ returns the transpose of $A$ or, if $A$ is complex, the conjugate transpose.

## Syntax

$$
A^{\prime}
$$

## Remarks and examples

The ' postfix operator may be used on any type of matrix or vector: real, complex, string, or pointer:

```
: a
            1 2 3
    1
                1
a'
            1 2
                    l|ll
: s
1}\begin{array}{cc}{1}&{2}\\{\hline\mathrm{ alpha }}&{\mathrm{ beta }}\\{\hline}
S'
            1
        alpha
            beta
p
            1
    1
    2
        0x4367e0a8
        0x45e00518
    p'
        1
        2
    1
        0x4367e0a8 0x45e00518
```



When ' is applied to a complex, returned is the conjugate transpose. If you do not want this, code $\operatorname{conj}\left(z^{\prime}\right)$ or conj $(z)^{\prime}$-it makes no difference; see $[\mathrm{M}-5]$ conj(),

```
    : conj(z')
```

|  | 1 | 2 |
| :--- | ---: | ---: |
| 1 |  |  |
| 2 | $1+2 i$ $5+6 i$ <br> $3+4 i$ $7+8 i$ |  |
|  |  |  |

Or use the transposeonly () function; see [M-5] transposeonly():

transposeonly() executes slightly faster than $\operatorname{conj}\left(z^{\prime}\right)$.
For real and complex $A$, also see [M-5] _transpose( ), which provides a way to transpose a matrix in place and so saves memory.

## Conformability

$A^{\prime}:$

| $A:$ | $r \times c$ |
| ---: | :--- |
| result: |  |
| $c \times r$ |  |

## Diagnostics

The transpose operator cannot fail, but it is easy to use it incorrectly when working with complex quantities.

A user wanted to form $A * x$ but when he tried, got a conformability error. He thought $x$ was a column vector, but it turned out to be a row vector, or perhaps it was the other way around. Anyway, he then coded $A * x^{\prime}$, and the program worked and, even better, produced the correct answers. In his test, $x$ was real.

Later, the user ran the program with complex $x$, and the program generated incorrect results, although it took him a while to notice. Study and study his code he did, before he thought about the innocuous $A * x^{\prime}$. The transpose operator not only had changed $x$ from being a row into being a column but also had taken the conjugate of each element of $x$ ! He changed the code to read $A * \operatorname{transposeonly}(x)$.

The user no doubt wondered why the ' transpose operator was not defined at the outset to be equivalent to transposeonly (). If it had been, then rather than telling the story of the man who was bitten by conjugate transpose when he only wanted the transpose, we would have told the story of the woman who was bitten by the transpose when she needed the conjugate transpose. There are, in fact, more of the latter stories than there are of the former.

## Also see

[M-2] exp - Expressions
[M-5] conj() - Complex conjugate
[M-5] _transpose( ) - Transposition in place
[M-5] transposeonly () - Transposition without conjugation
[M-2] Intro - Language definition

## Title

[M-2] optargs - Optional arguments
Description Syntax Remarks and examples Also see

## Description

Mata functions may have various numbers of arguments. How you write programs that allow these optional arguments is described below.

## Syntax

```
function functionname \((\mid \arg [, \arg [, \ldots]])\{\ldots\}\)
function functionname (arg, |arg \([, \ldots]\) ) \{...\}
function functionname (arg, arg , |...) \{...\}
```

The vertical (or) bar separates required arguments from optional arguments in function declarations. The bar may appear at most once.

## Remarks and examples

Remarks are presented under the following headings:
What are optional arguments?
How to code optional arguments
Examples revisited

## What are optional arguments?

$>$ Example 1
You write a function named $\operatorname{ditty}()$. Function $\operatorname{ditty}()$ allows the caller to specify two or three arguments:
real matrix $\operatorname{ditty}($ real matrix $A$, real matrix $B$, real scalar scale) real matrix $\operatorname{ditty}($ real matrix $A$, real matrix $B$ )

If the caller specifies only two arguments, results are as if the caller had specified the third argument equal to missing; that is, $\operatorname{ditty}(A, B)$ is equivalent to $\operatorname{ditty}(A, B,$.
> Example 2
You write function gash(). Function gash() allows the caller to specify one or two arguments: real matrix gash (real matrix $A$, real matrix $B$ ) real matrix gash (real matrix $A$ )

If the caller specifies only one argument, results are as if $J(0,0,$.$) were specified for the second.$

## > Example 3

You write function easygoing(). Function easygoing() takes three arguments but allows the caller to specify three, two, one, or even no arguments:

```
real scalar easygoing(real matrix A, real matrix B, real scalar scale)
real scalar easygoing(real matrix A, real matrix B)
real scalar easygoing(real matrix A)
real scalar easygoing()
```

If scale is not specified, results are as if scale $=1$ were specified. If $B$ is not specified, results are as if $B=A$ were specified. If $A$ is not specified, results are as if $A=\mathrm{I}(2)$ were specified.

## > Example 4

You write function midsection(). midsection() takes three arguments, but users may specify only two-the first and last-if they wish.
real matrix midsection (real matrix $A$, real vector $w$, real matrix $B$ )
real matrix midsection(real matrix $A$, real matrix $B$ )
If $w$ is not specified, results are as if $w=\mathrm{J}(1, \operatorname{cols}(A), 1)$ was specified.

## How to code optional arguments

When you code

```
function nebulous(a, b, c)
{
}
```

you are stating that function nebulous() requires three arguments. If the caller specifies fewer or more, execution will abort.

If you code

```
function nebulous( }a,b,|c
```

\{
\}
you are stating that the last argument is optional. Note the vertical or bar in front of $c$.
If you code

```
function nebulous(a, | b, c)
{
}
```

you are stating that the last two arguments are optional; the user may specify one, two, or three arguments.

If you code

```
function nebulous(|a, b, c)
{
}
```

you are stating that all arguments are optional; the user may specify zero, one, two, or three arguments.

The arguments that the user does not specify will be filled in according to the arguments' type,

| If the argument type is | The default value will be ... |
| :---: | :---: |
| undeclared | $\mathrm{J}(0,0,$. |
| transmorphic matrix | $J(0,0,$. |
| real matrix | $J(0,0,$. |
| complex matrix | $J(0,0,1 i)$ |
| string matrix | J(0, 0, "'") |
| pointer matrix | J (0, 0, NULL) |
| transmorphic rowvector | $J(1,0,$. |
| real rowvector | $\mathrm{J}(1,0,$. |
| complex rowvector | $J(1,0,1 i)$ |
| string rowvector | J(1, 0, "") |
| pointer rowvector | $J(1,0, N U L L)$ |
| transmorphic colvector | $\mathrm{J}(0,1,$. |
| real colvector | $\mathrm{J}(0,1,$. |
| complex colvector | $\mathrm{J}(0,1,1 i)$ |
| string colvector | J(0, 1, "") |
| pointer colvector | J (0, 1, NULL) |
| transmorphic vector | $J(1,0,$. |
| real vector | $J(1,0,$. |
| complex vector | $J(1,0,1 i)$ |
| string vector | J(1, 0, "") |
| pointer vector | J (1, 0, NULL) |
| transmorphic scalar | $\mathrm{J}(1,1,$. |
| real scalar | $\mathrm{J}(1,1,$. |
| complex scalar | $\mathrm{J}(1,1, \mathrm{C}()$. |
| string scalar | J(1, 1, "") |
| pointer scalar | J (1, 1, NULL) |

Also, the function $\operatorname{args}()$ (see $[\mathrm{M}-5] \operatorname{args}())$ will return the number of arguments that the user specified.

The vertical bar can be specified only once. That is sufficient, as we will show.

## Examples revisited

## $>$ Example 1

In this example, real matrix function $\operatorname{ditty}(A, B$, scale $)$ allowed real scalar scale to be optional. If scale was not specified, results were as if scale $=$. had been specified. This can be coded

```
real matrix ditty(real matrix A, real matrix B, |real scalar scale)
{
}
```

The body of the code is written just as if scale were not optional because, if the caller does not specify the argument, the missing argument is automatically filled in with missing, per the table above.

## > Example 2

Real matrix function gash $(A, B)$ allowed real matrix $B$ to be optional, and if not specified, $B=$ $\mathrm{J}(0,0,$.$) was assumed. Hence, this is coded just as example 1$ was coded:

```
real matrix gash(real matrix A, |real matrix B)
{
}
```

$>$ Example 3
Real scalar function easygoing ( $A, B$, scale) allowed all arguments to be optional. scale $=1$ was assumed, $B=A$, and if necessary, $A=\mathrm{I}(2)$.

```
real scalar easygoing(|real matrix A, real matrix B,
                                    real scalar scale)
{
    if (args()==2) scale = 1
        else if (args==1) {
            B = A
            scale = 1
        }
        else if (args()==0) {
            A = B = I(2)
            scale = 1 ;
    }
}
```


## $>$ Example 4

Real matrix function midsection $(A, w, B)$ allowed $w$-its middle argument-to be omitted. If $w$ was not specified, $J(1, \operatorname{cols}(A), 1)$ was assumed. Here is one solution:

```
real matrix midsection(a1, a2, |a3)
{
    if (args()==3) return(midsection_u(a1, a2, a3))
    else return(midsection_u(a1, J(1,cols(a1),1), a2))
}
real matrix midsection_u(real matrix A, real vector w, real matrix B)
{
}
```

We will never tell callers about the existence of midsection_u() even though midsection_u() is our real program.

What we did above was write midsection() to take two or three arguments, and then we called midsection_u() with the arguments in the correct position.

## Also see

[M-2] Intro - Language definition

## Title

[M-2] pointers — Pointers

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| References | Also see |  |

## Description

Pointers are objects that contain the addresses of other objects. The $*$ prefix operator obtains the contents of an address. Thus if $p$ is a pointer, $* p$ refers to the contents of the object to which $p$ points. Pointers are an advanced programming concept. Most programs, including involved and complicated ones, can be written without them.

In Mata, pointers are commonly used to

1. put a collection of objects under one name and
2. pass functions to functions.

One need not understand everything about pointers merely to pass functions to functions; see [M-2] ftof.
For a more detailed description of pointers, see Gould (2018).

## Syntax

$$
\text { pointer }[(\text { totype })][\text { orgtype }][\text { function }] \ldots
$$

where totype is
[eltype] [orgtype] [function]
and where eltype and orgtype are

| eltype |  | orgtype |
| :--- | :--- | :--- |
| transmorphic |  | matrix |
| numeric |  | vector |
| real |  | rowvector |
| complex |  | colvector |
| string |  | scalar |
| pointer[(towhat) $]$ |  |  |

pointer $[$ (totype $)][$ orgtype $]$ can be used in front of declarations, be they function declarations, argument declarations, or variable definitions.

## Remarks and examples

Remarks are presented under the following headings:

What is a pointer?<br>Pointers to variables<br>Pointers to expressions<br>Pointers to functions<br>Pointers to pointers<br>Pointer arrays<br>Mixed pointer arrays<br>Definition of NULL<br>Use of parentheses<br>Pointer arithmetic<br>Listing pointers<br>Declaration of pointers<br>Use of pointers to collect objects<br>Efficiency

## What is a pointer?

A pointer is the address of a variable or a function. Say that variable $X$ contains a matrix. Another variable $p$ might contain $137,799,016$, and if $137,799,016$ were the address at which $X$ were stored, then $p$ would be said to point to $X$. Addresses are seldom written in base 10 , so rather than saying $p$ contains $137,799,016$, we would be more likely to say that $p$ contains 0 x 836 a 568 , which is the way we write numbers in base 16. Regardless of how we write addresses, however, $p$ contains a number and that number corresponds to the address of another variable.

In our program, if we refer to $p$, we are referring to $p$ 's contents, the number $0 \times 836 \mathrm{a} 568$. The monadic operator $*$ is defined as "refer to the contents of the address" or "dereference": *p means $X$. We could code $\mathrm{Y}=* \mathrm{p}$ or $\mathrm{Y}=\mathrm{X}$, and either way, we would obtain the same result. In our program, we could refer to $X[i, j]$ or ( $* p$ ) $[i, j]$, and either way, we would obtain the $i, j$ element of $X$.

The monadic operator \& is how we put addresses into $p$. To load $p$ with the address of $X$, we code $p=\& X$.

The special address 0 (zero, written in hexadecimal as $0 \times 0$ ), also known as NULL, is how we record that a pointer variable points to nothing. A pointer variable contains NULL or it contains the address of another variable.

Or it contains the address of a function. Say that $p$ contains $0 \times 836 \mathrm{a} 568$ and that $0 \times 836 \mathrm{a} 568$, rather than being the address of matrix $X$, is the address of function $f()$. To get the address of $f()$ into $p$, just as we coded $p=\& X$ previously, we code $p=\& f()$. The () at the end tells $\&$ that we want the address of a function. We code () on the end regardless of the number of arguments $f()$ requires because we are not executing $f()$, we are just obtaining its address.

To execute the function at $0 x 836 \mathrm{a} 568$-now we will assume that $f()$ takes two arguments and call them $i$ and $j$-we code $(* p)(i, j)$ just as we coded ( $* p$ ) $[i, j]$ when $p$ contained the address of matrix $X$.

## Pointers to variables

To create a pointer $p$ to a variable, you code

$$
p=\text { \&varname }
$$

For instance, if $X$ is a matrix,

$$
p=\& X
$$

stores in $p$ the address of $X$. Subsequently, referring to $* p$ and referring to $X$ amount to the same thing. That is, if $X$ contained a $3 \times 3$ identity matrix and you coded

$$
* p=\text { Hilbert (4) }
$$

then after that you would find that $X$ contained the $4 \times 4$ Hilbert matrix. $X$ and $* p$ are the same matrix.

If $X$ contained a $3 \times 3$ identity matrix and you coded

$$
(* p)[2,3]=4
$$

you would then find $X[2,3]$ equal to 4 .
You cannot, however, point to the interior of objects. That is, you cannot code

$$
p=\& X[2,3]
$$

and get a pointer that is equivalent to $X[2,3]$ in the sense that if you later coded $* p=2$, you would see the change reflected in $X[2,3]$. The statement $p=\& X[2,3]$ is valid, but what it does, we will explain in Pointers to expressions below.

By the way, variables can be sustained by being pointed to. Consider the program

```
pointer(real matrix) scalar example(real scalar n)
{
    real matrix tmp
        tmp = I(3)
        return(&tmp)
}
```

Ordinarily, variable tmp would be destroyed when example() concluded execution. Here, however, tmp's existence will be sustained because of the pointer to it. We might code

$$
p=\text { example(3) }
$$

and the result will be to create $* p$ containing the $3 \times 3$ identity matrix. The memory consumed by that matrix will be freed when it is no longer being pointed to, which will occur when $p$ itself is freed, or, before that, when the value of $p$ is changed, perhaps by

$$
p=\text { NULL }
$$

For a discussion of pointers to structures, see [M-2] struct. For a discussion of pointers to classes, see [M-2] class.

## Pointers to expressions

You can code

$$
p=\&(2+3)
$$

and the result will be to create $* p$ containing 5 . Mata creates a temporary variable to contain the evaluation of the expression and sets $p$ to the address of the temporary variable. That temporary variable will be freed when $p$ is freed or, before that, when the value of $p$ is changed, just as tmp was freed in the example in the previous section.

When you code

$$
p=\& X[2,3]
$$

the result is the same. The expression is evaluated and the result of the expression stored in a temporary variable. That is why subsequently coding $* p=2$ does not change $X[2,3]$. All $* p=2$ does is change the value of the temporary variable.

Setting pointers equal to the value of expressions can be useful. In the following code fragment, we create $n 5 \times 5$ matrices for later use:

```
pvec = J(1, n, NULL)
for (i=1; i<=n; i++) pvec[i] = &(J(5, 5, .))
```


## Pointers to functions

When you code

$$
p=\& f u n c t i o n n a m e()
$$

the address of the function is stored in $p$. You can later execute the function by coding

$$
\ldots(* p)(\ldots)
$$

Distinguish carefully between

$$
p=\& f u n c t i o n n a m e()
$$

and

$$
p=\&(\text { functionname }())
$$

The latter would execute functionname () with no arguments and then assign the returned result to a temporary variable.

For instance, assume that you wish to write a function neat () that will calculate the derivative of another function, which function you will pass to neat(). Your function, we will pretend, returns a real scalar.

You could do that as follows

```
real scalar neat(pointer(function) p, other args...)
{
}
```

although you could be more explicit as to the characteristics of the function you are passed:

```
real scalar neat(pointer(real scalar function) p, other args...)
{
}
```

In any case, inside the body of your function, where you want to call the passed function, you code (*p) (arguments)

For instance, you might code

```
approx = ( (*p)(x+delta)-(*p)(x) ) / delta
```

The caller of your neat() function, wanting to use it with, say, function zeta_i_just_wrote(), would code

```
result = neat(&zeta_i_just_wrote(), other args...)
```

For an example of the use of pointers in sophisticated numerical calculations, see Støvring (2007).

## Pointers to pointers

Pointers to pointers (to pointers ...) are allowed, for instance, if $X$ is a matrix,

$$
\begin{aligned}
& p 1=\& X \\
& p 2=\& p 1
\end{aligned}
$$

Here $* p 2$ is equivalent to $p 1$, and $* * p 2$ is equivalent to $X$.
Similarly, we can construct a pointer to a pointer to a function:

$$
\begin{aligned}
& q 1=\& f() \\
& q 2=\& q 1
\end{aligned}
$$

Here $* q 2$ is equivalent to $q 1$, and $* * q 2$ is equivalent to $f()$.
When constructing pointers to pointers, never type \&\&-such as \&\&x-to obtain the address of the address of $x$. Type $\&(\& x)$ or $\& \& x$. \&\& is a synonym for $\&$, included for those used to coding in C.

## Pointer arrays

You may create an array of pointers, such as

$$
P=(\& X 1, \& X 2, \& X 3)
$$

or

$$
Q=(\& f l(), \& f 2(), \& f 3())
$$

Here $* P$ [2] is equivalent to $X 2$ and $* Q$ [2] is equivalent to $f 2()$.

## Mixed pointer arrays

You may create mixed pointer arrays, such as

$$
R=(\& X, \& f())
$$

Here $* R[2]$ is equivalent to $f()$.
You may not, however, create arrays of pointers mixed with real, complex, or string elements. Mata will abort with a type-mismatch error.

## Definition of NULL

NULL is the special pointer value that means "points to nothing" or undefined. NULL is like 0 in many ways-for instance, coding if ( X ) is equivalent to coding if ( $\mathrm{X}!=$ NULL), but NULL is distinct from zero in other ways. 0 is a numeric value; NULL is a pointer value.

## Use of parentheses

Use parentheses to make your meaning clear.
In the table below, we assume

$$
\begin{aligned}
& p=\& X \\
& P=(\& X 11, \& X 12 \backslash \& X 21, \& X 22) \\
& q=\& f() \\
& Q=(\& f 11(), \& f 12() \backslash \& f 21(), \& f 22())
\end{aligned}
$$

where $X, X 11, X 12, X 21$, and $X 22$ are matrices and $f(), f 11(), f 12(), f 21()$, and $f 22()$ are functions.

| Expression | Meaning |
| :--- | :--- |
| $* p$ | $X$ |
| $* p[1,1]$ | $X$ |
| $(* p)[1,1]$ | $X[1,1]$ |
| $* P[1,2]$ | $X 12$ |
| $(* P[1,2])[3,4]$ | $X 12[3,4]$ |
| $* q(\mathrm{a}, \mathrm{b})$ | execute function $q()$ of $\mathrm{a}, \mathrm{b} ;$ |
|  | $\quad$ dereference that |
| $(* q)(\mathrm{a}, \mathrm{b})$ | $f(a, b)$ |
| $(* q[1,1])(\mathrm{a}, \mathrm{b})$ | $f(a, b)$ |
| $* Q[1,2](\mathrm{a}, \mathrm{b})$ | nonsense |
| $(* Q[1,2])(\mathrm{a}, \mathrm{b})$ | $f 12(a, b)$ |

## Pointer arithmetic

Arithmetic with pointers (which is to say, with addresses) is not allowed:

```
: y = 2
: x = &y
: x+2
r(3012);
```

    <stmt>: 3012 undefined operation on pointer (e.g., p1>p2)
    Do not confuse the expression $\mathrm{x}+2$ with the expression $* \mathrm{x}+2$, which is allowed and in fact evaluates to 4 .

You may use the equality and inequality operators $==$ and $!=$ with pairs of pointer values:

```
if (p1 == p2) {
    }
    if (p1 != p2) {
    }
```

Also pointer values may be assigned and compared with the value NULL, which is much like, but still different from, zero: NULL is a $1 \times 1$ scalar containing an address value of 0 . An unassigned pointer has the value NULL, and you may assign the value NULL to pointers:

$$
\mathrm{p}=\mathrm{NULL}
$$

Pointer values may be compared with NULL,

```
if (p1 == NULL) {
}
if (p1 != NULL) {
}
```

but if you attempt to dereference a NULL pointer, you will get an error:

```
: x = NULL
: *x + 2
    <stmt>: 3010 attempt to dereference NULL pointer
r(3010);
```

Concerning logical expressions, you may directly examine pointer values:

```
if (p1) {
}
```

The above is interpreted as if if ( $\mathrm{p} 1!=\mathrm{NULL}$ ) were coded.

## Listing pointers

You may list pointers:

```
y = 2
x = &y
x
0x8359e80
```

What is shown, $0 \times 8359 \mathrm{e} 80$, is the memory address of y during our Stata session. If you typed the above lines, the address you would see could differ, but that does not matter.

Listing the value of pointers often helps in debugging because, by comparing addresses, you can determine where pointers are pointing and whether some are pointing to the same thing.

In listings, NULL is presented as 0 x 0 .

## Declaration of pointers

Declaration of pointers, as with all declarations (see [M-2] Declarations), is optional. That basic syntax is

```
pointer[(totype)] orgtype [function]...
```

For instance,

```
pointer(real matrix) scalar pl
```

declares that $p l$ is a pointer scalar and that it points to a real matrix, and

```
pointer(complex colvector) rowvector p2
```

declares that $p 2$ is a rowvector of pointers and that each pointer points to a complex colvector, and pointer(real scalar function) scalar p3
declares that $p 3$ is a pointer scalar, and that it points to a function that returns a real scalar, and

```
pointer(pointer(real matrix function) rowvector) colvector p4
```

declares that $p 4$ is a column vector of pointers to pointers, the pointers to which each element points are rowvectors, and each of those elements points to a function returning a real matrix.

You can omit the pieces you wish.

```
pointer() scalar p5
```

declares that $p 5$ is a pointer scalar-to what being uncertain.

```
pointer scalar p5
```

means the same thing.
pointer $p 6$
declares that $p 6$ is a pointer, but whether it is a matrix, vector, or scalar, is unsaid.

## Use of pointers to collect objects

Assume that you wish to write a function in two parts: result_setup() and repeated_result ().
In the first part, result_setup(), you will be passed a matrix and a function by the user, and you will make a private calculation that you will use later, each time repeated_result() is called. When repeated_result () is called, you will need to know the matrix, the function, and the value of the private calculation that you previously made.

One solution is to adopt the following design. You request the user code

```
resultinfo = result_setup(setup args...)
```

on the first call, and

```
value \(=\) repeated_result(resultinfo, other args...)
```

on subsequent calls. The design is that you will pass the information between the two functions in resultinfo. Here resultinfo will need to contain three things: the original matrix, the original function, and the private result you calculated. The user, however, never need know the details, and you will simply request that the user declare resultinfo as a pointer vector.

Filling in the details, you code

```
pointer vector result_setup(real matrix X, pointer(function) f)
{
    real matrix privmat
    pointer vector info
    privmat = ...
    info = (&X, f, &privmat)
    return(info)
}
real matrix repeated_result(pointer vector info, ...)
{
    pointer(function) scalar f
    pointer(matrix) scalar X
    pointer(matrix) scalar privmat
    f = info[2]
    X = info[1]
    privmat = info[3]
    ... (*f)(...) ...
    ... (*X) ...
    ... (*privmat)...
}
```

It was not necessary to unload info [] into the individual scalars. The lines using the passed values could just as well have read

```
... (*info[2])(...) ...
... (*info[1]) ...
... (*info[3]) ...
```


## Efficiency

When calling subroutines, it is better to pass the evaluation of pointer scalar arguments rather than the pointer scalar itself, because then the subroutine can run a little faster. Say that $p$ points to a real matrix. It is better to code

```
... mysub(*p) ...
```

rather than

```
... mysub(p)...
```

and then to write mysub() as

```
    function mysub(real matrix X)
    {
        ... X ...
    }
```

rather than

```
function mysub(pointer(real matrix) scalar p)
{
    ... (*р)...
}
```

Dereferencing a pointer (obtaining $* p$ from $p$ ) does not take long, but it does take time. Passing $* p$ rather than $p$ can be important if mysub() loops and performs the evaluation of $* p$ hundreds of thousands or millions of times.

## Diagnostics

The prefix operator * (called the dereferencing operator) aborts with error if it is applied to a nonpointer object.

Arithmetic may not be performed on undereferenced pointers. Arithmetic operators abort with error. The prefix operator \& aborts with error if it is applied to a built-in function.

## References

Gould, W. W. 2011. Advanced Mata: Pointers. The Stata Blog: Not Elsewhere Classified. http://blog.stata.com/2011/09/11/advanced-mata-pointers/.
__. 2018. The Mata Book: A Book for Serious Programmers and Those Who Want to Be. College Station, TX: Stata Press.

Støvring, H. 2007. A generic function evaluator implemented in Mata. Stata Journal 7: 542-555.

## Also see

[M-2] Intro - Language definition

## Title

[M-2] pragma - Suppressing warning messages

Description Syntax Remarks and examples Also see

## Description

pragma informs the compiler of your intentions so that the compiler can avoid presenting misleading warning messages and so that the compiler can better optimize the code.

## Syntax

pragma unset varname
pragma unused varname

## Remarks and examples

Remarks are presented under the following headings:

```
pragma unset
```

pragma unused

## pragma unset

The pragma
pragma unset $X$
suppresses the warning message
note: variable $X$ may be used before set.
The pragma has no effect on the resulting compiled code.
In general, the warning message flags logical errors in your program, such as

```
: function problem(real matrix a, real scalar j)
> {
> real scalar i
>
> j = i
> ...
> }
note: variable i may be used before set.
```

Sometimes, however, the message is misleading:

```
function notaproblem(real matrix a, real scalar j)
> {
> real matrix V
>
> st_view(V, ...)
> ...
> }
note: variable V may be used before set.
```

In the above, function st_view() (see [M-5] st_view()) defines V, but the compiler does not know that.

The warning message causes no problem but, if you wish to suppress it, change the code to read

```
function notaproblem(real matrix a, real scalar j)
{
    real matrix V
    pragma unset V
    st_view(V, ...)
    ...
}
```

pragma unset V states that you know V is unset and that, for warning messages, the compiler should act as if V were set at this point in your code.

## pragma unused

The pragma
pragma unused $X$
suppresses the warning messages
note: argument $X$ unused.
note: variable $X$ unused.
note: variable $X$ set but not used.
The pragma has no effect on the resulting compiled code.
Intentionally unused variables most often arise with respect to function arguments. You code

```
: function resolve(A, B, C)
> {
> ...
> }
note: argument C unused.
```

and you know well that you are not using C. You include the unnecessary argument because you are attempting to fit into a standard or you know that, later, you may wish to change the function to include $C$. To suppress the warning message, change the code to read

```
: function resolve(A, B, C)
> {
> ...
> pragma unused C
> ...
> }
```

The pragma states that you know $C$ is unused and, for the purposes of warning messages, the compiler should act as if $C$ were used at this point in your code.

Unused variables can also arise, and in general, they should simply be removed,

```
: function resin(X, Y)
> {
> real scalar i
> ...
> ... code in which i never appears
> ...
> }
note: variable i unused.
```

Rather than using the pragma to suppress the message, you should remove the line real scalar i.
Warnings are also given for variables that are set and not used:

```
: function thwart(X, Y)
> {
> real scalar i
> ...
> i = 1
> ...
> ... code in which i never appears
> ...
> }
note: variable i set but unused.
```

Here you should remove both the real scalar $i$ and $i=1$ lines.
It is possible, however, that the set-but-unused variable was intentional:

```
: function thwart(X, Y)
> {
> real scalar i
> ...
> i = somefunction(...)
> ...
> ... code in which i never appears
> ...
> }
note: variable i set but not used.
```

You assigned the value of somefunction() to i to prevent the result from being displayed. Here you could use pragma unused i to suppress the warning message, but a better alternative would be

```
: function thwart(X, Y)
> {
> ...
> (void) somefunction(...)
> ...
> }
```

See Assignment suppresses display, as does (void) in [M-2] exp.

## Also see

[M-2] Intro - Language definition

## Title

[M-2] reswords - Reserved words
Description Syntax Remarks and examples Also see

## Description

Reserved words are words reserved by the Mata compiler; they may not be used to name either variables or functions.

## Syntax

Reserved words are

| aggregate | float | pointer | union |
| :---: | :---: | :---: | :---: |
| array | for | polymorphic | unsigned |
|  | friend | pragma | using |
| boolean | function | private |  |
| break |  | protected | vector |
| byte | global | public | version |
|  | goto |  | virtual |
| case |  | quad | void |
| catch | if |  | volatile |
| class | inline | real |  |
| colvector | int | return | while |
| complex |  | rowvector |  |
| const | local |  |  |
| continue | long | scalar |  |
|  |  | short |  |
| default | mata | signed |  |
| delegate | matrix | static |  |
| delete |  | string |  |
| do | namespace | strL |  |
| double | new | struct |  |
|  | NULL | super |  |
| else | numeric | switch |  |
| eltypedef |  |  |  |
| end | operator | template |  |
| enum | orgtypedef | this |  |
| explicit |  | throw |  |
| export |  | transmorphic |  |
| external |  | try |  |
|  |  | typedef |  |
|  |  | typename |  |

## Remarks and examples

Remarks are presented under the following headings:
Future developments
Version control

## Future developments

Many of the words above are reserved for the future implementation of new features. For instance, the words aggregate, array, boolean, byte, etc., currently play no role in Mata, but they are reserved.

You cannot infer much about short-run development plans from the presence or absence of a word from the list. The list was constructed by including words that would be needed to add certain features, but we have erred on the side of reserving too many words because it is better to give back a word than to take it away later. Taking away a word can cause previously written code to break.

Also, features can be added without reserving words if the word will be used only within a specific context. Our original list was much longer, but then we struck from it such context-specific words.

## Version control

Even if we should need to reserve new words in the future, you can ensure that you need not modify your programs by engaging in version control; see [M-2] version.

## Also see

[M-1] Naming - Advice on naming functions and variables
[M-2] version - Version control
[M-2] Intro - Language definition

## Title

[ $\mathrm{M}-2$ ] return — return and return(exp)

## Description Syntax Remarks and examples Also see

## Description

return causes the function to stop execution and return to the caller, returning nothing.
return (exp) causes the function to stop execution and return to the caller, returning the evaluation of exp.

## Syntax

```
return
return(exp)
```


## Remarks and examples

Remarks are presented under the following headings:
Functions that return results
Functions that return nothing (void functions)

## Functions that return results

return (exp) specifies the value to be returned. For instance, you have written a program to return the sum of two numbers:

```
function mysum(a, b)
{
        return(a+b)
}
```

return (exp) may appear multiple times in the program. The following program calculates $x$ factorial; it assumes $x$ is an integer greater than 0 :

```
real scalar myfactorial(real scalar x)
{
    if (x<=0) return(1)
    return(x*factorial(x-1))
}
```

If $x \leq 0$, the function returns 1 ; execution does not continue to the next line.
Functions that return a result always include one or more return (exp) statements.

## Functions that return nothing (void functions)

A function is said to be void if it returns nothing. The following program changes the diagonal of a matrix to be 1 :

```
function fixdiag(matrix A)
{
    real scalar i
    for (i=1; i<=rows(A); i++) A[i,i] = 1
}
```

This function does not even include a return statement; execution just ends. That is fine, although the function could just as well read

```
function fixdiag(matrix A)
{
    real scalar i
    for (i=1; i<=rows(A); i++) A[i,i] = 1
    return
}
```

The use of return is when the function has reason to end early:

```
void fixmatrix(matrix A, scalar how)
{
    real scalar i, j
    for (i=1; i<=rows(A); i++) A[i,i] = 1
    if (how==0) return
    for (i=1; i<=rows(A); i++) {
        for (j=1; j<i; j++) A[i,j] = 0
    }
}
```


## Also see

[M-5] exit() - Terminate execution
[M-2] Intro - Language definition

## Title

[M-2] Semicolons - Use of semicolons
Description Syntax Remarks and examples Also see

## Description

Mata allows, but does not require, semicolons.
Use of semicolons is discussed below, along with advice on the possible interactions of Stata's \#delimit instruction; see [P] \#delimit.

## Syntax

## stmt

```
stmt ;
```


## Remarks and examples

Remarks are presented under the following headings:
Optional use of semicolons
You cannot break a statement anywhere even if you use semicolons
Use of semicolons to create multistatement lines
Significant semicolons
Do not use \#delimit ;

## Optional use of semicolons

You can code your program to look like this

```
real scalar foo(real matrix A)
{
    real scalar i, sum
    sum = 0
    for (i=1; i<=rows(A); i++) {
        sum = sum + A[i,i]
    }
    return(sum)
}
```

or you can code your program to look like this:

```
real scalar foo(real matrix A)
{
        real scalar i, sum ;
        sum = 0 ;
        for (i=1; i<=rows(A); i++) {
            sum = sum + A[i,i] ;
    }
    return(sum) ;
}
```

That is, you may omit or include semicolons at the end of statements. It makes no difference. You can even mix the two styles:

```
real scalar foo(real matrix A)
{
    real scalar i, sum ;
    sum = 0 ;
    for (i=1; i<=rows(A); i++) {
        sum = sum + A[i,i]
    }
    return(sum)
}
```


## You cannot break a statement anywhere even if you use semicolons

Most languages that use semicolons follow the rule that a statement continues up to the semicolon.
Mata follows a different rule: a statement continues across lines until it looks to be complete, and semicolons force the end of statements.

For instance, consider the statement $\mathrm{x}=\mathrm{b}-\mathrm{c}$ appearing in some program. In the code, might appear

$$
\begin{aligned}
& \mathrm{x}=\mathrm{b}- \\
& \mathrm{c}
\end{aligned}
$$

or

$$
\begin{aligned}
& x=b- \\
& c ;
\end{aligned}
$$

and, either way, Mata will understand the statement to be $\mathrm{x}=\mathrm{b}-\mathrm{c}$, because the statement could not possibly end at the minus: $\mathrm{x}=\mathrm{b}-$ makes no sense.

On the other hand,

$$
\begin{aligned}
& x=b \\
& -c
\end{aligned}
$$

would be interpreted by Mata as two statements: $\mathrm{x}=\mathrm{b}$ and -c because $\mathrm{x}=\mathrm{b}$ looks like a completed statement to Mata. The first statement will assign b to $x$, and the second statement will display the negative value of $c$.

Adding a semicolon will not help:

$$
\begin{aligned}
& x=b \\
& -c ;
\end{aligned}
$$

$\mathrm{x}=\mathrm{b}$ is still, by itself, a complete statement. All that has changed is that the second statement ends in a semicolon, and that does not matter.

Thus remember always to break multiline statements at places where the statement could not possibly be interpreted as being complete, such as

$$
\begin{aligned}
\mathrm{x}=\mathrm{b}- & \\
& \\
& \\
& +\mathrm{e})
\end{aligned}
$$

myfunction(A,

$$
B, C,
$$

)
Do this whether or not you use semicolons.

## Use of semicolons to create multistatement lines

Semicolons allow you to put more than one statement on a line. Rather than coding

$$
\begin{aligned}
& \mathrm{a}=2 \\
& \mathrm{~b}=3
\end{aligned}
$$

you can code

$$
\mathrm{a}=2 ; \quad \mathrm{b}=3 ;
$$

and you can even omit the trailing semicolon:

$$
a=2 ; \quad b=3
$$

Whether you code separate statements on separate lines or the same line is just a matter of style; it does not change the meaning. Coding

```
for (i=1; i<n; i++) a[i] = -a[i] ; sum = sum + a[i] ;
```

still means

```
for (i=1; i<n; i++) a[i] = -a[i] ;
sum = sum + a[i] ;
```

and, without doubt, the programmer intended to code

```
for (i=1; i<n; i++) {
    a[i] = -a[i] ;
    sum = sum + a[i] ;
}
```

which has a different meaning.

## Significant semicolons

Semicolons are not all style. The syntax for the for statement is (see [M-2] for)

$$
\text { for }\left(\exp _{1} ; \exp _{2} ; \exp _{3}\right) \text { stmt }
$$

Say that the complete for loop that we want to code is

```
for (x=init(); !converged(x); iterate(x))
```

and that there is no stmt following it. Then we must code

```
for (x=init(); !converged(x); iterate(x)) ;
```

Here we use the semicolon to force the end of the statement. Say we omitted it, and the code read

```
for (x=init(); !converged(x); iterate(x))
x = -x
```

The for statement would look incomplete to Mata, so it would interpret our code as if we had coded

```
for (x=init(); !converged(x); iterate(x)) {
    x = -x
}
```

Here the semicolon is significant.
Significant semicolons only happen following for and while.

## Do not use \#delimit ;

What follows has to do with Stata's \#delimit ; mode. If you do not know what it is or if you never use it, you can skip what follows.

Stata has an optional ability to allow its lines to continue up to semicolons. In Stata, you code

```
. #delimit ;
```

and the delimiter is changed to semicolon until your do-file or ado-file ends, or until you code

```
. #delimit cr
```

We recommend that you do not use Mata when Stata is in \#delimit ; mode. Mata will not mind, but you will confuse yourself.

When Mata gets control, if \#delimit ; is on, Mata turns it off temporarily, and then Mata applies its own rules, which we have summarized above.

## Also see

[M-2] Intro - Language definition
[P] \#delimit - Change delimiter

## Title

[M-2] struct - Structures
Description Syntax Remarks and examples References Also see

## Description

A structure contains a set of variables tied together under one name. For a more detailed description of structures and how they are used, see Gould (2018).

## Syntax

```
struct structname {
    declaration(s)
}
```

such as

```
struct mystruct {
        real scalar n1, n2
        real matrix X
}
```


## Remarks and examples

Remarks are presented under the following headings:

[^4]
## Introduction

Here is an overview of the use of structures:

```
struct mystruct {
    real scalar n1, n2
    real matrix X
}
function myfunc()
{
        struct mystruct scalar e
    e.n1 = ...
    e.n2 = ...
    e.X = ...
    ... mysubroutine(e, ...)
}
function mysubroutine(struct mystruct scalar x, ...)
{
    struct mystruct scalar y
    ... x.n1 ... x.n2 ... x.X ...
    y = mysubfcn(x)
    ... y.n1 ... y.n2 ... y.X ...
    ... x.n1 ... x.n2 ... x.X ...
}
struct mystruct scalar mysubfcn(struct mystruct scalar x)
{
    struct mystruct scalar result
    result = x
    ... result.n1 ... result.n2 ... result.X ...
    return(result)
}
```

Note the following:

1. We first defined the structure. Definition does not create variables; definition defines what we mean when we refer to a struct mystruct in the future. This definition is done outside and separately from the definition of the functions that will use it. The structure is defined before the functions.
2. In myfunc(), we declared that variable e is a struct mystruct scalar. We then used variables e.n1, e.n2, and e.X just as we would use any other variable. In the call to mysubroutine(), however, we passed the entire e structure.
3. In mysubroutine(), we declared that the first argument received is a struct mystruct scalar. We chose to call it x rather than e to emphasize that names are not important. y is also a struct mystruct scalar.
4. mysubfen() not only accepts a struct mystruct as an argument but also returns a struct mystruct. One of the best uses of structures is as a way to return multiple, related values.

The line result=x copied all the values in the structure; we did not need to code result.n1=x.n1, result.n2=x.n2, and result. $\mathrm{X}=\mathrm{x} . \mathrm{X}$.

## Structures and functions must have different names

You define structures much as you define functions, at the colon prompt, with the definition enclosed in braces:

```
struct twopart {
    real scalar n1, n2
}
function setuphistory()
{
}
```

Structures and functions may not have the same names. If you call a structure twopart, then you cannot have a function named twopart (), and vice versa.

## Structure variables must be explicitly declared

Declarations are usually optional in Mata. You can code

```
real matrix swaprows(real matrix A, real scalar i1, real scalar i2)
{
        real matrix B
        real rowvector v
        B = A
        v = B[i1, .]
        B[i1, .] = B[i2, .]
        B[i2, .] = v
        return(B)
}
```

or you can code

```
function swaprows(A, i1, i2)
{
        B = A
        v = B[i1, .]
        B[i1, .] = B[i2, .]
        B[i2, .] = v
        return(B)
}
```

When a variable, argument, or returned value is a structure, however, you must explicitly declare it:

```
function makecalc()
{
    struct twopart scalar t
    t.n1 = t.n2 = 0
}
function clear_twopart(struct twopart scalar t)
{
    t.n1 = t.n2 = 0
}
struct twopart scalar new_twopart()
{
    struct twopart scalar t
    t.n1 = t.n2 = 0
    return(t)
}
```

In the functions above, we refer to variables t.n1 and t.n2. The Mata compiler cannot interpret those names unless it knows the definition of the structure.

Aside: All structure references are resolved at compile time, not run time. That is, terms like t.n1 are not stored in the compiled code and resolved during execution. Instead, the Mata compiler accesses the structure definition when it compiles your code. The compiler knows that t.n1 refers to the first element of the structure and generates efficient code to access it.

## Declare structure variables to be scalars whenever possible

In our declarations, we code things like

```
struct twopart scalar t
```

and do not simply code
struct twopart t
although the simpler statement would be valid.
Structure variables can be scalars, vectors, or matrices; when you do not say which, matrix is assumed.
Most uses of structures are as scalars, and the compiler will generate more efficient code if you tell it that the structures are scalars. Also, when you use structure vectors or matrices, there is an extra step you need to fill in, as described in the next section.

## Vectors and matrices of structures

Just as you can have real scalars, vectors, or matrices, you can have structure scalars, vectors, or matrices. The following are all valid:

```
struct twopart scalar t
struct twopart vector t
struct twopart rowvector t
struct twopart colvector t
struct twopart matrix t
```

In a struct twopart matrix, every element of the matrix is a separate structure. Say that the matrix were $2 \times 3$. Then you could refer to any of the following variables,
$\mathrm{t}[1,1] . \mathrm{n} 1$
$\mathrm{t}[1,2] . \mathrm{n} 1$
$\mathrm{t}[1,3] . \mathrm{n} 1$
$\mathrm{t}[2,1] . \mathrm{n} 1$
$\mathrm{t}[2,2] . \mathrm{n} 1$
$\mathrm{t}[2,3] . \mathrm{n} 1$
and similarly for $\mathrm{t}[i, j] . \mathrm{n} 2$.
If struct twopart also contained a matrix X , then
$\mathrm{t}[i, j] . \mathrm{X}$
would refer to the $(i, j)$ th matrix.

$$
\mathrm{t}[i, j] . \mathrm{X}[k, l]
$$

would refer to the $(k, l)$ th element of the $(i, j)$ th matrix.
If $t$ is to be a $2 \times 3$ matrix, you must arrange to make it $2 \times 3$. After the declaration
struct twopart matrix $t$
t is $0 \times 0$. This result is no different from the situation where t is a real matrix and after declaration, t is $0 \times 0$.

Whether $t$ is a real matrix or a struct twopart matrix, you allocate $t$ by assignment. Let's pretend that $t$ is a real matrix. There are three solutions to the allocation problem:
(1) $t=x$
(2) $t=$ somefunction(...)
(3) $\mathrm{t}=\mathrm{J}(r, c, v)$

All three are so natural that you do not even think of them as allocation; you think of them as definition.

The situation with structures is the same.
Let's take each in turn.

1. x contains a $2 \times 3$ struct twopart. You code

$$
t=x
$$

and now t contains a copy of x . t is $2 \times 3$.
2. somefunction(...) returns a $2 \times 3$ struct twopart. You code

$$
\mathrm{t}=\text { somefunction }(\ldots)
$$

and now $t$ contains the $2 \times 3$ result.
3. Mata function $\mathrm{J}(r, c, v)$ returns an $r \times c$ matrix, every element of which is set to $v$. So pretend that variable tpc contains a struct twopart scalar. You code

$$
\mathrm{t}=\mathrm{J}(2,3, \mathrm{tpc})
$$

and now $t$ is $2 \times 3$, every element of which is a copy of tpc. Here is how you might do that:

```
function ...(...)
{
    struct twopart scalar tpc
    struct twopart matrix t
    t = J (2, 3, tpc)
}
```

Finally, there is a fourth way to create structure vectors and matrices. When you defined

```
struct twopart {
    real scalar n1, n2
}
```

Mata not only recorded the structure definition but also created a function named twopart () that returns a struct twopart. Thus, rather than enduring all the rigmarole of creating a matrix from a preallocated scalar, you could simply code

$$
\mathrm{t}=\mathrm{J}(2,3, \text { twopart }())
$$

In fact, the function twopart () that Mata creates for you allows zero, one, or two arguments:

```
twopart() returns a 1 < 1 struct twopart
twopart(r) returns an r}\times1\mathrm{ struct twopart
twopart(r,c) returns an r}\timesc\mathrm{ struct twopart
```

so you could code

$$
\mathrm{t}=\mathrm{twopart}(2,3)
$$

or you could code

```
t = J(2, 3, twopart())
```

and whichever you code makes no difference.
Either way, what is in $t$ ? Each element contains a separate struct twopart. In each struct twopart, the scalars have been set to missing (., " ", or NULL, as appropriate), the vectors and row vectors have been made $1 \times 0$, the column vectors $0 \times 1$, and the matrices $0 \times 0$.

## Structures of structures

Structures may contain other structures:

```
struct twopart {
    real scalar n1, n2
}
struct pair_of_twoparts {
    struct twopart scalar t1, t2
}
```

If $t$ were a struct pair_of_twoparts scalar, then the members of $t$ would be

```
t.t1 a struct twopart scalar
t.t2 a struct twopart scalar
t.t1.n1 a real scalar
t.t1.n2 a real scalar
t.t2.n1 a real scalar
t.t2.n2 a real scalar
```

You may also create structures of structures of structures, structures of structures of structures of structures, and so on. You may not, however, include a structure in itself:

```
struct recursive {
    struct recursive scalar r
}
```

Do you see how, even in the scalar case, struct recursive would require an infinite amount of memory to store?

## Pointers to structures

What you can do is this:

```
struct recursive {
    pointer(struct recursive scalar) scalar r
}
```

Thus, if $r$ were a struct recursive scalar, then $* r$.r would be the next structure, or $r . r$ would be NULL if there were no next structure. Immediately after allocation, r.r would equal NULL.

In this way, you may create linked lists.
Mata provides operator $\rightarrow$ for accessing members of pointers to structures.
Let rec be a struct recursive, and assume that struct recursive also had member real scalar $n$, so that rec.n would be rec's $n$ value. The value of the next structure's $n$ would be rec.r->n (assuming rec.r!=NULL).

The syntax of $->$ is

$$
\exp 1->\exp 2
$$

where $\exp 1$ evaluates to a structure pointer and $\exp 2$ indexes the structure.

## Operators and functions for use with structure members

All operators, all functions, and all features of Mata work with members of structures. That is, given

```
struct example {
    real scalar n
    real matrix X
}
function ...(...)
{
    real scalar rs
    real matrix rm
    struct example scalar ex
}
```

then ex.n and ex.X may be used anyplace rs and rm would be valid.

## Operators and functions for use with entire structures

Some operators and functions can be used with entire structures, not just the structure's elements. Given

```
struct mystruct scalar ex1, ex2, ex3, ex4
struct mystruct matrix E, F, G
```

1. You may use $==$ and $!=$ to test for equality:

$$
\begin{aligned}
& \text { if (ex1==ex2) ... } \\
& \text { if (ex1!=ex2) ... }
\end{aligned}
$$

Two structures are equal if their members are equal.
In the example, struct mystruct itself contains no substructures. If it did, the definition of equality would include checking the equality of substructures, sub-substructures, etc.

In the example, ex1 and ex2 are scalars. If they were matrices, each element would be compared, and the matrices would be equal if the corresponding elements were equal.
2. You may use $:==$ and $:!=$ to form pattern matrices of equality and inequality.
3. You may use the comma and backslash operators to form vectors and matrices of structures:

$$
e x=e x 1, \text { ex2 } \backslash \text { ex3, ex4 }
$$

4. You may use \& to obtain pointers to structures:
```
ptr_to_ex1 = &ex1
```

5. You may use subscripting to access and copy structure members:

$$
\begin{aligned}
& \operatorname{ex} 1=\mathrm{E}[1,2] \\
& \mathrm{E}[1,2]=\mathrm{ex} 1 \\
& \mathrm{~F}=\mathrm{E}[2, .] \\
& \mathrm{E}[2, .]=\mathrm{F} \\
& \mathrm{G}=\mathrm{E}[|1,1 \backslash 2,2|] \\
& \mathrm{E}[|1,1 \backslash 2,2|]=\mathrm{G}
\end{aligned}
$$

6. You may use the rows () and cols() functions to obtain the number of rows and columns of a matrix of structures.
7. You may use eltype() and orgtype() with structures. eltype() returns struct; orgtype() returns the usual results.
8. You may use most functions that start with the letters is, as in isreal(), iscomplex(), isstring(), etc. These functions return 1 if true and 0 if false and with structures, usually return 0 .
9. You may use $\operatorname{swap}()$ with structures.

## Listing structures

To list the contents of a structure variable, as for debugging purposes, use function liststruct(); see [M-5] liststruct().

Using the default, unassigned-expression method to list structures is not recommended, because all that is shown is a pointer value instead of the structure itself.

## Use of transmorphics as passthrus

A transmorphic matrix can theoretically hold anything, so when we told you that structures had to be explicitly declared, that was not exactly right. Say that function twopart () returns a struct twopart scalar. You could code

```
x = twopart()
```

without declaring x (or declaring it transmorphic), and that would not be an error. What you could not do would be to then refer to $\mathrm{x} . \mathrm{n} 1$ or $\mathrm{x} . \mathrm{n} 2$, because the compiler would not know that x contains a struct twopart and so would have no way of interpreting the variable references.

This property can be put to good use in implementing handles and passthrus.
Say that you are implementing a complicated system. Just to fix ideas, we'll pretend that the system finds the maximum of user-specified functions and that the system has many bells and whistles. To track a given problem, let's assume that your system needs many variables. One variable records the method to be used. Another records whether numerical derivatives are to be used. Another records the current gradient vector. Another records the iteration count, and so on. There might be hundreds of these variables.

You bind all of these variables together in one structure:

```
struct maxvariables {
    real scalar method
    real scalar use_numeric_d
    real vector gradient
    real scalar iteration
}
```

You design a system with many functions, and some functions call others, but because all the status variables are bound together in one structure, it is easy to pass the values from one function to another.

You also design a system that is easy to use. It starts by the user "opening" a problem,

```
handle = maximize_open()
```

and from that point on the user passes the handle around to the other maximize routines:

```
maximize_set_use_numeric_d(handle, 1)
maximize_set_function_to_max(handle, &myfunc())
maximize_maximize_my_function(handle)
```

In this way, you, the programmer of this system, can hold on to values from one call to the next, and you can change the values, too.

What you never do, however, is tell the user that the handle is a struct maxvariables. You just tell the user to open a problem by typing

```
handle = maximize_open()
```

and then to pass the handle returned to the other maximize_*() routines. If the user feels that he must explicitly declare the handle, you tell him to declare it:

```
transmorphic scalar handle
```

What is the advantage of this secrecy? You can be certain that the user never changes any of the values in your struct maxvariables because the compiler does not even know what they are.

Thus you have made your system more robust to user errors.

## Saving compiled structure definitions

You save compiled structure definitions just as you save compiled function definitions; see [M-3] mata mosave and [M-3] mata mlib.

When you define a structure, such as twopart,

```
struct twopart {
    real scalar n1, n2
}
```

that also creates a function, twopart (), that creates instances of the structure.
Saving twopart () in a .mo file, or in a .mlib library, saves the compiled definition as well. Once twopart() has been saved, you may write future programs without bothering to define struct twopart. The definition will be automatically found.

## Saving structure variables

Variables containing structures may be saved on disk just as you would save any other variable. No special action is required. See [M-3] mata matsave and see the function fputmatrix() in [M-5] fopen( ). mata matsave and fputmatrix() both work with structure variables, although their entries do not mention them.

## References

Gould, W. W. 2007. Mata Matters: Structures. Stata Journal 7: 556-570.
_-. 2018. The Mata Book: A Book for Serious Programmers and Those Who Want to Be. College Station, TX: Stata Press.

## Also see

[M-2] Declarations - Declarations and types
[M-2] Intro - Language definition

## Title

[M-2] Subscripts - Use of subscripts

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Reference | Also see |  |

## Description

Subscripts come in two styles.
In [subscript] syntax-called list subscripts-an element or a matrix is specified:

$$
\begin{array}{ll}
\mathrm{x}[1,2] & \text { the } 1,2 \text { element of } x ; \text { a scalar } \\
\mathrm{x}[(1 \backslash 3 \backslash 2),(4,5)] & \text { the } 3 \times 2 \text { matrix composed of rows } 1,3, \text { and } 2 \\
& \text { and columns } 4 \text { and } 5 \text { of } x:
\end{array}
$$

$$
\left[\begin{array}{ll}
x_{14} & x_{15} \\
x_{34} & x_{35} \\
x_{24} & x_{25}
\end{array}\right]
$$

In [|subscript $\mid$ ] syntax—called range subscripts—an element or a contiguous submatrix is specified:

$$
\begin{array}{ll}
\mathrm{x}[|1,2|] & \text { same as } \mathrm{x}[1,2] ; \text { a scalar } \\
\mathrm{x}[|2,3 \backslash 4,7|] & 3 \times 4 \text { submatrix of } x: \\
& {\left[\begin{array}{lllll}
x_{23} & x_{24} & x_{25} & x_{26} & x_{27} \\
x_{33} & x_{34} & x_{35} & x_{36} & x_{37} \\
x_{43} & x_{44} & x_{45} & x_{46} & x_{47}
\end{array}\right]}
\end{array}
$$

Both style subscripts may be used in expressions and may be used on the left-hand side of the equal-assignment operator.

## Syntax

$x[$ real vector $r$, real vector $c]$
$x[\mid$ real matrix sub|]

Subscripts may be used on the left or right of the equal-assignment operator.

## Remarks and examples

Remarks are presented under the following headings:

List subscripts<br>Range subscripts<br>When to use list subscripts and when to use range subscripts<br>A fine distinction

## List subscripts

List subscripts-also known simply as subscripts-are obtained when you enclose the subscripts in square brackets, [ and ]. List subscripts come in two basic forms:

$$
\begin{array}{ll}
x[\text { ivec }, j v e c] & \text { matrix composed of rows } i v e c \text { and columns jvec of matrix } x \\
v[\text { kvec }] & \text { vector composed of elements } k v e c \text { of vector } v
\end{array}
$$

where ivec, jvec, kvec may be a vector or a scalar, so the two basic forms include

$$
\begin{array}{ll}
x[i, j] & \text { scalar } i, j \text { element } \\
x[i, j v e c] & \text { row vector of row } i, \text { elements } j v e c \\
x[i v e c, j] & \text { column vector of column } j, \text { elements ivec } \\
v[k] & \text { scalar } k \text { th element of vector } v
\end{array}
$$

Also missing value may be specified to mean all the rows or all the columns:

$$
\begin{array}{ll}
x[i, .] & \text { row vector of row } i \text { of } x \\
x[., j] & \text { column vector of column } j \text { of } x \\
x[\text { ivec }, .] & \text { matrix of rows ivec, all columns } \\
x[., j v e c] & \text { matrix of columns jvec, all rows } \\
x[., .] & \text { the entire matrix }
\end{array}
$$

Finally, Mata assumes missing value when you omit the argument entirely:

| $x[i]$, | same as $x[i,]$. |
| :--- | :--- |
| $x[$ ivec,$]$ | same as $x[$ ivec , .] |
| $x[, j]$ | same as $x[., j]$ |
| $x[, j v e c]$ | same as $x[., j v e c]$ |
| $x[]$, | same as $x[.,]$. |

Good style is to specify ivec as a column vector and jvec as a row vector, but that is not required:

```
x[(1\2\3), (1,2,3)] good style
x[(1,2,3),(1,2,3)] same as x[(1\2\3),(1,2,3)]
x[(1\2\3),(1\2\3)] same as x[(1\2\3), (1,2,3)]
x[(1,2,3),(1\2\3)] same as x[(1\2\3),(1,2,3)]
```

Similarly, good style is to specify kvec as a column when $v$ is a column vector and to specify kvec as a row when $v$ is a row vector, but that is not required and what is returned is a column vector if $v$ is a column and a row vector if $v$ is a row:

```
rowv}[(1,2,3)
rowv[(1\2\3)]
colv[(1\2\3)]
colv[(1,2,3)]
good style for specifying row vector
same as \(\operatorname{rowv}[(1,2,3)]\)
good style for specifying column vector
same as \(\operatorname{colv}[(1 \backslash 2 \backslash 3)]\)
```

Subscripts may be used in expressions following a variable name:

```
first = list[1]
multiplier = x[3,4]
result = colsum(x[,j])
```

Subscripts may be used following an expression to extract a submatrix from a result:

```
allneeded = invsym(x)[(1::4), .] * multiplier
```

Subscripts may be used on the left-hand side of the equal-assignment operator:

```
x[1,1] = 1
x[1,.] = y[3,.]
x[(1::4), (1..4)] = I(4)
```


## Range subscripts

Range subscripts appear inside the difficult to type [| and |] brackets. Range subscripts come in four basic forms:

$$
\begin{array}{ll}
x[|i, j|] & i, j \text { element; same result as } x[i, j] \\
v[|k|] & k \text { th element of vector; same result as } v[k] \\
x[|i, j \backslash k, l|] & \begin{array}{l}
\text { submatrix, vector, or scalar formed using }(i, j) \text { as top-left } \\
\text { corner and }(k, l) \text { as bottom-right corner }
\end{array} \\
v[|i \backslash k|] & \begin{array}{l}
\text { subvector or scalar of elements } i \text { through } k \text {; result is } \\
\text { row vector if } v \text { is row vector, column vector if } v \text { is column } \\
\text { vector }
\end{array}
\end{array}
$$

Missing value may be specified for a row or column to mean all rows or all columns when a $1 \times 2$ or $1 \times 1$ subscript is specified:

```
x[| i, .|]
x[|.,j|] column j of x; same as x[.,j]
x[l.,.|] entire matrix; same as x[.,.]
v[|.|] entire vector; same as v[.]
```

Also missing may be specified to mean the number of rows or the number of columns of the matrix being subscripted when a $2 \times 2$ subscript is specified:

```
x[|1,2\4,.|] equivalent to }x[|1,2\4,\operatorname{cols}(x)|
x[|1,2\.,3|] equivalent to x[|1,2\ rows (x),3|]
x[|1,2\.,.|] equivalent to x[|1,2\ rows (x),\operatorname{cols}(x|]
```

With range subscripts, what appears inside the square brackets is in all cases interpreted as a matrix expression, so in

```
sub = (1,2)
... x[lsubl] ...
```

x [sub] refers to $\mathrm{x}[1,2]$.

Range subscripts may be used in all the same contexts as list subscripts; they may be used in expressions following a variable name

$$
\text { submat }=\operatorname{result}[|1,1 \backslash 3,3|]
$$

they may be used to extract a submatrix from a calculated result

$$
\text { allneeded }=\operatorname{invsym}(x)[|1,1 \backslash 4,4|]
$$

and they may be used on the left-hand side of the equal-assignment operator:

$$
\mathrm{x}[|1,1 \backslash 4,4|]=I(4)
$$

## When to use list subscripts and when to use range subscripts

Everything a range subscript can do, a list subscript can also do. The one seemingly unique feature of a range subscript,

$$
x[|i 1, j 1 \backslash i 2, j 2|]
$$

is perfectly mimicked by

$$
x[(i 1:: i 2),(j 1 \ldots j 2)]
$$

The range-subscript construction, however, executes more quickly, and so that is the purpose of range subscripts: to provide a fast way to extract contiguous submatrices. In all other cases, use list subscripts because they are faster.

Use list subscripts to refer to scalar values:

$$
\begin{aligned}
& \text { result }=x[1,3] \\
& x[1,3]=2
\end{aligned}
$$

Use list subscripts to extract entire rows or columns:

```
obs = x[., 3]
var = x[4, .]
```

Use list subscripts to permute the rows and columns of matrices:

```
: x = (1,2,3,4 \ 5,6,7,8\ \,10,11,12)
y = x[(1\3\2), .]
y
\begin{tabular}{l|rrrr|}
\multicolumn{1}{c}{1} & 2 & 3 & \multicolumn{1}{c}{4} \\
\cline { 2 - 5 } 1 & 1 & 2 & 3 & 4 \\
2 & 9 & 10 & 11 & 12 \\
3 & 5 & 6 & 7 & 8 \\
\cline { 2 - 5 } & & &
\end{tabular}
: y = x[., (1,3,2,4)]
y
\begin{tabular}{l|rrrr|}
\multicolumn{1}{c}{1} & 2 & 3 & \multicolumn{1}{c}{4} \\
\cline { 2 - 5 } 1 & 1 & 3 & 2 & 4 \\
2 & 5 & 7 & 6 & 8 \\
9 & 11 & 10 & 12 \\
\cline { 2 - 5 } & & & &
\end{tabular}
: y=x[(1\3\2), (1,3,2,4)]
```

| y |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: |
| 1 | 2 | 3 | 4 |  |
| 1 | 1 | 3 | 2 | 4 |
| 2 | 9 | 11 | 10 | 12 |
| 3 | 5 | 7 | 6 | 8 |
|  |  |  |  |  |

Use list subscripts to duplicate rows or columns:

```
: x = (1,2,3,4 \ 5,6,7,8 \ 9,10,11,12)
: y = x[(1\2\3\1), .]
y
\begin{tabular}{|rrrr|}
\multicolumn{1}{|c}{1} & 2 & 3 & 4 \\
\hline 1 & 2 & 3 & 4 \\
5 & 6 & 7 & 8 \\
9 & 10 & 11 & 12 \\
1 & 2 & 3 & 4 \\
\hline
\end{tabular}
y = x[., (1,2,3,4,2)]
y
\begin{tabular}{l|rrrrr|}
\multicolumn{1}{c}{} & 1 & 2 & 3 & 4 & \multicolumn{1}{c}{5} \\
\cline { 2 - 6 } 1 & 1 & 2 & 3 & 4 & 2 \\
2 & 5 & 6 & 7 & 8 & 6 \\
3 & 9 & 10 & 11 & 12 & 10 \\
\hline
\end{tabular}
y = x[(1\2\3\1), (1,2,3,4,2)]
y
\begin{tabular}{l|rrrrr|}
\multicolumn{1}{c}{} & 1 & 2 & 3 & 4 & 5 \\
\cline { 2 - 6 } 1 & 1 & 2 & 3 & 4 & 2 \\
2 & 5 & 6 & 7 & 8 & 6 \\
3 & 9 & 10 & 11 & 12 & 10 \\
4 & 1 & 2 & 3 & 4 & 2 \\
\cline { 2 - 6 } & & &
\end{tabular}
```


## A fine distinction

There is a fine distinction between $x[i, j]$ and $x[|i, j|]$. In $x[i, j]$, there are two arguments, $i$ and $j$. The comma separates the arguments. In $x[|i, j|]$, there is one argument: $i, j$. The comma is the column-join operator.

In Mata, comma means mostly the column-join operator:

```
newvec = oldvec, addedvalues
qsum = (x,1)'(x,1)
```

There are, in fact, only two exceptions. When you type the arguments for a function, the comma separates one argument from the next:

```
result = f(a,b,c)
```

In the above example, $f()$ receives three arguments: $a, b$, and $c$. If we wanted $f()$ to receive one argument, ( $a, b, c$ ), we would have to enclose the calculation in parentheses:

```
result = f((a,b,c))
```

That is the first exception. When you type the arguments inside a function, comma means argument separation. You get back to the usual meaning of comma-the column-join operator-by opening another set of parentheses.

The second exception is in list subscripting:

$$
x[i, j]
$$

Inside the list-subscript brackets, comma means argument separation. That is why you have seen us type vectors inside parentheses:

$$
x[(1 \backslash 2 \backslash 3),(1,2,3)]
$$

These are the two exceptions. Range subscripting is not an exception. Thus in

$$
x[|i, j|]
$$

there is one argument, $i, j$. With range subscripts, you may program constructs such as

```
IJ = (i,j)
RANGE = (1,2 \ 4,4)
... x[|IJ|] ... x[|RANGE|] ...
```

You may not code in this way with list subscripts. In particular, $x[I J]$ would be interpreted as a request to extract elements $i$ and $j$ from vector $x$, and would be an error otherwise, $x$ [RANGE] would always be an error.

We said earlier that list subscripts $x[i, j]$ are a little faster than range subscripts $x[|i, j|]$. That is true, but if $\mathrm{IJ}=(i, j)$ already, $x[|I J|]$ is faster than $x[i, j]$. You would, however, have to execute many millions of references to $x[|I J|]$ before you could measure the difference.

## Conformability

$x[i, j]:$

| $x:$ | $r \times c$ |  |  |  |
| ---: | :--- | :--- | :--- | :--- |
| $i:$ | $m \times 1$ | or | $1 \times m$ | (does not matter which) |
| $j:$ | $1 \times n$ | or | $n \times 1$ | (does not matter which) |
| result: | $m \times n$ |  |  |  |

$x[i,]:$.
$x: \quad r \times c$
$i$ : $\quad m \times 1$ or $1 \times m \quad$ (does not matter which)
result: $\quad m \times c$
$x[., j]:$

| $x:$ | $r \times c$ |  |  |
| ---: | :--- | :--- | :--- |
| $j:$ | $1 \times n$ | or | $n \times 1 \quad$ (does not matter which) |
| result: | $r \times n$ |  |  |

$x[.,]:$.

```
        x: }\quadr\times
    result: }r\times
```

$x[i]:$

| $x:$ | $n \times 1$ |  | $1 \times n$ |
| ---: | :--- | :--- | :--- | :--- | :--- |
| $i:$ | $m \times 1$ |  |  | or $1 \times m \quad 1 \times m \quad$ or $\quad m \times 1$

$x[]:$.

| $x:$ | $n \times 1$ | $1 \times n$ |
| ---: | :--- | :--- |
| result: | $n \times 1$ | $1 \times n$ |

$x[|k|]:$

| $x:$ | $r \times c$ |  |
| ---: | :--- | :--- |
| $k:$ | $1 \times 2$ |  |
| result: | $1 \times 1$ | if $k[1]<$. |
|  | $r \times 1$ | if $k[1]>=$. and $k[2]<$. |
|  | $1 \times c$ | if $k[1]<$. | and $k[2]>=$.

$x[|k|]:$
$x: \quad r \times c$
$k: \quad 2 \times 2$
result: $\quad k[2,1]-k[1,1]+1 \times k[2,2]-k[1,2]+1$
(in the above formula, if $k[2,1]>=$., treat as if $k[2,1]=r$, and similarly, $\quad$ if $k[2,2]>=$. , treat as if $k[2,2]=c$ )
$x[|k|]:$

| $x:$ | $r \times 1$ | $1 \times c$ |
| ---: | :--- | :--- |
| $k:$ | $2 \times 1$ | $2 \times 1$ |
| result: | $k[2]-k[1]+1 \times 1$ | $1 \times k[2]-k[1]+1$ |
|  | (if $k[2]>=.$, treat as | (if $k[2]>=.$, treat as |
|  | if $k[2]=r)$ | if $k[2]=c$ ) |

## Diagnostics

Both styles of subscripts abort with error if the subscript is out of range, if a reference is made to a nonexisting row or column.

## Reference

Gould, W. W. 2007. Mata Matters: Subscripting. Stata Journal 7: 106-116.

## Also see

[M-2] Intro - Language definition

## Title

[M-2] Syntax - Mata language grammar and syntax
Description Syntax Remarks and examples Reference Also see

## Description

Mata is a C-like compiled-into-pseudocode language with matrix extensions and run-time linking.

## Syntax

The basic language syntax is
istmt
where

```
    istmt := stmt
        function name(farglist) fstmt
            ftype name(farglist) fstmt
            ftype function name(farglist) fstmt
    stmt := nothing
            ;
        (meaning nothing)
            version number
            { stmt ...}
            exp
            pragma pstmt
            if (exp) stmt
            if (exp) stmt else stmt
            for (exp;exp;exp) stmt
            while (exp) stmt
            do stmt while (exp)
            break
            continue
            label:
            goto label
            return
            return(exp)
fstmt := stmt
            type arglist
            external type arglist
arglist := name
            name()
            name, arglist
            name(), arglist
farglist := nothing
            efarglist
```

```
efarglist := felement
    felement, efarglist
    | felement
    | felement, efarglist
felement := name
    type name
    name()
    type name()
    ftype := type
        void
        type := eltype
        orgtype
        eltype orgtype
    eltype := transmorphic
        string
        numeric
        real
        complex
        pointer
        pointer(ptrtype)
orgtype := matrix
        vector
        rowvector
        colvector
        scalar
ptrtype := nothing
        type
        type function
        function
    pstmt := unset name
        unused name
    name := identifier up to 32 characters long
    label := identifier up to 8 characters long
        exp := expression as defined in [M-2] exp
```


## Remarks and examples

Remarks are presented under the following headings:

```
Treatment of semicolons
Types and declarations
Void matrices
Void functions
Operators
Subscripts
Implied input tokens
Function argument-passing convention
Passing functions to functions
Optional arguments
```

After reading [M-2] Syntax, see [M-2] Intro for a list of entries that give more explanation of what is discussed here.

## Treatment of semicolons

Semicolon (;) is treated as a line separator. It is not required, but it may be used to place two statements on the same physical line:

$$
\mathrm{x}=1 ; \mathrm{y}=2 \text {; }
$$

The last semicolon in the above example is unnecessary but allowed.
Single statements may continue onto more than one line if the continuation is obvious. Take "obvious" to mean that there is a hanging open parenthesis or a hanging dyadic operator; for example,

```
x = (
            3)
x = x +
```

2
See [M-2] Semicolons for more information.

## Types and declarations

The type of a variable or function is described by

```
eltype orgtype
```

where eltype and orgtype are each one of

| eltype | orgtype |  |
| :--- | :--- | :--- |
| transmorphic |  | matrix |
| numeric |  | vector |
| real | rowvector |  |
| complex | colvector |  |
| string |  | scalar |
| pointer |  |  |

For example, a variable might be real scalar, or complex matrix, or string vector.
Mata also has structures-the eltype is struct name-but these are not discussed here. For a discussion of structures, see [M-2] struct.

Mata also has classes-the eltype is class name-but these are not discussed here. For a discussion of classes, see [M-2] class.

Declarations are optional. When the type of a variable or function is not declared, it is assumed to be a transmorphic matrix. In particular:

1. eltype specifies the type of the elements. When eltype is not specified, transmorphic is assumed.
2. orgtype specifies the organization of the elements. When orgtype is not specified, matrix is assumed.

All types are special cases of transmorphic matrix.
The nesting of eltypes is

orgtypes amount to nothing more than a constraint on the number of rows and columns of a matrix:

| orgtype | Constraint |
| :--- | :--- |
| matrix | $r \geq 0$ and $c \geq 0$ |
| vector | $r=1$ and $c \geq 0$ or $r \geq 0$ and $c=1$ |
| rowvector | $r=1$ and $c \geq 0$ |
| colvector | $r \geq 0$ and $c=1$ |
| scalar | $r=1$ and $c=1$ |

See [M-2] Declarations.

## Void matrices

A matrix (vector, row vector, or column vector) that is $0 \times 0, r \times 0$, or $0 \times c$ is said to be void; see [M-2] void.

The function $\mathrm{J}(r, c, v a l)$ returns an $r \times c$ matrix with each element containing val; see [M-5] $\mathbf{J}()$. J() can be used to create void matrices.

See [M-2] void.

## Void functions

Rather than eltype orgtype, a function can be declared to return nothing by being declared to return void:

```
void function example(matrix A)
{
        real scalar i
        for (i=1; i<=rows(A); i++) A[i,i] = 1
}
```

A function that returns nothing (does not include a return (exp) statement), in fact returns $\mathrm{J}(0,0,$.$) ,$ and the above function could equally well be coded as

```
void function example(matrix A)
{
        real scalar i
        for (i=1; i<=rows(A); i++) A[i,i] = 1
        return(J(0, 0, .))
}
```

or

```
void function example(matrix A)
{
    real scalar i
    for (i=1; i<=rows(A); i++) A[i,i] = 1
    return(J(0,0,.))
}
```

Therefore, void also is a special case of transmorphic matrix (it is in fact a $0 \times 0$ real matrix). Because declarations are optional (but recommended both for reasons of style and for reasons of efficiency), the above function could also be coded as

```
function example(A)
{
    for (i=1; i<=rows(A); i++) A[i,i] = 1
}
```

See [M-2] Declarations.

## Operators

Mata provides the usual assortment of operators; see [M-2] exp.
The monadic prefix operators are

Prefix operators \& and $*$ have to do with pointers; see [M-2] pointers.

The monadic postfix operators are

Note the inclusion of postfix operator ' for transposition. Also, for $Z$ complex, $Z$ ' returns the conjugate transpose. If you want the transposition without conjugation, see [M-5] transposeonly().

The dyadic operators are

$$
\begin{array}{llllllllll}
= & ? & : & , & \cdots & \mid & \& & == & >= & <= \\
!= & - & * & \# & - &
\end{array}
$$

Also, \&\& and || are included as synonyms for \& and |.
The operators $==$ and $!=$ do not require conformability, nor do they require that the matrices be of the same type. In such cases, the matrices are unequal ( $==$ is false and $!=$ is true). For complex arguments, $\langle,\langle=$,$\rangle , and \rangle=$ refer to length of the complex vector. $==$ and $!=$, however, refer not to length but to actual components. See [M-2] op_logical.

The operators , and $\backslash$ are the row-join and column-join operators. $(1,2,3)$ constructs the row vector $(1,2,3)$. ( $1 \backslash 2 \backslash 3$ ) constructs the column vector $(1,2,3)^{\prime}$. $(1,2 \backslash 3,4)$ constructs the matrix with first row ( 1,2 ) and second row (3,4). $a, b$ joins two scalars, vectors, or matrices rowwise. $a \backslash b$ joins two scalars, vectors, or matrices columnwise. See [M-2] op_join.
. . and : : refer to the row-to and column-to operators. $1 . .5$ is $(1,2,3,4,5) .1:: 5$ is $(1 \backslash 2 \backslash 3 \backslash 4 \backslash 5)$. $5 . .1$ is $(5,4,3,2,1) .5:: 1$ is $(5 \backslash 4 \backslash 3 \backslash 2 \backslash 1)$. See [M-2] op_range.

For $\mid, \&,==,>=,<=,<,>,!=,+,-, *, /$, and ${ }^{\wedge}$, there is :op at precedence just below $o p$. These operators perform the elementwise operation. For instance, $A * B$ refers to matrix multiplication; $A: * B$ refers to elementwise multiplication. Moreover, elementwise is generalized to cases where $A$ and $B$ do not have the same number of rows and the same number of columns. For instance, if $A$ is a $1 \times$ $c$ row vector and $B$ is a $r \times c$ matrix, then $\left\|C_{i j}\right\|=\left\|A_{j}\right\| *\left\|B_{i j}\right\|$ is returned. See [M-2] op_colon.

## Subscripts

$A[i, j]$ returns the $i, j$ element of $A$.
$A[k]$ returns $A[1, k]$ if $A$ is $1 \times c$ and $A[k, 1]$ if $A$ is $r \times 1$. That is, in addition to declared vectors, any $1 \times c$ matrix or $r \times 1$ matrix may be subscripted by one index. Similarly, any vector can be subscripted by two indices.
$i, j$, and $k$ may be vectors as well as scalars. For instance, $A[(3 \backslash 4 \backslash 5), 4]$ returns a $3 \times 1$ column vector containing rows 3 to 5 of the 4th column.
$i, j$, and $k$ may be missing value. $A[., 4]$ returns a column vector of the 4 th column of $A$.
The above subscripts are called list-style subscripts. Mata provides a second format called range-style subscripts that is especially useful for selecting submatrices. $A[|3,3 \backslash 5,5|]$ returns the $3 \times 3$ submatrix of $A$ starting at $A[3,3]$.

See [M-2] Subscripts.

## Implied input tokens

Before interpreting and compiling a line, Mata makes the following substitutions to what it sees:

| Input sequence | Interpretation |
| :--- | :--- |
| 'name | '*name |
| $[$, | $[.$, |
| ,$]$ | $,]$. |

Hence, coding $X^{\prime} Z$ is equivalent to coding $X^{\prime} * Z$, and coding $x=z[1$,$] is equivalent to coding x=$ $z[1,$.$] .$

## Function argument-passing convention

Arguments are passed to functions by address, also known as by name or by reference. They are not passed by value. When you code

$$
\ldots f(A) \ldots
$$

it is the address of A that is passed to $f()$, not a copy of the values in A. $f()$ can modify A.
Most functions do not modify their arguments, but some do. $\operatorname{lud}(A, L, U, p)$, for instance, calculates the LU decomposition of $A$. The function replaces the contents of $L, U$, and $p$ with matrices such that $L[p] * U=$,$A .$

Oldtimers will have heard of the FORTRAN programmer who called a subroutine and passed to it a second argument of 1 . Unbeknownst to him, the subroutine changed its second argument, with the result that the constant 1 was changed throughout the rest of his code. That cannot happen in Mata. When an expression is passed as an argument (and constants are expressions), a temporary variable containing the evaluation is passed to the function. Modifications to the temporary variable are irrelevant because the temporary variable is discarded once the function returns. Thus if $f()$ modifies its second argument and you call it by coding $f(A, 2)$, because 2 is copied to a temporary variable, the value of the literal 2 will remain unchanged on the next call.

If you call a function with an expression that includes the assignment operator, it is the left-hand side of the expression that is passed. That is, coding

$$
f(a, b=c)
$$

has the same result as coding

$$
\begin{aligned}
& b=c \\
& f(a, b)
\end{aligned}
$$

If function $f()$ changes its second argument, it will be $b$ and not $c$ that is modified.
Also, Mata attempts not to create unnecessary copies of matrices. For instance, consider

$$
\text { function changearg }(x) x[1,1]=1
$$

changearg (mymat) changes the 1,1 element of mymat to 1 . Now let us define

```
function cp(x) return(x)
```

Coding changearg(cp(mymat)) would still change mymat because cp () returned x itself. On the other hand, if we defined cp() as

```
function cp(x)
{
        matrix t
        t = x
        return(t)
}
```

then coding changearg (cp(mymat)) would not change mymat. It would change a temporary matrix which would be discarded once changearg() returned.

## Passing functions to functions

One function may receive another function as an argument using pointers. One codes

```
function myfunc(pointer(function) f, a, b)
{
    ... (*f)(a) ... (*f)(b) ...
}
```

although the pointer (function) declaration, like all declarations, is optional. To call myfunc() and tell it to use function prima() for $f()$, and 2 and 3 for a and b, one codes

```
myfunc(&prima(), 2, 3)
```

See [M-2] ftof and [M-2] pointers.

## Optional arguments

Functions may be coded to allow receiving a variable number of arguments. This is done by placing a vertical or bar (l) in front of the first argument that is optional. For instance,

```
function mynorm(matrix A, |scalar power)
{
}
```

The above function may be called with one matrix or with a matrix followed by a scalar.
The function $\operatorname{args}()$ (see [M-5] $\operatorname{args}()$ ) can be used to determine the number of arguments received and to set defaults:

```
function mynorm(matrix A, |scalar power)
{
    (args()==1) power = 2
}
```

See [M-2] optargs.

## Reference

Gould, W. W. 2005. Mata Matters: Translating Fortran. Stata Journal 5: 421-441.

## Also see

[M-2] Intro - Language definition

## Title

[ M -2] version - Version control
Description Syntax Remarks and examples Also see

## Description

In syntax 1, Stata's version command (see [P] version) sets the version before entering Mata. This specifies both the compiler and library versions to be used. Syntax 1 is recommended.

In syntax 2, Mata's version command sets the version of the library functions that are to be used. Syntax 2 is rarely used.

## Syntax

Syntax 1

```
version #[.#]
mata:
...
function name(...)
{
}
...
end
```

Syntax 2

```
function name(...)
: {
: version #[.#]
}
```


## Remarks and examples

Remarks are presented under the following headings:

Purpose of version control Recommendations for do-files Recommendations for ado-files Compile-time and run-time versioning

## Purpose of version control

Mata is under continual development, which means not only that new features are being added but also that old features sometimes change how they work. Old features changing how they work is supposedly an improvement-it generally is-but that also means old programs might stop working or, worse, work differently.
version provides the solution.
If you are working interactively, nothing said here matters.
If you use Mata in do-files or ado-files, we recommend that you set version before entering Mata.

## Recommendations for do-files

The recommendation for do-files that use Mata is the same as for do-files that do not use Mata: specify the version number of the Stata you are using on the top line:


To determine the number that should appear after version, type about at the Stata prompt:

```
. about
Stata/SE 18.0
    (output omitted)
```

We are using Stata 18.0. If we were using StataNow, we would instead see StataNow/SE 18.5.
Coding version 18.0 will not benefit us today but, in the future, we will be able to rerun our do-file and obtain the same results.

By the way, a do-file is any file that you intend to execute using Stata's do or run commands (see $[\mathrm{R}]$ do), regardless of the file suffix. Many users (us included) save Mata source code in .mata files and then type do myfile.mata to compile. .mata files are do-files; we include the version line:

| version 18.0 | $/ /$ (or version 18.5 for StataNow) |
| :--- | :--- |
| mata: |  |
| end |  |

## Recommendations for ado-files

Mata functions may be included in ado-files; see [M-1] Ado. In such files, set version before entering Mata along with, as usual, setting the version at the top of your program:
$\qquad$

```
program myfile
    version 18.0}\leftarrow as usua
end
version 18.0 \leftarrow new
mata:
end
```

end myfile.ado
In StataNow, we would replace each version 18.0 statement with version 18.5.

## Compile-time and run-time versioning

What follows is detail. We recommend always following the recommendations above.
There are actually two version numbers that matter-the version number set at the time of compilation, which affects how the source code is interpreted, and the version of the libraries used to supply subroutines at the time of execution.

The version command that we used in the previous sections is in fact Stata's version command (see $[P]$ version), and it sets both versions:

```
version 18.0
mata:
function example()
{
}
end
```

In the above, we compile example() by using the version 18.0 syntax of the Mata language, and any functions example() calls will be the 18.0 version of those functions. Setting version 18.0 before entering Mata ensured all of that.

In the following example, we compile using version 18.0 syntax and use version 18.2 functions:

```
. version 18.0
mata:
function example()
{
    version 18.2
}
end
```

In the following example, we compile using version 18.2 syntax and use version 18.0 functions:

```
version 18.2
mata:
function example()
{
    version 18.0
}
end
```

It is, however, very rare that you will want to compile and execute at different version levels.

## Also see

[M-5] callersversion( ) - Obtain version number of caller
[M-2] Intro - Language definition

## Title

[M-2] void - Void matrices

Description Syntax Remarks and examples Also see

## Description

Mata allows $0 \times 0, r \times 0$, and $0 \times c$ matrices. These matrices are called void matrices.

## Syntax

| $\mathrm{J}(0,0,)$. | $0 \times 0$ real matrix |
| :---: | :--- |
| $\mathrm{J}(r, 0,)$. | $r \times 0$ real matrix |
| $\mathrm{J}(0, c,)$. | $0 \times c$ real matrix |
|  |  |
| $\mathrm{J}(0,0,1 \mathrm{i})$ | $0 \times 0$ complex matrix |
| $\mathrm{J}(r, 0,1 \mathrm{i})$ | $r \times 0$ complex matrix |
| $\mathrm{J}(0, c, 1 \mathrm{i})$ | $0 \times c$ complex matrix |
|  |  |
| $\mathrm{J}(0,0, " ")$ | $0 \times 0$ string matrix |
| $\mathrm{J}(r, 0, " ")$ | $r \times 0$ string matrix |
| $\mathrm{J}(0, c, " ")$ | $0 \times c$ string matrix |
| $\mathrm{J}(0,0, \mathrm{NULL})$ | $0 \times 0$ pointer matrix |
| $\mathrm{J}(r, 0, \mathrm{NULL})$ | $r \times 0$ pointer matrix |
| $\mathrm{J}(0, c, \mathrm{NULL})$ | $0 \times c$ pointer matrix |

## Remarks and examples

Remarks are presented under the following headings:
Void matrices, vectors, row vectors, and column vectors How to read conformability charts

## Void matrices, vectors, row vectors, and column vectors

Void matrices contain nothing, but they have dimension information (they are $0 \times 0, r \times 0$, or $0 \times$ c) and have an eltype (which is real, complex, string, or pointer):

1. A matrix is said to be void if it is $0 \times 0, r \times 0$, or $0 \times c$.
2. A vector is said to be void if it is $0 \times 1$ or $1 \times 0$.
3. A column vector is said to be void if it is $0 \times 1$.
4. A row vector is said to be void if it is $1 \times 0$.
5. A scalar cannot be void because it is, by definition, $1 \times 1$.

The function $\mathrm{J}(r, c, v a l)$ creates $r \times c$ matrices containing val; see [M-5] J() . J() can be used to manufacture void matrices by specifying $r$ and/or $c$ as 0 . The value of the third argument does not matter, but its eltype does:

1. $J(0,0,$.$) creates a real 0 \times 0$ matrix, as will $J(0,0,1)$ and as will $J()$ with any real third argument.
2. $\mathrm{J}(0,0,1 i)$ creates a $0 \times 0$ complex matrix, as will J() with any complex third argument.
3. $J(0,0, " ")$ creates $0 \times 0$ string matrices, as will $J()$ with any string third argument.
4. $\mathrm{J}(0,0, \mathrm{NULL})$ creates $0 \times 0$ pointer matrices, as will J() with any pointer third argument.

In fact, one rarely needs to manufacture such matrices because they arise naturally in extreme cases. Similarly, one rarely needs to include special code to handle void matrices because such matrices handle themselves. Loops vanish when the number of rows or columns are zero.

## How to read conformability charts

In general, no emphasis placed on how functions and operators deal with void matrices; moreover, no mention is even made of the fact. Instead, the information is buried in the Conformability section located near the end of the function's or operator's manual entry.

For instance, the conformability chart for some function might read
somefunction $(A, B, v)$ :

| $A:$ | $r \times c$ |  |  |
| ---: | :--- | :--- | :--- |
| $B:$ | $c \times k$ |  |  |
| $v:$ | $1 \times k$ | or | $k \times 1$ |
| result $:$ | $r \times k$ |  |  |

Among other things, the chart above is stating how somefunction() handles void matrices. A must be $r \times c$. That chart does not say

$$
A: \quad r \times c, r>0, c>0
$$

and that is what it would have said if somefunction() did not allow $A$ to be void. Hence, $A$ may be $0 \times 0,0 \times c$, or $r \times 0$.

Similarly, $B$ may be void as long as rows $(B)=\operatorname{cols}(A) . v$ may be void if $\operatorname{cols}(B)=0$. The returned result will be void if rows $(A)=0$ or $\operatorname{cols}(B)=0$.

Interestingly, somefunction() can produce a nonvoid result from void input. For instance, if $A$ were $5 \times 0$ and $B, 0 \times 3$, a $5 \times 3$ result would be produced. It is interesting to speculate what would be in that $5 \times 3$ result. Probably, if we knew what somefunction() did, it would be obvious to us, but if it were not, the following section, Diagnostics, would state what the surprising result would be.

As a real example, see [M-5] trace(). trace() will take the trace of a $0 \times 0$ matrix. The result is 0 . Or see multiplication ( $*$ ) in [M-2] op_arith. One can multiply a $k \times 0$ matrix by a $0 \times m$ matrix to produce a $k \times m$ result. The matrix will contain zeros.

[^5]
## Title

[M-2] while - while (exp) stmt

## Description Syntax Remarks and examples Also see

## Description

while executes stmt or stmts zero or more times. The loop continues as long as exp is not equal to zero.

## Syntax

```
while (exp) stmt
while (exp) {
    stmts
}
```

where $\exp$ must evaluate to a real scalar.

## Remarks and examples

To understand while, enter the following program

```
function example(n)
{
    i = 1
    while (i<=n) {
        printf("i=%g\n", i)
        i++
    }
    printf("done\n")
}
```

and run example(3), example(2), example(1), example(0), and example(-1).
One common use of while is to loop until convergence:

```
while (mreldif(a, lasta)>1e-10) {
    lasta = a
    a = ...
}
```


## Also see

[M-2] break - Break out of for, while, or do loop
[M-2] continue - Continue with next iteration of for, while, or do loop
[M-2] do - do ... while (exp)
[M-2] for - for $(\exp 1 ; \exp 2 ; \exp 3)$ stmt
[M-2] Semicolons - Use of semicolons
[M-2] Intro - Language definition
[M-3] Commands for controlling Mata

## Title

[M-3] Intro - Commands for controlling Mata
Contents Description Remarks and examples Also see

## Contents

Command for invoking Mata from Stata:

| $[$ M-3] Entry | Command | Description |
| :--- | :--- | :--- |
| mata | . mata | invoke Mata |

Once you are running Mata, you can use the following commands from the colon prompt:

| [M-3] Entry | Command | Description |
| :--- | :--- | :--- |
| mata help | : mata help | execute help command |
| mata clear | $:$ mata clear | clear Mata |
| mata describe | $:$ mata describe | describe contents of Mata's memory <br> mata memory |
| mata rename | $:$ mata memory | display memory-usage report |
| mata drop | : mata rename | rename matrix or function |
| remove from memory matrix or function |  |  |
| mata mosave | $:$ mata mosave | create object file |
| mata mlib | $:$ mata mlib | create function library |
| lmbuild | : lmbuild | easily create function library |
| mata matsave | $:$ mata matsave | save matrices |
| mata matsave | $:$ mata matuse | restore matrices |
| mata matsave | $:$ mata matdescribe | describe contents of matrix file |
| mata which | $:$ mata which | identify function |
| mata set | $:$ mata query | display values of settable parameters |
| mata set | $:$ mata set | set parameters |
| mata stata | $:$ mata stata | execute Stata command |
| end | $:$ end | exit Mata and return to Stata |

## Description

When you type something at the Mata prompt, it is assumed to be a Mata statement-something that can be compiled and executed-such as
$2+3$
5
The mata command, however, is different. When what you type is prefixed by the word mata, think of yourself as standing outside of Mata and giving an instruction that affects the Mata environment and the way Mata works. For instance, typing

```
: mata clear
```

says that Mata is to be cleared. Typing

```
: mata set matastrict on
```

says that Mata is to require that programs explicitly declare their arguments and their working variables; see [M-2] Declarations.

## Remarks and examples

The mata command cannot be used inside functions. It would make no sense to code

```
function foo(...)
{
        mata query
}
```

because mata query is something that can be typed only at the Mata colon prompt:

```
: mata query
        (output omitted)
```

See [M-1] How.

## Also see

[M-0] Intro - Introduction to the Mata manual

## Title

[M-3] end - Exit Mata and return to Stata

Description Syntax Remarks and examples Also see

## Description

end exits Mata and returns to Stata.

## Syntax

: end

## Remarks and examples

When you exit from Mata back into Stata, Mata does not clear itself; so if you later return to Mata, you will be right back where you were. See [M-3] mata.

## Also see

[M-3] Intro - Commands for controlling Mata

## Title

[M-3] Imbuild — Easily create function library
Description Syntax Options Remarks and examples Also see

## Description

lmbuild builds Mata function libraries just as mata mlib does. Even though lmbuild virtually requires the creation of a do-file, it is easier to use and is therefore a better alternative than mata mlib.

Why two commands to do the same thing? mata mlib existed first to create Mata libraries, but it was complicated to use. lmbuild was added later and makes it easier to create those libraries.

You type lmbuild after Stata's dot prompt, not Mata's colon prompt.

## Syntax

. Imbuild libname $[$, [new $\mid$ replace $\mid$ add $] \operatorname{dir}($ dirname) size(\#)]
libname is the name of a Mata library, such as lexample.mlib. Library names must start with 1 and end in .mlib. Library names may be specified with or without the extension.
lmbuild is a Stata command that you use from Stata's dot prompt, not from Mata's colon prompt.

## Options

new, replace, and add are alternatives and indicate whether the library file is new, should be replaced, or should be added to.
new is the default. It specifies that libname does not already exist and is to be created. replace specifies that libname might already exist, and if it does, the library is to be replaced. add specifies that libname already exists and functions are to be added to the existing library. We advise you not to use this option except in carefully constructed do-files that create the library from start to finish. If you choose to use add interactively, you run the risk of creating irreproducible libraries.
dir (dirname) specifies the directory in which libname exists or is to be created. dirname may be one of the following:
$\operatorname{dir}$ (PERSONAL) is the default. Libraries will be created or updated in your personal directory. You can type the sysdir command to find out where your personal directory is. Libraries in your personal directory will be automatically found by Mata. If you do not already have a personal directory, lmbuild will create one for you.
dir (SITE) specifies that the library be created or updated in the site directory. This directory is shared across Stata users at your location. You can type the sysdir command to find out where your site directory is. You will probably need administrator privileges to write to this directory.
$\operatorname{dir}($.$) specifies that the library exists or is to be created in the current directory. The only reasons$ to specify $\operatorname{dir}($.$) are that you intend to copy the library to another directory later, to send the$ library to someone, or to include the library in a package for distribution to other users.
dir (directory) specifies that the library exists or is to be created in directory. Specifying this option is not recommended because Mata will not find such libraries unless they are added to Mata's search path.
size(\#) specifies the maximum number of functions to be allowed in the library. Libraries allow up to 1,024 functions by default. \# may be a number from 2 to 2,048 . size() may be specified only with new libraries or libraries that are being replaced.

## Remarks and examples

Remarks are presented under the following headings:

Background<br>Version control

## Background

Mata functions that you write and then store in libraries are placed on the same footing as Mata's built-in functions. They can be used in code that you write without being preloaded, whether that code is in do-files, ado-files, or Mata.

You can have as many Mata libraries as you wish. Each library may contain up to 1,024 functions, or up to 2,048 if you specify lmbuild's size() option.

Libraries store the compiled version of functions, not the source code. We recommend that you place your source code in do-files that look like the following:

```
version #
mata:
void hello()
{
    printf("hello world\n")
}
end
```

end hello.mata
You can load the function into Mata by typing do hello.mata at the Stata prompt. You can test the function. When you want to place the function in a library, you type

```
. clear all
. do hello.mata
lmbuild lmylib
```

lmbuild lmylib creates a Mata library named lmylib.mlib containing all the Mata functions loaded into memory since the last clear all. Thus, this library will contain just one function, namely, hello().

If you had other functions stored in other .mata files, you could load each of them and then create the library:

```
clear all
. do hello.mata
. do havelunch.mata
. do goodbye.mata
. lmbuild lmylib
```

The Mata library would contain three functions, assuming the three .mata files defined three Mata functions. The three functions defined are probably named hello(), havelunch(), and goodbye(), but it is not required that the function name match the filename. Each .mata file, in fact, can define as many functions as you wish. If the file hello.mata contained

and you typed

- clear all
. do hello.mata
. lmbuild lmylib
then there would be two functions in the library: hello() and goodbye().
Usually, you will have multiple .mata files and define multiple functions in some of them. Each file will define a function and its subroutines. Sometimes, however, you will define related functions in the same file. Regardless of the situation, libraries should not be built interactively because someday code will change and you will need to rebuild the library. The right way to proceed is to make a do-file that will make it easy for you to create and re-create the library:

```
```

* version number intentionally omitted

```
```

* version number intentionally omitted
clear all
clear all
do hello.mata
do hello.mata
do bigfcn.mata
do bigfcn.mata
do utilityfunctions.mata
do utilityfunctions.mata
.
.
lmbuild lmylib, replace

```
```

lmbuild lmylib, replace

```
```

With this do-file written, all we have to type to create the library for the first time is
$\qquad$

All we have to type to re-create the library later is

```
. do make_lmylib
```

Why would we need to re-create the library? One reason would be that we need to re-create the library after fixing a bug in bigfen.mata.

Notice the comment at the top of make_lmylib.do,

```
* version number intentionally omitted
```

and notice that we included version numbers in each of the .mata files. That is how you handle version control with libraries.

## Version control

The version number appearing in each .mata file is the version number under which the code was written. If hello.mata was written back in the days of Stata 11, it would read

```
version 11
mata:
void hello()
{
    printf("hello world\n")
}
end
```

The first line of the file sets the version of Mata in which the code is written. It is called the compile-time version number. Specifying the compile-time version number ensures that the code is backdated to retain its original behavior should the meaning or requirements of some aspect of Mata's programming language or some feature of Mata's compiler change.

In file make_mylib.do, there is nothing we want backdated. The entire purpose of make_mylib.do is to make a modern Mata library, even as new versions of Stata are released. Thus, the version number is intentionally omitted.

One more type of version number where Mata is concerned is called the run-time version number. It is not directly relevant for building libraries, but it is relevant when you want to change the way functions work for different versions of Stata just as we do at StataCorp with the functions we write. We do not preserve bugs, of course, but we do add features, and sometimes new features get in the way of old ones. If we did not write our code in a certain way, old do-files would not continue to work until the user had updated them to new syntax and calling sequences. We write code in such a way that users do not have to do that.

Let's consider a case where you wrote bigfon() back in the days of Stata 13. In Stata 18, you rewrote the function, changed what the arguments did, and increased the number of arguments from one to two. Your original code looked like this:
begin bigfcn.mata

```
version 13
mata:
real matrix bigfcn(real matrix A)
{
}
end
```

Here is how your updated code might look if you wanted to preserve old behavior and allow new features:


Notice that we specified version 18 for part of the file and version 13 for the other part. That is how we made sure that the old code compiled as intended, just in case the compiler changed.

In the new bigfon() function, we make the second argument optional by specifying |real scalar style. Notice the vertical bar and see [M-2] optargs.

Finally, notice that we used Mata built-in function callersversion() to call the new or old code as appropriate.

With bigfen() defined this way, old do-files continue to work, such as

```
                                    begin oldfile.do
version 13
... bigfcn(X) ...
...
```

Do-files specifying version 14 through 17 would continue to work, too.
A modern do-file set to version 18 or later, however, would use the improved bigfcn() and its two arguments:
begin modernfile.do $\qquad$

```
version 18
... bigfcn(X, 1) ...
...
```

The version number specified in the do-file is known as its run-time setting.

## Also see

[M-3] mata mlib - Create function library
[M-3] Intro - Commands for controlling Mata

## Title

[M-3] mata - Mata invocation command

## Description Syntax Remarks and examples Also see

## Description

The mata command invokes Mata. An istmt is something Mata understands; istmt stands for interactive statement of Mata.

## Syntax

The mata command documented here is for use from Stata. It is how you enter Mata. You type mata at a Stata dot prompt, not a Mata colon prompt.

| Syntax 1 | Comment |
| :--- | :--- |
| mata | no colon following mata |
| istmt | if an error occurs, you stay in |
| istmt | mata mode |
| $\quad$. | you exit when you type end |
| istmt | y. |

Syntax 1 is the best way to use Mata interactively.

| Syntax 2 | Comment |
| :--- | :--- |
| mata: | colon following mata |
| istmt <br> istmt | if an error occurs, you are |
| .. | dumped from mata |
| istmt | otherwise, you exit when you type end |
| end |  |

Syntax 2 is mostly used by programmers in ado-files.
Programmers want errors to stop everything.

| Syntax 3 | Comment |
| :--- | :--- |
| mata istmt | rarely used |

Syntax 3 is the single-line variant of syntax 1, but it is not useful.

Syntax 4
Comment
mata: istmt
for use by programmers
Syntax 4 is the single-line variant of syntax 2, and it exists for the same reason as syntax 2: for use by programmers in ado-files.

## Remarks and examples

Remarks are presented under the following headings:

```
Introduction
The fine distinction between syntaxes 3 and 4
The fine distinction between syntaxes 1 and 2
```


## Introduction

For interactive use, use syntax 1. Type mata (no colon), press Enter, and then use Mata freely. Type end to return to Stata. (When you exit from Mata back into Stata, Mata does not clear itself; so if you later type mata-followed-by-enter again, you will be right back where you were.)

For programming use, use syntax 2 or syntax 4. Inside a program or an ado-file, you can just call a Mata function

```
program myprog
    mata: utility("'varlist'")
end
```

and you can even include that Mata function in your ado-file

$\qquad$
or you could separately compile utility () and put it in a .mo file or in a Mata library.

## The fine distinction between syntaxes 3 and 4

Syntaxes 3 and 4 are both single-line syntaxes. You type mata, perhaps a colon, and follow that with the Mata istmt.

The differences between the two syntaxes is whether they allow continuation lines. With a colon, no continuation line is allowed. Without a colon, you may have continuation lines.

For instance, let's consider

```
function renorm(scalar a, scalar b)
{
}
```

No matter how long the function, it is one istmt. Using mata:, if you were to try to enter that istmt, here is what would happen:

```
. mata: function renorm(scalar a, scalar b)
<istmt> incomplete
r(197);
```

When you got to the end of the first line and pressed Enter, you got an error message. Using the mata: command, the istmt must all fit on one line.

Now try the same thing using mata without the colon:

```
. mata function renorm(scalar a, scalar b)
> {
>
}
```

That worked! Single-line mata without the colon allows continuation lines and, on this score at least, seems better than single-line mata with the colon. In programming contexts, however, this feature can bite. Consider the following program fragment:

```
program example
    mata utility("'varlist'"
    replace 'x' = ...
end
```

We used mata without the colon, and we made an error: we forgot the close parenthesis mata without the colon will be looking for that close parenthesis and so will eat the next line-a line not intended for Mata. Here we will get an error message because "replace ' $x$ ' $=\ldots$ " will make no sense to Mata, but that error will be different from the one we should have gotten. In the unlikely worse case, that next line will make sense to Mata.

Ergo, programmers want to include the colon. It will make your programs easier to debug.
There is, however, a programmer's use for single-line mata without the colon. In our sample ado-file above when we included the routine utility(), we bound it in mata: and end. It would be satisfactory if instead we coded

```
program myprog
    mata: utility("'varlist'")
end
mata function utility(string scalar varlist)
{
}
```

end myprog.ado
Using mata without the colon, we can omit the end. We admit we sometimes do that.

## The fine distinction between syntaxes 1 and 2

Nothing said above about continuation lines applies to syntaxes 1 and 2. The multiline mata, with or without colon, always allows continuation lines because where the Mata session ends is clear enough: end.

The difference between the two multiline syntaxes is whether Mata tolerates errors or instead dumps you back into Stata. Interactive users appreciate tolerance. Programmers want strictness. Programmers, consider the following (using mata without the colon):

```
program example2
    mata
        result = myfunc("'varlist'")
            st_local("n" result) /* <- mistake here */
            result = J(0,0,"")
    end
end
```

In the above example, we omitted the comma between " n " and result. We also used multiline mata without the colon. Therefore, the incorrect line will be tolerated by Mata, which will merrily continue executing our program until the end statement, at which point Mata will return control to Stata and not tell Stata that anything went wrong! This could have serious consequences, all of which could be avoided by substituting multiline mata with the colon.

## Also see

[M-3] Intro - Commands for controlling Mata

## Title

[M-3] mata clear - Clear Mata's memory

## Description Syntax Remarks and examples Also see

## Description

mata clear clears Mata's memory, in effect resetting Mata. All functions, matrices, etc., are freed.

## Syntax

: mata clear

This command is for use in Mata mode following Mata's colon prompt. To use this command from Stata's dot prompt, type
. mata: mata clear

## Remarks and examples

Stata can call Mata which can call Stata, which can call Mata, etc. In such cases, mata clear releases only resources that are not in use by prior invocations of Mata.

Stata's clear all command (see [D] clear) performs a mata clear, among other things.
See [M-3] mata drop for clearing individual matrices or functions.

## Also see

[M-3] mata drop - Drop matrix or function
[M-3] Intro - Commands for controlling Mata

## Title

[M-3] mata describe - Describe contents of Mata's memory

Description Syntax Option Remarks and examples Diagnostics Also see

## Description

mata describe lists the names of the matrices and functions in memory, including the amount of memory consumed by each.
mata describe using libname describes the contents of the specified .mlib library; see [M-3] mata mlib.

## Syntax

```
: mata describe \([\) namelist \(][\), all \(]\)
: mata describe using libname
```

where namelist is as defined in [M-3] namelists. If namelist is not specified, " $* *()$ " is assumed.
This command is for use in Mata mode following Mata's colon prompt. To use this command from Stata's dot prompt, type

```
. mata: mata describe ...
```


## Option

all specifies that automatically loaded library functions that happen to be in memory are to be included in the output.

## Remarks and examples

mata describe is often issued without arguments, and then everything in memory is described:

| mata describe <br> \# bytes | type |  |
| ---: | :--- | :--- |
| 50 | real matrix | name and extent |
| 1,600 | real matrix | foo() |
| 8 | real scalar | $\mathrm{x}[10,20]$ |

mata describe using libname lists the functions stored in a .mlib library:

| : mata describe using lmatabase |  |  |
| ---: | :--- | :--- |
| \# bytes | type | name and extent |
| 508 | auto structdef scalar | AsArray_char() |
| 188 | auto structdef scalar | AsArray_dup() |
| 312 | auto structdef scalar | AsArray_top() |
| 984 | auto numeric vector | Corr() |
| 864 | auto numeric vector | Corrslowly() |
| 400 | auto real matrix | Dmatrix() |
| 340 | auto real matrix | Hilbert() |
| (output omitted) |  |  |
| 672 | auto transmorphic colvector | vech() |
| 184 | auto real scalar | whether_ssd() |

## Diagnostics

The reported memory usage does not include overhead, which usually amounts to 64 bytes, but can be less (as small as zero for recently used scalars).

The reported memory usage in the case of pointer matrices reflects the memory used to store the matrix itself and does not include memory consumed by siblings.

## Also see

[M-5] sizeof( ) - Number of bytes consumed by object
[M-3] Intro - Commands for controlling Mata

## Title

[ M -3] mata drop - Drop matrix or function

## Description Syntax Remarks and examples Also see

## Description

mata drop clears from memory the specified matrices and functions.

## Syntax

: mata drop namelist
where namelist is as defined in [M-3] namelists.
This command is for use in Mata mode following Mata's colon prompt. To use this command from Stata's dot prompt, type

```
. mata: mata drop ...
```


## Remarks and examples

Use mata describe (see [M-3] mata describe) to determine what is in memory. Use mata clear (see [M-3] mata clear) to drop all matrices and functions, or use Stata's clear mata command (see [D] clear).

To drop a matrix named A, type
: mata drop A

To drop a function named foo(), type

```
: mata drop foo()
```

To drop a matrix named A and a function named foo(), type

```
: mata drop A foo()
```


## Also see

[M-3] mata clear - Clear Mata's memory
[M-3] Intro - Commands for controlling Mata

## Title

[M-3] mata help - Obtain help in Stata

## Description Syntax Remarks and examples Also see

## Description

mata help is Stata's help command. Thus you do not have to exit Mata to use help.

## Syntax

: mata help...
: help...
help need not be preceded by mata.

## Remarks and examples

See [M-1] help.
You need not type the Mata prefix:

```
: mata help mata
(output appears in Stata's Viewer)
: help mata
(same result)
```

help can be used to obtain help for Mata or Stata:

```
: help mata missing()
: help missing()
```


## Also see

[M-1] help - Obtaining help in Stata
[R] help - Display help in Stata
[M-3] Intro - Commands for controlling Mata

## Title

[M-3] mata matsave - Save and restore matrices

## Description

Option for mata matuse Also see

Syntax
Remarks and examples

Option for mata matsave
Diagnostics

## Description

mata matsave saves the specified global matrices in filename.
mata matuse loads the matrices stored in filename.
mata matdescribe describes the contents of filename.

## Syntax

: mata matsave filename namelist [, replace]
: mata matuse filename [, replace]
: mata matdescribe filename
where namelist is a list of matrix names as defined in [M-3] namelists.
If filename is specified without a suffix, .mmat is assumed.
These commands are for use in Mata mode following Mata's colon prompt. To use these commands from Stata's dot prompt, type

```
. mata: mata matsave ...
```


## Option for mata matsave

replace specifies that filename may be replaced if it already exists.

## Option for mata matuse

replace specifies that any matrices in memory with the same name as those stored in filename can be replaced.

## Remarks and examples

These commands are for interactive use; they are not for use inside programs. See [M-5] fopen() for Mata's programming functions for reading and writing files. In the programming environment, if you have a matrix $X$ and want to write it to file mymatrix.myfile, you code

```
fh = fopen("mymatrix.myfile", "w")
fputmatrix(fh, X)
fclose(fh)
```

Later, you can read it back by coding

```
fh = fopen("mymatrix.myfile", "r")
X = fgetmatrix(fh)
fclose(fh)
```

mata matsave, mata matuse, and mata matdescribe are for use outside programs, when you are working interactively. You can save your global matrices

```
: mata matsave mywork *
(saving A, X, Z, beta)
file mywork.mmat saved
```

and then later get them back:
: mata matuse mywork
(loading A, X, Z, beta)
mata matdescribe will tell you the contents of a file:
: mata matdescribe mywork
file mywork.mmat saved on 4 Apr 2022 08:46:39 contains
$\mathrm{X}, \mathrm{X}, \mathrm{Z}$, beta

## Diagnostics

mata matsave saves the contents of view matrices. Thus when they are restored by mata matuse, the contents will be correct regardless of the data Stata has loaded in memory.

## Also see

[M-3] Intro - Commands for controlling Mata

## Title

[M-3] mata memory - Report on Mata's memory usage

## Description Syntax Remarks and examples Also see

## Description

mata memory provides a summary of Mata's current memory utilization.

## Syntax

```
: mata memory
```

This command is for use in Mata mode following Mata's colon prompt. To use this command from Stata's dot prompt, type

```
. mata: mata memory
```


## Remarks and examples

| mata memory | $\#$ | bytes |
| :--- | ---: | ---: |
| autoloaded functions | 15 | 5,514 |
| ado functions | 0 | 0 |
| defined functions | 0 | 0 |
| matrices \& scalars | 14 | 8,256 |
| overhead |  | 1,972 |
| total | 29 | 15,742 |

Functions are divided into three types:

1. Autoloaded functions, which refer to library functions currently loaded (and which will be automatically unloaded) from .mlib library files and .mo object files.
2. Ado-functions, which refer to functions currently loaded whose source appears in ado-files themselves. These functions will be cleared when the ado-file is automatically cleared.
3. Defined functions, which refer to functions that have been defined either interactively or from do-files and which will be cleared only when a mata clear, mata drop, Stata clear mata, or Stata clear all command is executed; see [M-3] mata clear, [M-3] mata drop, or [D] clear.

## Also see

[M-1] Limits - Limits and memory utilization
[M-3] mata clear - Clear Mata's memory
[M-3] Intro - Commands for controlling Mata

## Title

[M-3] mata mlib - Create function library
Description Syntax Options Remarks and examples Also see

## Description

Mata libraries are useful. You can put your functions in them. If you do that, you can use your functions just as if they were built in to Mata. Your functions and Mata's are put on equal footing. The footing really is equal because Mata's built-in functions are also stored in libraries. The only difference is that those libraries are created and maintained by StataCorp.
mata mlib creates, adds to, and causes Mata to index .mlib files, which are libraries containing the object-code functions. The lmbuild command also creates and maintains Mata function libraries, but lmbuild is easier to use than mata mlib create or mata mlib add. Therefore, we suggest you use lmbuild (see [M-3] lmbuild) instead of these commands.
mata mlib has two other features that are sometimes useful. Mata maintains a list for itself of the libraries it is to search when looking for functions. Mata builds that list when Stata is launched. Type mata mlib query to see the list. Mata tries to keep the list up to date as you work, but if you create a new library and do not use lmbuild, Mata will not know about it. Or if you copy a library from a colleague, Mata will not know about it until Stata is relaunched. Type mata mlib index in such cases, and Mata will rebuild the list.
mata mlib create creates a new, empty library.
mata mlib add adds new members to a library.
mata mlib index causes Mata to build a new list of libraries to be searched.
mata mlib query lists the libraries to be searched.

## Syntax

```
: mata mlib create libname [, dir(path) replace size(#)]
: mata mlib add libname fcnlist() [, dir(path) complete]
: mata mlib index
: mata mlib query
```

where $f_{c n l i s t()}$ is a namelist containing only function names, such as

```
fcnlist() examples
myfunc()
myfunc() myotherfunc() foo()
f*() g*()
*()
```

see $[\mathrm{M}-3]$ namelists
and where libname is the name of a library. You must start libname with the letter 1 and do not add the .mlib suffix as it will be added for you. Examples of libnames include

| libname | Corresponding filename |
| :--- | :--- |
| lmath | lmath.mlib |
| lmoremath | lmoremath.mlib |
| lnjc | lnjc.mlib |

Also libnames that begin with the letters lmata, such as lmatabase, are reserved for the names of official libraries.

This command is for use in Mata mode following Mata's colon prompt. To use this command from Stata's dot prompt, type

```
. mata: mata mlib ...
```


## Options

dir (path) specifies the directory (folder) into which the file should be written. dir(.) is the default, meaning that if $\operatorname{dir}()$ is not specified, the file is written into the current (working) directory. path may be a directory name or may be the sysdir shorthand STATA, BASE, SITE, PLUS, PERSONAL, or OLDPLACE; see [P] sysdir. dir (PERSONAL) is recommended.
complete is for use when saving class definitions. It specifies that the definition be saved only if it is complete; otherwise, an error message is to be issued. See [M-2] class.
replace specifies that the file may be replaced if it already exists.
size(\#), used with mlib create, specifies the maximum number of members the newly created library will be able to contain, $2 \leq \# \leq 2048$. The default is size(1024).

## Remarks and examples

Remarks are presented under the following headings:

```
Background
Outline of the procedure for dealing with libraries
Creating a .mlib library
Adding members to a .mlib library
Listing the contents of a library
Making it so Mata knows to search your libraries
Advice on organizing your source code
```

Also see [M-1] How for an explanation of object code.

## Background

.mlib files contain the object code for one or more functions. Functions which happen to be stored in libraries are called library functions, and Mata's library functions are also stored in .mlib libraries. You can create your own libraries, too.

Mata provides two ways to store object code:

1. In a .mo file, which contains the code for one function
2. In a .mlib library file, which may contain the code for up to 2,048 functions
.mo files are easier to use and work just as well as .mlib libraries; see [M-3] mata mosave. .mlib libraries, however, are easier to distribute to others when you have many functions, because they are combined into one file.

## Outline of the procedure for dealing with libraries

Working with libraries is easy:

1. First, choose a name for your library. We will choose the name lpersonal.
2. Next, create an empty library by using the mata mlib create command.
3. After that, you can add new members to the library at any time, using mata mlib add.
.mlib libraries contain object code, not the original source code, so you need to keep track of the source code yourself. Also, if you want to update the object code in a function stored in a library, you must re-create the entire library; there is no way to replace or delete a member once it is added.

We begin by showing you the mechanical steps, and then we will tell you how we manage libraries and source code.

## Creating a .mlib library

If you have not read [M-3] mata mosave, please do so.
To create a new, empty library named lpersonal.mlib in the current directory, type

```
: mata mlib create lpersonal
(file lpersonal.mlib created)
```

If lpersonal.mlib already exists and you want to replace it, either erase the existing file first or type

```
: mata mlib create lpersonal, replace
(file lpersonal.mlib created)
```

To create a new, empty library named lpersonal.mlib in your PERSONAL (see [P] sysdir) directory, type

```
: mata mlib create lpersonal, dir(PERSONAL)
(file c:\ado\personal\lpersonal.mlib created)
```

or

```
: mata mlib create lpersonal, dir(PERSONAL) replace
(file c:\ado\personal\lpersonal.mlib created)
```


## Adding members to a .mlib library

Once a library exists, whether you have just created it and it is empty, or it already existed and contains some functions, you can add new functions to it by typing

```
: mata mlib add libname fcnname()
```

So, if we wanted to add function example() to library lpersonal.mlib, we would type

```
: mata mlib add lpersonal example()
(1 function added)
```

In doing this, we do not have to say where lpersonal.mlib is stored; Mata searches for it along the ado-path.

Before you can add example() to the library, however, you must compile it:

```
: function example(...)
> {
>
> }
```

: mata mlib add lpersonal example()
(1 function added)

You can add many functions to a library in one command:

```
: mata mlib add lpersonal example2() example3()
(2 functions added)
```

You can add all the functions currently in memory by typing

```
: mata mlib add lanother *()
(3 functions added)
```

In the above example, we added to lanother.mlib because we had already added example(), example2(), and example3() to lpersonal.mlib and trying to add them again would result in an error. (Before adding $*()$, we could verify that we are adding what we want to add by typing mata describe $*()$; see [M-3] mata describe.)

## Listing the contents of a library

Once a library exists, you can list its contents (the names of the functions it contains) by typing

```
: mata describe using libname
```

Here we would type

```
: mata describe using lpersonal
(library contains 3 members)
    # bytes type name and extent
    32 auto transmorphic matrix example()
    32 auto transmorphic matrix example2()
    32 auto transmorphic matrix example3()
```

mata describe usually lists the contents of memory, but mata describe using lists the contents of a library.

## Making it so Mata knows to search your libraries

Mata automatically finds the .mlib libraries on your ado-path. It does this when Mata is invoked for the first time during a session. Thus everything is automatic except that Mata will know nothing about any new libraries created during the Stata session, so after creating a new library, you must tell Mata about it. You do this by asking Mata to rebuild its library index:

```
: mata mlib index
```

You do not specify the name of your new library. That name does not matter because Mata rebuilds its entire library index.

You can issue the mata mlib index command right after creating the new library

```
: mata mlib create lpersonal, dir(PERSONAL)
: mata mlib index
```

or after you have created and added to the library:

```
: mata mlib create lpersonal, dir(PERSONAL)
: mata mlib add lpersonal *()
: mata mlib index
```

It does not matter. Mata does not need to rebuild its index after a known library is updated; Mata needs to be told to rebuild only when a new library is added during the session.

## Advice on organizing your source code

Say you wish to create and maintain lpersonal.mlib. Our preferred way is to use a do-file:

```
                                    begin lpersonal.do
```

$\qquad$

```
mata:
```

mata:
mata clear
mata clear
function definitions appear here
function definitions appear here
mata mlib create lpersonal, dir(PERSONAL) replace
mata mlib create lpersonal, dir(PERSONAL) replace
mata mlib add lpersonal *()
mata mlib add lpersonal *()
mata mlib index
mata mlib index
end

```
end
```

This way, all we have to do to create or re-create the library is enter Stata, change to the directory containing our source code, and type

```
. do lpersonal
```

For large libraries, we like to put the source code for different parts in different files:

```
_ begin lpersonal.do ___
```

```
mata: mata clear
```

mata: mata clear
do function1.mata
do function1.mata
do function2.mata
do function2.mata
...
...
mata:
mata:
mata mlib create lpersonal, dir(PERSONAL) replace
mata mlib create lpersonal, dir(PERSONAL) replace
mata mlib add lpersonal *()
mata mlib add lpersonal *()
mata mlib index
mata mlib index
end

```
end
```

end lpersonal.do $\qquad$
The function files contain the source code, which might include one function, or it might include more than one function if the primary function had subroutines:
mata:
function definitions appear here
end
$\qquad$
We name our component files ending in .mata, but they are still just do-files.

[^6]
## Title

[M-3] mata mosave - Save function's compiled code in object file

Description Syntax Options Remarks and examples Also see

## Description

mata mosave saves the object code for the specified function in the file fcnname.mo.

## Syntax

: mata mosave fcnname() [, dir (path) complete replace]
This command is for use in Mata mode following Mata's colon prompt. To use this command from Stata's dot prompt, type

```
. mata: mata mosave ...
```


## Options

$\operatorname{dir}$ (path) specifies the directory (folder) into which the file should be written. dir(.) is the default, meaning that if $\operatorname{dir}()$ is not specified, the file is written into the current (working) directory. path may be a directory name or may be the sysdir shorthand STATA, BASE, SITE, PLUS, PERSONAL, or OLDPLACE; see [ P ] sysdir. dir (PERSONAL) is recommended.
complete is for use when saving class definitions. It specifies that the definition be saved only if it is complete; otherwise, an error message is to be issued. See [M-2] class.
replace specifies that the file may be replaced if it already exists.

## Remarks and examples

See [M-1] How for an explanation of object code.
Remarks are presented under the following headings:
Example of use
Where to store .mo files
Use of .mo files versus .mlib files

## Example of use

.mo files contain the object code for one function. If you store a function's object code in a .mo file, then in future Mata sessions, you can use the function without recompiling the source. The function will appear to become a part of Mata just as all the other functions documented in this manual are. The function can be used because the object code will be automatically found and loaded when needed.

For example,

```
: function add(a,b) return(a+b)
: add(1,2)
    3
```

```
: mata mosave add()
(file add.mo created)
: mata clear
: add(1,2)
    3
```

In the example above, function add() was saved in file add.mo stored in the current directory. After clearing Mata, we could still use the function because Mata found the stored object code.

## Where to store .mo files

Mata could find add() because file add.mo was in the current directory, and our ado-path included .:

```
. adopath
[1] (BASE) "C:\Program Files\Stata18\ado\base\"
[2] (SITE) "C:\Program Files\Stata18\ado\site\"
[3] "."
[4] (PERSONAL) "C:\ado\personal\"
[5] (PLUS) "C:\ado\plus\"
[6] (OLDPLACE) "C:\ado\"
```

If later we were to change our current directory

```
. cd ..\otherdir
```

Mata would no longer be able to find the file add.mo. Thus the best place to store your personal .mo files is in your PERSONAL directory. Thus rather than typing
: mata mosave example()
we would have been better off typing

```
: mata mosave example(), dir(PERSONAL)
```


## Use of .mo files versus .mlib files

Use of .mo files is heartily recommended. The alternative for saving compiled object code are .mlib libraries; see [M-3] mata mlib and [M-1] Ado.

Libraries are useful when you have many functions and want to tie them together into one file, especially if you want to share those functions with others, because then you have only one file to distribute. The disadvantage of libraries is that you must rebuild them whenever you wish to remove or change the code of one. If you have only a few object files, or if you have many but sharing is not an issue, .mo libraries are easier to manage.

## Also see

[M-3] mata mlib - Create function library
[M-3] Intro - Commands for controlling Mata

## Title

[M-3] mata rename - Rename matrix or function

## Description

mata rename changes the names of functions and global matrices.

## Syntax

: mata rename oldmatrixname newmatrixname
: mata rename oldfcnname() newfcnname()

This command is for use in Mata mode following Mata's colon prompt. To use this command from Stata's dot prompt, type
. mata: mata rename ...

## Also see

[M-3] Intro - Commands for controlling Mata

## Title

[M-3] mata set — Set and display Mata system parameters

## Description Syntax Option Remarks and examples Also see

## Description

mata query shows the values of Mata's system parameters.
mata set sets the value of the system parameters:
mata set matacache specifies the maximum amount of memory, in kilobytes, that may be consumed before Mata starts looking to drop autoloaded functions that are not currently being used. The default value is 2000, meaning 2000 kilobytes. This parameter affects the efficiency with which Stata runs. Larger values cannot hurt, but once matacache is large enough, larger values will not improve performance.
mata set matalnum turns program line-number tracing on or off. The default setting is off. This setting modifies how programs are compiled. Programs compiled when matalnum is turned on include code so that, if an error occurs during execution of the program, the line number is also reported. Turning matalnum on prevents Mata from being able to optimize programs, so they will run more slowly. Except when debugging, the recommended setting for this is off.
mata set mataoptimize turns compile-time code optimization on or off. The default setting is on. Programs compiled when mataoptimize is switched off will run more slowly and, sometimes, much more slowly. The only reason to set mataoptimize off is if a bug in the optimizer is suspected.
mata set matafavor specifies whether, when executing code, Mata should favor conserving memory (space) or running quickly (speed). The default setting is space. Switching to speed will make Mata, in a few instances, run a little quicker but consume more memory. Also see [M-5] favorspeed( ).
mata set matastrict sets whether declarations can be omitted inside the body of a program. The default is off. If matastrict is switched on, compiling programs that omit the declarations will result in a compile-time error; see [M-2] Declarations. matastrict acts unexpectedly but pleasingly when set/reset inside ado-files; see [M-1] Ado.
mata set matalibs sets the names and order of the .mlib libraries to be searched; see [M-1] How. matalibs usually is set to "lmatabase;lmataado". However it is set, it is probably set correctly, because Mata automatically searches for libraries the first time it is invoked in a Stata session. If, during a session, you erase or copy new libraries along the ado-path, the best way to reset matalibs is with the mata mlib index command; see [M-3] mata mlib. The only reason to set matalibs by hand is to modify the order in which libraries are searched.
mata set matamofirst states whether .mo files or .mlib libraries are searched first. The default is off, meaning libraries are searched first.
mata set matasolvetol sets the scalar factor in the computation of the tolerance used by many Mata solvers and inverters. See [M-5] solve_tol() for details.

## Syntax

```
: mata query
: mata set matacache # [, permanently]
: mata set matalnum {off|on}
: mata set mataoptimize {on|off }
: mata set matafavor {space|speed }[, permanently]
: mata set matastrict {off|on}[, permanently]
: mata set matalibs "libname;libname;..."
: mata set matamofirst {off|on}[, permanently]
: mata set matasolvetol #
```

\# in mata set matasolvetol may be . or any double-precision number.

These commands are for use in Mata mode following Mata's colon prompt. To use these commands from Stata's dot prompt, type
. mata: mata query
. mata: mata set ...

## Option

permanently specifies that, in addition to making the change right now, the setting be remembered and become the default setting when you invoke Stata in the future.

## Remarks and examples

Remarks are presented under the following headings:

## Relationship between Mata's mata set and Stata's set commands

The command

```
: mata set ...
```

issued from Mata's colon prompt and the command

```
. set ...
```

issued from Stata's dot prompt are the same command, so you may set Mata's (or even Stata's) system parameters either way.

The command
issued from Mata's colon prompt and the command . query mata
issued from Stata's dot prompt are also the same command.

## c() values

The following concerns Stata more than Mata.
Stata's c-class, c(), contains the values of system parameters and settings along with certain other constants. c() values may be referred to in Stata, either via macro substitution ('c (current_date) ', for example) or in expressions (in which case the macro quoting characters may be omitted). Stata's $c()$ is also available in Mata via Mata's c() function; see [M-5] c().

Most everything set by set is available via c(), including Mata's set parameters:

```
c(matacache) returns a numeric scalar equal to the cache size.
c(matalnum) returns a string equal to "on" or "off".
c(mataoptimize) returns a string equal to "on" or "off".
c(matafavor) returns a string equal to "space" or "speed".
c(matastrict) returns a string equal to "on" or "off".
c(matalibs) returns a string of library names separated by semicolons.
c(matamofirst) returns a string equal to "on" or "off".
c(matasolvetol) returns a numeric scalar containing . or #.
```

The above is in Stataspeak. Rather than referring to $c$ (matacache), we would refer to $c$ ("matacache") if we were using Mata's function. The real use of these values, however, is in Stata.

## Also see

[R] query - Display system parameters
[R] set - Overview of system parameters
[P] creturn - Return c-class values
[M-3] Intro - Commands for controlling Mata

## Title

[M-3] mata stata - Execute Stata command
Description Syntax Remarks and examples Also see

## Description

mata stata stata_command passes stata_command to Stata for execution.

## Syntax

: mata stata stata_command

This command is for use in Mata mode following Mata's colon prompt.

## Remarks and examples

mata stata is a convenience tool to keep you from having to exit Mata:

```
: st_view(V=., 1\5, ("mpg", "weight"))
            st_view(): 3598 Stata returned error
                        <istmt>: - function returned error
r(3598);
: mata stata sysuse auto
(1978 automobile data)
: st_view(V=., 1\5, ("mpg", "weight"))
```

mata stata is for interactive use. If you wish to execute a Stata command from a function, see [M-5] stata().

## Also see

[M-5] stata() - Execute Stata command
[M-3] Intro - Commands for controlling Mata

## Title

[M-3] mata which - Identify function

## Description Syntax Remarks and examples Also see

## Description

mata which fcnname looks for fcnname() and reports whether it is built in, stored in a .mlib library, or stored in a .mo file.

## Syntax

```
: mata which fcnname()
```

This command is for use in Mata mode following Mata's colon prompt. To use this command from Stata's dot prompt, type

```
. mata: mata which ...
```


## Remarks and examples

mata which fcnname() looks for fcnname() and reports where it is found:

```
: mata which I()
    I(): built-in
: mata which assert()
    assert(): lmatabase
: mata which myfcn()
    userfunction(): .\myfcn.mo
: mata which nosuchfunction()
function nosuchfunction() not found
r(111);
```

Function I() is built in; it was written in C and is a part of Mata itself.
Function assert () is a library function and, as a matter of fact, its executable object code is located in the official function library lmatabase.mlib.

Function myfcn() exists and has its executable object code stored in file myfcn.mo, located in the current directory.

Function nosuchfunction() does not exist.
Going back to mata which assert(), which was found in lmatabase.mlib, if you wanted to know where lmatabase.mlib was stored, you could type findfile lmatabase.mlib at the Stata prompt; see [P] findfile.

## Also see

[M-3] Intro - Commands for controlling Mata

## Title

[M-3] namelists - Specifying matrix and function names
Description Syntax Remarks and examples Also see

## Description

Namelists appear in syntax diagrams.

## Syntax

Many mata commands allow or require a namelist, such as : mata describe $[$ namelist $][$, all $]$

A namelist is defined as a list of matrix and/or function names, such as alpha beta foo()

The above namelist refers to the matrices alpha and beta along with the function named foo().
Function names always end in (), hence
alpha refers to the matrix named alpha
alpha() refers to the function of the same name
Names may also be specified using the $*$ and ? wildcard characters:

* means zero or more characters go here
? means exactly one character goes here
hence,

| * | means all matrices |
| :---: | :---: |
| *() | means all functions |
| * * () | means all matrices and all functions |
| s* | means all matrices that start with $s$ |
| s*() | means all functions that start with $s$ |
| *e | means all matrices that end with $e$ |
| *e() | means all functions that end with $e$ |
| s*e | means all matrices that start with $s$ and end with $e$ |
| s*e() | means all functions that start with $s$ and end with $e$ |
| s?e | means all matrices that start with $s$ and end with $e$ and have one character in between |
| s?e() | means all functions that start with $s$ and end with and have one character in between |

## Remarks and examples

Some namelists allow only matrices, and some allow only functions. Even when only functions are allowed, you must include the () suffix.

## Also see

[M-3] Intro - Commands for controlling Mata

## [M-4]Categorical guide to Mata functions

## Title

[M-4] Intro - Categorical guide to Mata functions

Contents Description Remarks and examples Also see

## Contents

[M-4] Entry
Description

|  | Mathematical |
| :--- | :--- |
|  |  |
| Matrix | Matrix functions |
| Solvers | Matrix solvers and inverters |
| Scalar | Scalar functions |
| Statistical | Statistical functions |
| Mathematical | Other important functions |


|  |  |
| :--- | :--- |
| Utility \& manipulation |  |
| Standard | Standard matrices |
| Utility | Matrix utility functions |
| Manipulation | Matrix manipulation functions |

Stata interface

Stata Stata interface functions

|  | Date and time |  |
| :--- | :--- | :--- |
| Dates |  |  |
|  | Date and time functions |  |


|  | String, I/O, \& programming |
| :--- | :---: |
| String | String manipulation functions |
| IO | I/O functions |
| Programming | Programming functions |

## Description

The entries in this section provides an index to the functions, grouped according to purpose.

## Remarks and examples

The next section, section [M-5], presents all the Mata functions, in alphabetical order.

## Also see

[M-0] Intro - Introduction to the Mata manual

## Title

[M-4] Dates - Date and time functions
Contents Description Also see

## Contents

[M-5] Manual entryFunction

Dates
clock()
mdyhms()
dhms()
hms()
hh ()
mm ()
ss()
dofc()
Cofc ()
Clock()
Cmdyhms()
Cdhms()
Chms()
hhC()
mmC()
ssC()
dofC()
date()
mdy ()
dmy ()
yw()
ym()
yq()
yh()
cofd()
Cofd()

## Purpose

\%tc of string
$\% t c$ of month, day, year, hour, minute, and second
$\% \mathrm{tc}$ of $\% \mathrm{td}$, hour, minute, and second
\%tc of hour, minute, and second
hour of \%tc
minute of \%tc
second of \%tc
\%td of \%tc
\%tc of \%tc
$\% \mathrm{tC}$ of string
$\% \mathrm{tC}$ of month, day, year, hour, minute, and second
$\% \mathrm{tC}$ of $\% \mathrm{td}$, hour, minute, and second
$\% \mathrm{tC}$ of hour, minute, and second
hour of \%tC
minute of $\% \mathrm{tC}$
second of $\% \mathrm{tc}$
\%td of \%tC
$\% \mathrm{td}$ of string
\%td of month, day, and year
\%td of day, month, and year
$\%$ tw of year and week
$\% \mathrm{tm}$ of year and month
\%tq of year and quarter
$\%$ th of year and half
$\% \mathrm{tc}$ of $\% \mathrm{td}$
$\% \mathrm{tC}$ of $\% \mathrm{td}$
date(), continued dofb()
bofd()
month (
day()
year()
dow()
week()
quarter()
halfyear()
doy()
yearly()
yofd()
dofy()
halfyearly()
hofd()
dofh()
quarterly()
qofd()
dofq()
monthly()
mofd()
dofm()
weekly()
wofd()
dofw()
hours()
minutes()
seconds()
msofhours()
msofminutes()
msofseconds()
age()
age_frac()
Clockdiff()
clockdiff()
Clockdiff_frac()
clockdiff_frac()
datediff()
datediff_frac()
$\% \mathrm{td}$ of $\% \mathrm{tb}$
$\%$ tb of \%td
month of \%td
day-of-month of $\% t d$
year of \%td
day-of-week of \%td
week of $\%$ td
quarter of \%td
half-of-year of \%td
day-of-year of \%td
\%ty of string
\%ty of \%td
$\% t d$ of \%ty
\%th of string
$\%$ th of $\%$ td
$\%$ td of \%th
\%tq of string
$\% t q$ of $\% t d$
$\% t d$ of $\% t q$
\%tm of string
$\% t m$ of $\% t d$
$\% t d$ of $\%$ tm
\%tw of string
\%tw of \%td
\%td of \%tw
hours of milliseconds
minutes of milliseconds
seconds of milliseconds
milliseconds of hours
milliseconds of minutes
milliseconds of seconds
integer age on \%td
age on \%td with fractional part
integer \%tC difference
integer \%tc difference
$\% \mathrm{tC}$ difference with fractional part
\%tc difference with fractional part
integer \%td difference
\%td difference with fractional part

| date(), continued | $\begin{aligned} & \text { birthday() } \\ & \text { previousbirthday() } \\ & \text { nextbirthday() } \end{aligned}$ | \%td birthday in year <br> \%td birthday immediately before \%td <br> \%td first birthday after \%td |
| :---: | :---: | :---: |
|  | isleapyear() | 1 if leap year; 0 otherwise |
|  | previousleapyear() | leap year immediately before year |
|  | nextleapyear () | first leap year after year |
|  | daysinmonth() | number of days in month of \%td |
|  | firstdayofmonth() | \%td first day of month of \%td |
|  | lastdayofmonth() | \%td last day of month of \%td |
|  | datepart() | part of \%td corresponding to time unit |
|  | clockpart() | part of \%tc corresponding to time unit |
|  | Clockpart() | part of \%tC corresponding to time unit |
|  | isleapsecond() | 1 if \%tC is leap second; 0 otherwise |
|  | today () | \%td today's date |
|  | now() | \%tc current datetime |
|  | dayssinceweekday(), dayssincedow() | number of days since previous day of week |
|  | daysuntilweekday(), daysuntildow() | number of days until next day of week |
|  | ```firstweekdayofmonth() firstdowofmonth()``` | $\% \mathrm{td}$ of first day of week of month and year |
|  | ```lastweekdayofmonth(), lastdowofmonth()``` | \%td of last day of week of month and year |
|  | previousweekday(), <br> previousdow() | \%td of last day of week before \% td |
|  | nextweekday(), nextdow() | \%td of next day of week after \%td |

## Description

The above functions allow you to work with dates and times in Mata. They are what most people would consider scalar functions, although in fact they will work with matrices, in an element-by-element fashion.

## Also see <br> [M-4] Intro - Categorical guide to Mata functions

## Title

[M-4] IO - I/O functions

Contents Description Remarks and examples Reference Also see

## Contents

| [M-5] Manual entry | Function | Purpose |
| :---: | :---: | :---: |
| Console output |  |  |
| printf() | $\begin{aligned} & \text { printf() } \\ & \text { sprintf() } \end{aligned}$ | display display into string |
| errprintf() | errprintf() | display error message |
| display() | display() | display text interpreting SMCL |
| displayas() | displayas() | set whether output is displayed |
| displayflush() | displayflush() | flush terminal output buffer |
| liststruct() | liststruct() | list structure's contents |
| more() | ```more() setmore() setmoreonexit()``` | create -more- condition query or set more on or off set more on or off on exit |
| File directories |  |  |
| direxists() | direxists() | whether directory exists |
| $\operatorname{dir}()$ | dir () | file list |
| chdir() | pwd() <br> chdir() <br> mkdir() <br> rmdir() | obtain current working directory change current working directory make new directory remove directory |


|  | File management |  |
| :--- | :--- | :--- |
| findfile() | fileexists () | whether file exists |
| fileexists() | cat() | read file into string matrix |
| $\operatorname{cat()}$ | unlink() | erase file |
| unlink() | adosubdir() | obtain ado-subdirectory for file |
| adosubdir() | issamefile() | whether two file paths point to the same file |


| fopen() | fopen() | open file |
| :---: | :---: | :---: |
|  | fclose() | close file |
|  | fget() | read line of text file |
|  | fgetnl() | same, but include newline character |
|  | fread() | read $k$ bytes of binary file |
|  | fput() | write line into text file |
|  | fwrite() | write $k$ bytes into binary file |
|  | fgetmatrix() | read matrix |
|  | fputmatrix() | write matrix |
|  | fstatus() | status of last I/O command |
|  | ftell() | report location in file |
|  | fseek() | seek to location in file |
|  | ftruncate() | truncate file at current position |
| ferrortext() | ferrortext() | error text of file error code |
|  | freturncode() | return code of file error code |
| bufio() | bufio() | initialize buffer |
|  | bufbyteorder() | reset (specify) byte order |
|  | bufmissingvalue() | reset (specify) missing-value encoding |
|  | bufput() | copy into buffer |
|  | bufget() | copy from buffer |
|  | fbufput() | copy into and write buffer |
|  | fbufget() | read and copy from buffer |
|  | bufbfmtlen() | utility routine |
|  | bufbfmtisnum() | utility routine |
| $\mathbf{x l}$ () | xl() | Excel file I/O class |
| _docx*() | _docx*() | generate Office Open XML file |
| Pdf*() | Pdf* () | create a PDF file |

## Filename \& path manipulation

pathjoin()

```
pathjoin() join paths
pathsplit() split paths
pathbasename() path basename
pathsuffix() file suffix
pathrmsuffix() remove file suffix
pathisurl() whether path is URL
pathisabs() whether path is absolute
pathasciisuffix() whether file is text
pathstatasuffix() whether file is Stata
pathlist() process path list
pathsubsysdir()
pathsearchlist()
pathresolve()
pathgetparent()
```

join paths
split paths
path basename
file suffix
remove file suffix
whether path is URL
whether path is absolute
whether file is text
whether file is Stata
process path list
substitute for system directories
path list to search for file
resolve a relative path
get the parent path

## Description

The above functions have to do with

1. Displaying output at the terminal.
2. Reading and writing data in a file.

## Remarks and examples

To display the contents of a scalar, vector, or matrix, it is sufficient merely to code the identity of the scalar, vector, or matrix:


You can follow this approach even in programs:

```
function example()
{
    "i am about to calculate the result"
    "the result is"
    b
}
```

On the other hand, display() and printf() (see [M-5] display() and [M-5] printf()) will allow you to exercise more control over how the output looks.

Changing the subject: you will find that many I/O functions come in two varieties: with and without an underscore in front of the name, such as _fopen() and fopen(). As always, functions that begin with an underscore are generally silent about their work and return flags indicating their success or failure. Functions that omit the underscore abort and issue the appropriate error message when things go wrong.

## Reference

Gould, W. W. 2009. Mata Matters: File processing. Stata Journal 9: 599-620.

## Also see

[M-4] Intro - Categorical guide to Mata functions

## Title

[M-4] Manipulation - Matrix manipulation
Contents Description Remarks and examples Also see

## Contents

## $\frac{\text { [M-5] Manual entry }}{} \begin{array}{r}\text { Transposition }\end{array}$

| transposeonly() | transposeonly () | transposition without conjugation |
| :--- | :--- | :--- |
| _transpose () | _transpose () | transposition in place |


|  | Diagonals |  |
| :--- | :--- | :--- |
| $\operatorname{diag}()$ |  |  |
| _diag() | create diagonal matrix from vector |  |
| $\operatorname{diagonal()}$ | _diag() | replace diagonal of matrix |


| lowertriangle() | lowertriangle() <br> uppertriangle() | extract lower triangle extract upper triangle |
| :---: | :---: | :---: |
| sublowertriangle() | sublowertriangle() | generalized lowertriangle() |
| makesymmetric() | makesymmetric() | make matrix symmetric (Hermitian) |
| Sorting |  |  |
| sort() | sort() <br> jumble() <br> order() <br> unorder() <br> _collate() | sort rows of matrix <br> randomize order of rows of matrix <br> permutation vector for ordered rows <br> permutation vector for randomized rows <br> order matrix on permutation vector |
| uniqrows() | uniqrows() | sorted, unique rows |


| _fillmissing() | -fillmissing() |
| :--- | :--- |
| editmissing() | editmissing() |
| editvalue() | editvalue() |
| edittozero() | edittozero() <br> edittozerotol() <br> edittoint() |
|  | edittoint() <br> edittointtol () |



## Description

The above functions manipulate matrices, such as extracting the diagonal and sorting.

## Remarks and examples

There is a thin line between manipulation and utility; also see
[M-4] Utility Matrix utility functions

## Also see

[M-4] Intro - Categorical guide to Mata functions

## Title

[M-4] Mathematical - Important mathematical functions
Contents Description Remarks and examples Also see

## Contents

[M-5]
Manual entry Function Purpose


| moptimize() | ```moptimize() moptimize_ado_cleanup() moptimize_evaluate() moptimize_init() moptimize_init_*() moptimize_result_*() moptimize_query() moptimize_util_*()``` | function optimization perform cleanup after ado evaluate function at initial values begin setup of optimization problem set details access moptimize() results report settings utility functions for writing evaluators and processing results |
| :---: | :---: | :---: |
| solvenl() | ```solvenl_init() solvenl_init_*() solvenl_solve() solvenl_result_*() solvenl_dump()``` | begin solver <br> set details <br> solve equations <br> access results <br> report detailed settings |
| LinearProgram() | LinearProgram() | linear programming |
| Quadrature() | Quadrature() <br> QuadratureVec() | numerical integration vector of numerical integration |

fft()

```
fft()
    invfft()
    convolve()
    deconvolve()
    Corr()
    ftperiodogram()
    ftpad()
    ftwrap()
    ftunwrap()
    ftretime()
    ftfreqs()
```

fast Fourier transform
inverse fast Fourier transform
convolution
inverse of convolve()
correlation
power spectrum
pad to power-of-2 length
convert to frequency-wraparound order convert from frequency-wraparound order
change time scale of signal
frequencies of transform

Cubic splines

| spline3() | spline3() |
| :--- | :--- |
| spline3eval() |  |

fit cubic spline
evaluate cubic spline

polyeval() | polyeval() |  |
| :--- | :--- |
| polysolve() |  |
|  | polytrim() |
|  | polyderiv() |
|  | polyinteg() |
|  | polyadd() |
|  | polymult() |
|  | polydiv() |
|  | polyroots() |

evaluate polynomial solve for polynomial trim polynomial derivative of polynomial integral of polynomial add polynomials multiply polynomials divide polynomials find roots of polynomial

## Number-theoretic point sets

halton( )

```
halton()
ghalton()
```

generate a Halton or Hammersley set generate a generalized Halton sequence

Base conversion

| inbase( ) | inbase() |
| :--- | :--- |
| frombase () | convert to specified base <br> convert from specified base |

## Description

The above functions are important mathematical functions that most people would not call either matrix functions or scalar functions, but that use matrices and scalars.

## Remarks and examples

For other mathematical functions, see

| [M-4] Matrix | Matrix mathematical functions |
| :--- | :--- |
| [M-4] Scalar | Scalar mathematical functions |
| [M-4] Statistical | Statistical functions |

## Also see

[M-4] Intro - Categorical guide to Mata functions

## Title

[M-4] Matrix - Matrix functions
Contents Description Remarks and examples Also see

## Contents

[M-5] Manual entry Function

\begin{tabular}{|c|c|c|}
\hline \& Characteristics \& <br>
\hline trace() \& trace() \& trace of matrix <br>
\hline \multirow[t]{2}{*}{$\operatorname{det}()$} \& det() \& determinant <br>
\hline \& dettriangular() \& determinant of triangular matrix <br>
\hline norm() \& norm() \& matrix and vector norms <br>
\hline cond() \& cond() \& matrix condition number <br>
\hline \multirow[t]{2}{*}{$\operatorname{rank}()$

B} \& rank() \& rank of matrix <br>
\hline \& Bunch-Kaufman decomposition \& <br>
\hline ldl( ) \& ldl () \& Bunch-Kaufman decomposition $A=L D L^{\prime}$ <br>

\hline $$
-\sqrt{\mathrm{Cl}}
$$ \& Cholesky decomposition, solvers, \& inverters \& nverters <br>

\hline cholesky() \& ) cholesky() \& Cholesky square-root decomposition $A=G G^{\prime}$ <br>
\hline \multirow[t]{2}{*}{cholsolve()} \& ) cholsolve() \& solve $A X=B$ for $X$ <br>
\hline \& cholsolvelapacke() \& solve $A X=B$ for $X$ using LAPACK routines <br>
\hline \multirow[t]{2}{*}{cholinv()} \& cholinv() \& inverse of positive-definite symmetric matrix <br>
\hline \& cholinvlapacke() \& inverse of positive-definite symmetric matrix using LAPACK routines <br>
\hline invsym( ) \& invsym() \& real symmetric matrix inversion <br>
\hline
\end{tabular}

LU decomposition, solvers, \& inverters

| lud( ) | lud() | LU decomposition $A=P L U$ |
| :--- | :--- | :--- |
| lusolve() | lusolve() | solve $A X=B$ for $X$ |
| luinv() | luinv() | inverse of square matrix |

QR decomposition, solvers, \& inverters

| qrd() | qrd() | QR decomposition $A=Q R$ |
| :---: | :---: | :---: |
|  | qrdp() | QR decomposition $A=Q R P^{\prime}$ |
|  | hqrd() | QR decomposition $A=f(H) R_{1}$ |
|  | hqrdp() | QR decomposition $A=f(H$, tau $) R_{1} P^{\prime}$ |
|  | hqrdmultq() | return $Q X$ or $Q^{\prime} X, Q=f(H$, tau $)$ |
|  | hqrdmultq1t() | return $Q_{1}^{\prime} X, Q_{1}=f(H$, tau $)$ |
|  | hqrdq() | return $Q=f(H$, tau $)$ |
|  | hqrdq1() | return $Q_{1}=f(H$, tau $)$ |
|  | hqrdr () | return $R$ |
|  | hqrdr1() | return $R_{1}$ |
| qrsolve() | qrsolve() | solve $A X=B$ for $X$ |
| qrinv() | qrinv() | generalized inverse of matrix |

Hessenberg decomposition \& generalized Hessenberg decomposition

| hessenbergd() | hessenbergd() | Hessenberg decomposition $T=Q^{\prime} X Q$ |
| :--- | :--- | :--- |
| ghessenbergd() | ghessenbergd() | generalized Hessenberg decomposition $T=Q^{\prime} X Q$ |



Triangular solvers

solvelower() | solvelower() |
| :--- |
| solvelowerlapacke() |
| solveupper() |
| solveupperlapacke() |

Eigensystems, powers, \& transcendental

| eigensystem() | eigensystem() <br> eigenvalues() <br> lefteigensystem() <br> symeigensystem() <br> symeigenvalues() | right eigenvectors and eigenvalues <br> eigenvalues <br> left eigenvectors and eigenvalues <br> eigenvectors and eigenvalues of symmetric matrix <br> eigenvalues of symmetric matrix |
| :--- | :--- | :--- |
| eigensystemselect()eigensystemselect*() <br> etc. | selected eigenvectors and eigenvalues |  |
| geigensystem() | geigensystem() <br> etc. | generalized eigenvectors and eigenvalues |
| matpowersym() | matpowersym() | powers of symmetric matrix |
| matexpsym() | matexpsym() <br> matlogsym() | exponentiation of symmetric matrix <br> logarithm of symmetric matrix |


| _equilre() | _equilrc() | row/column equilibration |
| :---: | :---: | :---: |
|  | _equilr() | row equilibration |
|  | _equilc() | column equilibration |
|  | _perhapsequilrc() | row/column equilibration if necessary |
|  | _perhapsequilr() | row equilibration if necessary |
|  | _perhapsequilc() | column equilibration if necessary |
|  | rowscalefactors() | row-scaling factors for equilibration |
|  | colscalefactors() | column-scaling factors for equilibration |


|  | LAPACK |  |
| :--- | :--- | :--- |
| lapack() | LA_*() <br> _flopin() | LAPACK linear-algebra functions <br> convert matrix order from row major to <br> column major |
|  | _flopout () | convert matrix order from column major to <br> row major |

## Description

The above functions are what most people would call mathematical matrix functions.

## Remarks and examples

For other mathematical functions, see

[M-4] Scalar Scalar mathematical functions<br>[M-4] Mathematical Important mathematical functions

## Also see

[M-4] Intro - Categorical guide to Mata functions

## Title

[M-4] Programming — Programming functions

## Contents Also see

## Contents

| [M-5] Manual entry | Function | Purpose |
| :---: | :---: | :---: |
| Argument and caller-preference processing |  |  |
| $\operatorname{args}()$ | args () | number of arguments |
| isfleeting() | isfleeting() | whether argument is temporary |
| callersversion( ) | callersversion() | obtain version number of caller |
| favorspeed() | favorspeed() | whether speed or space is to be favored |
| Advanced parsing |  |  |
| tokenget() | ```tokeninit() tokeninitstata() tokenset() tokengetall() tokenget() tokenpeek() tokenrest() tokenoffset() tokenwchars() tokenpchars() tokenqchars() tokenallownum() tokenallowhex()``` | initialize parsing environment initialize environment as Stata would set/reset string to be parsed parse entire string parse next element of string peek at next tokenget () result return yet-to-be-parsed portion query/reset offset in string query/reset whitespace characters query/reset parsing characters query/reset quote characters query/reset number parsing query/reset hex-number parsing |

Accessing externals

| findexternal() | findexternal() <br> crexternal() <br> rmexternal() <br> nameexternal() |
| :--- | :--- |
| direxternal() | direxternal() |
| valofexternal() | valofexternal() |

find global create global
remove global name of external
obtain list of existing globals
obtain value of global

| Break key |  |  |
| :---: | :---: | :---: |
| setbreakintr() | ```setbreakintr() querybreakintr() breakkey() breakkeyreset()``` | turn off/on break-key interrupt whether break-key interrupt is off/on whether break key has been pressed reset break key |
| Associative arrays |  |  |
| asarray () | $\begin{aligned} & \text { asarray() } \\ & \text { asarray_*() } \end{aligned}$ | store or retrieve element in array utility routines |
| AssociativeArray( ) |  | class interface into asarray () |
|  | A.put() | store element |
|  | $A \cdot \operatorname{get}()$ etc. | get element |
| hash1() | hash1() | Jenkins's one-at-a-time hash |
| Miscellaneous |  |  |
| assert( ) | $\begin{aligned} & \text { assert() } \\ & \text { asserteq() } \end{aligned}$ | abort execution if not true abort execution if not equal |
| c() | c () | access c () value |
| sizeof() | sizeof() | number of bytes consumed by object |
| swap() | swap() | interchange contents of variables |
| System info |  |  |
| byteorder() <br> stataversion() | ```byteorder() stataversion() statasetversion()``` | byte order used by computer version of Stata being used version of Stata set |
|  |  |  |
| Exiting |  |  |
| exit() | exit() | terminate execution |
| error() | error() | issue standard Stata error message |
|  | _error() | issue error message with traceback log |

## Also see

[M-4] Intro - Categorical guide to Mata functions

## Title

[M-4] Scalar - Scalar mathematical functions
Contents Description Remarks and examples Also see

## Contents

[M-5] Manual entry
Function

Complex

| $\operatorname{Re}()$ | $\operatorname{Re}()$ | real part |
| :--- | :--- | :--- |
| $\operatorname{Im}()$ | imaginary part |  |
| $\mathbf{C}()$ | C() | make complex |


|  | Sign related |  |  |
| :---: | :---: | :---: | :---: |
| abs() |  | abs () | absolute value (length if complex) |
| $\boldsymbol{\operatorname { s i g n }}$ ( ) |  | $\begin{aligned} & \text { sign() } \\ & \text { quadrant() } \end{aligned}$ | sign function quadrant of value |
| dsign( ) |  | dsign() | FORTRAN-like DSIGN function |
| conj() |  | conj() | complex conjugate |

Transcendental \& square root
$\exp ()$
sqrt()
$\exp ()$
$\ln (), \log ()$
$\log 10()$
expm1()
$\ln 1 p(), \log 1 p()$
$\ln 1 \mathrm{~m}(), \log 1 \mathrm{~m}()$
sqrt()
exponentiation
natural logarithm
base-10 logarithm
$\exp ()-1$
natural logarithm of $(1+x)$
natural logarithm of $(1-x)$
square root

Transcendental \& square root, continued

| $\sin ()$ | $\sin ()$ | sine |
| :--- | :--- | :--- |
|  | $\cos ()$ | cosine |
| $\tan ()$ | tangent |  |
|  | $\operatorname{asin}()$ | arcsine |
|  | $\operatorname{acos}()$ | arccosine |
|  | $\operatorname{atan}()$ | arctangent |
|  | $\arg ()$ | arctangent of complex |
|  | $\operatorname{atan2}()$ | two-argument arctangent |
|  | $\sinh ()$ | hyperbolic sine |
|  | $\cosh ()$ | hyperbolic cosine |
|  | $\tanh ()$ | hyperbolic tangent |
|  | $\operatorname{asinh}()$ | inverse-hyperbolic sine |
|  | $\operatorname{acosh}()$ | inverse-hyperbolic cosine |
|  | $\operatorname{atanh}()$ | inverse-hyperbolic tangent |
| $\operatorname{pi}()$ | value of $\pi$ |  |


| factorial() | $\begin{aligned} & \text { factorial() } \\ & \text { lnfactorial() } \\ & \text { gamma() } \\ & \text { lngamma() } \\ & \text { digamma() } \\ & \text { trigamma() } \end{aligned}$ | factorial <br> natural logarithm of factorial <br> gamma function <br> natural logarithm of gamma function <br> derivative of Ingamma() <br> second derivative of Ingamma() |
| :---: | :---: | :---: |
|  | Modulus \& integer rounding |  |
| $\bmod ()$ | $\bmod ()$ | modulus |
| trunc() | trunc () | truncate to integer |
|  | floor() | round down to integer |
|  | ceil() | round up to integer |
|  | round() | round to closest integer or multiple |

## Description

With a few exceptions, the above functions are what most people would consider scalar functions, although in fact all will work with matrices, in an element-by-element fashion.

## Remarks and examples

For other mathematical functions, see

| $[\mathrm{M}-4]$ Matrix | Matrix functions |
| :--- | :--- |
| $[\mathrm{M}-4]$ Mathematical | Important mathematical functions |
| $[\mathrm{M}-4]$ Statistical | Statistical functions |

## Also see

[M-4] Intro - Categorical guide to Mata functions

## Title

[M-4] Solvers - Functions to solve $\mathrm{AX}=\mathrm{B}$ and to obtain A inverse
Contents Description Remarks and examples Also see

## Contents

[M-5] Manual entry
Function

Solvers

| cholsolve() | cholsolve() <br> cholsolvelapacke() | A positive definite; symmetric or Hermitian <br> A positive definite using LAPACK routines; <br> symmetric or Hermitian |
| :--- | :--- | :--- |
| lusolve() | lusolve() | A full rank, square, real or complex |
| qrsolve() | qrsolve() | A general; $m \times n, m \geq n$, real or complex; <br> least-squares generalized solution |
| svsolve() | svsolve() | generalized; $m \times n$, real or complex; <br> minimum norm, least-squares solution |


|  | Inverters |  |  |
| :---: | :---: | :---: | :---: |
| invsym() |  | invsym() | generalized; real symmetric |
| cholinv() |  | ```cholinv() cholinvlapacke()``` | positive definite; symmetric or Hermitian positive definite using LAPACK routines; symmetric or Hermitian |
| luinv() |  | luinv() | full rank; square; real or complex |
| qrinv() |  | qrinv() | generalized; $m \times n, m \geq n$; real or complex |
| $\operatorname{pinv}()$ |  | pinv() | generalized; $m \times n$, real or complex Moore-Penrose pseudoinverse |

## Description

The above functions solve $A X=B$ for $X$ and solve for $A^{-1}$.

## Remarks and examples

Matrix solvers can be used to implement matrix inverters, and so the two nearly always come as a pair.

Solvers solve $A X=B$ for $X$. One way to obtain $A^{-1}$ is to solve $A X=I$. If $f(A, B)$ solves $A X=B$, then $f(A$, I (rows $(A)$ ) solves for the inverse. Some matrix inverters are in fact implemented this way, although usually custom code is written because memory savings are possible when it is known that $B=I$.

The pairings of inverter and solver are

| inverter | solver |
| :--- | :--- |
| invsym() | (none) |
| cholinv() | cholsolve() |
| cholinvlapacke() | cholsolvelapacke() |
| luinv() | lusolve() |
| qrinv() | qrsolve() |
| pinv() | svsolve() |

## Also see

[M-4] Intro - Categorical guide to Mata functions

## Title

[M-4] Standard - Functions to create standard matrices
Contents Description Remarks and examples Also see

## Contents

[M-5] Manual entry Function Purpose


| I() | I() | identity matrix |
| :--- | :--- | :--- |
| $\mathbf{e}()$ | e() | unit vectors |
| $\mathbf{J}()$ | J() | matrix of constants |
| designmatrix() | designmatrix() | design matrices |



| range( ) | range () <br> rangen() | vector over specified range <br> vector of $n$ over specified range |
| :--- | :--- | :--- |
| unitcircle() | unitcircle() | unit circle on complex plane |

## Random

| runiform() | runiform() <br> rnormal() | uniform random variates <br> normal (Gaussian) random variates |
| :--- | :--- | :--- |
|  | rbeta() |  |
| rbinomial() | beta random variates |  |
| rchi2() | binomial random variates |  |
| rdiscrete() | $\chi^{2}$ random variates |  |
| rexponential() | discrete random variates |  |
| exponential random variates |  |  |
|  | rgamma() | gamma random variates |

Named matrices

| Hilbert() | Hilbert() <br> invHilbert() | Hilbert matrices <br> inverse Hilbert matrices |
| :--- | :--- | :--- |
| Toeplitz() | Toeplitz() | Toeplitz matrices |
| Vandermonde() | Vandermonde() | Vandermonde matrices |

$\operatorname{vec}() \& \operatorname{vech}()$ transforms

| Dmatrix() | Dmatrix() | duplication matrices |
| :--- | :--- | :--- |
| Kmatrix() | Kmatrix() | commutation matrices |
| Lmatrix() | Lmatrix() | elimination matrices |

## Description

The functions above create standard matrices such as the identity matrix, etc.

## Remarks and examples

For other mathematical functions, see

[M-4] Matrix Matrix mathematical functions<br>[M-4] Scalar Scalar mathematical functions<br>[M-4] Mathematical Important mathematical functions

## Also see

[M-4] Intro - Categorical guide to Mata functions

## Title

[M-4] Stata - Stata interface functions

Contents Description Remarks and examples Reference Also see

## Contents

$\frac{[\mathrm{M}-5] \text { Manual entry }}{\square \text { Access to data }}$

| st_nvar() | st_nvar() <br> st_nobs() | number of variables <br> number of observations |
| :--- | :--- | :--- |
| st_data() | st_data() <br> st_sdata() | load numeric data from Stata into matrix <br> load string data from Stata into matrix |
| st_store() | st_store() <br> st_sstore() | store numeric data in Stata dataset <br> store string data in Stata dataset |
| st_view() | st_view() <br> st_sview() | make view onto Stata dataset <br> same; string variables |
| st_subview() | st_subview() | make view from view |
| st_viewvars() | st_viewvars() <br> st_viewobs() | identify variables and observations <br> corresponding to view |
|  |  |  |

Variable names \& indices
st_varindex() st_varindex()
st_varname() st_varname()
variable indices from variable names
variable names from variable indices

Variable characteristics

| st_varrename() | st_varrename() | rename Stata variable |
| :---: | :---: | :---: |
| st_vartype() | ```st_vartype() st_isnumvar() st_isstrvar()``` | storage type of Stata variable whether variable is numeric whether variable is string |
| st_isalias() | ```st_isalias() st_aliasframe() st_aliaslinkname() st_aliasvarname()``` | whether variable is alias linked frame for alias link variable for alias target variable for alias |
| st_varformat () | $\begin{aligned} & \text { st_varformat() } \\ & \text { st_varlabel() } \\ & \text { st_varvaluelabel() } \end{aligned}$ | obtain/set format of Stata variable obtain/set variable label obtain/set value label |
| st_vlexists() | $\begin{aligned} & \text { st_vlexists() } \\ & \text { st_vldrop() } \\ & \text { st_vlmap() } \\ & \text { st_vlsearch() } \\ & \text { st_vlload() } \\ & \text { st_vlmodify() } \end{aligned}$ | whether value label exists drop value <br> map values <br> map text <br> load value label <br> create or modify value label |

Temporary variables \& time-series operators

| st_tempname() | st_tempname () <br> st_tempfilename () | temporary variable name <br> temporary filename |
| :--- | :--- | :--- |
| st_tsrevar() | st_tsrevar () <br>  | create time-series op.varname |

Adding \& removing variables \& observations

| st_addobs() | st_addobs () | add observations to Stata dataset |
| :--- | :--- | :--- |
| st_addvar() | st_addvar() | add variable to Stata dataset |
| st_addalias() | st_addalias() | add alias to Stata dataset |
|  | _st_addalias() | same |
| st_dropvar() | st_dropvar() | drop variables |
|  | st_dropobsin() | drop specified observations |
|  | st_dropobsif () | drop selected observations |
|  | st_keepvar() | keep variables |
|  | st_keepobsin() | keep specified observations |
|  | st_keepobsif() | keep selected observations |



Accessing e()$, \mathrm{r}(), \mathrm{s}()$, macros, matrices, etc.

| st_global() | ```st_global() st_global_hcat()``` | obtain/set Stata global obtain hidden/historical status |
| :---: | :---: | :---: |
| st_local() | st_local() | obtain/set local Stata macro |
| st_numscalar() | ```st_numscalar() st_numscalar_hcat() st_strscalar()``` | obtain/set Stata numeric scalar obtain hidden/historical status obtain/set Stata string scalar |
| st_matrix () | ```st_matrix() st_matrix_hcat() st_matrixrowstripe() st_matrixcolstripe() st_replacematrix()``` | obtain/set Stata matrix obtain hidden/historical status obtain/set row labels obtain/set column labels replace existing Stata matrix |
| st_dir() | st_dir() | obtain list of Stata objects |
| st_rclear() | $\begin{aligned} & \text { st_rclear() } \\ & \text { st_eclear() } \\ & \text { st_sclear() } \end{aligned}$ | clear $\mathrm{r}($ ) <br> clear e() <br> clear s() |

Parsing \& verification

| st_isname() | st_isname() <br> st_islmname() | whether valid Stata name <br> whether valid local macro name |
| :--- | :--- | :--- |
| st_isfmt() | st_isfmt() <br> st_isnumfmt() <br> st_isstrfmt() | whether valid \%fmt <br> whether valid numeric \%fmt <br> whether valid string \% fmt |
| abbrev() | abbrev() | abbreviate strings |

## Data frames

st_frame*()
st_framecurrent() return or change current frame
st_framecreate() make new frame
st_framedrop() drop (eliminate) existing frame
st_framedropabc() drop all but current frame
st_framerename() rename frame
st_framecopy () copy contents of one frame to another
st_framereset() reset to empty default frame
st_frameexists() whether frame name already exists
st_framedir() obtain vector of existing frame names

## Description

The above functions interface with Stata.

## Remarks and examples

The following manual entries have to do with getting data from or putting data into Stata:

| $[\mathrm{M}-5]$ st_data() | Load copy of current Stata dataset |
| :--- | :--- |
| $[\mathrm{M}-5]$ st_view() | Make matrix that is a view onto current Stata dataset |
| $[\mathrm{M}-5]$ st_store( ) | Modify values stored in current Stata dataset |
| $[\mathrm{M}-5]$ st_nvar() | Numbers of variables and observations |

In some cases, you may find yourself needing to translate variable names into variable indices and vice versa:
[M-5] st_varname() Obtain variable names from variable indices
[M-5] st_varindex() Obtain variable indices from variable names
[M-5] st_tsrevar() Create time-series op.varname variables

The other functions mostly have to do with getting and putting Stata's scalars, matrices, and returned results:
[M-5] st_local() Obtain strings from and put strings into Stata
[M-5] st_global() Obtain strings from and put strings into global macros
[M-5] st_numscalar() Obtain values from and put values into Stata scalars
[M-5] st_matrix() Obtain and put Stata matrices

The stata() function, documented in
[M-5] stata() Execute Stata command
allows you to cause Stata to execute a command that you construct in a string.

## Reference

Gould, W. W. 2008. Mata Matters: Macros. Stata Journal 8: 401-412.

## Also see

[M-4] Intro - Categorical guide to Mata functions

## Title

[M-4] Statistical - Statistical functions
Contents Description Remarks and examples Also see

## Contents

[M-5] Manual entry Function
Pseudorandom variates
runiform()

| runiform() | uniform random variates |
| :--- | :--- |
| rnormal() | normal (Gaussian) random variates |
| rseed() | obtain or set the random-variate seed <br> obtain or set the random-number generator state |
| rngstate() |  |
| rbeta() | beta random variates |
| rbinomial() | binomial random variates |
| rcauchy() | Cauchy random variates |
| rchi2() | $\chi^{2}$ random variates |
| rdiscrete() | discrete random variates |
| rexponential() | exponential random variates |
| rgamma() | gamma random variates |
| rhypergeometric() | hypergeometric random variates |
| rigaussian() | inverse Gaussian random variates |
| rlaplace() | Laplace random variates |
| rlogistic() | logistic random variates |
| rnbinomial() | negative binomial random variates |
| rpoisson() | Poisson random variates |
| rt() | Student's $t$ random variates |
| runiformint() | uniform random integer variates |
| rweibull() | Weibull random variates |
| rweibullph() | Weibull (proportional hazards) random variates |

runiform() uniform random variates rnormal()
rseed()
rngstate()
rbeta() rbinomial()
rcauchy()
rchi2()
rdiscrete()
rexponential()
rgamma()
rhypergeometric()
rigaussian()
rlaplace()
rlogistic()
rnbinomial()
rpoisson()
rt()
runiformint() rweibull()
rweibullph()

Purpose
normal (Gaussian) random variates
obtain or set the random-variate seed
obtain or set the random-number generator state
beta random variates
binomial random variates
Cauchy random variates
$\chi^{2}$ random variates
discrete random variates
exponential random variates
gamma random variates
hypergeometric random variates
inverse Gaussian random variates
Laplace random variates
logistic random variates
negative binomial random variates
Poisson random variates
Student's $t$ random variates
uniform random integer variates
Weibull random variates
Weibull (proportional hazards) random variates


## Densities \& distributions

normal()

| normalden() | normal density |
| :---: | :---: |
| normal () | cumulative normal |
| invnormal() | inverse cumulative normal |
| Innormalden() | logarithm of the normal density |
| lnnormal() | logarithm of the cumulative normal |
| binormal() | cumulative binormal |
| Inmvnormalden() | logarithm of the multivariate normal density |
| betaden() | beta density |
| ibeta() | cumulative beta; <br> a.k.a. incomplete beta function |
| ibetatail() | reverse cumulative beta |
| invibeta() | inverse cumulative beta |
| invibetatail() | inverse reverse cumulative beta |
| binomialp() | binomial probability |
| binomial() | cumulative binomial |
| binomialtail() | reverse cumulative binomial |
| invbinomial() | inverse cumulative binomial |
| invbinomialtail() | inverse reverse cumulative binomial |
| cauchyden() | Cauchy density |
| cauchy() | cumulative Cauchy |
| cauchytail() | reverse cumulative Cauchy |
| invcauchy() | inverse cumulative Cauchy |
| invcauchytail() | inverse reverse cumulative Cauchy |
| Incauchyden() | logarithm of the Cauchy density |
| chi2() | cumulative $\chi^{2}$ |
| chi2den() | $\chi^{2}$ density |
| chi2tail() | reverse cumulative $\chi^{2}$ |
| invchi2() | inverse cumulative $\chi^{2}$ |
| invchi2tail() | inverse reverse cumulative $\chi^{2}$ |
| dunnettprob() | cumulative multiple range; used in Dunnett's multiple comparison |
| invdunnettprob() | inverse cumulative multiple range; used in Dunnett's multiple comparison |

normal density
cumulative normal
inverse cumulative normal
logarithm of the normal density
logarithm of the cumulative normal
cumulative binormal
logarithm of the multivariate normal density
beta density
cumulative beta;
a.k.a. incomplete beta function
reverse cumulative beta
inverse cumulative beta
inverse reverse cumulative beta
binomial probability
cumulative binomial
reverse cumulative binomial
inverse cumulative binomial
inverse reverse cumulative binomial

Cauchy density
cumulative Cauchy
reverse cumulative Cauchy
inverse cumulative Cauchy
inverse reverse cumulative Cauchy
logarithm of the Cauchy density
cumulative $\chi^{2}$
$\chi^{2}$ density
reverse cumulative $\chi^{2}$
inverse cumulative $\chi^{2}$
inverse reverse cumulative $\chi^{2}$
cumulative multiple range;
used in Dunnett's multiple comparison
inverse cumulative multiple range; used in Dunnett's multiple comparison
normal(), continued exponentialden()
exponential()
exponentialtail()
invexponential()
invexponentialtail()
Fden()
F()
Ftail()
invF()
invFtail()
gammaden()
gammap()
gammaptail()
invgammap()
invgammaptail()
dgammapda()
dgammapdx()
dgammapdada()
dgammapdadx()
dgammapdxdx()
lnigammaden()
hypergeometricp() hypergeometric()
igaussianden()
igaussian()
igaussiantail()
invigaussian()
invigaussiantail()
lnigaussianden()
laplaceden()
laplace()
laplacetail()
invlaplace()
invlaplacetail()
lnlaplaceden()
exponential density
cumulative exponential reverse cumulative exponential inverse cumulative exponential inverse reverse cumulative exponential
$F$ density
cumulative $F$
reverse cumulative $F$
inverse cumulative $F$
inverse reverse cumulative $F$
gamma density
cumulative gamma;
a.k.a. incomplete gamma function
reverse cumulative gamma;
inverse cumulative gamma
inverse reverse cumulative gamma
$\partial P(a, x) / \partial a$, where $P(a, x)=\operatorname{gammap}(a, x)$
$\partial P(a, x) / \partial x$, where $P(a, x)=\operatorname{gammap}(a, x)$
$\partial^{2} P(a, x) / \partial a^{2}$, where $P(a, x)=\operatorname{gammap}(a, x)$
$\partial^{2} P(a, x) / \partial a \partial x$, where $P(a, x)=\operatorname{gammap}(a, x)$
$\partial^{2} P(a, x) / \partial x^{2}$, where $P(a, x)=\operatorname{gammap}(a, x)$
logarithm of the inverse gamma density
hypergeometric probability
cumulative hypergeometric
inverse Gaussian density
cumulative inverse Gaussian
reverse cumulative inverse Gaussian
inverse cumulative of inverse Gaussian inverse reverse cumulative of inverse Gaussian logarithm of the inverse Gaussian density

Laplace density
cumulative Laplace
reverse cumulative Laplace
inverse cumulative Laplace
inverse reverse cumulative Laplace
logarithm of the Laplace density
normal(), continued logisticden()
logistic()
logistictail()
invlogistic()
invlogistictail()
nbetaden()
nibeta()
invnibeta()
$\qquad$
nbinomialp()
nbinomial()
nbinomialtail()
invnbinomial()
invnbinomialtail()
nchi2()
nchi2den()
nchi2tail()
invnchi2() invnchi2tail()
npnchi2()
nF ()
nFden()
nFtail()
invnF()
invnFtail()
npnF()
nt ()
ntden()
nttail()
invnt()
invnttail()
npnt()
poissonp()
poisson()
poissontail()
invpoisson()
invpoissontail()
logistic density
cumulative logistic
reverse cumulative logistic
inverse cumulative logistic
inverse reverse cumulative logistic
noncentral beta density
cumulative noncentral beta
inverse cumulative noncentral beta
negative binomial probability
cumulative negative binomial reverse cumulative negative binomial inverse cumulative negative binomial inverse reverse cumulative negative binomial
cumulative noncentral $\chi^{2}$
noncentral $\chi^{2}$ density
reverse cumulative noncentral $\chi^{2}$
inverse cumulative noncentral $\chi^{2}$
inverse reverse cumulative noncentral $\chi^{2}$
noncentrality parameter of nchi2()
cumulative noncentral $F$
noncentral $F$ density
reverse cumulative noncentral $F$
inverse cumulative noncentral $F$
inverse reverse cumulative noncentral $F$
noncentrality parameter of nF()
cumulative noncentral Student's $t$
noncentral Student's $t$ density
reverse cumulative noncentral $t$
inverse cumulative noncentral $t$
inverse reverse cumulative noncentral $t$
noncentrality parameter of nt ()

Poisson probability
cumulative Poisson
reverse cumulative Poisson
inverse cumulative Poisson
inverse reverse cumulative Poisson

| normal(), continued |  | cumulative Student's $t$ |
| :---: | :---: | :---: |
|  | tden() | Student's $t$ density |
|  | ttail() | reverse cumulative Student's $t$ |
|  | invt() | inverse cumulative Student's $t$ |
|  | invttail() | inverse reverse cumulative Student's $t$ |
|  |  |  |
|  | tukeyprob() | cumulative multiple range; used in Tukey's multiple comparison |
|  | invtukeyprob() | inverse cumulative multiple range; used in Tukey's multiple comparison |
|  | weibullden() | Weibull density |
|  | weibull() | cumulative Weibull |
|  | weibulltail() | reverse cumulative Weibull |
|  | invweibull() | inverse cumulative Weibull |
|  | invweibulltail() | inverse reverse cumulative Weibull |
|  |  |  |
|  | ```weibullphden() weibullph()``` | Weibull (proportional hazards) density cumulative Weibull (proportional hazards) |
|  | weibullphtail() | reverse cumulative Weibull (proportional hazards) |
|  | invweibullph() | inverse cumulative Weibull (proportional hazards) |
|  | invweibullphtail() | inverse reverse cumulative Weibull (proportional hazards) |
|  |  |  |
|  | lnwishartden() | logarithm of the Wishart density |
|  | lniwishartden() | logarithm of the inverse-Wishart density |
| mvnormal() | mvnormal() | multivariate normal probabilities (correlation specified) |
|  | mvnormalcv() | multivariate normal probabilities (covariance specified) |
|  | mvnormalqp() | mvnormal () with specified quadrature points |
|  | mvnormalcvqp() | mvnormalcv() with specified quadrature points |
|  | mvnormalderiv() | derivatives of mvnormal () |
|  | mvnormalcvderiv() | derivatives of mvnormalcv() |
|  | mvnormaldervqp() | mvnormalderiv() with specified quadrature points |
|  | mvnormalcvderivqp() | mvnormalcvderiv() with specified quadrature points |

Maximization \& minimization

| optimize() | $\begin{aligned} & \text { optimize() } \\ & \text { optimize_evaluate() } \\ & \text { optimize_init() } \\ & \text { optimize_init_*() } \\ & \text { optimize_result_*() } \\ & \text { optimize_query() } \end{aligned}$ | function maximization and minimization evaluate function at initial values begin optimization set details access results report settings |
| :---: | :---: | :---: |
| moptimize() | ```moptimize() moptimize_evaluate() moptimize_init() moptimize_init_*() moptimize_result_*() moptimize_ado_cleanup() moptimize_query() moptimize_util_*()``` | function optimization evaluate function at initial values begin setup of optimization problem set details access moptimize() results perform cleanup after ado report settings utility functions for writing evaluators and processing results |
| LinearProgram( ) | LinearProgram() | linear programming |
| Logits, odds, \& related |  |  |
| $\operatorname{logit}()$ | ```logit() invlogit() cloglog() invcloglog()``` | $\log$ of the odds ratio inverse $\log$ of the odds ratio complementary log-log inverse complementary log-log |
| Multivariate normal |  |  |
| ghk() | ```ghk() ghk_init() ghk_init_*() ghk() ghk_query_npts()``` | GHK multivariate normal (MVN) simulator GHK MVN initialization <br> set details <br> perform simulation return number of simulation points |
| ghkfast() | ```ghkfast() ghkfast_init() ghkfast_init_*() ghkfast() ghkfast_i() ghk_query_*()``` | GHK MVN simulator GHK MVN initialization set details perform simulation results for the $i$ th observation display settings |

## Description

The above functions are statistical, probabilistic, or designed to work with data matrices.

## Remarks and examples

Concerning data matrices, see
[M-4] Stata Stata interface functions
and especially
[M-5] st_data() Load copy of current Stata dataset
[M-5] st_view() Make matrix that is a view onto current Stata dataset
For other mathematical functions, see
[M-4] Matrix Matrix mathematical functions
[M-4] Scalar Scalar mathematical functions
[M-4] Mathematical Important mathematical functions

## Also see

[M-4] Intro - Categorical guide to Mata functions

## Title

[M-4] String - String manipulation functions
Contents Description Remarks and examples Also see

## Contents

[M-5] Manual entryFunction

|  |  |
| :--- | :--- |
| tokens() Parsing |  |
| invtokens() | tokens() |
| invtokens() |  |
| ustrword() | ustrword() <br> ustrwordcount () <br> strmatch() |
| strmatch() |  |
| tokenget() | $\ldots$ |
| ustrsplit() | ustrsplit() |

Purpose
obtain tokens (words) from string
concatenate string vector into string scalar
return $n$th Unicode word
return the number of Unicode words
pattern matching
advanced parsing
split string into parts based on a Unicode regular expression

| strlen() | strlen() | length of string in bytes |
| :---: | :---: | :---: |
| ustrlen() | ustrlen() | length of string in Unicode characters |
| udstrlen() | udstrlen() | length of string in display columns |
| fmtwidth() | fmtwidth() | width of \%fmt |
| strpos() | $\begin{aligned} & \text { strpos() } \\ & \text { strrpos() } \end{aligned}$ | find substring within string from left find substring within string from right |
| ustrpos() | ustrpos() <br> ustrrpos() | find Unicode substring within string, first occurrence find Unicode substring within string, last occurrence |
| indexnot() | indexnot() | find character not in list |


|  | Editing |  |
| :--- | :--- | :--- |
|  |  |  |
| substr() | substr () | extract substring |
| usubstr() | usubstr () | extract Unicode substring |
| udsubstr() | udsubstr() | extract Unicode substring based on display columns |


| strupper( ) | strupper () |
| :---: | :---: |
|  | strlower () |
|  | strproper() |
| ustrupper( ) | ustrupper() |
|  | ustrlower() |
|  | ustrtitle() |
| strtrim( ) | stritrim() |
|  | strltrim() |
|  | strrtrim() |
|  | strtrim() |
| ustrtrim( ) | ustrtrim() |
|  | ustrltrim() |
|  | ustrrtrim() |
| subinstr( ) | subinstr () |
|  | subinword() |
| usubinstr( ) | usubinstr() |
| _substr( ) | _substr () |
| _usubstr ( ) | _usubstr () |
| strdup( ) | * |
| strreverse( ) | strreverse() |
| ustrreverse( ) | ustrreverse() |
| soundex() | soundex () |
|  | soundex_nara() |

convert to uppercase convert to lowercase convert to proper case
convert Unicode characters to uppercase convert Unicode characters to lowercase convert Unicode characters to titlecase
replace multiple, consecutive internal blanks with one blank
remove leading blanks
remove trailing blanks
remove leading and trailing blanks
remove leading and trailing Unicode whitespace characters and blanks
remove leading Unicode whitespace characters and blanks
remove trailing Unicode whitespace characters and blanks
substitute text
substitute word
replace Unicode substring
substitute into string
substitute into Unicode string
duplicate string
reverse string in bytes
reverse string in Unicode characters
convert to soundex code
convert to U.S. Census soundex code

## Stata

| $\operatorname{abbrev}()$ | $\operatorname{abbrev}()$ | abbreviate Unicode strings to display columns |
| :--- | :--- | :--- |
| strtoname() | strtoname() | translate strings to Stata 13 compatible names |
| ustrtoname() | ustrtoname() | translate Unicode strings to Stata names |



## Description

The above functions are for manipulating strings. Strings in Mata are strings of Unicode characters in UTF-8 encoding, usually the printable characters, but Mata enforces no such restriction. In particular, strings may contain binary 0 .

## Remarks and examples

In addition to the above functions, two operators are especially useful for dealing with strings.
The first is + . Addition is how you concatenate strings:

```
: "abc" + "def"
abcdef
: "Café " + "de Flore"
Café de Flore
: command = "list"
: args = "mpg weight"
: result = command + " " + args
: result
list mpg weight
```

The second is $*$. Multiplication is how you duplicate strings:

```
: 5*"a"
aaaaa
: "Allô"*2
AllôAllô
: indent = 20
: title = indent*" " + "My Title"
: title
    My Title
```


## Also see

[M-4] Intro - Categorical guide to Mata functions

## Title

[M-4] Utility — Matrix utility functions
Contents Description Remarks and examples Also see

## Contents

$\frac{\text { [M-5] Manual entry }}{\text { Complex }}$

| $\operatorname{Re}()$ | $\operatorname{Re}()$ <br> $\operatorname{Im}()$ | real part <br> imaginary part <br> C() |
| :--- | :--- | :--- |
|  |  | C() |


| rows() | rows() | number of rows |
| :---: | :---: | :---: |
|  | cols() | number of columns |
|  | length() | number of elements of vector |
| eltype() | eltype() | element type of object |
|  | orgtype() | organizational type of object |
|  | classname() | class name of a Mata class scalar |
|  | structname() | struct name of a Mata struct scalar |
| isreal() | isreal() | object is real matrix |
|  | iscomplex() | object is complex matrix |
|  | isstring() | object is string matrix |
|  | ispointer() | object is pointer matrix |
| isrealvalues() | isrealvalues() | whether matrix contains only real values |
| isview( ) | isview() | whether matrix is view |

Properties

| issymmetric() | issymmetric() <br> issymmetriconly() |
| :--- | :--- |
| isdiagonal() | isdiagonal() |
| diag0cnt() | diag0cnt() |

whether matrix is symmetric (Hermitian) whether matrix is mechanically symmetric whether matrix is diagonal count 0s on diagonal

## Selection

| select() | select() |
| :--- | :--- |
|  | st_select () |
|  | selectindex () |

select rows or columns select rows or columns of view select indices

Missing values

| $\operatorname{missing}()$ | missing() | count of missing values |
| :--- | :--- | :--- |
| rowmissing() | count of missing values, by row |  |
| colmissing() | count of missing values, by column |  |
|  | nonmissing() | count of nonmissing values |
|  | rownonmissing() | count of nonmissing values, by row |
| colnonmissing() | count of nonmissing values, by column <br> whether matrix has missing values |  |
| masmissing() | missingof() | appropriate missing value |

Range, sums, \& cross products

| minmax () | rowmin() |
| :---: | :---: |
|  | colmin() |
|  | min() |
|  | rowmax () |
|  | colmax () |
|  | $\max ()$ |
|  | rowminmax() |
|  | colminmax() |
|  | minmax () |
|  | rowmaxabs() |
|  | colmaxabs() |
| minindex() | minindex() |
|  | maxindex() |
| $\operatorname{sum}()$ | rowsum() |
|  | colsum() |
|  | sum() |
|  | quadrowsum() |
|  | quadcolsum() |
|  | quadsum() |

minimum, by row
minimum, by column
minimum, overall
maximum, by row
maximum, by column
maximum, overall
minimum and maximum, by row
minimum and maximum, by column
minimum and maximum, overall rowmax (abs())
colmax (abs())
indices of minimums
indices of maximums
sum of each row
sum of each column
overall sum
quad-precision sum of each row quad-precision sum of each column quad-precision overall sum

Range, sums, \& cross products, continued

| runningsum() | runningsum() <br> quadrunningsum() | running sum of vector quad-precision runningsum() |
| :---: | :---: | :---: |
| panelsum() | panelsum() | within-panel sum of each column |
| cross() | cross() | $X^{\prime} X, X^{\prime} Z$, etc. |
| crossdev() | crossdev() | $(X:-x)^{\prime}(X:-x),(X:-x)^{\prime}(Z:-z)$, etc. |
| quadcross( ) | quadcross() <br> quadcrossdev() | quad-precision cross () quad-precision crossdev() |
| Programming |  |  |
| reldif() | reldif() | relative difference |
|  | mreldif() | max. relative difference between matrices |
|  | mreldifsym() | max. relative difference from symmetry |
|  | mreldifre() | max. relative difference from real |
| all() | all() | $\operatorname{sum}(!L)==0$ |
|  | any() | $\operatorname{sum}(L)!=0$ |
|  | allof() | $\operatorname{all}(P:==s)$ |
|  | anyof() | any ( $P:==s$ ) |
| panelsetup() | panelsetup() | initialize panel-data processing |
|  | panelstats() | summary statistics on panels |
|  | panelsubmatrix() | obtain matrix for panel $i$ |
|  | panelsubview() | obtain view matrix for panel $i$ |
| _negate() | _negate() | fast negation of matrix |

Constants \& tolerances

| mindouble() | mindouble() <br> maxdouble() <br> smallestdouble() | minimum nonmissing value <br> maximum nonmissing value <br> smallest $e>0$ |
| :--- | :--- | :--- |
| epsilon() | epsilon() | unit roundoff error |
| floatround() | floatround() | round to float precision |
| solve_tol() | solve_tol() | tolerance used by solvers and inverters |

## Description

Matrix utility functions tell you something about the matrix, such as the number of rows or whether it is diagonal.

## Remarks and examples

There is a thin line between utility and manipulation; also see
[M-4] Manipulation Matrix manipulation functions

## Also see

[M-4] Intro - Categorical guide to Mata functions

# [M-5] Alphabetical index to Mata functions 

## Title

[M-5] Intro - Alphabetical index to Mata functions
Contents Description Remarks and examples Also see

## Contents

See [M-4] Intro

## Description

The following entries, alphabetically arranged, document all the Mata functions.

## Remarks and examples

See [M-4] Intro for an index, grouped logically, of the functions presented in this section.

## Also see

[M-4] Intro - Categorical guide to Mata functions
[M-0] Intro - Introduction to the Mata manual

## Title

[ $\mathrm{M}-5$ ] abbrev() - Abbreviate strings

```
Description Syntax Conformability Diagnostics Also see
```


## Description

$\operatorname{abbrev}(s, n)$ returns $s$ abbreviated to $n$ display columns. Usually, this means it will be abbreviated to $n$ characters, but if $s$ contains characters requiring more than one display column, such as Chinese, Japanese, and Korean (CJK), $s$ will be abbreviated such that it does not exceed $n$ display columns.

1. $n$ is the abbreviation length and is assumed to contain integer values in the range $5,6, \ldots$, 32.
2. If $s$ contains a period, ., and $n<8$, then the value $n$ defaults to 8 . Otherwise, if $n<5$, then $n$ defaults to 5 .
3. If $n$ is missing, the entire string (up to the first binary 0 ) is returned.

If there is a binary 0 in $s$, the abbreviation is derived from the beginning of the string up to but not including the binary 0 .

When arguments are not scalar, abbrev() returns element-by-element results.

## Syntax

string matrix abbrev(string matrix $s$, real matrix $n$ )

## Conformability

```
abbrev (s, n):
    s: }\quad\mp@subsup{r}{1}{}\times\mp@subsup{c}{1}{
    n:}\quad\mp@subsup{r}{2}{}\times\mp@subsup{c}{2}{};s\mathrm{ and }n\mathrm{ r-conformable
    result: }\quad\operatorname{max}(\mp@subsup{r}{1}{},\mp@subsup{r}{2}{})\times\operatorname{max}(\mp@subsup{c}{1}{},\mp@subsup{c}{2}{}
```


## Diagnostics

abbrev() returns "" if $s$ is " ". abbrev() aborts with error if $s$ is not a string.

## Also see

[M-4] String - String manipulation functions

## Title

[M-5] abs() — Absolute value (length)
Description Syntax Conformability Diagnostics Also see

## Description

For $Z$ real, abs $(Z)$ returns the elementwise absolute values of $Z$.
For $Z$ complex, abs $(Z)$ returns the elementwise length of each element. If $Z=a+b i$, returned is sqrt $\left(a^{2}+b^{2}\right)$, although the calculation is not made in that way. The method actually used prevents overflow.

## Syntax

real matrix abs (numeric matrix $Z$ )

## Conformability

```
abs(Z):
```

| Z: | $r \times c$ |
| ---: | :--- |
| result: | $r \times c$ |

## Diagnostics

abs (.) returns . (missing).

## Also see

[M-4] Scalar - Scalar mathematical functions

## Title

[M-5] adosubdir( ) - Determine ado-subdirectory for file

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

adosubdir (filename) returns the subdirectory in which Stata would search for filename. Typically, the subdirectory will be simply the first letter of filename. However, certain files may result in a different subdirectory, depending on their extension.

## Syntax

string scalar adosubdir (string scalar filename)

## Remarks and examples

adosubdir("xyz.ado") returns "x" because Stata ado-files are located in subdirectories with name given by their first letter.
adosubdir("xyz.style") returns "style" because Stata style files are located in subdirectories named style.

## Conformability

```
adosubdir(filename):
    filename: }1\times
        result: }1\times
```


## Diagnostics

adosubdir() returns the first letter of the filename if the filetype is unknown to Stata, thus treating unknown filetypes as if they were ado-files.
adosubdir () aborts with error if the filename is too long for the operating system; nothing else causes abort with error.

## Also see

[M-4] IO - I/O functions

## Title

[M-5] all() — Element comparisons

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

$\operatorname{all}(L)$ is equivalent to $\operatorname{sum}(!L)==0$ but is significantly faster.
any $(L)$ is equivalent to $\operatorname{sum}(L)!=0$ but is slightly faster.
allof $(P, s)$ returns 1 if every element of $P$ equals $s$ and returns 0 otherwise. allof $(P, s)$ is faster and consumes less memory than the equivalent construction all ( $P:==s$ ).
anyof $(P, s)$ returns 1 if any element of $P$ equals $s$ and returns 0 otherwise. anyof $(P, s)$ is faster and consumes less memory than the equivalent any $(P:==s)$.

## Syntax

> real scalar all $($ real matrix $L)$
> real scalar any $($ real matrix $L)$
real scalar allof (transmorphic matrix $P$, transmorphic scalar $s$ )
real scalar anyof(transmorphic matrix $P$, transmorphic scalar $s$ )

## Remarks and examples

These functions are fast, so their use is encouraged over alternative constructions.
all () and any () are typically used with logical expressions to detect special cases, such as

```
    if (any(x :< 0)) {
    }
```

or
if (all(x :>= 0)) \{
\}
allof() and anyof() are used to look for special values:

```
    if (allof(x, 0)) {
    }
```

or

```
if (anyof(x, 0)) {
}
```

Do not use allof() and anyof() to check for missing values-for example, anyof (x, .) -because to really check, you would have to check not only . but also .a, .b, .., . z. Instead use missing (); see [M-5] missing().

## Conformability

all ( $L$ ), any ( $L$ ):

| $L:$ | $r \times c$ |
| ---: | :--- |
| result $:$ |  |
|  | $1 \times 1$ |

allof $(P, s)$, anyof $(P, s)$ :
$P: \quad r \times c$
$s: \quad 1 \times 1$
result: $\quad 1 \times 1$

## Diagnostics

all ( $L$ ) and any $(L)$ treat missing values in $L$ as true.
all ( $L$ ) and any ( $L$ ) return 0 (false) if $L$ is $r \times 0,0 \times c$, or $0 \times 0$.
allof $(P, s)$ and anyof $(P, s)$ return 0 (false) if $P$ is $r \times 0,0 \times c$, or $0 \times 0$.

## Also see

[M-4] Utility - Matrix utility functions

## Title

[M-5] args() — Number of arguments

Description Syntax Conformability Diagnostics Also see

## Description

$\operatorname{args}()$ returns the number of arguments actually passed to the function; see [M-2] optargs.

## Syntax

```
real scalar args()
```


## Conformability

```
args():
result: }1\times
```


## Diagnostics

None.

## Also see

[M-4] Programming — Programming functions

## Title

[M-5] asarray( ) - Associative arrays

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

asarray () provides one- and multi-dimensional associative arrays, also known as containers, maps, dictionaries, indices, and hash tables.

Also see [M-5] AssociativeArray() for a class-based interface into the functions documented here.

## Syntax

```
            A= asarray_create(})[{,\mathrm{ keytype declare A
        asarray(A, key, a)
a= asarray(A,key)
    asarray_remove(A, key)
    bool = asarray_contains(A, key)
    N = asarray_elements(A)
    keys = asarray_keys(A) all keys in A
    loc = asarray_first(A) location of first element
        or NULL
    loc= asarray_next (A,loc)
    key = asarray_key (A,loc)
    a= asarray_contents(A,loc)
        asarray_notfound(A, notfound) set notfound value
notfound = asarray_notfound (A) query notfound value
```

where
A: Associative array A[key]. Created by asarray_create() and passed to the other functions. If $A$ is declared, it is declared transmorphic.
keytype: Element type of keys; "string", "real", "complex", or "pointer". Optional; default "string".
keydim: Dimension of key; $1 \leq$ keydim $\leq 50$. Optional; default 1 .
minsize: Initial size of hash table used to speed locating keys in A; real scalar; $5 \leq$ minsize $\leq 1,431,655,764$. Optional; default 100.
minratio: Fraction filled at which hash table is automatically downsized; real scalar; $0 \leq$ minratio $\leq 1$. Optional; default 0.5 .
maxratio: Fraction filled at which hash table is automatically upsized; real scalar; $1<$ maxratio $\leq$. (meaning infinity). Optional; default 1.5.
key: Key under which an element is stored in A; string scalar by default; type and dimension are declared using asarray_create().
$a$ : Element of $A$; transmorphic; may be anything of any size; different $A[k e y]$ elements may have different types of contents.
bool: Boolean logic value; real scalar; equal to 1 or 0 meaning true or false.
$N$ : Number of elements stored in $A$; real scalar; $0 \leq N \leq 2,147,483,647$.
keys: List of all keys that exist in $A$. Each row is a key. Thus keys is a string colvector if keys are string scalars, a string matrix if keys are string vectors, a real colvector if keys are real scalars, etc. Note that rows (keys) $=N$.
loc: A location in $A$; transmorphic. The first location is returned by asarray_first(), subsequent locations by asarray_next(). loc==NULL when there are no more elements.
notfound: Value returned by asarray ( $A$, key) when key does not exist in $A$. notfound $=\mathrm{J}(0,0,$.$) by default.$

## Remarks and examples

Before writing a program using asarray(), you should try it interactively. Remarks are presented under the following headings:

[^7]
## Detailed description

In associative arrays, rather than being dense integers, the indices can be anything, even strings. So you might have A["Frank Smith"] equal to something and A["Mary Jones"] equal to something else. In Mata, you write that as asarray ( $A$, "Frank Smith", something) and asarray ( $A$, "Mary Jones", somethingelse) to define the elements and asarray ( $A$, "Frank Smith") and asarray ( $A$, "Mary Jones") to obtain their values.
$A=$ asarray_create() declares (creates) an associative array. The function allows arguments, but they are optional. Without arguments, asarray_create() declares an associative array with string scalar keys, corresponding to the A["Frank Smith"] and A["Mary Jones"] example above.
$A=$ asarray_create(keytype, keydim) declares an associative array with keytype keys each of dimension $1 \times$ keydim. asarray_create("string", 1) is equivalent to asarray_create() without arguments. asarray_create("string", 2) declares the keys to be string, as before, but now they are $1 \times 2$ rather than $1 \times 1$, so array elements would be of the form A[ "Frank Smith", "Mary Jones"]. A["Mary Jones", "Frank Smith"] would be a different element. asarray_create ("real", 2) declares the keys to be real $1 \times 2$, which would somewhat correspond to our ordinary idea of a matrix, namely $A[i, j]$. The difference would be that to store, say, A[100, 980], it would not be necessary to store the interior elements, and in addition to storing A[100, 980], we could store $A[3.14159,2.71828]$.
asarray_create() has three more optional arguments: minsize, minratio, and maxratio. We recommend that you do not specify them. They are discussed in Setting the efficiency parameters under Remarks and examples below.
$\operatorname{asarray}(A, k e y, a)$ sets or resets element $A[k e y]=a$. Note that if you declare key to be $1 \times 2$, you must use the parentheses vector notation to specify key literals, such as asarray $(A,(100,980)$, $2.15)$. Alternatively, if $\mathrm{k}=(100,980)$, then you can omit the parentheses in asarray $(A, \mathrm{k}$, 2.15).
asarray ( $A$, key) returns element $A[k e y]$ or it returns notfound if the element does not exist. By default, notfound is $\mathrm{J}(0,0,$.$) , but you can change that using asarray_notfound(). If you$ redefined notfound to be 0 and defined keys to be real $1 \times 2$, you would be on your way to recording sparse matrices efficiently.
asarray_remove (A, key) removes A[key], or it does nothing if $A[k e y]$ is already undefined.
asarray_contains ( $A$, key) returns 1 if $A[k e y]$ is defined, and it returns 0 otherwise.
asarray_elements $(A)$ returns the number of elements stored in $A$.
asarray_keys $(A)$ returns a vector or matrix containing all the keys, one to a row. The keys are not in alphabetical or numerical order. If you want them that way, code sort (asarray_keys $(A), 1$ ) if your keys are scalar, or in general, code sort (asarray_keys (A), idx); see [M-5] sort().
asarray_first ( $A$ ) and asarray_next ( $A, l o c$ ) provide a way of obtaining the names one at a time. Code

```
for (loc=asarray_first(A); loc!=NULL; loc=asarray_next(A, loc)) {
```

asarray_key $(A, l o c)$ and asarray_contents $(A, l o c)$ return the key and contents at $l o c$, so the loop becomes

```
for (loc=asarray_first(A); loc!=NULL; loc=asarray_next(A, loc)) {
    ... asarray_key(A, loc) ...
    ...
    ... asarray_contents(A, loc) ...
    ...
}
```

asarray_notfound ( $A$, notfound) defines what asarray ( $A$, key) returns when the element does not exist. By default, notfound is $\mathrm{J}(0,0,$.$) , which is to say, a 0 \times 0$ real matrix. You can reset notfound at any time. asarray_notfound ( $A$ ) returns the current value of notfound.

## Example 1: Scalar keys and scalar contents

```
A = asarray_create()
asarray(A, "bill", 1.25)
asarray(A, "mary", 2.75)
asarray(A, "dan", 1.50)
asarray(A, "bill")
1.25
asarray(A, "mary")
2.75
asarray(A, "mary", 3.25)
asarray(A, "mary")
3.25
sum = 0
for (loc=asarray_first(A); loc!=NULL; loc=asarray_next(A, loc)) {
    sum = sum + asarray_contents(A, loc)
}
sum
6
sum/asarray_elements(A)
2
```


## Example 2: Scalar keys and matrix contents

```
: A = asarray_create()
: asarray(A, "Count", (1,2\3,4))
: asarray(A, "Hilbert", Hilbert(3))
asarray(A, "Count")
            1 2
    \begin{array} { | l l } { \hline 1 } & { 2 } \\ { 3 } & { 4 } \\ { \hline } \end{array}
: asarray(A, "Hilbert")
[symmetric]
```

1
2
1
. 5
. 3333333333
.3333333333 . 25
. 2

## Example 3: Vector keys and scalar contents; sparse matrix

```
A = asarray_create("real", 2)
asarray_notfound(A, 0)
asarray(A, (1, 1), 1)
asarray(A, (1000, 999), .5)
asarray(A, (1000, 1000), 1)
asarray(A, (1000, 1001), .5)
asarray(A, (1,1))
1
asarray(A, (2,2))
O
// one way to get the trace:
trace = 0
for (i=1; i<=1000; i++) trace = trace + asarray(A, (i,i))
trace
2
// another way to get the trace
trace = 0
for (loc=asarray_first(A); loc!=NULL; loc=asarray_next(A, loc)) {
    index = asarray_key(A, loc)
    if (index[1]==index[2]) {
        trace = trace + asarray_contents(A, loc)
    }
}
trace
2
```


## Setting the efficiency parameters

The syntax asarray_create() is

$$
A=\text { asarray_create(keytype, keydim, minsize, minratio, maxratio) }
$$

All arguments are optional. The first two specify the characteristics of the key and their use has already been illustrated. The last three are efficiency parameters. In most circumstances, we recommend you do not specify them. The default values have been chosen to produce reasonable execution times with reasonable memory consumption.
asarray () works via hash tables. Say we wish to record $n$ entries. The idea is to allocate a hash table of $N$ rows, where $N$ can be less than, equal to, or greater than $n$. When one needs to find the element corresponding to a key, one calculates a function of the key, called a hash function, that returns an integer $h$ from 1 to $N$. One first looks in row $h$. If row $h$ is already in use and the keys are different, we have a collision. In that case, we have to allocate a duplicates list for the $h$ th row and put the duplicate keys and contents into that list. Collisions are bad because, when they occur, asarray () has to allocate a duplicates list, requiring both time and memory, though it does not require much. When fetching results, if row $h$ has a duplicates list, asarray() has to search the list, which it does sequentially, and that takes extra time, too. Hash tables work best when collisions happen rarely.

Obviously, collisions are certain to occur if $N<n$. Note, however, that although performance suffers, the method does not break. A hash table of $N$ can hold any number of entries, even if $N<n$.

Performance depends on details of implementation. We have examined the behavior of asarray() and discovered that collisions rarely occur when $n / N \leq 0.75$. When $n / N=1.5$, performance suffers, but not by as much as you might expect. Around $n / N=2$, performance degrades considerably.

When you add or remove an element, asarray () examines $n / N$ and considers rebuilding the table with a larger or smaller $N$; it rebuilds the table when $n / N$ is large to preserve efficiency. It rebuilds the table when $n / N$ is small to conserve memory. Rebuilding the table is a computer-intensive operation, and so should not be performed too often.

In making these decisions, asarray() uses three parameters:
maxratio: When $n / N \geq$ maxratio, the table is upsized to $N=1.5 n$.
minratio: When $n / N \leq$ minratio 1.5 , the table is downsized to $N=1.5 n$. (For an exception, see minsize.)
minsize: If the new $N<1.5$ minsize, the table is downsized to $N=1.5$ minsize if it is not already that size.

The default values of the three parameters are $1.5,0.5$, and 100 . You can reset them, though you are unlikely to improve on the default values of minratio and maxratio.

You can improve on minsize when you know the number of elements that will be in the table and that number is greater than 100 . For instance, if you know the table will contain at least 1,000 elements, starting minsize at 1,000 , which implies $N=1,500$, will prevent two rescalings, namely, from 150 to 451 , and from 451 to 1,354 . This saves a little time.

You can also turn off the resizing features. Setting minratio to 0 turns off downsizing. Setting maxratio to . (missing) turns off upsizing. You might want to turn off both downsizing and upsizing if you set minsize sufficiently large for your problem.

We would never recommend turning off upsizing alone, and we seldom would recommend turning off downsizing alone. In a program where it is known that the array will exist for only a short time, however, turning off downsizing can be efficient. In a program where the array might exist for a considerable time, turning off downsizing is dangerous because then the array could only grow (and probably will).

## Conformability

```
asarray_create(keytype, keydim, minsize, minratio, maxratio):
    keytype: 1 1 1 (optional)
    keydim: }1\times1\quad\mathrm{ (optional)
    minsize: }1\times1\quad\mathrm{ (optional)
    minratio: }1\times1\quad\mathrm{ (optional)
    maxratio: }1\times1\quad\mathrm{ (optional)
        result: transmorphic
asarray(A, key, a):
            A: transmorphic
            key: }1\times\mathrm{ keydim
            a: }\quad\mp@subsup{r}{\mathrm{ key }}{}\times\mp@subsup{c}{\mathrm{ key }}{
            result: void
asarray(A, key):
            A: transmorphic
            key: }1\times\mathrm{ keydim
            result: }\quad\mp@subsup{r}{key}{}\times\mp@subsup{c}{key}{
asarray_remove(A, key):
                            A: transmorphic
            key: }1\times\mathrm{ keydim
    result: void
asarray_contains(A, key), asarray_elements(A, key):
            A: transmorphic
            key: }1\times\mathrm{ keydim
    result: }\quad1\times
asarray_keys(A, key):
            A: transmorphic
            key: }1\times\mathrm{ keydim
            result: }n\times\mathrm{ keydim
asarray_first(A):
A: transmorphic
result: transmorphic
asarray_first ( \(A, l o c\) ):
A: transmorphic
loc: transmorphic
result: transmorphic
asarray_key \((A, l o c)\) :
A: transmorphic
loc: transmorphic
result: \(1 \times\) keydim
```

asarray_contents ( $A, l o c$ ):
A: transmorphic
loc: transmorphic
result: $\quad r_{\text {key }} \times c_{\text {key }}$
asarray_notfound ( $A$, notfound):
A: transmorphic
notfound: $\quad r \times c$
result: void
asarray_notfound ( $A$ ):
A: transmorphic
result: $\quad r \times c$

## Diagnostics

None.

## Also see

[M-5] AssociativeArray ( ) - Associative arrays (class)
[M-5] hash1() - Jenkins's one-at-a-time hash function
[M-4] Manipulation - Matrix manipulation
[M-4] Programming — Programming functions

## Title

[M-5] AssociativeArray() - Associative arrays (class)

Description Syntax Remarks and examples Conformability

## Description

AssociativeArray provides a class-based interface into the associative arrays provided by asarray (); see [M-5] asarray( ). The class-based interface provides more tersely named functions, making code written with it easier to read.

Associative arrays are also known as containers, maps, dictionaries, indices, and hash tables.

## Syntax

| class AssociativeArray <br> or <br> A = AssociativeArray () | create array with string scalar keys create array with string scalar keys |
| :---: | :---: |
| A.reinit([ keytype | "string", "real", "complex", |
| [, keydim | 1 to 50 |
| [, minsize | tuning parameter |
| [, minratio | tuning parameter |
| [, maxratio ]]]]]) | tuning parameter |
| A.put (key, val) | A[key] = val |
| val $=A . \operatorname{get}(k e y)$ | val $=$ A[key] or val $=$ notfound |
| A.notfound (notfound) | change notfound value |
| notfound $=A$. notfound () | query notfound value |
| A.remove (key) | delete A[key] if it exists |
| bool $=$ A. $\mathrm{exists}(k e y)$ | A[key] exists? |
| val $=$ A.firstval () | first val or notfound |
| val $=A$. nextval () | next val or notfound |
| $k e y=A . \operatorname{key}()$ | key corresponding to val |
| $v a l=A . v a l()$ | val yet again |
| $l o c=A . f i r s t l o c() ~$ | first location or NULL |
| $l o c=A . n e x t l o c() ~$ | next location or NULL |
| $k e y=A . \operatorname{key}(l o c)$ | key at location |
| $v a l=A . v a l(l o c)$ | val at location |


| keys $=A \cdot$ keys () | $N \times$ keydim matrix of defined keys |
| :--- | :--- |
| $N=A \cdot N()$ | $N$, number of defined keys |
| $A . c l e a r()$ | clear array; set $N$ equal to 0 |

## Remarks and examples

Array is computer jargon. A one-dimensional array is a vector with elements $A[i]$. A two-dimensional array is a matrix with elements $A[i, j]$. A three-dimensional array generalizes a matrix to three dimensions with elements $A[i, j, k]$, and so on.

An associative array is an array where the indices are not necessarily integers. Most commonly, the indices are strings, so you might have A["bill"], A["bill", "bob"], A["bill", "bob", "mary"], etc.

Associative arrays created by AssociativeArray are one-dimensional, and the keys are string by default. At the same time, the elements may be of any type and even vary element by element. Such an array is created when you code

```
function foo(...)
{
    class AssociativeArray scalar A
}
```

or, if you are working interactively, you type

```
: A = AssociativeArray()
```

Either way, $A$ is now a one-dimensional array with string keys. This is the style of associative array most users will want, but if you want a different style, say, a two-dimensional array with real-number keys, you next use $A$.reinit () to change the style. You code

```
function foo(...)
{
    class AssociativeArray scalar A
    A.reinit("real", 2)
}
```

or you interactively type
: $A=$ AssociativeArray()
: A.reinit("real", 2)
This associative array will be like a matrix in that you can store elements such as $A[1,2], A[2,1]$ :

```
:A.put((1,2), 5)
:A.put((2,1), -2)
```

A.put() is how we define elements or change the contents of elements that are already defined. If we typed

```
:A.put((2,1), -5)
```

$A[2,1]$ would change from -2 to -5 . The first argument of $A . p u t()$ is the key (think indices), and the second argument the value to be assigned. The first argument is enclosed in parentheses because $A$ is a two-dimensional array, and thus keys are $1 \times 2$.

If we now coded

$$
\begin{aligned}
& \mathrm{x}=A \cdot \operatorname{get}((1,2)) \\
& \mathrm{y}=A \cdot \operatorname{get}((2,1))
\end{aligned}
$$

then x would equal 5 and y would equal -5 . A may seem as if it were a regular matrix, but it is not. One difference is that only $A[1,2]$ and $A[2,1])$ are defined and $A[1,1]$ and $A[2,2]$ are not defined. If we fetched the value of $A[1,1]$ by typing

$$
z=A \cdot \operatorname{get}((1,1))
$$

that would not be an error, but we are in for a surprise, because $z$ will equal $J(0,0,$.$) , a real 0 \times 0$ matrix. That is AssociativeArray's way of saying that $A[1,1]$ has never been defined. We can change what AssociativeArray returns when an element is not defined. Let's change it to be zero:

```
A.notfound(0)
```

Now if we fetched the value of $A[1,1]$ by typing

$$
z=A \cdot \operatorname{get}((1,1))
$$

z would equal zero. We are on our way to creating sparse matrices! In fact, we have created a sparse matrix. I do not know whether our matrix is $2 \times 2$ or $1000 \times 1000$ because $A[i, j]$ is 0 for all ( $i, j$ ) not equal to $(1,2)$ and $(2,1)$. We will just have to keep track of the overall dimension separately. If I defined

$$
\text { A.put }((1000,1000), 6)
$$

then the sparse matrix would be at least $1000 \times 1000$. And our matrix really is sparse and stored efficiently in that $A$ contains only three elements.

Creating sparse matrices is one use of associative arrays. The typical use, however, involves the one-dimensional arrays with string keys, and these associative arrays are usually the converse of sparse matrices in that, rather than storing just a few elements, they store lots of elements. One can imagine an associative array

```
: Dictionary = AssociativeArray()
```

in which the elements are string colvectors, with the result:

```
: Dictionary.get("aback")
    [1, 1] = (archaic) toward or situation to the rear.
    [2, 1] = (sailing) with the sail pressed backward against the
        mast by the head.
```

I stored the definition for "aback" by coding

```
: Dictionary.put("aback",
```

    (
        "(archaic) toward or situation to the rear."
        \}
        "(sailing) with the sail pressed backward
        against the mast by the head."
    ))
    The great feature of associative arrays is that I could enter definitions for 25,591 other words and still Dictionary.get () could find the definition for any of the words, including "wombat", almost instantly. Performance would not depend on entering words alphabetically. They could be defined in any order. A user once complained that we slowed down somewhere between 500,000 and 1,000,000 elements, but that was due to a bug, and we fixed it.

Here is a summary of AssociativeArray's features.

## Initialization:

Declare $A$ in functions you write,

$$
\text { class AssociativeArray scalar } A
$$

or if working interactively, create $A$ using the creator function:

$$
A=\text { AssociativeArray() }
$$

$A$ is now an associative array indexed by string scalar keys. string scalar is the default.

## Reinitialization:

After initialization, the associative array is ready for use with string scalar keys. Use A.reinit() if you want to change the type of the keys or set tuning parameters. Keys can be scalar or rowvector and can be real, complex, string, or even pointer.


Do not specify tuning parameters. Treat A.reinit() as if it allowed only two arguments. You are unlikely to improve over the default values unless you understand how the parameters work. Tuning parameters are described in [M-5] asarray ().

Add or replace elements in the array:
Add or replace elements in the array using $A \cdot p u t()$ :

$$
A \cdot p u t(k e y, v a l) \quad A[k e y]=\text { val }
$$

Values can be of any element type, real, complex, string, or pointer. They can even be structure or class instances. Values can be scalars, vectors, or matrices. Value types do not need to be declared. Value types may even vary from one element to the next.

Retrieve elements from the array:
Retrieve values using $A$.get ():

$$
v a l=A . \operatorname{get}(k e y) \quad v a l=A[k e y] \text { or val }=\text { notfound }
$$

Retrieving a value for a key that has never been defined is not an error. A special value called notfound is returned in that case. The default value of notfound is $J(0,0,$.$) . You$ can change that:

Users of associative arrays containing numeric values often change notfound to zero or missing by coding $A$. notfound (0) or $A$. notfound (.).

You can use $A$.notfound() without arguments to query the current notfound value:

$$
\text { notfound }=A . \text { notfound }() \quad \text { query notfound value }
$$

Delete elements in the array:
Delete elements using A.remove():

## A.remove (key) delete A[key] if it exists

Function A.exists() will tell you whether an element exists:

$$
\text { bool }=A . \text { exists }(k e y) \quad A[k e y] \text { exists? }
$$

The function returns 1 or $0 ; 1$ means the element exists. You may wonder about the necessity of this function because $A$.get () returns notfound when an element does not exist. Why are there two ways to do one task? A.exists() is useful because you could store the notfound value in an element. You should not do that, of course.

Iterating through all elements of the array:
There are three ways to iterate through the elements.
Method 1 is

```
for (val=A.firstval(); val!=notfound; val=A.nextval()) {
    key = A.key() // if you need it
}
```

Inside the loop, val contains the element's value. If you need to know the element's key, use A.key ().

Method 2 is

```
transmorphic loc
for (loc=A.firstloc(); loc!=NULL; loc=A.nextloc()) {
    val = A.val(loc)
    key = A.key(loc) // if you need it
}
```

Method 2 allows for recursion. Use method 2 if you call a subroutine inside the loop that itself might iterate through the elements of the array.

Method 3 is an entirely different approach. You fetch the full set of defined keys and loop through them. Function A.keys () returns the keys as a matrix. Each row of the matrix is a key.

```
K = A.keys()
for (i=1; i<=length(K); i++) {
    val = A.get(K[i,.])
}
```

The keys returned by A.keys () are in no particular order. For some loops, order matters. Use Mata's sort () function to order them. If the keys were of dimension 1 and thus K were $N \times 1$, you could code

$$
\mathrm{K}=\operatorname{sort}(A \cdot \operatorname{keys}(), 1)
$$

If $A$ were $N \times k$, you could code

$$
\mathrm{K}=\operatorname{sort}(A . \operatorname{keys}(),(1 . . k))
$$

Vector operator $1 . . k$ produces the row vector $(1,2, \ldots, k)$.
Miscellany:
A.N() returns the number of defined elements in $A$.
A.clear () clears the array $A$. The array's characteristics-key type and dimension, notfound value, and tuning parameters-remain unchanged.

## Conformability

A.reinit(keytype, keydim, minsize, minratio, maxratio):

| keytype: | $1 \times 1$ | (optional) |
| :---: | :---: | :---: |
| keydim: | $1 \times 1$ | (optional) |
| minsize: | $1 \times 1$ | (optional) |
| minratio: | $1 \times 1$ | (optional) |
| maxratio: | $1 \times 1$ | (optional) |

A.put (key, a):

$$
\begin{aligned}
\text { key: } & 1 \times \text { keydim } \\
a: & r_{k e y} \times c_{k e y} ; r_{k e y} \text { and } c_{k e y} \text { your choice } \\
\text { result }: & \text { void }
\end{aligned}
$$

A.get (key):

$$
\begin{aligned}
\text { key: } & 1 \times \text { keydim } \\
\text { result: } & r_{\text {key }} \times c_{k e y}
\end{aligned}
$$

A.remove (key):

| key: | $1 \times$ keydim |
| ---: | :--- |
| result: | void |

A.clear():
result: void
A.exists (key):
$\begin{aligned} & \text { key: } \\ & \text { result }: \\ & 1 \times \text { keydim } \\ &\end{aligned}$
A.N():
result: $\quad 1 \times 1$
A.keys():
result: $\quad N \times$ keydim
A.firstval():
result: transmorphic
A.firstloc():
result: transmorphic
A.nextval():
result: transmorphic
A.nextloc():
result: transmorphic
A.key():
result: $\quad 1 \times$ keydim
A.key (loc):
loc: transmorphic
result: $\quad 1 \times$ keydim
A.val():
result: $\quad r_{\text {key }} \times c_{\text {key }}$
A.val (loc):
loc: transmorphic
result: $\quad r_{\text {key }} \times c_{\text {key }}$
A.notfound(notfound):
notfound: $\quad r \times c$; your choice
result: void
A.notfound():
result: $\quad r \times c$

## Diagnostics

None.

## Also see

[M-5] asarray ( ) - Associative arrays
[M-4] Manipulation - Matrix manipulation
[M-4] Programming — Programming functions

## Title

[M-5] ascii() — Manipulate ASCII and byte codes
Description Syntax Conformability Diagnostics Also see

## Description

ascii ( $s$ ) returns a row vector containing the ASCII codes ( $0-127$ ) and byte codes (128-255) corresponding to $s$. For instance, ascii("abc") returns (97, 98, 99); ascii("café") returns ( $99,97,102,195,169$ ). Note that the Unicode character "é" is beyond ASCII range. Its UTF-8 encoding requires 2 bytes and their byte values are 195 and 169 .
char ( $c$ ) returns a UTF-8 encoded string consisting of the specified ASCII and byte codes. For instance, $\operatorname{char}((97,98,99))$ returns "abc", and $\operatorname{char}((99,97,102,195,169))$ returns "café".

## Syntax

real rowvector ascii(string scalar $s$ )
string scalar char (real rowvector $c$ )

## Conformability

ascii ( $s$ ):

| $s:$ | $1 \times 1$ |
| ---: | :--- |
| result: | $1 \times \operatorname{strlen}(s)$ |

char (c):

| $c:$ | $1 \times n, \quad n \geq 0$ |
| ---: | :--- |
| result: | $1 \times 1$ |

## Diagnostics

$\operatorname{ascii}(s)$ returns $\mathrm{J}(1,0,$.$) if strlen (s)==0$.
In char $(c)$, if any element of $c$ is outside the range 0 to 255 , the returned string is terminated at that point. For instance, $\operatorname{char}((97,98,99,1000,97,98,99))==" a b c "$.
char(J(1,0,.)) returns "".

## Also see

[M-5] isascii() - Whether string scalar contains only ASCII codes
[M-5] uchar( ) - Convert code point to Unicode character
[M-4] String - String manipulation functions

## Title

[M-5] uchar( ) — Convert code point to Unicode character
Description Syntax Remarks and examples Conformability Diagnostics
Also see

## Description

uchar (c) returns the Unicode character in UTF-8 encoding corresponding to Unicode code point $c$.
It returns an empty string if $c$ is beyond the Unicode code-point range.
When $c$ is not a scalar, this function returns element-by-element results.

## Syntax

string matrix uchar (real matrix $c$ )

## Remarks and examples

uchar() returns the same results as char() for code points $0-127$.

## Conformability

$\operatorname{uchar}(c)$ :<br>$c: r \times c$<br>result: $r \times c$

## Diagnostics

None.

## Also see

[M-5] ascii( ) - Manipulate ASCII and byte codes
[M-4] String - String manipulation functions
[U] 12.4.2 Handling Unicode strings

## Title

[M-5] assert( ) - Abort execution if false

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

assert ( $r$ ) produces the error message "assertion is false" and aborts with error if $r==0$.
asserteq $(A, B)$ is logically equivalent to assert $(A==B)$. If the assertion is false, however, information is presented on the number of mismatches.

## Syntax

```
void assert(real scalar r)
void asserteq(transmorphic matrix A, transmorphic matrix B)
```


## Remarks and examples

In the midst of complicated code, you know that a certain calculation must produce a result greater than 0 , but you worry that perhaps you have an error in your code:

```
assert(n>0)
```

In another spot, you have produced matrix A and know every element of A should be positive or zero:

```
assert(all(A:>=0))
```

Once you are convinced that your function works, these verifications should be removed. In a third part of your code, however, the problem is different if the number of rows $r$ exceed the number of columns $c$. In all the cases you need to use it, however, $r$ will be less than $c$, so you are not much interested in programming the alternative solution:

```
assert(rows(PROBLEM) < cols(PROBLEM))
```

Leave that one in.

## Conformability

assert $(r)$ :
$\begin{aligned} r: & 1 \times 1 \\ \text { result: } & \text { void }\end{aligned}$
resuln void
asserteq $(A, B)$ :
A: $\quad r_{1} \times c_{1}$
B: $\quad r_{2} \times c_{2}$
result: void

## Diagnostics

assert ( $r$ ) aborts with error if $r==0$.
asserteq $(A, B)$ aborts with error if $A \neq B$.

## Also see

[M-4] Programming - Programming functions

## Title

[M-5] base64encode() — Encode string into Base64 format
Description Syntax Conformability Diagnostics Also see

## Description

base64encode ( $s$ ) encodes string $s$ into Base64 format.
base64decode $(s)$ decodes the string obtained from base64encode() and returns the original string; that is, base64decode ( $s$ ) decodes Base64-encoded string $s$.
base64encodefile (srcfile, destfile [, basedir]) encodes the text in file srcfile into Base64 format and saves the results in file destfile. If srcfile or destfile is a relative path, then basedir is used to resolve the relative path. If basedir is not specified, the current working directory is used. base64encodefile() returns 0 if successful and returns an error or -1 otherwise.
base64decodefile (srcfile, destfile [, basedir]) decodes Base64-encoded text in file srcfile and saves the results in file destfile. If srcfile or destfile is a relative path, then basedir is used to resolve the relative path. If basedir is not specified, the current working directory is used. base64decodefile() returns 0 if successful and returns an error or -1 otherwise.

## Syntax

string scalar base64encode (string scalar s)
string scalar base64decode (string scalar $s$ )
real scalar base64encodefile(string scalar srcfile, string scalar destfile [, string scalar basedir])
real scalar base64decodefile(string scalar srcfile, string scalar destfile [, string scalar basedir])

## Conformability

base64encode ( $s$ ), base64decode ( $s$ ):
$s: \quad 1 \times 1$
result: $\quad 1 \times 1$
base64encodefile(srcfile, destfile [, basedir]),
base64decodefile(srcfile, destfile [, basedir]):

| srcfile: | $1 \times 1$ |
| ---: | :--- |
| destfile: | $1 \times 1$ |
| basedir: | $1 \times 1$ |
| result: | $1 \times 1$ |

## Diagnostics

None.

## Also see

[M-4] String - String manipulation functions

## Title

[M-5] blockdiag( ) - Block-diagonal matrix

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

blockdiag $\left(Z_{1}, Z_{2}\right)$ returns a block-diagonal matrix with $Z_{1}$ in the upper-left corner and $Z_{2}$ in the lower right, that is,

$$
\left[\begin{array}{cc}
Z_{1} & \mathbf{0} \\
\mathbf{0} & Z_{2}
\end{array}\right]
$$

$Z_{1}$ and $Z_{2}$ may be either real or complex and need not be of the same type.

## Syntax

numeric matrix blockdiag (numeric matrix $Z_{1}$, numeric matrix $Z_{2}$ )

## Remarks and examples

To create a block diagonal matrix of $Z_{1}, Z_{2}, Z_{3}$, code
: blockdiag(Z1, blockdiag(Z2,Z3))

## Conformability

```
blockdiag \(\left(Z_{1}, Z_{2}\right)\) :
    \(Z_{1}: \quad r_{1} \times c_{1}\)
    \(Z_{2}: \quad r_{2} \times c_{2}\)
    result: \(\quad r_{1}+r_{2} \times c_{1}+c_{2}\)
```


## Diagnostics

None. Either or both $Z_{1}$ and $Z_{2}$ may be void.

## Also see

[M-4] Standard - Functions to create standard matrices

## Title

[M-5] bufio() — Buffered (binary) I/O

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

These functions manipulate buffers (string scalars) containing binary data and, optionally, perform I/O.
bufio() returns a control vector, $C$, that you pass to the other buffer functions. $C$ specifies the byte order of the buffer and specifies how missing values are to be encoded. Despite its name, bufio() opens no files and performs no I/O. bufio() merely returns a vector of default values for use with the remaining buffer functions.
bufbyteorder() and bufmissingvalue() allow changing the defaults in $C$.
bufput() and bufget () copy elements into and out of buffers. No I/O is performed. Buffers can then be written by using fwrite() and read by using fread(); see [M-5] fopen().
fbufput() and fbufget () do the same, and they perform the corresponding I/O by using fwrite() or fread().
bufbfmtlen (bfmt) and bufbfmtisnum (bfmt) are utility routines for processing bfmts; they are rarely used. bufbfmtlen (bfmt) returns the implied length, in bytes, of the specified bfmt, and bufbfmtisnum (bfmt) returns 1 if the bfmt is numeric, 0 if string.

## Syntax

```
colvector C = bufio()
real scalar bufbyteorder(C)
void bufbyteorder (C, real scalar byteorder)
real scalar bufmissingvalue(C)
void bufmissingvalue(C, real scalar version)
void bufput(C,B, offset, bfmt,X)
scalar bufget(C,B,offset, bfmt)
rowvector bufget (C,B, offset, bfmt, c)
matrix bufget(C, B, offset, bfmt,r, c)
void fbufput(C,fh, bfmt,X)
scalar fbufget(C,fh, bfmt)
rowvector fbufget(C,fh, bfmt, c)
matrix fbufget(C,fh, bfmt,r,c)
real scalar bufbfmtlen(string scalar bfmt)
real scalar bufbfmtisnum(string scalar bfmt)
```

where
$C: \quad$ colvector returned by bufio()
$B$ : string scalar (buffer)
offset: real scalar (buffer position, starts at 0)
$f h$ : file handle returned by fopen()
bfint: string scalar (binary format; see below)
$r: \quad$ string scalar
$c$ : string scalar
$X: \quad$ value to be written; see Remarks and examples
bfint may contain

| bfmt | meaning |
| :--- | :--- |
| $\%\{8 \mid 4\} \mathrm{z}$ | 8-byte floating point or 4-byte floating point <br> $\%\{4\|2\| 1\} \mathrm{b}[\mathrm{s} \mid \mathrm{u}]$ |
| 4-, 2-, or 1-byte integer; Stata, signed or unsigned |  |
| $\%$ \#s | text string |
| $\%$ \#S | binary string |

## Remarks and examples

If you wish simply to read and write matrices, etc., see fgetmatrix() and fputmatrix() and the other functions in [M-5] fopen( ).

The functions documented here are of interest if

1. you wish to create your own binary-data format because you are writing routines in low-level languages such as FORTRAN or C and need to transfer data between your new routines and Stata, or
2. you wish to write a program to read and write the binary format of another software package.

These are advanced and tedious programming projects.
Remarks are presented under the following headings:

```
Basics
Argument C
Arguments B and offset
Argument fh
Argument bfmt
bfmts for numeric data
bfmts for string data
Argument X
Arguments r and c
Advanced issues
```


## Basics

Let's assume that you wish to write a matrix to disk so you can move it back and forth from FORTRAN. You settle on a file format in which the number of rows and number of columns are first written as 4 -byte integers, and then the values of the matrix are written as 8 -byte doubles, by row:

| \# rows | $\#$ cols | $X[1,1]$ | $X[1,2]$ | $\ldots$ |
| :--- | :---: | :---: | :---: | :---: |${ }^{4}$ bytes 4 bytes

One solution to writing matrices in such a format is

```
fh = fopen("filename", "w")
C = bufio()
fbufput(C, fh, "%4b", rows(X))
fbufput(C, fh, "%4b", cols(X))
fbufput(C, fh, "%8z", X)
fclose(fh)
```

The code to read the matrix back is

```
fh = fopen("filename", "r")
C = bufio()
rows = fbufget(C, fh, "%4b")
cols = fbufget(C, fh, "%4b")
X = fbufget(C, fh, "%%z", rows, cols)
fclose(fh)
```

Another solution, which would be slightly more efficient, is

```
fh = fopen("filename", "w")
C = bufio()
buf = 8*char(0)
bufput(C, buf, 0, "%4b", rows(X))
bufput(C, buf, 4, "%4b", cols(X))
fwrite(C, buf)
fbufput(C, fh, "%%z", X)
fclose(fh)
```

and

```
fh = fopen("filename", "r")
C = bufio()
buf = fread(fh, 8)
rows = bufget(C, buf, 0, "%4b")
cols = bufget(C, buf, 4, "%4b")
X = fbufget(C, fh, "%8z", rows, cols)
fclose(fh)
```

What makes the above approach more efficient is that, rather than writing 4 bytes (the number of rows), and 4 bytes again (the number of columns), we created one 8 -byte buffer and put the two 4-byte fields in it, and then we wrote all 8 bytes at once. We did the reverse when we read back the data: we read all 8 bytes and then broke out the fields. The benefit is minuscule here but, in general, writing longer buffers results in faster I/O.

In all the above examples, we wrote and read the entire matrix with one function call,

```
fbufput(C, fh, "%%z", X)
```

and

```
X = fbufget(C, fh, "%8z", rows, cols)
```

Perhaps you would have preferred our having coded

```
for (i=1; i<=rows(X); i++) {
    for (j=1; j<=cols(X); j++) {
        fbufput(C, fh, "%8z", X[i,j])
    }
}
```

and perhaps you would have preferred our having coded something similar to read back the matrix. Had we done so, the results would have been the same.

If you are familiar with FORTRAN, you know that it records matrices in column-dominant order, rather than the row-dominant order used by Mata. It would be a little easier to code the FORTRAN side of things if we changed our file-format design to write columns first:

| \# rows | \# cols | $\mathrm{X}[1,1]$ | $\mathrm{X}[2,1]$ | $\ldots$ |
| :--- | :---: | :---: | :---: | :---: |
| 4 bytes 4 bytes | 8 bytes | 8 bytes |  |  |

One way we could do that would be to write the loops ourselves:

```
fh = fopen("filename", "w")
C = bufio()
fbufput(C, fh, "%4b", rows(X))
fbufput(C, fh, "%4b", cols(X))
for (j=1; j<=cols(X); i++) {
        for (i=1; i<=rows(X); j++) {
        fbufput(C, fh, "%8z", X[i,j])
        }
}
```

and

```
fh = fopen("filename", "r")
C = bufio()
rows = fbufget(C, fh, "%4b")
cols = fbufget(C, fh, "%4b")
X = J(rows, cols, .)
for (j=1; j<=cols(X); i++) {
    for (i=1; i<=rows(X); j++) {
        X[i,j] = fbufget(C, fh, "%8z")
        }
}
```

We could do that, but there are more efficient and easier ways to proceed. For instance, we could simply transpose the matrix before writing and after reading, and if we do that transposition in place, our code will use no extra memory:

```
fh = fopen("filename", "w")
C = bufio()
fbufput(C, fh, "%4b", rows(X))
fbufput(C, fh, "%4b", cols(X))
_transpose(X)
fbufput(C, fh, "%8z", X)
_transpose(X)
fclose(fh)
```

The code to read the matrices back is

```
fh = fopen("filename", "r")
C = bufio()
rows = fbufget(C, fh, "%4b")
cols = fbufget(C, fh, "%4b")
X = fbufget(C, fh, "%8z", cols, rows)
_transpose(X)
fclose(fh)
```


## Argument C

Argument $C$ in

```
bufput(C, B, offset, bfmt, X),
bufget(C, B, offset, bfmt,...),
fbufput(C,fh, bfmt, X), and
fbufget(C,fh, bfmt,...)
```

specifies the control vector. You obtain $C$ by coding

$$
C=\text { bufio() }
$$

bufio() returns $C$, which is nothing more than a vector filled in with default values. The other buffer routines know how to interpret the vector. The vector contains two pieces of information:

1. The byte order to be used.
2. The missing-value coding scheme to be used.

Some computer hardware writes numbers left to right (for example, Sun), and other computer hardware writes numbers right to left (for example, Intel); see [M-5] byteorder(). If you are going to write binary files, and if you want your files to be readable on all computers, you must write code to deal with this issue.

Many programmers ignore the issue because the programs they write are intended for one computer or on computers like the one they use. If that is the case, you can ignore the issue, too. The default byte order filled in by bufio() is the byte order of the computer you are using.

If you intend to read and write files across different computers, however, you will need to concern yourself with byte order, and how you do that is described in Advanced issues below.

The other issue you may need to consider is missing values. If you are writing a binary format that is intended to be used outside Stata, it is best if the values you write simply do not include missing values. Not all packages have them, and the packages that do don't agree on how they are encoded. In such cases, if the data you are writing include missing values, change the values to another value such as $-1,99,999$, or -9999 .

If, however, you are writing binary files in Stata to be read back in Stata, you can allow Stata's missing values ., .a, .b, ..., .z. No special action is required. The missing-value scheme in $C$ specifies how those missing values are encoded, and there is only one way right now, so there is in fact no issue at all. $C$ includes the extra information in case Stata ever changes the way it encodes missing values so that you will have a way to read and write old-format files. How this process works is described in Advanced issues.

## Arguments B and offset

Functions

```
bufput(C, B, offset, bfmt,X) and
bufget(C, B, offset, bfmt,...)
```

do not perform I/O; they copy values into and out of the buffer. $B$ specifies the buffer, and offset specifies the position within it.
$B$ is a string scalar.
offset is an integer scalar specifying the position within $B$. Offset 0 specifies the first byte of $B$.
For bufput(), $B$ must already exist and be long enough to receive the result, and it is usual to code something like

```
B = (4 + 4 + rows(X)*cols(X)*8) * char(0)
bufput(C, B, 0, "%4b", rows(X))
bufput(C, B, 4, "%4b", cols(X))
bufput(C, B, 8, "%8z", X)
```


## Argument fh

Argument $f h$ in

```
fbufput(C, fh, bfmt, X) and
fbufget(C,fh, bfmt,...)
```

plays the role of arguments $B$ and offset in bufput() and bufget(). Rather than copy into or out of a buffer, data are written to, or read from, file $f h$. $f$ h is obtained from fopen(); see [M-5] fopen().

## Argument bfmt

Argument bfmt in

```
bufput(C, B, offset, bfmt, X),
bufget(C, B, offset, bfmt,...),
fbufput(C,fh, bfmt, X), and
fbufget(C,fh, bfmt,...)
```

specifies how the elements are to be written or read.

## bfmts for numeric data

The numeric bfints are

| bfmt | Interpretation |
| :--- | :--- |
| $\% 8 \mathrm{z}$ | 8-byte floating point |
| $\% 4 \mathrm{z}$ | 4-byte floating point |
| $\% 4 \mathrm{bu}$ | 4-byte unsigned integer |
| $\% 4 \mathrm{bs}$ | 4-byte signed integer |
| $\% 4 \mathrm{~b}$ | 4-byte Stata integer |
| $\% 2 \mathrm{bu}$ | 2-byte unsigned integer |
| $\% 2 \mathrm{bs}$ | 2-byte signed integer |
| $\% 2 \mathrm{~b}$ | 2-byte Stata integer |
| $\% 1 \mathrm{bu}$ | 1-byte unsigned integer |
| $\% 1 \mathrm{bs}$ | 1-byte signed integer |
| $\% 1 \mathrm{~b}$ | 1-byte Stata integer |

A Stata integer is the same as a signed integer, except that the largest 27 values are given the interpretation ., .a, .b, ..., .z.

## bfmts for string data

The string bfints are
bfint Interpretation
\%\#s text string
\%\#S binary string
where \# represents the length of the string field. Examples include \%8s and \%639876S. When writing, it does not matter whether you use $\%$ \#s or $\%$ \#S, the same actions are taken:

1. If the string being written is shorter than \#, the field is padded with char (0).
2. If the string being written is longer than \#, only the first \# bytes of the string are written.

When reading, the distinction between \%\#s and \%\#S is important:

1. When reading with \%\#s, if char (0) appears within the first \# bytes, the returned result is truncated at that point.
2. When reading with $\%$ \#S, a full \# bytes are returned in all cases.

## Argument X

Argument $X$ in
bufput( $C, B$, offset, bfimt, $X$ ) and
fbufput ( $C, f h$, bfint, $X$ )
specifies the value to be written. $X$ may be real or string and may be a scalar, vector, or matrix. If $X$ is a vector, the elements are written one after the other. If $X$ is a matrix, the elements of the first row are written one after the other, followed by the elements of the second row, and so on.

In

$$
\begin{aligned}
& X=\operatorname{bufget}(C, B, \text { offset }, \text { bfmt }, \ldots) \text { and } \\
& X=\operatorname{fbufget}(C, f h, \text { bfmt }, \ldots)
\end{aligned}
$$

$X$ is returned.

## Arguments rand c

Arguments $r$ and $c$ are optional in the following:

$$
\begin{aligned}
& X=\operatorname{bufget}(C, B, \text { offset }, \text { bfmt }), \\
& X=\operatorname{bufget}(C, B, \text { offset }, \text { bfmt }, c), \\
& X=\operatorname{bufget}(C, B, \text { offset }, \text { bfmt }, r, c), \\
& X=\operatorname{fbufget}(C, f h, \text { bfmt }), \\
& X=\operatorname{fbufget}(C, f h, \text { bfmt }, c), \text { and } \\
& X=f \operatorname{bufget}(C, f h, \text { bfmt }, r, c) .
\end{aligned}
$$

If $r$ is not specified, results are as if $r=1$.
If $c$ is not specified, results are as if $c=1$.
Thus

$$
\begin{aligned}
& X=\operatorname{bufget}(C, B, \text { offset }, \text { bfmt }) \text { and } \\
& X=\operatorname{fbufget}(C, f h, \text { bfmt })
\end{aligned}
$$

read one element and return it, whereas

$$
\begin{aligned}
& X=\operatorname{bufget}(C, B, \text { offset }, \text { bfmt }, c) \text { and } \\
& X=\operatorname{fbufget}(C, f h, \text { bfmt }, c)
\end{aligned}
$$

read $c$ elements and return them in a column vector, and

$$
\begin{aligned}
& X=\operatorname{bufget}(C, B, \text { offset }, \text { bfint }, r, c) \text { and } \\
& X=\operatorname{fbufget}(C, f h, \text { bfmt }, r, c)
\end{aligned}
$$

read $r * c$ elements and return them in an $r \times c$ matrix.

## Advanced issues

A properly designed binary-file format includes a signature line first thing in the file:

```
fh = fopen(filename, "w")
fwrite(fh, "MyFormat For Mats v. 1.0")
    /* ----+----1----+-----2---- */
```

and

```
fh = fopen(filename, "r")
if (fread(fh, 24) != "MyFormat For Mats v. 1.0") {
    errprintf("%s not My-Format file\n", filename)
    exit(610)
}
```

If you are concerned with byte order and mapping of missing values, you should write the byte order and missing-value mapping in the file, write in natural byte order, and be prepared to read back in either byte order.

The code for writing is

```
fh = fopen(filename, "w")
fwrite(fh, "MyFormat For Mats v. 1.0")
C = bufio()
fbufput(C, fh, "%1bu", bufbyteorder(C))
fbufput(C, fh, "%2bu", bufmissingvalue(C))
```

and the corresponding code for reading the file is

```
fh = fopen(filename, "r")
if (fread(fh, 24) != "MyFormat For Mats v. 1.0") {
    errprintf("%s not My-Format file\n", filename)
    exit(610)
}
C = bufio()
bufbyteorder(C, fbufget(C, "%1bu"))
bufmissingvalue(C, fbufget(C, "%2bu"))
```

All we write in the file before recording the byte order are strings and bytes. This way, when we read the file later, we can set the byte order before reading any $2-, 4$-, or 8 -byte fields.
bufbyteorder ( $C$ )—bufbyteorder () with one argument—returns the byte-order encoding recorded in $C$. It returns 1 (meaning HILO) or 2 (meaning LOHI).
bufbyteorder ( $C$, value) —bufbyteorder () with two arguments-resets the byte order recorded in $C$. Once reset, all buffer functions will automatically reverse bytes if necessary.
bufmissingvalue() works the same way. With one argument, it returns a code for the encoding scheme recorded in $C$ (said code being the Stata release number multiplied by 100). With two arguments, it resets the code. Once the code is reset, all buffer routines used will automatically take the appropriate action.

## Conformability

bufio():
result: colvector
bufbyteorder ( $C$ ):
C: colvector made by bufio()
result: $1 \times 1 \quad$ containing 1 (HILO) or 2 (LOHI)
bufbyteorder ( $C$, byteorder):
C: colvector made by bufio()
byteorder: $\quad 1 \times 1 \quad$ containing 1 (HILO) or 2 (LOHI)
bufmissingvalue ( $C$ ):
C: colvector made by bufio()
result: $1 \times 1$
bufmissingvalue( $C$, version):
$C$ : colvector made by bufio()
version: $1 \times 1$
result: void
bufput( $C, B$, offset, bfint, $X$ ):
$C$ : colvector made by bufio()
B: $\quad 1 \times 1$
offset: $\quad 1 \times 1$
bfmt: $\quad 1 \times 1$
$X: \quad r \times c$
result: void
bufget ( $C, B$, offset, bfmt):
C: colvector made by bufio()
B: $\quad 1 \times 1$
offset: $\quad 1 \times 1$
bfint: $\quad 1 \times 1$
result: $\quad 1 \times 1$
bufget ( $C, B$, offset, bfint, $r$ ):
C: colvector made by bufio()
B: $\quad 1 \times 1$
offset: $\quad 1 \times 1$
bfimt: $\quad 1 \times 1$
$r: \quad 1 \times 1$
result: $1 \times c$
bufget ( $C, B$, offset, bfint, $r, c$ ):
$C$ : colvector made by bufio()
B: $\quad 1 \times 1$
offset: $\quad 1 \times 1$
bfmt: $\quad 1 \times 1$
$r: \quad 1 \times 1$
c: $\quad 1 \times 1$
result: $\quad r \times c$
fbufput ( $C, f h$, bfint,$X$ ):
C: colvector made by bufio()
fh: $\quad 1 \times 1$
bfmt: $\quad 1 \times 1$
$X: \quad r \times c$
result: void
fbufget ( $C, f h, b f m t$ ):
C: colvector made by bufio()
fh: $\quad 1 \times 1$
bfmt: $\quad 1 \times 1$
result: $\quad 1 \times 1$
fbufget ( $C, f h, b f m t, r$ ):
C: colvector made by bufio()
fh: $\quad 1 \times 1$
bfmt: $\quad 1 \times 1$
$r: \quad 1 \times 1$
result: $1 \times c$
fbufget ( $C, f h, b f m t, r, c$ ):
C: colvector made by bufio()
fh: $\quad 1 \times 1$
bfint: $\quad 1 \times 1$
$r: \quad 1 \times 1$
$c: \quad 1 \times 1$
result: $\quad r \times c$
bufbfmtlen (bfmt):

$$
\begin{aligned}
& \text { bfint: } \\
& \text { result: } \\
& 1 \times 1 \times 1
\end{aligned}
$$

bufbfmtisnum (bfmt):
bfmt: $\quad 1 \times 1$
result: $\quad 1 \times 1$

## Diagnostics

bufio() cannot fail.
bufbyteorder ( $C$ ) cannot fail. bufbyteorder ( $C$, byteorder) aborts with error if byteorder is not 1 or 2.
bufmissingvalue ( $C$ ) cannot fail. bufmissingvalue ( $C$, version) aborts with error if version $<$ 100 or version $>$ stataversion().
bufput ( $C, B$, offset, bfmt, $X$ ) aborts with error if $B$ is too short to receive the result, offset is out of range, bfint is invalid, or bfmt is a string format and $X$ is numeric or vice versa. Putting a void matrix results in 0 bytes being inserted into the buffer and is not an error.
bufget ( $C, B$, offset, bfmt, ...) aborts with error if $B$ is too short, offset is out of range, or bfmt is invalid. Reading zero rows or columns results in a void returned result and is not an error.
fbufput ( $C, f h, b f m t, X$ ) aborts with error if $f h$ is invalid, bfmt is invalid, or bfmt is a string format and $X$ is numeric or vice versa. Putting a void matrix results in 0 bytes being written and is not an error. I/O errors are possible; use fstatus () to detect them.
fbufget ( $C, f h, b f m t, \ldots$ ) aborts with error if $f h$ is invalid or bfmt is invalid. Reading zero rows or columns results in a void returned result and is not an error. End-of-file and I/O errors are possible; use fstatus() to detect them.
bufbfmtlen (bfmt) and bufbfmtisnum(bfmt) abort with error if bfmt is invalid.

## Also see

[M-5] fopen() - File I/O
[M-4] IO - I/O functions

## Title

[M-5] byteorder() — Byte order used by computer

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

byteorder () returns 1 if the computer is HILO (records most significant byte first) and returns 2 if LOHI (records least significant byte first).

## Syntax

real scalar byteorder()

## Remarks and examples

Pretend that the values 00 and 01 are recorded at memory positions 58 and 59 and that you know what is recorded there is a 2-byte integer. How should the 2-byte number be interpreted: as 0001 (meaning 1) or 0100 (meaning 256 in decimal)? For different computers, the answer varies. For HILO computers, the number is to be interpreted as 0001 . For LOHI computers, the number is interpreted as 0100 .

Regardless, it does not matter because the computer is consistent with itself. An issue arises, however, when we write binary files that may be read on computers using a different byte order or when we read files from computers that used a different byte order.

Stata and Mata automatically handle these problems for you, so you may wish to stop reading. byteorder (), however, is the tool on which the solution is based. If you intend to write code based on your own binary-file format or to write code to process the binary files of others, then you may need to use it.

There are two popular solutions to the byte-order problem: 1) write the file in a known byte order or 2) write the file by using whichever byte order is convenient and record the byte order used in the file. StataCorp tends to use the second, but others who have worried about this problem have used both solutions.

In solution 1, it is usually agreed that the file will be written in HILO order. If you are using a HILO computer, you write and read files the ordinary way, ignoring the problem altogether. If you are on a LOHI computer, however, you must reverse the bytes before placing them in the file. If you are writing code designed to execute on both kinds of computers, you must write code for both circumstances, and you must consider the problem when both reading and writing the files.

In solution 2, files are written LOHI or HILO, depending solely on the computer being used. Early in the file, however, the byte order is recorded. When reading the file, you compare the order in which the file is recorded with the order of the computer and, if they are different, you reverse bytes.

Mata-buffered I/O utilities will automatically reverse bytes for you. See [M-5] bufio( ).

## Conformability

byteorder():
result: $\quad 1 \times 1$

## Diagnostics

None.

## Also see

[M-5] bufio( ) - Buffered (binary) I/O
[M-5] stataversion( ) - Version of Stata being used
[M-4] Programming - Programming functions

## Title

[M-5] C( ) — Make complex

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

$\mathrm{C}(A)$ returns $A$ converted to complex. $\mathrm{C}(A)$ returns $A$ if $A$ is already complex. If $A$ is real, $\mathrm{C}(A)$ returns $A+0 i-A$ cast up to complex. Coding $C(A)$ is thus how you ensure that the matrix is treated as complex.
$\mathrm{C}(R, I)$ returns the complex matrix $R+I i$ and is faster than the alternative $R+I: * 1 \mathrm{i}$.

## Syntax

```
complex matrix C(numeric matrix A)
complex matrix C(real matrix R, real matrix I)
```


## Remarks and examples

Many of Mata's functions are overloaded, meaning they return a real when given real arguments and a complex when given complex arguments. Given real arguments, if the result cannot be expressed as a real, missing value is returned. Thus sqrt ( -1 ) evaluates to missing, whereas sqrt $(-1+0 i)$ is $1 i$.
$C()$ is the fast way to make arguments that might be real into complex. You can code

$$
\text { result }=\operatorname{sqrt}(C(x))
$$

If $x$ already is complex, C() does nothing; if $x$ is real, $\mathrm{C}(x)$ returns the complex equivalent.
The two-argument version of C() is less frequently used. $\mathrm{C}(R, I)$ is literally equivalent to $R:+I * 1 \mathrm{i}$, meaning that $R$ and $I$ need only be c-conformable.

For instance, $\mathrm{C}(1,(1,2,3))$ evaluates to $(1+1 i, 1+2 i, 1+3 i)$.

## Conformability

C(A):

$$
\begin{aligned}
A: & r \times c \\
\text { result: } & r \times c
\end{aligned}
$$

$\mathrm{C}(R, I)$ :

| $R:$ | $r_{1} \times c_{1}$ |
| ---: | :--- |
| $I:$ | $r_{2} \times c_{2}, \quad R$ and $I$ c-conformable |
| result: | $\max \left(r_{1}, r_{2}\right) \times \max \left(c_{1}, c_{2}\right)$ |

## Diagnostics

$C(Z)$, if $Z$ is complex, literally returns $Z$ and not a copy of $Z$. This makes execution of $C()$ applied to complex arguments instant.

In $C(R, I)$, the $i, j$ element of the result will be missing anywhere $R[i, j]$ or $I[i, j]$ is missing. For instance, $C((1,3,),.(., 2,4))$ results in (., 3+2i, .). If $R[i, j]$ and $I[i, j]$ are both missing, then the $R[i, j]$ value will be used; for example, $\mathrm{C}(. \mathrm{a}, . \mathrm{b})$ results in .a.

## Also see

[M-5] $\mathbf{R e}()$ - Extract real or imaginary part
[M-4] Scalar - Scalar mathematical functions
[M-4] Utility - Matrix utility functions

## Title

[M-5] c() - Access c() value

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

$c$ (name) returns Stata's c-class value; see [P] creturn.
Do not confuse $c()$ with $C()$, which makes complex out of real arguments; see [M-5] C().

## Syntax

scalar c (string scalar name)
returned is either a real or string scalar, depending on the value of name.

## Remarks and examples

See [P] creturn or, in Stata, type
. creturn list
to see what is stored in $c()$. Among the especially useful $c()$ values are

```
string c("current_date")
string c("current_time")
string c("os")
string c("dirsep")
```


## Conformability

$c$ (name):

| name: | $1 \times 1$ |
| :--- | :--- |
| result: | $1 \times 1$ |

## Diagnostics

c (name) returns a string or real depending on the particular $\mathrm{c}($ ) value requested. If name is an invalid name or contains a name for which no $c()$ value is defined, returned is "".

## Also see

[M-4] Programming - Programming functions

## Title

[M-5] callersversion() - Obtain version number of caller

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

callersversion() returns the version set by the caller (see [M-2] version), or if the caller did not set the version, it returns the version of Stata under which the caller was compiled.

## Syntax

real scalar callersversion()

## Remarks and examples

callersversion() is how [M-2] version is made to work. Say that you have written function real matrix useful (real matrix $A$, real scalar $k$ )
and assume that useful() aborts with error if $A$ is void. You wrote useful() in the days of Stata 17. For Stata 18, you want to change useful() so that it returns $J(0,0,$.$) if A$ is void, but you want to maintain the current behavior for old Stata 17 callers and programs. You do that as follows:

```
real matrix useful(real matrix A, real scalar k)
{
    if (callersversion()>=18) {
    if (rows(A)==0 | cols(A)==0) return(J(0,0,.))
    }
}
```


## Conformability

callersversion():
result: $1 \times 1$

## Diagnostics

None.

## Also see

[M-2] version - Version control
[M-4] Programming - Programming functions

## Title

[M-5] cat() — Load file into string matrix

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

cat (filename) returns a column vector containing the lines from text file filename.
cat (filename, line1) returns a column vector containing the lines from text file filename starting with line number linel.
cat(filename, line1, line2) returns a column vector containing the lines from text file filename starting with line number linel and ending with line number line 2 .

## Syntax

string colvector cat(string scalar filename $[$, real scalar line1 $[$, real scalar line 2$]]$ )

## Remarks and examples

cat (filename) removes new-line characters at the end of lines.

## Conformability

cat (filename, line1, line2):
filename: $\quad 1 \times 1$
line1: $\quad 1 \times 1$ (optional)
line2: $1 \times 1$ (optional)
result: $\quad r \times 1, \quad r \geq 0$

## Diagnostics

cat (filename) aborts with error if filename does not exist.
cat() returns a $0 \times 1$ result if filename contains 0 bytes.

## Also see

[M-4] IO - I/O functions

## Title

[M-5] chdir() — Manipulate directories
Description Syntax Conformability Diagnostics Also see

## Description

pwd() returns the full name (path) of the current working directory.
chdir (dirpath) changes the current working directory to dirpath. chdir () aborts with error if the directory does not exist or the operating system cannot change to it.
_chdir (dirpath) does the same thing but returns 170 (a return code) when chdir() would abort. _chdir () returns 0 if it is successful.
mkdir (dirpath) and mkdir (dirpath, public) create directory dirpath. mkdir() aborts with error if the directory already exists or cannot be created. If public $\neq 0$ is specified, the directory is given permissions so that everyone can read it; otherwise, it is given the usual permissions.
_mkdir (dirpath) and _mkdir (dirpath, public) do the same thing but return 693 (a return code) when mkdir () would abort. _mkdir () returns 0 if it is successful.
rmdir (dirpath) removes directory dirpath. rmdir () aborts with error if the directory does not exist, is not empty, or the operating system refuses to remove it.
_rmdir (dirpath) does the same thing but returns 693 (a return code) when rmdir() would abort. _rmdir() returns 0 if it is successful.

## Syntax

| string scalar | pwd() |
| :--- | :--- |
| void | chdir (string scalar dirpath) |
| real scalar | _chdir (string scalar dirpath) |
| void | mkdir (string scalar dirpath) |
| void | mkdir (string scalar dirpath, real scalar public) |
| real scalar | _mkdir (string scalar dirpath) |
| real scalar | _mkdir (string scalar dirpath, real scalar public) |
|  |  |
| void | rmdir (string scalar dirpath) |
| real scalar | _rmdir (string scalar dirpath) |

## Conformability

pwd():
result: $\quad 1 \times 1$
chdir(dirpath):

| dirpath: | $1 \times 1$ |
| ---: | :--- |
| result: | void |

_chdir (dirpath):
dirpath: $1 \times 1$
result: $\quad 1 \times 1$
mkdir(dirpath, public):
dirpath: $\quad 1 \times 1$
public: $1 \times 1 \quad$ (optional)
result: void
_mkdir (dirpath, public):
dirpath: $1 \times 1$
public: $1 \times 1 \quad$ (optional)
result: $\quad 1 \times 1$
rmdir(dirpath):
dirpath: $\quad 1 \times 1$
result: void
_rmdir (dirpath):
dirpath: $\quad 1 \times 1$
result: $\quad 1 \times 1$

## Diagnostics

pwd() never aborts with error, but it can return " " if the operating system does not know or does not have a name for the current directory (which happens when another process removes the directory in which you are working).
chdir (dirpath) aborts with error if the directory does not exist or the operating system cannot change to it.
_chdir (dirpath) never aborts with error; it returns 0 on success and 170 on failure.
mkdir (dirpath) and mkdir (dirpath, public) abort with error if the directory already exists or the operating system cannot change to it.
_mkdir (dirpath) and _mkdir (dirpath, public) never abort with error; they return 0 on success and 693 on failure.
rmdir (dirpath) aborts with error if the directory does not exist, is not empty, or the operating system cannot remove it.
_rmdir (dirpath) never aborts with error; it returns 0 on success and 693 on failure.

## Also see

[M-4] IO - I/O functions

## Title

[M-5] cholesky( ) - Cholesky square-root decomposition

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Reference | Also see |  |

## Description

cholesky ( $A$ ) returns the Cholesky decomposition $G$ of symmetric (Hermitian), positive-definite matrix $A$. cholesky () returns a lower-triangular matrix of missing values if $A$ is not positive definite.
_cholesky $(A)$ does the same thing, except that it overwrites $A$ with the Cholesky result.

## Syntax

```
numeric matrix cholesky(numeric matrix A)
void __cholesky(numeric matrix A)
```


## Remarks and examples

The Cholesky decomposition $G$ of a symmetric, positive-definite matrix $A$ is

$$
A=G G^{\prime}
$$

where $G$ is lower triangular. When $A$ is complex, $A$ must be Hermitian, and $G^{\prime}$, of course, is the conjugate transpose of $G$.

Decomposition is performed via [M-1] LAPACK.

## Conformability

cholesky (A):
$A: n \times n$
result: $n \times n$
_cholesky (A):
input:

$$
A: n \times n
$$

output:
A: $n \times n$

## Diagnostics

cholesky () returns a lower-triangular matrix of missing values if $A$ contains missing values or if $A$ is not positive definite.
_cholesky ( $A$ ) overwrites $A$ with a lower-triangular matrix of missing values if $A$ contains missing values or if $A$ is not positive definite.

Both functions use the elements from the lower triangle of $A$ without checking whether $A$ is symmetric or, in the complex case, Hermitian.

André-Louis Cholesky (1875-1918) was born near Bordeaux in France. He studied at the Ecole Polytechnique and then joined the French army. Cholesky served in Tunisia and Algeria and then worked in the Geodesic Section of the Army Geographic Service, where he invented his now-famous method. In the war of 1914-1918, he served in the Vosges and in Romania but after returning to the Western front was fatally wounded. Cholesky's method was written up posthumously by one of his fellow officers but attracted little attention until the 1940s.

## Reference

Chabert, J.-L., É. Barbin, J. Borowczyk, M. Guillemot, and A. Michel-Pajus. 1999. A History of Algorithms: From the Pebble to the Microchip. Trans. C. Weeks. Berlin: Springer.

## Also see

[M-5] ldl( ) - Bunch-Kaufman decomposition
[M-5] lud( ) - LU decomposition
[M-4] Matrix - Matrix functions

## Title

[M-5] cholinv() — Symmetric, positive-definite matrix inversion

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

cholinv ( $A$ ) and cholinv ( $A$, tol) return the inverse of real or complex, symmetric (Hermitian), positive-definite, square matrix $A$.
_cholinv ( $A$ ) and _cholinv ( $A$, tol) do the same thing except that, rather than returning the inverse matrix, they overwrite the original matrix $A$ with the inverse.
cholinvlapacke ( $A$ ), cholinvlapacke ( $A$, tol), _cholinvlapacke ( $A$ ), and _cholinvlapacke ( $A$, tol) are similar to their correspondent functions without lapacke endings, but instead they use interfaces to the LAPACK routines to compute the inverse.

In all cases, optional argument tol specifies the tolerance for determining singularity; see Remarks and examples below.

## Syntax

| numeric matrix cholinv(numeric matrix A) <br> numeric matrix  | cholinv(numeric matrix A, real scalar tol) |
| :--- | :--- |
| void | _cholinv(numeric matrix A) |
| void | _cholinv(numeric matrix A, real scalar tol) |
| numeric matrix | cholinvlapacke (numeric matrix A) |
| numeric matrix | cholinvlapacke(numeric matrix A, real scalar tol) |
| void |  |
| void | _cholinvlapacke(numeric matrix A) |
|  | _cholinvlapacke(numeric matrix A, real scalar tol) |

## Remarks and examples

These routines calculate the inverse of a symmetric, positive-definite square matrix $A$. See $[\mathrm{M}-5] \operatorname{luinv}()$ for the inverse of a general square matrix.
$A$ is required to be square and positive definite. See [M-5] qrinv() and [M-5] pinv() for generalized inverses of nonsquare or rank-deficient matrices. See [M-5] invsym( ) for generalized inverses of real, symmetric matrices.
cholinv(A) is logically equivalent to cholsolve( $A$, I (rows ( $A$ )) ); see [M-5] cholsolve() for details and for use of the optional tol argument.

## Conformability

cholinv ( $A$, tol):

| A: |  | $n \times n$ |
| ---: | :--- | :--- |
| tol: | $1 \times 1$ |  |
| result: |  | $n \times n$ |

_cholinv ( $A$, tol):
input:

$$
A: \quad n \times n
$$

tol: $1 \times 1 \quad$ (optional)
output:
A: $\quad n \times n$
cholinvlapacke( $A$, tol):

$$
\begin{array}{rll}
A: & & n \times n \\
\text { tol: } & & 1 \times 1 \\
\text { result: } & & n \times n
\end{array}
$$

_cholinvlapacke ( $A$, tol):
input:

$$
\begin{array}{rlll}
\text { A: } & & n \times n & \\
\text { tol: } & & 1 \times 1 & \text { (optional) }
\end{array}
$$

output:

$$
A: \quad n \times n
$$

## Diagnostics

The inverse returned by these functions is real if $A$ is real and is complex if $A$ is complex. If you use these functions with a non-positive-definite matrix, or a matrix that is too close to singularity, returned will be a matrix of missing values. The determination of singularity is made relative to tol. See Tolerance under Remarks and examples in [M-5] cholsolve() for details.
cholinv ( $A$ ) and _cholinv ( $A$ ) return a result containing all missing values if $A$ is not positive definite or if $A$ contains missing values.
_cholinv ( $A$ ) aborts with error if $A$ is a view.
cholinvlapacke ( $A$ ) and _cholinvlapacke ( $A$ ) return a result containing all missing values if $A$ is not positive definite or if $A$ contains missing values.
_cholinvlapacke ( $A$ ) aborts with error if $A$ is a view.
See [M-5] cholsolve() and [M-1] Tolerance for information on the optional tol argument.
All functions use the elements from the lower triangle of $A$ without checking whether $A$ is symmetric or, in the complex case, Hermitian.

## Also see

[M-5] invsym( ) - Symmetric real matrix inversion
[M-5] luinv( ) - Square matrix inversion
[M-5] pinv() - Moore-Penrose pseudoinverse
[M-5] qrinv( ) - Generalized inverse of matrix via QR decomposition
[M-5] cholsolve( ) - Solve AX=B for X using Cholesky decomposition
[M-5] solve_tol( ) - Tolerance used by solvers and inverters
[M-4] Matrix - Matrix functions
[M-4] Solvers - Functions to solve $A X=B$ and to obtain A inverse

## Title

[M-5] cholsolve( ) - Solve AX=B for X using Cholesky decomposition

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

cholsolve $(A, B)$ solves $A X=B$ and returns $X$ for symmetric (Hermitian), positive-definite $A$. cholsolve() returns a matrix of missing values if $A$ is not positive definite or if $A$ is singular.
cholsolve ( $A, B$, tol ) does the same thing; it allows you to specify the tolerance for declaring that $A$ is singular; see Tolerance under Remarks and examples below.
_cholsolve $(A, B)$ and _cholsolve $(A, B, t o l)$ do the same thing, except that, rather than returning the solution $X$, they overwrite $B$ with the solution, and in the process of making the calculation, they destroy the contents of $A$.
cholsolvelapacke ( $A$ ), cholsolvelapacke ( $A$, tol), _cholsolvelapacke ( $A$ ), and _cholsolvelapacke ( $A, t o l$ ) are similar to their correspondent functions without lapacke endings, but instead they use interfaces to the LAPACK routines to compute the solutions.

## Syntax

| numeric matrix | cholsolve (numeric matrix $A$, numeric matrix $B$, real scalar tol) |
| :---: | :---: |
| void | _cholsolve(numeric matrix $A$, numeric matrix $B$ ) |
| void | _cholsolve (numeric matrix $A$, numeric matrix $B$, real scalar tol) |
| numeric matrix | cholsolvelapacke (numeric matrix $A$, numeric matrix $B$ ) |
| numeric matrix | cholsolvelapacke(numeric matrix $A$, numeric matrix $B$, real scalar tol) |
| void | _cholsolvelapacke(numeric matrix $A$, numeric matrix $B$ ) |
| void | _cholsolvelapacke (numeric matrix $A$, numeric matrix $B$, real scalar tol) |

## Remarks and examples

The above functions solve $A X=B$ via Cholesky decomposition and are accurate. When $A$ is not symmetric and positive definite, [M-5] lusolve( ), [M-5] qrsolve( ), and [M-5] svsolve() are alternatives based on the LU decomposition, the QR decomposition, and the singular value decomposition (SVD). The alternatives differ in how they handle singular $A$. Then, the LU-based routines return missing values, whereas the QR-based and SVD-based routines return generalized (least-squares) solutions.

Remarks are presented under the following headings:
Derivation
Relationship to inversion
Tolerance

## Derivation

We wish to solve for $X$

$$
\begin{equation*}
A X=B \tag{1}
\end{equation*}
$$

when $A$ is symmetric and positive definite. Perform the Cholesky decomposition of $A$ so that we have $A=G G^{\prime}$. Then, (1) can be written as

$$
\begin{equation*}
G G^{\prime} X=B \tag{2}
\end{equation*}
$$

Define

$$
\begin{equation*}
Z=G^{\prime} X \tag{3}
\end{equation*}
$$

Then, (2) can be rewritten as

$$
\begin{equation*}
G Z=B \tag{4}
\end{equation*}
$$

It is easy to solve (4) for $Z$ because $G$ is a lower-triangular matrix. Once $Z$ is known, it is easy to solve (3) for $X$ because $G^{\prime}$ is upper triangular.

## Relationship to inversion

See Relationship to inversion in [M-5] lusolve() for a discussion of the relationship between solving the linear system and matrix inversion.

## Tolerance

The default tolerance used is

$$
\eta=\frac{(1 \mathrm{e}-13) * \operatorname{trace}(\operatorname{abs}(G))}{n}
$$

where $G$ is the lower-triangular Cholesky factor of $A: n \times n . A$ is declared to be singular if cholesky () (see [M-5] cholesky ()) finds that $A$ is not positive definite or, if $A$ is found to be positive definite, if any diagonal element of $G$ is less than or equal to $\eta$. Mathematically, positive definiteness implies that the matrix is not singular. In the numerical method used, two checks are made: cholesky () makes one, and then the $\eta$ rule is applied to ensure numerical stability in the use of the result cholesky () returns.

If you specify tol $>0$, the value you specify is used to multiply $\eta$. You may instead specify tol $\leq$ 0 , and then the negative of the value you specify is used in place of $\eta$; see [M-1] Tolerance.

See [M-5] lusolve( ) for a detailed discussion of the issues surrounding solving nearly singular systems. The main point to keep in mind is that if $A$ is ill conditioned, then small changes in $A$ or $B$ can lead to radically large differences in the solution for $X$.

## Conformability

## cholsolve ( $A, B$, tol):

input:

$$
\begin{array}{rll}
A: & n \times n & \\
B: & n \times k & \\
\text { tol: } & 1 \times 1 & \text { (optional) } \\
\text { result: } & n \times k &
\end{array}
$$

_cholsolve ( $A, B$, tol):
input:

| A: |  | $n \times n$ |
| ---: | :--- | ---: |
| $B:$ |  | $n \times k$ |
| $t o l$ |  | $1 \times 1$ |

tol: $\quad 1 \times 1 \quad$ (optional)
output:

$$
\begin{array}{ll}
A: & \\
B: & \\
B \times k
\end{array}
$$

cholsolvelapacke( $A, B$, tol):
input:

$$
\begin{array}{rll}
A: & n \times n & \\
B: & n \times k & \\
\text { tol: } & 1 \times 1 & \text { (optional) } \\
\text { result: } & n \times k &
\end{array}
$$

_cholsolvelapacke ( $A, B$, tol):
input:

$$
\begin{array}{rll}
A: & n \times n & \\
B: & n \times k & \\
\text { tol: } & 1 \times 1 & \text { (optional) }
\end{array}
$$

output:
A: $\quad 0 \times 0$
B: $\quad n \times k$

## Diagnostics

cholsolve $(A, B, \ldots)$ and _cholsolve $(A, B, \ldots)$ return a result of all missing values if $A$ is not positive definite or if $A$ contains missing values.
_cholsolve $(A, B, \ldots)$ also aborts with error if $A$ or $B$ is a view.
cholsolvelapacke $(A, B, \ldots)$ and _cholsolvelapacke $(A, B, \ldots)$ return a result of all missing values if $A$ is not positive definite or if $A$ contains missing values.
_cholsolvelapacke $(A, B, \ldots)$ also aborts with error if $A$ or $B$ is a view.
All functions use the elements from the lower triangle of $A$ without checking whether $A$ is symmetric or, in the complex case, Hermitian.

## Also see

[M-5] cholesky () - Cholesky square-root decomposition
[M-5] cholinv( ) - Symmetric, positive-definite matrix inversion
[M-5] lusolve( ) - Solve AX=B for X using LU decomposition
[M-5] qrsolve() - Solve AX=B for X using QR decomposition
[M-5] solvelower() - Solve AX=B for X, A triangular
[M-5] svsolve( ) - Solve AX=B for X using singular value decomposition
[M-5] solve_tol() - Tolerance used by solvers and inverters
[M-4] Matrix - Matrix functions
[M-4] Solvers - Functions to solve $\mathrm{AX}=\mathrm{B}$ and to obtain A inverse

## Title

[M-5] comb( ) - Combinatorial function

```
Description Syntax Conformability Diagnostics Also see
```


## Description

$\operatorname{comb}(n, k)$ returns the elementwise combinatorial function $n$-choose- $k$, the number of ways to choose $k$ items from $n$ items, regardless of order.

## Syntax

real matrix $\operatorname{comb}($ real matrix $n$, real matrix $k$ )

## Conformability

```
comb (n, k):
    n:
    k: }\quad\mp@subsup{r}{2}{}\times\mp@subsup{c}{2}{},\quadn\mathrm{ and }k\mathrm{ r-conformable
result: max (r},\mp@subsup{r}{2}{})\times\operatorname{max}(\mp@subsup{c}{1}{},\mp@subsup{c}{2}{}
```


## Diagnostics

$\operatorname{comb}(n, k)$ returns missing when either argument is missing or when the result would be larger than $10^{300}$.

## Also see

[M-4] Statistical - Statistical functions

## Title

[M-5] cond() - Condition number

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

cond ( $A$ ) returns cond ( $A, 2$ ).
cond $(A, p)$ returns the value of the condition number of $A$ for the specified norm $p$, where $p$ may be $0,1,2$, or . (missing).

## Syntax

real scalar cond(numeric matrix $A$ )
real scalar cond(numeric matrix $A$, real scalar $p$ )

## Remarks and examples

The condition number of a matrix $A$ is

$$
\text { cond }=\operatorname{norm}(A, p) \times \operatorname{norm}\left(A^{-1}, p\right)
$$

These functions return missing when $A$ is singular.
Values near 1 indicate that the matrix is well conditioned, and large values indicate ill conditioning.

## Conformability

```
cond(A):
```

| $A:$ |  |
| ---: | :--- |
| result: |  |
|  | $1 \times c$ |

cond $(A, p)$ :

| $A:$ | $r \times c$ |
| ---: | :--- |
| $p:$ | $1 \times 1$ |
| result: | $1 \times 1$ |

## Diagnostics

cond ( $A, p$ ) aborts with error if $p$ is not $0,1,2$, or . (missing).
cond $(A)$ and cond $(A, p)$ return missing when $A$ is singular or if $A$ contains missing values. cond $(A)$ and cond $(A, p)$ return 1 when $A$ is void.
cond $(A)$ and cond $(A, 2)$ return missing if the SVD algorithm fails to converge, which is highly unlikely; see $[\mathrm{M}-5] \operatorname{svd}()$.

## Also see

[M-5] norm( ) - Matrix and vector norms
[M-4] Matrix - Matrix functions

## Title

[M-5] conj() - Complex conjugate

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

$\operatorname{conj}(Z)$ returns the elementwise complex conjugate of $Z$, that is, $\operatorname{conj}(a+b i)=a-b i . \operatorname{conj}()$ may be used with real or complex matrices. If $Z$ is real, $Z$ is returned unmodified.
$\quad \operatorname{conj}(A)$ replaces $A$ with $\operatorname{conj}(A)$. Coding $\quad \operatorname{conj}(A)$ is equivalent to coding $A=\operatorname{conj}(A)$, except that less memory is used.

## Syntax

```
numeric matrix conj(numeric matrix Z)
void __conj(numeric matrix A)
```


## Remarks and examples

Given $m \times n$ matrix $Z$, $\operatorname{conj}(Z)$ returns an $m \times n$ matrix; it does not return the transpose. To obtain the conjugate transpose matrix, also known as the adjoint matrix, adjugate matrix, Hermitian adjoin, or Hermitian transpose, code

$$
Z^{\prime}
$$

See [M-2] op_transpose.
A matrix equal to its conjugate transpose is called Hermitian or self-adjoint, although in this manual, we often use the term symmetric.

## Conformability

conj $(Z)$ :

$$
\begin{aligned}
& Z: \\
& \text { result: } \\
& r \times c
\end{aligned}
$$

_conj (A):
input:
A: $\quad r \times c$
output:
A: $\quad r \times c$

## Diagnostics

$\operatorname{conj}(Z)$ returns a real matrix if $Z$ is real and a complex matrix if $Z$ is complex.
$\operatorname{conj}(Z)$, if $Z$ is real, returns $Z$ itself and not a copy. This makes conj() execute instantly when applied to real matrices.
$\_\operatorname{conj}(A)$ does nothing if $A$ is real (and hence, does not abort if $A$ is a view).

## Also see

[M-5] _transpose() - Transposition in place
[M-4] Scalar - Scalar mathematical functions

## Title

[M-5] corr() - Make correlation matrix from variance matrix

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

$\operatorname{corr}(V)$ returns the correlation matrix corresponding to variance matrix $V$.
_corr $(V)$ changes the contents of $V$ from being a variance matrix to being a correlation matrix.

## Syntax

real matrix $\operatorname{corr}($ real matrix $V$ )
void $\quad$ _corr (real matrix $V$ )

## Remarks and examples

See function variance() in [M-5] mean() for obtaining a variance matrix from data.

## Conformability

## corr ( $V$ ):

input:

$$
\begin{aligned}
V: & k \times k \\
\text { result: } & k \times k
\end{aligned}
$$

_corr ( $V$ ):
input:

$$
V: \quad k \times k
$$

output:

$$
V: \quad k \times k
$$

## Diagnostics

corr () and _corr () abort with error if $V$ is not square. $V$ should also be symmetric, but this is not checked.

## Also see

[M-5] mean() - Means, variances, and correlations
[M-4] Statistical - Statistical functions

## Title

[M-5] cross() - Cross products

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

cross() makes calculations of the form

$$
\begin{array}{r}
X^{\prime} X \\
X^{\prime} Z \\
X^{\prime} \operatorname{diag}(w) X \\
X^{\prime} \operatorname{diag}(w) Z
\end{array}
$$

cross() is designed for making calculations that often arise in statistical formulas. In one sense, cross () does nothing that you cannot easily write out in standard matrix notation. For instance, cross ( $X, Z$ ) calculates $X^{\prime} Z$. cross (), however, has the following differences and advantages over the standard matrix-notation approach:

1. cross () omits the rows in $X$ and $Z$ that contain missing values, which amounts to dropping observations with missing values.
2. cross () uses less memory and is especially efficient when used with views.
3. cross() watches for special cases and makes calculations in those special cases more efficiently. For instance, if you code cross ( $X, X$ ), cross () observes that the two matrices are the same and makes the calculation for a symmetric matrix result.
cross $(X, Z)$ returns $X^{\prime} Z$. Usually rows $(X)==r o w s(Z)$, but $X$ is also allowed to be a scalar, which is then treated as if J (rows $(Z), 1,1$ ) were specified. Thus cross $(1, Z)$ is equivalent to colsum ( $Z$ ).
$\operatorname{cross}(X, w, Z)$ returns $X^{\prime} \operatorname{diag}(w) Z$. Usually, rows $(w)==\operatorname{rows}(Z)$ or cols $(w)==r o w s(Z)$, but $w$ is also allowed to be a scalar, which is treated as if J (rows $(Z), 1, w$ ) were specified. Thus $\operatorname{cross}(X, 1, Z)$ is the same as $\operatorname{cross}(X, Z) . Z$ may also be a scalar, just as in the two-argument case.
$\operatorname{cross}(X, x c, Z, z c)$ is similar to $\operatorname{cross}(X, Z)$ in that $X^{\prime} Z$ is returned. In the four-argument case, however, $X$ is augmented on the right with a column of 1 s if $x c!=0$ and $Z$ is similarly augmented if $z c!=0 . \operatorname{cross}(X, 0, Z, 0)$ is equivalent to $\operatorname{cross}(X, Z) . Z$ may be specified as a scalar.
$\operatorname{cross}(X, x c, w, Z, z c)$ is similar to $\operatorname{cross}(X, w, Z)$ in that $X^{\prime} \operatorname{diag}(w) Z$ is returned. As with the four-argument cross(), $X$ is augmented on the right with a column of 1 s if $x c!=0$ and $Z$ is similarly augmented if $z c!=0$. Both $Z$ and $w$ may be specified as scalars. cross $(X, 0,1, Z, 0)$ is equivalent to $\operatorname{cross}(X, Z)$.

## Syntax

```
real matrix \(\operatorname{cross}(X, Z)\)
real matrix \(\operatorname{cross}(X, w, Z)\)
real matrix \(\operatorname{cross}(X, x c, Z, z c)\)
real matrix \(\operatorname{cross}(X, x c, w, Z, z c)\)
```

where
$X: \quad$ real matrix $X$
$x c: \quad$ real scalar $x c$
$w: \quad$ real vector $w$
$Z: \quad$ real matrix $Z$
$z c: \quad$ real scalar $z c$

## Remarks and examples

In the following examples, we are going to calculate linear regression coefficients using $b=$ $\left(X^{\prime} X\right)^{-1} X^{\prime} y$, means using $\sum x / n$, and variances using $n /(n-1) \times\left(\sum x^{2} / n-\right.$ mean $\left.^{2}\right)$. See [M-5] crossdev() for examples of the same calculations made in a more numerically stable way.

The examples use the automobile data. Because we are using the absolute form of the calculation equations, it would be better if all variables had values near 1 (in which case the absolute form of the calculation equations are perfectly adequate). Thus we suggest

```
. sysuse auto
. replace weight = weight/1000
```

Some of the examples use a weight w. For that, you might try

```
generate w = int(4*runiform())+1
```

> Example 1: Linear regression, the traditional way

```
y = X = .
st_view(y, ., "mpg")
st_view(X, ., "weight foreign")
X = X, J(rows(X),1,1)
b = invsym(X'X)*X'y
```

Comments: Does not handle missing values and uses much memory if X is large.
> Example 2: Linear regression using cross()

```
y = X = .
st_view(y, ., "mpg")
st_view(X, ., "weight foreign")
XX = cross(X,1 , X,1)
Xy = cross(X,1 , y,0)
b = invsym(XX)*Xy
```

Comments: There is still an issue with missing values; mpg might not be missing everywhere weight and foreign are missing.

## > Example 3: Linear regression using cross() and one view

```
// We will form
//
// (y X)'(y x) = (y'y, y'x \ X'y, X'X)
M = .
st_view(M, ., "mpg weight foreign", 0)
CP = cross(M,1 , M,1)
XX = CP[|2,2 \ .,.|]
Xy = CP[|2,1 \ .,1|]
b = invsym(XX)*Xy
```

Comments: Using one view handles all missing-value issues (we specified fourth argument 0 to st_view(); see [M-5] st_view()).
> Example 4: Linear regression using cross() and subviews

```
M = X = y = .
st_view(M, ., "mpg weight foreign", 0)
st_subview(y, M, ., 1)
st_subview(X, M, ., (2\.))
XX = cross(X,1 , X,1)
Xy = cross(X,1 , y,0)
b = invsym(XX)*Xy
```

Comments: Using subviews also handles all missing-value issues; see [M-5] st_subview(). The subview approach is a little less efficient than the previous solution but is perhaps easier to understand. The efficiency issue concerns only the extra memory required by the subviews y and X , which is not much.

Also, this subview solution could be used to handle the missing-value problems of calculating linear regression coefficients the traditional way, shown in example 1:

```
M = X = y = .
st_view(M, ., "mpg weight foreign", 0)
st_subview(y, M, ., 1)
st_subview(X, M, ., (2\.))
X = X, J(rows(X), 1, 1)
b = invsym(X'X)*X'y
```

> Example 5: Weighted linear regression, the traditional way

```
M = w = y = X = .
st_view(M, ., "w mpg weight foreign", 0)
st_subview(w, M, ., 1)
st_subview(y, M, ., 2)
st_subview(X, M, ., (3\.))
X = X, J(rows(X), 1, 1)
b = invsym(X'diag(w)*X)*X'diag(w)'y
```

Comments: The memory requirements are now truly impressive because diag(w) is an $N \times N$ matrix! That is, the memory requirements are truly impressive when $N$ is large. Part of the power of Mata is that you can write things like invsym(X'diag(w)*X)*X'diag(w)'y and obtain solutions. We do not mean to be dismissive of the traditional approach; we merely wish to emphasize its memory requirements and note that there are alternatives.

## > Example 6: Weighted linear regression using cross()

```
M = w = y = X = .
st_view(M, ., "w mpg weight foreign", 0)
st_subview(w, M, ., 1)
st_subview(y, M, ., 2)
st_subview(X, M, ., (3\.))
XX = cross(X,1 ,w, X,1)
Xy = cross(X,1 ,w, y,0)
b = invsym(XX)*Xy
```

Comments: The memory requirements here are no greater than they were in example 4 , which this example closely mirrors. We could also have mirrored the logic of example 3:

```
M = w = .
st_view(M, ., "w mpg weight foreign", 0)
st_subview(w, M, ., 1)
CP = cross(M,1 ,w, M,1)
XX = CP[|3,3 \ .,.|]
Xy = CP[|3,2 \ .,2|]
b = invsym(XX)*Xy
```

Note how similar these solutions are to their unweighted counterparts. The only important difference is the appearance of w as the middle argument of cross(). Because specifying the middle argument as a scalar 1 is also allowed and produces unweighted estimates, the above code could be modified to produce unweighted or weighted estimates, depending on how w is defined.

## > Example 7: Mean of one variable

```
x = .
st_view(x, ., "mpg", 0)
CP = cross(1,0 , x,1)
mean = CP[1]/CP[2]
```

Comments: An easier and every bit as good a solution would be

```
x = .
st_view(x, ., "mpg", 0)
mean = mean(x,1)
```

mean() (see [M-5] mean()) is implemented in terms of cross (). Actually, mean() is implemented using the quad-precision version of cross(); see [M-5] quadcross(). We could implement our solution in terms of quadcross():

```
: x = .
st_view(x, ., "mpg", 0)
CP = quadcross(1,0 , x,1)
mean = CP[1]/CP[2]
```

quadcross() returns a double-precision result just as does cross(). The difference is that quadcross() uses quad precision internally in calculating sums.
> Example 8: Means of multiple variables

```
X = .
st_view(X, ., "mpg weight displ", 0)
CP = cross(1,0 , X,1)
n = cols(CP)
means = CP[|1\n-1|] :/ CP[n]
```

Comments: The above logic will work for one variable, too. With mean(), the solution would be

```
X = .
st_view(X, ., "mpg weight displ", 0)
means = mean(X, 1)
```


## > Example 9: Weighted means of multiple variables

```
M = w = X = .
st_view(M, ., "w mpg weight displ", O)
st_subview(w, M, ., 1)
st_subview(X, M, ., (2\.))
CP = cross(1,0, w, X,1)
n = cols(CP)
means = CP[|1\n-1|] :/ CP[n]
```

Comments: Note how similar this solution is to the unweighted solution: w now appears as the middle argument of cross (). The line $\mathrm{CP}=\operatorname{cross}(1,0, \mathrm{w}, \mathrm{X}, 1)$ could also be coded $\mathrm{CP}=\operatorname{cross}(\mathrm{w}, 0$, $\mathrm{X}, 1$ ); it would make no difference.

The mean() solution to the problem is

```
M = w = X = .
st_view(M, ., "w mpg weight displ", 0)
st_subview(w, M, ., 1)
st_subview(X, M, ., (2\.))
means = mean(X, w)
```

> Example 10: Variance matrix, traditional approach 1

```
: X = .
st_view(X, ., "mpg weight displ", 0)
n = rows(X)
means = mean(X, 1)
cov = (X'X/n - means'means)*(n/(n-1))
```

Comments: This above is not $100 \%$ traditional since we used mean() to obtain the means, but that does make the solution more understandable. The solution requires calculating $\mathrm{X}^{\prime}$, requiring that the data matrix be duplicated. Also, we have used a numerically poor calculation formula.
> Example 11: Variance matrix, traditional approach 2

```
: X = .
st_view(X, ., "mpg weight displ", 0)
n = rows(X)
means = mean(X, 1)
cov = (X:-means)'(X:-means) :/ (n-1)
```

Comments: We use a better calculation formula and, in the process, increase our memory usage substantially.
> Example 12: Variance matrix using cross()

```
X = .
st_view(X, ., "mpg weight displ", 0)
n = rows(X)
means = mean(X, 1)
XX = cross(X, X)
cov = ((XX:/n)-means'means)*(n/(n-1))
```

Comments: The above solution conserves memory but uses the numerically poor calculation formula. A related function, crossdev(), will calculate deviation crossproducts:

```
X = .
st_view(X, ., "mpg weight displ", 0)
n = rows(X)
means = mean(X, 1)
xx = crossdev(X,means, X,means)
cov = xx:/(n-1)
```

See [M-5] crossdev( ). The easiest solution, however, is

```
X = .
st_view(X, ., "mpg weight displ", 0)
cov = variance(X, 1)
```

See [M-5] mean() for a description of the variance() function. variance() is implemented in terms of crossdev().

## Example 13: Weighted variance matrix, traditional approaches

```
: M = w = X = .
st_view(M, ., "w mpg weight displ", 0)
st_subview(w, M, ., 1)
st_subview(X, M, ., (2\.))
n = colsum(w)
means = mean(X, w)
cov = (X'diag(w)*X:/n - means'means)*(n/(n-1))
```

Comments: Above we use the numerically poor formula. Using the better deviation formula, we would have

```
M = w = X = .
st_view(M, ., "w mpg weight displ", O)
st_subview(w, M, ., 1)
st_subview(X, M, ., (2\.))
n = colsum(w)
means = mean(X, w)
cov = (X:-means)'diag(w)*(X:-means) :/ (n-1)
```

The memory requirements include making a copy of the data with the means removed and making an $N \times N$ diagonal matrix.

## > Example 14: Weighted variance matrix using cross( )

```
\(\mathrm{M}=\mathrm{w}=\mathrm{X}=\).
st_view(M, ., "w mpg weight displ", 0)
st_subview(w, M, ., 1)
st_subview(X, M, ., (2\.))
\(\mathrm{n} \quad=\operatorname{col} \operatorname{sum}(\mathrm{w})\)
means \(=\) mean \((X, w)\)
\(\operatorname{cov}=(\operatorname{cross}(X, w, X): / n-\) means'means \() *(n /(n-1))\)
```

Comments: As in example 12, the above solution conserves memory but uses a numerically poor calculation formula. Better is to use crossdev():

```
M = w = X = .
st_view(M, ., "w mpg weight displ", 0)
st_subview(w, M, ., 1)
st_subview(X, M, ., (2\.))
n = colsum(w)
means = mean(X, w)
cov = crossdev(X,means, w, X,means) :/ (n-1)
```

and easiest is to use variance():

```
M = w = X = .
st_view(M, ., "w mpg weight displ", 0)
st_subview(w, M, ., 1)
st_subview(X, M, ., (2\.))
cov = variance(X, w)
```

See [M-5] crossdev() and [M-5] mean( ).

## Comment concerning cross() and missing values

cross () automatically omits rows containing missing values in making its calculation. Depending on this feature, however, is considered bad style because so many other Mata functions do not provide that feature and it is easy to make a mistake.

The right way to handle missing values is to exclude them when constructing views and subviews, as we have done above. When we constructed a view, we invariably specified fourth argument 0 to st_view(). In formal programming situations, you will probably specify the name of the touse variable you have previously constructed in your ado-file that calls your Mata function.

## Conformability

```
\(\operatorname{cross}(X, x c, w, Z, z c)\) :
        X: \(\quad n \times v_{1}\) or \(1 \times 1, \quad 1 \times 1\) treated as if \(n \times 1\)
        \(x c: 1 \times 1\) (optional)
        \(w: \quad n \times 1\) or \(1 \times n\) or \(1 \times 1\) (optional)
        \(Z: \quad n \times v_{2}\)
        \(z c: \quad 1 \times 1 \quad\) (optional)
        result: \(\quad\left(v_{1}+(x c \neq 0)\right) \times\left(v_{2}+(z c \neq 0)\right)\)
```


## Diagnostics

$\operatorname{cross}(X, x c, w, Z, z c)$ omits rows in $X$ and $Z$ that contain missing values.

## Also see

[M-5] crossdev( ) - Deviation cross products
[M-5] mean() - Means, variances, and correlations
[M-5] quadcross() - Quad-precision cross products
[M-4] Statistical - Statistical functions
[M-4] Utility - Matrix utility functions

## Title

[M-5] crossdev( ) — Deviation cross products

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

crossdev() makes calculations of the form

$$
\begin{aligned}
& (X:-x)^{\prime}(X:-x) \\
& (X:-x)^{\prime}(Z:-z) \\
& (X:-x)^{\prime} \operatorname{diag}(w)(X:-x) \\
& (X:-x)^{\prime} \operatorname{diag}(w)(Z:-z)
\end{aligned}
$$

crossdev() is a variation on [M-5] cross(). crossdev() mirrors cross() in every respect except that it has two additional arguments: $x$ and $z . x$ and $z$ record the amount by which $X$ and $Z$ are to be deviated. $x$ and $z$ usually contain the (appropriately weighted) column means of $X$ and $Z$.

## Syntax

```
real matrix crossdev (X, x, Z,z)
real matrix crossdev (X,x,w,Z,z)
real matrix crossdev (X, xc, x, Z, zc,z)
real matrix crossdev (X,xc, x,w,Z,zc,z)
```

where

$$
X: \quad \text { real matrix } X
$$

$x c$ : real scalar $x c$
$x$ : real rowvector $x$
$w$ : real vector $w$
Z: real matrix $Z$
$z c$ : real scalar $z c$
$z$ : real rowvector $z$

## Remarks and examples

$x$ usually contains the same number of rows as $X$ but, if $x c \neq 0, x$ may contain an extra element on the right recording the amount from which the constant 1 should be deviated.

The same applies to $z$ : it usually contains the same number of rows as $Z$ but, if $z c \neq 0, z$ may contain an extra element on the right.
> Example 1: Linear regression using one view

```
M = .
st_view(M, ., "mpg weight foreign", 0)
means = mean(M, 1)
CP = crossdev(M,means, M,means)
XX = CP[|2,2 \ .,.|]
Xy = CP[|2,1 \ .,1|]
b = invsym(XX)*Xy
b = b \ means[1] - means[|2\.|]*b
```

Compare this solution with example 3 in [M-5] cross().
> Example 2: Linear regression using subviews

```
M = X = y = .
st_view(M, ., "mpg weight foreign", 0)
st_subview(y, M, ., 1)
st_subview(X, M, ., (2\.))
xmean = mean(X, 1)
ymean = mean(y, 1)
XX = crossdev(X,xmean, X,xmean)
Xy = crossdev(X,xmean, y,ymean)
b = invsym(XX)*Xy
b = b \ ymean - xmean*b
```

Compare this solution with example 4 in [M-5] cross().
$>$ Example 3: Weighted linear regression

```
M = X = y = w = .
st_view(M, ., "w mpg weight foreign", 0)
st_subview(w, M, ., 1)
st_subview(y, M, ., 2)
st_subview(X, M, ., (3\.))
xmean = mean(X, w)
ymean = mean(y, w)
XX = crossdev(X,xmean, w, X,xmean)
Xy = crossdev(X,xmean, w, y,ymean)
b = invsym(XX)*Xy
b}=\textrm{b}\\mathrm{ \ymean - xmean*b
```

Compare this solution with example 6 in [M-5] cross().
> Example 4: Variance matrix

```
: X = .
st_view(X, ., "mpg weight displ", 0)
n = rows(X)
means = mean(X, 1)
xx = crossdev(X,means, X,means)
cov = xx:/(n-1)
```

This is exactly what variance() does; see [M-5] mean(). Compare this solution with example 12 in [M-5] cross( ).
> Example 5: Weighted variance matrix

```
M = w = X = .
st_view(M, ., "w mpg weight displ", 0)
st_subview(w, M, ., 1)
st_subview(X, M, ., (2\.))
n = colsum(w)
means = mean(X, w)
cov = crossdev(X,means, w, X,means) :/ (n-1)
```

This is exactly what variance() does with weighted data; see [M-5] mean(). Compare this solution with example 14 in [M-5] cross( ).

## Conformability

```
\(\operatorname{crossdev}(X, x c, x, w, Z, z c, z)\) :
    \(X: \quad n \times v_{1}\) or \(1 \times 1, \quad 1 \times 1\) treated as if \(n \times 1\)
    \(x c: \quad 1 \times 1 \quad\) (optional)
    \(x: \quad 1 \times v_{1} \quad\) or \(\quad 1 \times v_{1}+(x c \neq 0)\)
    \(w: \quad n \times 1\) or \(1 \times n\) or \(1 \times 1\) (optional)
    \(Z: \quad n \times v_{2}\)
    \(z c: \quad 1 \times 1 \quad\) (optional)
    \(z: \quad 1 \times v_{2} \quad\) or \(\quad 1 \times v_{2}+(z c \neq 0)\)
    result: \(\quad\left(v_{1}+(x c \neq 0)\right) \times\left(v_{2}+(z c \neq 0)\right)\)
```


## Diagnostics

crossdev $(X, x c, x, w, Z, z c, z)$ omits rows in $X$ and $Z$ that contain missing values.

## Also see

[M-5] cross() - Cross products
[M-5] quadcross( ) - Quad-precision cross products
[M-4] Utility - Matrix utility functions
[M-4] Statistical - Statistical functions

## Title

[M-5] cvpermute( ) — Obtain all permutations

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

cvpermute() returns all permutations of the values of column vector $V$, one at a time. If $V=(1 \backslash 2 \backslash 3)$, there are six permutations and they are $(1 \backslash 2 \backslash 3),(1 \backslash 3 \backslash 2),(2 \backslash 1 \backslash 3),(2 \backslash 3 \backslash 1),(3 \backslash 1 \backslash 2)$, and $(3 \backslash 2 \backslash 1)$. If $V=(1 \backslash 2 \backslash 1)$, there are three permutations and they are $(1 \backslash 1 \backslash 2)$, $(1 \backslash 2 \backslash 1)$, and ( $2 \backslash 1 \backslash 1$ ).

Vector $V$ is specified by calling cvpermutesetup(),

```
info = cvpermutesetup(V)
```

info holds information that is needed by cvpermute() and it is info, not $V$, that is passed to cvpermute(). To obtain the permutations, repeated calls are made to cvpermute() until it returns J ( $0,1,$. ):

```
info = cvpermutesetup(V)
while ((p=cvpermute(info)) != J(0,1,.)) {
        ... p ...
}
```

Column vector $p$ will contain a permutation of $V$.
cvpermutesetup() may be specified with one or two arguments:

```
info = cvpermutesetup(V)
info = cvpermutesetup ( }V\mathrm{ , unique)
```

unique is usually not specified. If unique is specified, it should be 0 or 1 . Not specifying unique is equivalent to specifying unique $=0$. Specifying unique $=1$ states that the elements of $V$ are unique or, at least, are to be treated that way.

When the arguments of $V$ are unique-for instance, $V=(1 \backslash 2 \backslash 3)$ —specifying unique $=1$ will make cvpermute() run faster. The same permutations will be returned, although usually in a different order.

When the arguments of $V$ are not unique-for instance, $V=(1 \backslash 2 \backslash 1)$-specifying unique $=1$ will make cvpermute () treat them as if they were unique. With unique $=0$, there are three permutations of $(1 \backslash 2 \backslash 1)$. With unique $=1$, there are six permutations, just as there are with $(1 \backslash 2 \backslash 3)$.

## Syntax

$$
\text { info }=\text { cvpermutesetup }(\text { real colvector } V[, \text { real scalar unique }])
$$

real colvector cvpermute (info)
where info should be declared transmorphic.

## Remarks and examples

## Example 1

You have the following data:

| v1 | v2 |
| :--- | :--- |
| 22 | 29 |
| 17 | 33 |
| 21 | 26 |
| 20 | 32 |
| 16 | 35 |

You wish to do an exact permutation test for the correlation between v 1 and v 2 .
That is, you wish to (1) calculate the correlation between v1 and v2-call that value r-and then (2) calculate the correlation between v 1 and v 2 for all permutations of v 1 , and count how many times the result is more extreme than $r$.

For the first step,

```
X = (22, 29 \
> 17, 33\
> 21, 26 \
> 20,32\
> 16, 35)
: correlation(X)
[symmetric]
```

1
2
symmetric] 1


The correlation is -0.846855 . For the second step,

```
V1 = X[,1]
V2 = X[,2]
num = den = 0
info = cvpermutesetup(V1)
while ((V1=cvpermute(info)) != J(0,1,.)) {
    rho = correlation((V1,V2))[2,1]
    if (rho<=-.846 | rho>=.846) num++
    den++
}
```

```
: (num, den, num/den)
```

    \(\begin{array}{lll}1 & 2 & 3\end{array}\)
    1

| 13 | 120 | .1083333333 |
| :--- | :--- | :--- |

Of the 120 permutations, $13(10.8 \%)$ were outside 0.846855 or -0.846855 .

## > Example 2

You now wish to do the same thing but using the Spearman rank-correlation coefficient. Mata has no function that will calculate that, but Stata has a command that does-see $[R]$ spearman-so we will use the Stata command as our subroutine.

This time, we will assume that the data have been loaded into a Stata dataset:
list
1.

| var1 | var2 |
| ---: | ---: |
| 22 | 29 |
| 17 | 33 |
| 21 | 26 |
| 20 | 32 |
| 16 | 35 |

For the first step,

```
. spearman var1 var2
Number of observations = 5
    Spearman's rho = -0.9000
Test of HO: var1 and var2 are independent
                        Prob = 0.0468
```

For the second step,

```
V1 = st_data(., "var1")
info = cvpermutesetup(V1)
num = den = 0
while ((V1=cvpermute(info)) != J(0,1,.)) {
    st_store(., "var1", V1)
    stata("quietly spearman var1 var2")
    rho = st_numscalar("r(rho)")
        if (rho<=-.9 | rho>=.9) num++
        den++
}
(num, den, num/den)
                            1 2 3
1
\begin{tabular}{|lll|}
\hline 2 & 120 & .0166666667 \\
\hline
\end{tabular}
```

Only two of the permutations resulted in a rank correlation of at least 0.9 in magnitude.

In the code above, we obtained the rank correlation from $r$ ( $r h o$ ) which, we learned from $[R]$ spearman, is where spearman stores it.

Also note how we replaced the contents of var1 by using st_store(). Our code leaves the dataset changed and so could be improved.

## Conformability

cvpermutesetup ( $V$, unique):

$$
\begin{array}{rll}
V: & n \times 1 & \\
\text { unique: } & 1 \times 1 & \text { (optional) } \\
\text { result: } & 1 \times L &
\end{array}
$$

cvpermute(info):

$$
\begin{aligned}
\text { info: } & 1 \times L \\
\text { result: } & n \times 1 \text { or } 0 \times 1
\end{aligned}
$$

where

$$
L= \begin{cases}3 & \text { if } n=0 \\ 4 & \text { if } n=1 \\ (n+3)(n+2) / 2-6 & \text { otherwise }\end{cases}
$$

The value of $L$ is not important except that the info vector returned by cvpermutesetup() and then passed to cvpermute() consumes memory. For instance,

| $n$ | $L$ | Total memory $(8 \times L)$ |
| ---: | ---: | :---: |
| 5 | 22 | 176 bytes |
| 10 | 72 | 576 |
| 50 | 1,372 | 10,560 |
| 100 | 5,247 | 41,976 |
| 1,000 | 502,497 | $4,019,976$ |

## Diagnostics

cvpermute() returns $J(0,1,$.$) when there are no more permutations.$

## Also see

[M-4] Statistical - Statistical functions

## Title

[M-5] date() - Date and time manipulation
Description Syntax Conformability Diagnostics Also see

## Description

These functions mirror Stata's date functions; see [D] Datetime.

## Syntax

```
            tc = clock(strdatetime, pattern [, year])
            tc = mdyhms(month, day, year, hour, minute, second)
            tc = dhms(td, hour, minute, second)
            tc = hms(hour, minute, second)
            hour = hh(tc)
minute = mm(tc)
second = ss(tc)
            td= dofc(tc)
    tC=Cofc(tc)
    tC = Clock(strdatetime, pattern [, year])
    tC = Cmdyhms(month, day, year, hour, minute, second)
    tC = Cdhms (td, hour, minute, second)
    tC = Chms(hour, minute, second)
    hour = hhC(tC)
minute = mmC (tC)
second }=\operatorname{ssC}(tC
    td= dofC(tC)
    tc}=\operatorname{cofC}(tC
```

```
            \(t d=\) date \((\) strdate, dpattern \([\), year \(])\)
            \(t d=\operatorname{mdy}(\) month, day, year)
            \(t d=\operatorname{dmy}(\) day, month, year \()\)
            \(t w=y w(y e a r\), week \()\)
            tm \(=\mathrm{ym}(\) year, month \()\)
            \(t q=\mathrm{yq}(\) year,\(q u a r t e r)\)
            th \(=\mathrm{yh}(\) year, half \()\)
            \(t c=\operatorname{cofd}(t d)\)
            \(t C=\operatorname{Cofd}(t d)\)
                    \(t d=\operatorname{dofb}(t b\), "calendar")
                    \(t b=\operatorname{bofd}(" c a l e n d a r ", t d)\)
    month \(=\) month \((t d)\)
            \(d a y=\operatorname{day}(t d)\)
            year \(=\) year \((t d)\)
            week \(=\) week \((t d)\)
    quarter \(=\) quarter \((t d)\)
    half \(=\) halfyear \((t d)\)
dayofyear \(=\operatorname{doy}(t d)\)
dayofweek \(=\operatorname{dow}(t d)\)
ty \(=\operatorname{yearly}(\) strydate, ypattern \([\), year \(])\)
\(t y=\operatorname{yofd}(t d)\)
\(t d=\operatorname{dofy}(t y)\)
th \(=\) halfyearly (strhdate, hpattern \([\), year \(])\)
\(t h=\operatorname{hofd}(t d)\)
\(t d=\operatorname{dofh}(t h)\)
tq \(=\) quarterly \((\) strqdate, qpattern \([\), year \(])\)
\(t q=q \circ f d(t d)\)
\(t d=\operatorname{dofq}(t q)\)
tm \(=\) monthly \((\) strmdate, mpattern \([\), year \(])\)
\(t m=\operatorname{mofd}(t d)\)
\(t d=\operatorname{dofm}(t m)\)
```

```
            tw = weekly (strwdate, wpattern [, year])
            tw = wofd (td)
            td= dofw(tw)
        hours = hours(ms)
minutes = minutes(ms)
seconds}=\mathrm{ seconds(ms)
    ms = msofhours(hours)
    ms = msofminutes(minutes)
    ms = msofseconds(seconds)
    int = age(td, td [, strleapday])
real= age_frac(td,td [, strleapday])
    int = Clockdiff (tC, tC, strtimeunit)
    int = clockdiff(tc, tc, strtimeunit)
real = Clockdiff_frac( }tC,tC\mathrm{ , strtimeunit )
real = clockdiff_frac(tc,tc, strtimeunit)
    int = datediff(td,td, strtimeunit [, strleapday])
real = datediff_frac(td,td, strtimeunit [, strleapday])
    td = birthday (td, year [, strleapday])
    td = previousbirthday(td, td [, strleapday])
    td = nextbirthday (td, td [, strleapday])
bool = isleapyear(year)
year = previousleapyear (year)
year = nextleapyear (year)
    int = daysinmonth(td)
    td = firstdayofmonth(td)
    td = lastdayofmonth (td)
```

```
    int = datepart(td, strtimeunit)
    int = clockpart(tc, strtimeunit)
    int = Clockpart(tC, strtimeunit)
bool = isleapsecond(tC)
    td = today ()
    tc = now()
int = dayssinceweekday(td, dayofweek)
int = dayssincedow(td, dayofweek)
int = daysuntilweekday(td, dayofweek)
int = daysuntildow(td, dayofweek)
td = firstweekdayofmonth(month, year, dayofweek)
td = firstdowofmonth(month, year, dayofweek)
td = lastweekdayofmonth(month, year, dayofweek)
td = lastdowofmonth(month, year, dayofweek)
td = previousweekday(td, dayofweek)
td = previousdow(td, dayofweek)
td = nextweekday(td, dayofweek)
td = nextdow(td, dayofweek)
```

where
$t c: \quad$ number of milliseconds from 01jan1960 00:00:00.000, unadjusted for leap seconds
$t C: \quad$ number of milliseconds from 01jan1960 00:00:00.000, adjusted for leap seconds
strdatetime: string-format date, time, or date/time, for example, "15jan1992",
"1/15/1992", "15-1-1992", "January 15, 1992",
"9:15", "13:42", "1:42 p.m.", "1:42:15.002 pm",
"15jan1992 9:15", "1/15/1992 13:42",
"15-1-1992 1:42 p.m.",
"January 15, 1992 1:42:15.002 pm"
pattern: order of month, day, year, hour, minute, and seconds in strdatetime, plus optional default century, for example, "DMYhms" (meaning day, month, year, hour, minute, second), "DMYhm", "MDYhm", "hmMDY", "hms","hm", "MDY", "MD19Y", "MDY20Yhm"
$t d: \quad$ number of days from 01jan1960
$t b: \quad$ business date (days)
calendar: string scalar containing calendar name or \%tb format
strdate: $\quad$ string-format date, for example, "15jan1992", "1/15/1992", "15-1-1992", "January 15, 1992"
dpattern:
order of month, day, and year in strdate, plus optional default century, for example, "DMY" (meaning day, month, year), "MDY", "MD19Y"
hour: hour, 0-23
minute: minute, 0-59
second: second, 0.000-59.999 (maximum 60.999 in case of leap second)
month: month number, 1-12
day: day-of-month number, 1-31
year: year, for example, 1942, 1995, 2008
week: week within year, 1-52
quarter: quarter within year, 1-4
half: half within year, 1-2
dayofyear: day of year, 1-366
dayofweek:
day of week, $0-6(0=$ Sunday, $1=$ Monday, $\ldots, 6=$ Saturday $)$; alternatively, when used as an argument, strings with the first two or more letters of the day of week (case insensitive)
ty: calendar year
strydate: string-format calendar year, for example, "1980", "80"
ypattern: pattern of strydate, for example, "Y", "19Y"
th: number of halves from 1960 h 1
strhdate: string-format hdate, for example, "1982-1", "1982h2", "2 1982" hpattern: pattern of strhdate, for example, "YH", "19YH", "HY"
$t q: \quad$ number of quarters from 1960q1
strqdate: $\quad$ string-format qdate, for example, "1982-3", "1982q2", "3 1982"
qpattern: pattern of strqdate, for example, "YQ", "19YQ", "QY"
tm: number of months from 1960 m 1
strmdate:
string-format mdate, for example, "1982-3", "1982m2", "3/1982"
mpattern: pattern of strmdate, for example, "YM", "19YM", "MR"
$t w: \quad$ number of weeks from 1960w1
strwdate: $\quad$ string-format wdate, for example, "1982-3", "1982w2", "1982-15"
wpattern: pattern of strwdate, for example, "YW", "19YW", "WY"
hours: interval of time in hours (positive or negative, real)
minutes: interval of time in minutes (positive or negative, real)
seconds: interval of time in seconds (positive or negative, real)
$m s: \quad$ interval of time in milliseconds (positive or negative, integer)
int: integer value
real: real value
bool: $\quad$ Boolean logic value, 0 (false) or 1 (true)
strleapday: string-format nonleap-year anniversary for 29 feb, for example, "feb28",
"28feb", "mar01", "mar1", "1mar"
strtimeunit: string-format unit of time, for example, "year", "y", "month", "mon",
"m", "day", "d", "hour", "h", "minute", "min",
"second", "s", "millisecond", "ms"
Functions return an element-by-element result. Functions are usually used with scalars.
All variables are real matrix except the str* and *pattern variables, which are string matrix.

## Conformability

clock(strdatetime, pattern, year), Clock(strdatetime, pattern, year):

```
strdatetime: }\quad\mp@subsup{r}{1}{}\times\mp@subsup{c}{1}{
    pattern: }\quad\mp@subsup{r}{2}{}\times\mp@subsup{c}{2}{}\quad\mathrm{ (c-conformable with strdatetime)
        year: }\quad\mp@subsup{r}{3}{}\times\mp@subsup{c}{3}{}\quad\mathrm{ (optional, c-conformable)
        result: }\quad\operatorname{max}(\mp@subsup{r}{1}{},\mp@subsup{r}{2}{},\mp@subsup{r}{3}{})\times\operatorname{max}(\mp@subsup{c}{1}{},\mp@subsup{c}{2}{},\mp@subsup{c}{3}{}
```

mdyhms(month, day, year, hour, minute, second),
Cmdyhms (month, day, year, hour, minute, second):

```
month: \(\quad r_{1} \times c_{1}\)
    day: \(\quad r_{2} \times c_{2}\)
    year: \(\quad r_{3} \times c_{3}\)
    hour: \(\quad r_{4} \times c_{4}\)
minute: \(\quad r_{5} \times c_{5}\)
second: \(\quad r_{6} \times c_{6} \quad\) (all variables c-conformable)
    result: \(\quad \max \left(r_{1}, r_{2}, r_{3}, r_{4}, r_{5}, r_{6}\right) \times \max \left(c_{1}, c_{2}, c_{3}, c_{4}, c_{5}, c_{6}\right)\)
```

hms(hour, minute, second), Chms (hour, minute, second):

```
    hour: }\quad\mp@subsup{r}{1}{}\times\mp@subsup{c}{1}{
    minute: }\quad\mp@subsup{r}{2}{}\times\mp@subsup{c}{2}{
    second: }\quad\mp@subsup{r}{3}{}\times\mp@subsup{c}{3}{
    result: }\quad\operatorname{max}(\mp@subsup{r}{1}{},\mp@subsup{r}{2}{},\mp@subsup{r}{3}{})\times\operatorname{max}(\mp@subsup{c}{1}{},\mp@subsup{c}{2}{},\mp@subsup{c}{3}{}
```

dhms ( $t d$, hour, minute, second), Cdhms ( $t d$, hour, minute, second):

| $t d:$ | $r_{1} \times c_{1}$ |
| ---: | :--- |
| hour: | $r_{2} \times c_{2}$ |
| minute: | $r_{3} \times c_{3}$ |
| second: | $r_{4} \times c_{4} \quad$ (all variables c-conformable) |
| result: | $\max \left(r_{1}, r_{2}, r_{3}, r_{4}\right) \times \max \left(c_{1}, c_{2}, c_{3}, c_{4}\right)$ |

$\operatorname{hh}(x), \operatorname{mm}(x), \operatorname{ss}(x), \operatorname{hhC}(x), \operatorname{mmC}(x), \operatorname{ssC}(x):$

$$
\begin{aligned}
x: & & r \times c \\
\text { result: } & & r \times c
\end{aligned}
$$

date (strdate, dpattern, year):
strdate: $\quad r_{1} \times c_{1}$
dpattern: $\quad r_{2} \times c_{2} \quad$ (c-conformable with strdate)
year: $\quad r_{3} \times c_{3} \quad$ (optional, c-conformable)
result: $\quad \max \left(r_{1}, r_{2}, r_{3}\right) \times \max \left(c_{1}, c_{2}, c_{3}\right)$
mdy (month, day, year), dmy (day, month, year):

| month: | $r_{1} \times c_{1}$ |
| ---: | :--- |
| day: | $r_{2} \times c_{2}$ |
| year: | $r_{3} \times c_{3} \quad$ (all variables c-conformable) |
| result: | $\max \left(r_{1}, r_{2}, r_{3}\right) \times \max \left(c_{1}, c_{2}, c_{3}\right)$ |

yw (year, detail), ym(year, detail), yq (year, detail), yh (year, detail):

| year: | $r_{1} \times c_{1}$ |
| ---: | :--- |
| detail: | $r_{2} \times c_{2} \quad$ (c-conformable with year) |
| result: | $\max \left(r_{1}, r_{2}\right) \times \max \left(c_{1}, c_{2}\right)$ |

month $(t d)$, day $(t d)$, year $(t d)$, dow $(t d)$, week $(t d)$, quarter $(t d)$, halfyear $(t d)$, doy $(t d)$ :

$$
\begin{aligned}
& t d: \\
& \text { result: } \\
& r \times c
\end{aligned}
$$

yearly (str, pat, year), halfyearly(str, pat, year), quarterly (str, pat, year), monthly(str, pat, year), weekly(str, pat, year):

| str: | $r_{1} \times c_{1}$ |  |
| ---: | :--- | :--- |
| pat: | $r_{2} \times c_{2}$ | $(\mathrm{c}$-conformable with str) |
| year: | $r_{3} \times c_{3}$ | (optional, c-conformable) |
| result: |  | $\max \left(r_{1}, r_{2}, r_{3}\right) \times \max \left(c_{1}, c_{2}, c_{3}\right)$ |

$\operatorname{Cofc}(x), \operatorname{cofC}(x), \operatorname{dofc}(x), \operatorname{dofC}(x), \operatorname{cofd}(x), \operatorname{Cofd}(x), \operatorname{yofd}(x), \operatorname{dofy}(x), \operatorname{hofd}(x)$, $\operatorname{dofh}(x), \operatorname{qofd}(x), \operatorname{dofq}(x), \operatorname{mofd}(x), \operatorname{dofm}(x), \operatorname{wofd}(x), \operatorname{dofw}(x)$ :

$$
\begin{aligned}
& x: \\
& \text { result: } \\
& r \times c
\end{aligned}
$$

dofb(tb, "calendar"):

$$
\begin{aligned}
t b: & r \times c \\
\text { calendar: } & 1 \times 1 \\
\text { result: } & r \times c
\end{aligned}
$$

```
bofd("calendar",td):
    calendar: }1\times
    td: }\quadr\times
    result: }\quadr\times
```

hours $(x)$, minutes $(x)$, seconds $(x)$, msofhours $(x)$, msofminutes $(x)$, msofseconds $(x)$ :
$\begin{aligned} x: & r \times c \\ \text { result }: & r \times c\end{aligned}$
age $(t d 1, t d 2, s t r)$, age_frac $(t d 1, t d 2, s t r)$ :
$t d 1: \quad r_{1} \times c_{1}$
$t d 2: \quad r_{2} \times c_{2} \quad$ (c-conformable with $t d 1$ )
str: $\quad r_{3} \times c_{3} \quad$ (optional, c-conformable)
result: $\quad \max \left(r_{1}, r_{2}, r_{3}\right) \times \max \left(c_{1}, c_{2}, c_{3}\right)$
datediff $(t d 1, t d 2, \operatorname{str} 1, s t r 2)$, datediff_frac $(t d 1, t d 2, \operatorname{str} 1, s t r 2)$ :
$t d 1: \quad r_{1} \times c_{1}$
$t d 2: \quad r_{2} \times c_{2}$
strl: $\quad r_{3} \times c_{3} \quad$ (all variables c-conformable)
str2: $\quad r_{4} \times c_{4} \quad$ (optional, c-conformable)
result: $\quad \max \left(r_{1}, r_{2}, r_{3}, r_{4}\right) \times \max \left(c_{1}, c_{2}, c_{3}, c_{4}\right)$
Clockdiff ( $t$ C1, $t C 2$, str), Clockdiff_frac ( $t C 1, t C 2$, str) :
$t C 1: \quad r_{1} \times c_{1}$
$t C 2: \quad r_{2} \times c_{2}$
str: $\quad r_{3} \times c_{3} \quad$ (all variables c-conformable)
result: $\quad \max \left(r_{1}, r_{2}, r_{3}\right) \times \max \left(c_{1}, c_{2}, c_{3}\right)$
clockdiff( $t c 1, t c 2, \operatorname{str})$, clockdiff_frac (tcl, tc2, str):
$t c 1: \quad r_{1} \times c_{1}$
$t c 2: \quad r_{2} \times c_{2}$
str: $\quad r_{3} \times c_{3} \quad$ (all variables c-conformable)
result: $\quad \max \left(r_{1}, r_{2}, r_{3}\right) \times \max \left(c_{1}, c_{2}, c_{3}\right)$
birthday ( $t d$, detail, str), previousbirthday ( $t d$, detail, str),
nextbirthday ( $t d$, detail, str):
$t d: \quad r_{1} \times c_{1}$
detail: $\quad r_{2} \times c_{2} \quad$ (c-conformable with $t d$ )
str: $\quad r_{3} \times c_{3} \quad$ (optional, c-conformable)
result: $\quad \max \left(r_{1}, r_{2}, r_{3}\right) \times \max \left(c_{1}, c_{2}, c_{3}\right)$
isleapyear (year), previousleapyear (year), nextleapyear (year):
$\begin{array}{rlr}\text { year: } & & r \times c \\ \text { result: } & & r \times c\end{array}$
daysinmonth $(t d)$, firstdayofmonth ( $t d$ ), lastdayofmonth ( $t d$ ):

$$
\begin{aligned}
t d: & r \times c \\
\text { result: } & r \times c
\end{aligned}
$$

Clockpart (detail, str), clockpart (detail, str), datepart (detail, str):

```
detail: \(\quad r_{1} \times c_{1}\)
    str: \(\quad r_{2} \times c_{2}\)
    result: \(\quad \max \left(r_{1}, r_{2}\right) \times \max \left(c_{1}, c_{2}\right)\)
```

isleapsecond ( $t C$ ):
$t C: \quad r \times c$
result: $\quad r \times c$
dayssinceweekday( $t$, dayofweek), dayssincedow( $t d$, dayofweek), daysuntilweekday( $t d$, dayofweek), daysuntildow( $t d$, dayofweek):
$t d: \quad r_{1} \times c_{1}$
dayofweek: $\quad r_{2} \times c_{2}$
result: $\quad \max \left(r_{1}, r_{2}\right) \times \max \left(c_{1}, c_{2}\right)$
firstweekdayofmonth(month, year, dayofweek), firstdowofmonth(month, year, dayofweek), lastweekdayofmonth(month, year, dayofweek), lastdowofmonth(month, year, dayofweek):
month: $\quad r_{1} \times c_{1}$
year: $\quad r_{2} \times c_{2}$
dayofweek: $\quad r_{3} \times c_{3}$
result: $\quad \max \left(r_{1}, r_{2}, r_{3}\right) \times \max \left(c_{1}, c_{2}, c_{3}\right)$
previousweekday ( $t d$, dayofweek), previousdow ( $t d$, dayofweek),
nextweekday ( $t d$, dayofweek), nextdow ( $t d$, dayofweek):
$t d: \quad r_{1} \times c_{1}$
dayofweek: $\quad r_{2} \times c_{2}$
result: $\quad \max \left(r_{1}, r_{2}\right) \times \max \left(c_{1}, c_{2}\right)$

## Diagnostics

None.

## Also see

[M-4] Dates - Date and time functions

## Title

[M-5] deriv() — Numerical derivatives

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Methods and formulas | References | Also see |

## Description

These functions compute derivatives of the real function $f(p)$ at the real parameter values $p$.
deriv_init() begins the definition of a problem and returns $D$, a problem-description handle set that contains default values.

The deriv_init_* $(D, \ldots)$ functions then allow you to modify those defaults. You use these functions to describe your particular problem: to set the identity of function $f()$, to set parameter values, and the like.
deriv( $D$, todo) then computes derivatives depending upon the value of todo.
deriv ( $D, 0$ ) returns the function value without computing derivatives.
deriv $(D, 1)$ returns the first derivatives, also known as the gradient vector for scalar-valued functions (type d and v) or the Jacobian matrix for vector-valued functions (type t ).
deriv ( $D, 2$ ) returns the matrix of second derivatives, also known as the Hessian matrix; the gradient vector is also computed. This syntax is not allowed for type $t$ evaluators.

The deriv_result_*( $D$ ) functions can then be used to access other values associated with the solution.

Usually you would stop there. In other cases, you could compute derivatives at other parameter values:

```
deriv_init_params(D, p_alt)
deriv(D, todo)
```

Aside: The deriv_init_* ( $D, \ldots$ ) functions have two modes of operation. Each has an optional argument that you specify to set the value and that you omit to query the value. For instance, the full syntax of deriv_init_params() is
void deriv_init_params ( $D$, real rowvector parameters)
real rowvector deriv_init_params ( $D$ )
The first syntax sets the parameter values and returns nothing. The second syntax returns the previously set (or default, if not set) parameter values.

All the deriv_init_* $(D, \ldots)$ functions work the same way.

## Syntax

$$
D=\text { deriv_init() }
$$

(varies) deriv_init_evaluator ( $D[$, \&function() $]$ )
(varies) deriv_init_evaluatortype (D [, evaluatortype $]$ )
(varies) deriv_init_params (D [, real rowvector parameters $]$ )
(varies) deriv_init_argument ( $D$, real scalar $k[, X]$ )
(varies) deriv_init_narguments ( $D[$, real scalar $K]$ )
(varies) deriv_init_weights ( $D$ [, real colvector weights $]$ )
(varies) deriv_init_h(D [, real rowvector $h])$
(varies) deriv_init_scale( $D$ [, real matrix scale $]$ )
(varies) deriv_init_bounds ( $D$ [, real rowvector minmax $]$ )
(varies) deriv_init_search ( $D[$, search $]$ )
(varies) deriv_init_usemin(D [, \{"off"|"on"\}])
(varies) deriv_init_min( $D$ [, real rowvector min $]$ )
(varies) deriv_init_verbose(D [, \{"on"|"off"\}])
(varies) deriv_init_technique( $D[$, technique $]$ )
(varies) $\quad \operatorname{deriv}(D,\{0|1| 2\})$
real scalar _deriv( $D,\{0|1| 2\}$ )
real scalar deriv_result_value( $D$ )
real vector deriv_result_values ( $D$ )
void __deriv_result_values $(D, v)$
real rowvector deriv_result_gradient ( $D$ )
void
real matrix
deriv_result_scores(D)
void __deriv_result_scores $(D, S)$
real matrix deriv_result_Jacobian(D)
void _deriv_result_Jacobian ( $D, J$ )
real matrix deriv_result_Hessian( $D$ )

```
void _deriv_result_Hessian(D,H)
real rowvector deriv_result_h(D)
real matrix deriv_result_scale(D)
real matrix deriv_result_delta(D)
real scalar deriv_result_errorcode(D)
string scalar deriv_result_errortext(D)
real scalar deriv_result_returncode(D)
void deriv_query(D)
```

where $D$, if it is declared, should be declared

```
transmorphic D
```

and where evaluatortype optionally specified in deriv_init_evaluatortype() is

| evaluatortype | Description |
| :--- | :--- |
| "d" | function () returns scalar value |
| "v" | function () returns colvector value |
| "t" | function () returns rowvector value |

The default is "d" if not set.
and where search optionally specified in deriv_init_search() is

| search | Description |
| :--- | :--- |
| "interpolate" | use linear and quadratic interpolation to search <br> for an optimal delta |
| "bracket" | use a bracketed quadratic formula to search for <br> an optimal delta <br> do not search for an optimal delta |
| "off" | The default is "interpolate" if not set. |

and where technique optionally specified in deriv_init_technique() is

| technique | Description |
| :--- | :--- |
| "numdiff" | numerical differentiation method <br> complex step method |

The default is "numdiff" if not set.

## Remarks and examples

Remarks are presented under the following headings:
First example
Notation and formulas
Notation
Numerical differentiation method
Complex step method
Type d evaluators
Example of a type d evaluator
Type $v$ evaluators
User-defined arguments
Example of a type $v$ evaluator
Type $t$ evaluators
Example of a type $t$ evaluator
Example of using step-size lower bounds
Example of complex step method
Functions

```
deriv_init()
deriv_init_evaluator() and deriv_init_evaluatortype()
deriv_init_argument() and deriv_init_narguments()
deriv_init_weights()
deriv_init_params()
Advanced init functions
deriv_init_h(), . . _ _scale( ), . . _bounds( ), and . . ._search()
deriv_init_usemin() and deriv_init_min()
deriv_init_verbose()
deriv_init_technique()
deriv()
_deriv()
deriv_result_value()
deriv_result_values() and _deriv_result_values()
deriv_result_gradient() and _deriv_result_gradient()
deriv_result_scores() and _deriv_result_scores()
deriv_result_Jacobian() and _deriv_result_Jacobian()
deriv_result_Hessian() and _deriv_result_Hessian()
deriv_result_h(), ..._scale( ), and ..._delta()
deriv_result_errorcode(), . . _errortext(), and . . . _returncode()
    deriv_query()
```


## First example

The derivative functions may be used interactively.
Below we use the functions to compute $f^{\prime}(x)$ at $x=0$, where the function is

```
f(x)}=\operatorname{exp}(-\mp@subsup{x}{}{2}+x-3
void myeval(x, y)
> {
> y = exp(-x^2 + x - 3)
> }
D = deriv_init()
deriv_init_evaluator(D, &myeval())
: deriv_init_params(D, 0)
: dydx = deriv(D, 1)
```

```
dydx
    .0497870683
: exp(-3)
    .0497870684
```

The derivative, given the above function, is $f^{\prime}(x)=(-2 \times x+1) \times \exp \left(-x^{2}+x-3\right)$, so $f^{\prime}(0)=\exp (-3)$.

## Notation and formulas

## Notation

We wrote the above in the way that mathematicians think, that is, differentiate $y=f(x)$. Statisticians, on the other hand, think differentiate $s=f(b)$. To avoid favoritism, we will write $v=f(p)$ and write the general problem with the following notation:

Differentiate $v=f(p)$ with respect to $p$, where

$$
\begin{array}{ll}
v: & \text { a scalar } \\
p: & 1 \times n p
\end{array}
$$

The gradient vector is $g=f^{\prime}(p)=d f / d p$, where

$$
g: \quad 1 \times n p
$$

and the Hessian matrix is $H=f^{\prime \prime}(p)=d^{2} f / d p d p^{\prime}$, where

$$
H: \quad n p \times n p
$$

deriv() can also work with vector-valued functions. Here is the notation for vector-valued functions:
Differentiate $v=f(p)$ with respect to $p$, where

$$
\begin{array}{ll}
v: & 1 \times n v, \text { a vector } \\
p: & 1 \times n p
\end{array}
$$

The Jacobian matrix is $J=f^{\prime}(p)=d f / d p$, where

$$
J: \quad n v \times n p
$$

and where

$$
J[i, j]=d v[i] / d p[j]
$$

Second-order derivatives are not computed by deriv() when used with vector-valued functions.

## Numerical differentiation method

The classical method to compute numerical derivatives is the numerical differentiation method, which is our default setting for technique. This section considers numerical derivatives using this method. deriv() uses the following formula for computing the numerical derivative of $f()$ at $p$

$$
f^{\prime}(p)=\frac{f(p+d)-f(p-d)}{2 d}
$$

where we refer to $d$ as the delta used for computing numerical derivatives. To search for an optimal delta, we decompose $d$ into two parts.

$$
d=h \times \text { scale }
$$

By default, $h$ is a fixed value that depends on the parameter value.

$$
h=(\operatorname{abs}(p)+1 \mathrm{e}-3) * 1 \mathrm{e}-3
$$

deriv() searches for a value of scale that will result in an optimal numerical derivative, that is, one where $d$ is as small as possible subject to the constraint that $f(x+d)-f(x-d)$ will be calculated to at least half the accuracy of a double-precision number. This is accomplished by searching for scale such that $|f(x)-f(x-d)|$ falls between $v_{0}$ and $v_{1}$, where

$$
\begin{aligned}
& v_{0}=(\operatorname{abs}(f(x))+1 \mathrm{e}-8) * 1 \mathrm{e}-8 \\
& v_{1}=(\operatorname{abs}(f(x))+1 \mathrm{e}-7) * 1 \mathrm{e}-7
\end{aligned}
$$

Use deriv_init_h() to change the default $h$ values. Use deriv_init_scale() to change the default initial scale values. Use deriv_init_bounds () to change the default bounds (1e-8, 1e-7) used for determining the optimal scale.

Sometimes, the computed delta $d$ used for calculating numerical derivatives may be too small, especially in the denominator. This may cause unstable computation results. You can avoid this by bounding $d$ with a minimum value. Use deriv_init_usemin() to set whether to use minimum values for $d$. When minimum values are requested, the default minimums are $1 \mathrm{e}-6$ for first-order derivatives and $1 e-4$ for second-order derivatives. Use deriv_init_min() to specify the minimum values as a row vector with two columns; the value in the first column is used for first-order derivatives, and the value in the second column is used for second-order derivatives. If this function is used, deriv_init_usemin() is automatically set to "on".

## Complex step method

Let us expand $f(p+i d)$ at $p$ using the Taylor series on a complex plane and get the following,

$$
f(p+i d)=f(p)+i d f^{\prime}(p)-d^{2} \frac{f^{\prime \prime}(p)}{2!}-i d^{3} \frac{f^{\prime \prime \prime}(p)}{3!}+\cdots
$$

where $d$ is the delta used for computing numerical derivatives and $i$ is the imaginary unit, $i^{2}=-1$. Considering only the imaginary part of the above equation on both sides, we get the following:

$$
\operatorname{Im}(f(p+i d))=d f^{\prime}(p)-d^{3} \frac{f^{\prime \prime \prime}(p)}{3!}+\cdots
$$

Therefore,

$$
f^{\prime}(p)=\frac{\operatorname{Im}(f(p+i d))}{d}+d^{2} \frac{f^{\prime \prime \prime}(p)}{3!}+\cdots
$$

deriv() uses the following formula for computing the numerical derivative of $f()$ at $p$ :

$$
f^{\prime}(p)=\frac{\operatorname{Im}(f(p+i d))}{d}
$$

Please note that there is no subtraction in the formula and hence no error caused by cancellation. In this method, $d$ can take a fairly small number.

The complex step method is a very accurate method, yet it works only with first-order derivatives with evaluator type "d" or "t". In addition, because the complex step method uses complex extention of the evaluator, the evaluator itself has to have and allow complex extension. See Martins, Sturdza, and Alonso (2003) or Squire and Trapp (1998) for more details about the complex step method.

## Type d evaluators

You must write an evaluator function to calculate $f()$ before you can use the derivative functions. The example we showed above was of what is called a type d evaluator. Let's stay with that.

The evaluator function we wrote was

```
void myeval(x, y)
{
    y = exp(-x^2 + x - 3)
}
```

All type d evaluators open the same way, void evaluator $(x, y)$
although what you name the arguments is up to you. We named the arguments the way that mathematicians think, although we could just as well have named them the way that statisticians think:

$$
\text { void evaluator }(b, s)
$$

To avoid favoritism, we will write them as

```
void evaluator ( }p,v
```

That is, we will think in terms of computing the derivative of $v=f(p)$ with respect to the elements of $p$.

Here is the full definition of a type $d$ evaluator:
void evaluator (real rowvector $p, v$ )
where $v$ is the value to be returned:
$v$ : real scalar
evaluator () is to fill in $v$ given the values in $p$.
evaluator() may return $v=$. if $f()$ cannot be evaluated at $p$.

## Example of a type d evaluator

We wish to compute the gradient of the following function at $p_{1}=1$ and $p_{2}=2$ :

$$
v=\exp \left(-p_{1}^{2}-p_{2}^{2}-p_{1} p_{2}+p_{1}-p_{2}-3\right)
$$

Our numerical solution to the problem is

```
: void eval_d(p, v)
> {
> v = exp(-p[1]^2 - p[2]^2 - p[1]*p[2] + p[1] - p[2] - 3)
> }
: D = deriv_init()
: deriv_init_evaluator(D, &eval_d())
: deriv_init_params(D, (1,2))
: grad = deriv(D, 1)
grad
    1 2
    1
        -.0000501051 -.0001002102
:(-2*1 - 2 + 1)*exp(-1^2 - 2^2 - 1*2 + 1 - 2 - 3)
    -.0000501051
:(-2*2 - 1 - 1)*exp(-1^2 - 2^2 - 1*2 + 1 - 2 - 3)
    -.0001002102
```

For this problem, the elements of the gradient function are given by the following formulas, and we see that deriv() computed values that are in agreement with the analytical results (to the number of significant digits being displayed).

$$
\begin{aligned}
\frac{d v}{d p_{1}} & =\left(-2 p_{1}-p_{2}+1\right) \exp \left(-p_{1}^{2}-p_{2}^{2}-p_{1} p_{2}+p_{1}-p_{2}-3\right) \\
\frac{d v}{d p_{2}} & =\left(-2 p_{2}-p_{1}-1\right) \exp \left(-p_{1}^{2}-p_{2}^{2}-p_{1} p_{2}+p_{1}-p_{2}-3\right)
\end{aligned}
$$

## Type v evaluators

In some statistical applications, you will find type v evaluators more convenient to code than type d evaluators.

In statistical applications, one tends to think of a dataset of values arranged in matrix $X$, the rows of which are observations. The function $h(p, X[i,]$.$) can be calculated for each row separately, and$ it is the sum of those resulting values that forms the function $f(p)$ from which we would like to compute derivatives.

Type v evaluators are for such cases.
In a type d evaluator, you return scalar $v=f(p)$.
In a type v evaluator, you return a column vector, $v$, such that colsum $(v)=f(p)$.
The code outline for type v evaluators is the same as those for d evaluators. All that differs is that $v$, which is a real scalar in the d case, is now a real colvector in the v case.

## User-defined arguments

The type v evaluators arise in statistical applications and, in such applications, there are data; that is, just knowing $p$ is not sufficient to calculate $v, g$, and $H$. Actually, that same problem can also arise when coding type d evaluators.

You can pass extra arguments to evaluators. The first line of all evaluators, regardless of type, is void evaluator $(p, v)$

If you code

```
deriv_init_argument(D, 1, X)
```

the first line becomes

```
void evaluator ( }p,X,v
```

If you code

```
deriv_init_argument(D, 1, X)
deriv_init_argument(D, 2, Y)
```

the first line becomes

```
void evaluator ( }p,X,Y,v
```

and so on, up to nine extra arguments. That is, you can specify extra arguments to be passed to your function.

## Example of a type $\mathbf{v}$ evaluator

You have the following data:

| $: \mathrm{x}$ | 1 |
| :--- | :--- |
|  | 1 |
|  | .35 |
| 2 | .29 |
| 3 | .3 |
| 4 | .3 |
| 5 | .65 |
| 6 | .56 |
| 7 | .37 |
| 8 | .16 |
| 9 | .26 |
| 10 | .19 |
|  |  |

You believe that the data are the result of a beta distribution process with fixed parameters alpha and beta, and you wish to compute the gradient vector and Hessian matrix associated with the log likelihood at some values of those parameters alpha and beta ( $a$ and $b$ in what follows). The formula for the density of the beta distribution is

$$
\operatorname{density}(x)=\frac{\Gamma(a+b)}{\Gamma(a) \Gamma(b)} x^{a-1}(1-x)^{b-1}
$$

In our type v solution to this problem, we compute the gradient and Hessian at $a=0.5$ and $b=2$.

```
void lnbetaden_v(p, x, lnf)
{
    a = p[1]
    b = p[2]
    lnf = lngamma(a+b) :- lngamma(a) :- lngamma(b) :+
                                    (a-1)*log(x) :+ (b-1)*\operatorname{log}(1:-x)
}
D = deriv_init()
deriv_init_evaluator(D, &lnbetaden_v())
deriv_init_evaluatortype(D, "v")
deriv_init_params(D, (0.5, 2))
deriv_init_argument(D, 1, x) }\leftarrow importan
deriv(D, 2)
[symmetric]
    1 2
    -116.4988089
    8.724410052 -1.715062542
deriv_result_gradient(D)
    1
                                2
1% 15.12578465 -1.701917722
```

Note the following:

1. Rather than calling the returned value v , we called it lnf . You can name the arguments as you please.
2. We arranged for an extra argument to be passed by coding deriv_init_argument (D, 1, x ). The extra argument is the vector x , which we listed previously for you. In our function, we received the argument as $x$, but we could have used a different name just as we used lnf rather than $v$.
3. We set the evaluator type to "v".

## Type t evaluators

Type t evaluators are for when you need to compute the Jacobian matrix from a vector-valued function.

Type t evaluators are different from type v evaluators in that the resulting vector of values should not be summed. One example is when the function $f()$ performs a nonlinear transformation from the domain of $p$ to the domain of $v$.

## Example of a type $t$ evaluator

Let's compute the Jacobian matrix for the following transformation:

$$
\begin{aligned}
& v_{1}=p_{1}+p_{2} \\
& v_{2}=p_{1}-p_{2}
\end{aligned}
$$

Here is our numerical solution, evaluating the Jacobian at $p=(0,0)$ :

```
void eval_t1(p, v)
{
v = J(1,2,.)
    v[1] = p[1] + p[2]
    v[2] = p[1] - p[2]
}
D = deriv_init()
deriv_init_evaluator(D, &eval_t1())
deriv_init_evaluatortype(D, "t")
deriv_init_params(D, (0,0))
deriv(D, 1)
[symmetric]
    1 2
1
1
1 -1
```

Now let's compute the Jacobian matrix for a less trivial transformation:

$$
\begin{gathered}
v_{1}=p_{1}^{2} \\
v_{2}=p_{1} p_{2}
\end{gathered}
$$

Here is our numerical solution, evaluating the Jacobian at $p=(1,2)$ :

```
void eval_t2(p, v)
{
> v = J (1,2,.)
v[1] = p[1] ~2
v[2] = p[1] * p[2]
}
D = deriv_init()
deriv_init_evaluator(D, &eval_t2())
deriv_init_evaluatortype(D, "t")
deriv_init_params(D, (1,2))
deriv(D, 1)
```

1
2

| 1 | 1.999999998 | 0 |
| ---: | ---: | ---: |
| 2 | 1 |  |

## Example of using step-size lower bounds

Sometimes, the numerical differentiation method gives unstable results when the computed step size, delta, is too small. In this case, we can set lower bounds for step sizes. Let's consider the derivative of the normal density function with respect to the mean and standard deviation at $m=0, s=0.1$, and $x=0.5$.

```
void f(real vector params, real scalar x, real scalar v)
{
m = params[1]
s = params[2]
v = normalden(x, m, s)
}
// exact derivatives
m = 0
s = 0.1
x = 0.5
```

The exact solution for the Hessian matrix is

```
: Hessian
[symmetric]
```

    \(1 \quad 2\)
    1
2
.0356812684
. 1635391466
.7463331964

Without setting the step-size lower bounds, we see the computed result with adaptive step length search is

```
: D = deriv_init()
: deriv_init_evaluator(D, &f())
deriv_init_evaluatortype(D, "d")
deriv_init_params(D, params)
deriv_init_argument(D, 1, x)
d2_nousemin = deriv(D, 2)
: d2_nousemin
[symmetric]
                    1
                            2
1
\begin{tabular}{|ll|}
\hline .0788226616 & \\
-.026272616 & 1.84061274 \\
\hline
\end{tabular}
```

After setting the step-size lower bounds, we see the result is much closer to the exact solution:

```
: deriv_init_usemin(D, "on")
: d2_usemin = deriv(D, 2)
: d2_usemin
[symmetric]
```

    \(1 \quad 2\)
    .0356813276
.1635502121 . 7463480108

The relative difference between this result and the exact solution is

```
: mreldif(d2_usemin, Hessian)
    9.51017e-06
```


## Example of complex step method

We can use the complex step method to compute first-order derivatives with complex extension. For example, let's consider the derivative of the following function:

$$
v=p^{2.5}
$$

Here we set up the problem and choose the complex step method in deriv_init_technique():

```
: void eval(p, v) {
    v = p - 2.5
}
D = deriv_init()
deriv_init_evaluator(D, &eval())
deriv_init_technique(D, "complex")
```

Now we evaluate the gradient at $p=0.5$ :

```
deriv_init_params(D, 0.5)
grad = deriv(D, 1)
grad
    .8838834765
```


## Functions

## deriv_init()

transmorphic deriv_init()
deriv_init() is used to begin a derivative problem. Store the returned result in a variable name of your choosing; we have used $D$ in this documentation. You pass $D$ as the first argument to the other deriv*() functions.
deriv_init() sets all deriv_init_*() values to their defaults. You may use the query form of deriv_init_*() to determine an individual default, or you can use deriv_query() to see them all.

The query form of deriv_init_*() can be used before or after calling deriv().

## deriv_init_evaluator() and deriv_init_evaluatortype()

```
void deriv_init_evaluator(D, pointer(function) scalar fptr)
void deriv_init_evaluatortype(D, evaluatortype)
```

pointer(function) scalar deriv_init_evaluator ( $D$ )
string scalar deriv_init_evaluatortype ( $D$ )
deriv_init_evaluator ( $D, f p t r$ ) specifies the function to be called to evaluate $f(p)$. Use of this function is required. If your function is named myfn(), you code deriv_init_evaluator ( $D$, \&myfcn()).
deriv_init_evaluatortype ( $D$, evaluatortype) specifies the capabilities of the function that has been set using deriv_init_evaluator(). Alternatives for evaluatortype are "d", "v", and "t". The default is " d " if you do not invoke this function.
deriv_init_evaluator $(D)$ returns a pointer to the function that has been set.
deriv_init_evaluatortype ( $D$ ) returns the evaluator type currently set.

## deriv_init_argument( ) and deriv_init_narguments()

```
    void deriv_init_argument ( \(D\), real scalar \(k, X\) )
    void deriv_init_narguments \((D\), real scalar \(K)\)
    pointer scalar deriv_init_argument ( \(D\), real scalar \(k\) )
    real scalar deriv_init_narguments \((D)\)
```

deriv_init_argument ( $D, k, X$ ) sets the $k$ th extra argument of the evaluator function to be $X . X$ can be anything, including a view matrix or even a pointer to a function. No copy of $X$ is made; it is a pointer to $X$ that is stored, so any changes you make to $X$ between setting it and $X$ being used will be reflected in what is passed to the evaluator function.
deriv_init_narguments $(D, K)$ sets the number of extra arguments to be passed to the evaluator function. This function is useless and included only for completeness. The number of extra arguments is automatically set when you use deriv_init_argument ().
deriv_init_argument ( $D, k$ ) returns a pointer to the object that was previously set.
deriv_init_narguments $(D)$ returns the number of extra arguments that were passed to the evaluator function.

## deriv_init_weights()

```
    void deriv_init_weights(D, real colvector weights)
    pointer scalar deriv_init_weights(D)
```

deriv_init_weights( $D$, weights) sets the weights used with type vevaluators to produce the function value. By default, deriv() with a type v evaluator uses colsum (v) to compute the function
value. With weights, deriv() uses cross(weights, v). weights must be row conformable with the column vector returned by the evaluator.
deriv_init_weights $(D)$ returns a pointer to the weight vector that was previously set.

## deriv_init_params()

```
void deriv_init_params(D, real rowvector params)
real rowvector deriv_init_params(D)
```

deriv_init_params( $D$, params) sets the parameter values at which the derivatives will be computed. Use of this function is required.
deriv_init_params $(D)$ returns the parameter values at which the derivatives were computed.

## Advanced init functions

The rest of the deriv_init_*() functions provide finer control of the numerical derivative taker.
deriv_init_h(), ..._scale( ), ... _bounds(), ..._search()

```
void deriv_init_h(D, real rowvector h)
void deriv_init_scale(D, real rowvector s)
void deriv_init_bounds(D, real rowvector minmax)
void deriv_init_search(D, search)
real rowvector deriv_init_h(D)
real rowvector deriv_init_scale(D)
real rowvector deriv_init_bounds(D)
string scalar deriv_init_search(D)
```

deriv_init_h $(D, h)$ sets the $h$ values used to compute numerical derivatives.
deriv_init_scale ( $D, s$ ) sets the starting scale values used to compute numerical derivatives.
deriv_init_bounds ( $D$, minmax) sets the minimum and maximum values used to search for optimal scale values. The default is minmax $=(1 e-8,1 e-7)$.
deriv_init_search( $D$, "interpolate") causes deriv() to use linear and quadratic interpolation to search for an optimal delta for computing the numerical derivatives. This is the default search method.
deriv_init_search( $D$, "bracket") causes deriv() to use a bracketed quadratic formula to search for an optimal delta for computing the numerical derivatives.
deriv_init_search( $D$, "off") prevents deriv() from searching for an optimal delta.
deriv_init_h( $D$ ) returns the user-specified $h$ values.
deriv_init_scale( $D$ ) returns the user-specified starting scale values.
deriv_init_bounds ( $D$ ) returns the user-specified search bounds.
deriv_init_search( $D$ ) returns the currently set search method.
These functions only work with the numerical differentiation method.

## deriv_init_usemin( ) and deriv_init_min( )

```
void deriv_init_usemin(D, {"off"|"on"})
void deriv_init_min(D, real rowvector min)
string scalar deriv_init_usemin(D)
real rowvector deriv_init_min(D)
```

deriv_init_usemin( $D,\{$ "off"|"on" $\}$ ) specifies whether to use the minimum values of the step size, delta, for computing numerical derivatives. The default is "off", meaning minimum values will not be used. If you specify "on", minimum values will be used; when minimum values are not specified by deriv_init_min(), default minimum values are $1 \mathrm{e}-6$ and $1 \mathrm{e}-4$ for first- and second-order derivatives, respectively.
deriv_init_min $(D, \min )$ sets the minimum values of the delta used for computing numerical derivatives. min is a $1 \times 2$ real row vector; the first column specifies the minimum for first-order derivatives, and the second column specifies the minimum for second-order derivatives. If there is a missing value in the row vector, the default values of $1 e-6$ for first-order derivatives and $1 e-4$ for second-order derivatives are used. If this function is used, deriv_init_usemin() is automatically set to "on".
deriv_init_usemin ( $D$ ) returns the current value "off" or "on", indicating whether to use minimum values of the step size.
deriv_init_min ( $D$ ) returns the current value min.
These functions only work with the numerical differentiation method.

## deriv_init_verbose()

```
void deriv_init_verbose(D, verbose)
string scalar deriv_init_verbose(D)
```

deriv_init_verbose ( $D$, verbose) sets whether error messages that arise during the execution of deriv() or _deriv() are to be displayed. Setting verbose to "on" means that they are displayed; "off" means that they are not displayed. The default is "on". Setting verbose to "off" is of interest only to users of _deriv().
deriv_init_verbose ( $D$ ) returns the current value of verbose.

## deriv_init_technique()

(varies) deriv_init_technique( $D$, technique)
deriv_init_technique(technique) specifies the technique used to compute the numerical derivatives.

The numerical differentiation method ("numdiff") is the default setting for technique.
The complex step method ("complex") requires that the evaluator be extended to complex numbers. This method can be used with first-order derivatives when the evaluator type is "d" or "t".
deriv_init_technique ( $D$ ) returns the current technique.

## deriv()

(varies) $\operatorname{deriv}(D$, todo)
deriv( $D$, todo) invokes the derivative process. If something goes wrong, deriv() aborts with error.

> deriv $(D, 0)$ returns the function value without computing derivatives.
> deriv $(D, 1)$ returns the gradient vector; the Hessian matrix is not computed.
> deriv $(D, 2)$ returns the Hessian matrix; the gradient vector is also computed.

Before you can invoke deriv(), you must have defined your evaluator function, evaluator(), and you must have set the parameter values at which deriv() is to compute derivatives:

```
\(D=\) deriv_init()
deriv_init_evaluator( \(D\), \&evaluator())
deriv_init_params(D, (...))
```

The above assumes that your evaluator function is type d. If your evaluator function type is v (that is, it returns a column vector of values instead of a scalar value), you will also have coded
deriv_init_evaluatortype (D, "v")
and you may have coded other deriv_init_*() functions as well.
Once deriv() completes, you may use the deriv_result_*() functions. You may also continue to use the deriv_init_*() functions to access initial settings, and you may use them to change settings and recompute derivatives (that is, invoke deriv() again) if you wish.
real scalar _deriv( $D$, todo)
_deriv( $D$ ) performs the same actions as $\operatorname{deriv}(D)$ except that, rather than returning the requested derivatives, _deriv() returns a real scalar and, rather than aborting if numerical issues arise, _deriv() returns a nonzero value._deriv() returns 0 if all went well. The returned value is called an error code.
deriv() returns the requested result. It can work that way because the numerical derivative calculation must have gone well. Had it not, deriv() would have aborted execution.
_deriv() returns an error code. If it is 0 , the numerical derivative calculation went well, and you can obtain the gradient vector by using deriv_result_gradient (). If things did not go well, you can use the error code to diagnose what went wrong and take the appropriate action.

Thus _deriv ( $D$ ) is an alternative to $\operatorname{deriv}(D)$. Both functions do the same thing. The difference is what happens when there are numerical difficulties.
deriv() and _deriv() work around most numerical difficulties. For instance, the evaluator function you write is allowed to return $v$ equal to missing if it cannot calculate the $f()$ at $p+d$. If that happens while computing the derivative, deriv() and _deriv() will search for a better $d$ for taking the derivative. deriv(), however, cannot tolerate that happening at $p$ (the parameter values you set using deriv_init_params()) because the function value must exist at the point when you want deriv() to compute the numerical derivative. deriv() issues an error message and aborts, meaning that execution is stopped. There can be advantages in that. The calling program need not include complicated code for such instances, figuring that stopping is good enough because a human will know to address the problem.
_deriv(), however, does not stop execution. Rather than aborting, _deriv() returns a nonzero value to the caller, identifying what went wrong. The only exception is that _deriv() will return a zero value to the caller even when the evaluator function returns $v$ equal to missing at $p$, allowing programmers to handle this special case without having to turn deriv_init_verbose() off.

Programmers implementing advanced systems will want to use _deriv() instead of deriv(). Everybody else should use deriv().

Programmers using _deriv() will also be interested in the functions deriv_init_verbose(), deriv_result_errorcode(), deriv_result_errortext(), and deriv_result_returncode().

The error codes returned by _deriv() are listed below, under the heading deriv_result_errorcode(), ... _errortext(), and ..._returncode().

## deriv_result_value()

real scalar deriv_result_value ( $D$ )
deriv_result_value $(D)$ returns the value of $f()$ evaluated at $p$.
deriv_result_values() and _deriv_result_values()
real matrix deriv_result_values ( $D$ )
void _deriv_result_values $(D, v)$
deriv_result_values $(D)$ returns the vector values returned by the evaluator. For type v evaluators, this is the column vector that sums to the value of $f()$ evaluated at $p$. For type t evaluators, this is the rowvector returned by the evaluator.
_deriv_result_values ( $D, v$ ) uses swap() (see [M-5] swap()) to interchange $v$ with the vector values stored in $D$. This destroys the vector values stored in $D$.

These functions should be called only with type v evaluators.
deriv_result_gradient() and _deriv_result_gradient()
real rowvector deriv_result_gradient ( $D$ )
void __deriv_result_gradient ( $D, g$ )
deriv_result_gradient $(D)$ returns the gradient vector evaluated at $p$.
_deriv_result_gradient ( $D, g$ ) uses swap() (see [M-5] swap()) to interchange $g$ with the gradient vector stored in $D$. This destroys the gradient vector stored in $D$.
deriv_result_scores( ) and _deriv_result_scores()
real matrix deriv_result_scores ( $D$ )
void _deriv_result_scores $(D, S)$
deriv_result_scores $(D)$ returns the matrix of the scores evaluated at $p$. The matrix of scores can be summed over the columns to produce the gradient vector.
_deriv_result_scores ( $D, S$ ) uses swap() (see [M-5] swap()) to interchange $S$ with the scores matrix stored in $D$. This destroys the scores matrix stored in $D$.

These functions should be called only with type v evaluators.

## deriv_result_Jacobian() and _deriv_result_Jacobian()

> real matrix deriv_result_Jacobian(D)
void _deriv_result_Jacobian ( $D, J$ )
deriv_result_Jacobian ( $D$ ) returns the Jacobian matrix evaluated at $p$.
_deriv_result_Jacobian( $D, J$ ) uses swap() (see [M-5] swap()) to interchange $J$ with the Jacobian matrix stored in $D$. This destroys the Jacobian matrix stored in $D$.

These functions should be called only with type $t$ evaluators.

## deriv_result_Hessian() and _deriv_result_Hessian()

real matrix deriv_result_Hessian ( $D$ )
void _deriv_result_Hessian ( $D, H$ )
deriv_result_Hessian ( $D$ ) returns the Hessian matrix evaluated at $p$.
_deriv_result_Hessian ( $D, H$ ) uses swap() (see [M-5] swap()) to interchange $H$ with the Hessian matrix stored in $D$. This destroys the Hessian matrix stored in $D$.

These functions should not be called with type $t$ evaluators.

```
deriv_result_h(), .. _scale(), and ... _delta()
```

    real rowvector deriv_result_h(D)
    real rowvector deriv_result_scale ( \(D\) )
    real rowvector deriv_result_delta(D)
    deriv_result_h(D) returns the vector of $h$ values that was used to compute the numerical derivatives.
deriv_result_scale $(D)$ returns the vector of scale values that was used to compute the numerical derivatives.
deriv_result_delta( $D$ ) returns the vector of delta values used to compute the numerical derivatives.

```
deriv_result_errorcode( ), . . _errortext(), and ..._returncode()
```

```
real scalar deriv_result_errorcode(D)
string scalar deriv_result_errortext(D)
real scalar deriv_result_returncode(D)
```

These functions are for use after _deriv().
deriv_result_errorcode ( $D$ ) returns the same error code as _deriv(). The value will be zero if there were no errors. The error codes are listed in the table directly below.
deriv_result_errortext ( $D$ ) returns a string containing the error message corresponding to the error code. If the error code is zero, the string will be "".
deriv_result_returncode ( $D$ ) returns the Stata return code corresponding to the error code. The mapping is listed in the table directly below.

In advanced code, these functions might be used as

```
(void) _deriv(D, todo)
... if (ec = deriv_result_code(D)) {
    errprintf("{p}\n")
    errprintf("%s\n", deriv_result_errortext(D))
    errprintf("{p_end}\n")
    exit(deriv_result_returncode(D))
    /*NOTREACHED*/
}
```

The error codes and their corresponding Stata return codes are
Error Return

| code | code | Error text |
| :---: | :---: | :--- |
| 1 | 198 | invalid todo argument |

2111 evaluator function required
3459 parameter values required
4459 parameter values not feasible
5459 could not calculate numerical derivatives-discontinuous region with missing values encountered

6459 could not calculate numerical derivatives-flat or discontinuous region encountered

16111 function() not found
17459 Hessian calculations not allowed with type t evaluators
Note: Error 4 can occur only when evaluating $f()$ at the parameter values.
This error occurs only with deriv().

## deriv_query()

void deriv_query ( $D$ )
deriv_query ( $D$ ) displays a report on the current deriv_init_*() values and some of the deriv_result_*() values. deriv_query ( $D$ ) may be used before or after deriv(), and it is useful when using deriv() interactively or when debugging a program that calls deriv() or _deriv().

## Conformability

All functions have $1 \times 1$ inputs and have $1 \times 1$ or void outputs, except the following: deriv_init_params( $D$, params):

D: transmorphic
params: $\quad 1 \times n p$
result: void
deriv_init_params(D):
D: transmorphic
result: $\quad 1 \times n p$
deriv_init_argument $(D, k, X)$ :
D: transmorphic
k: $\quad 1 \times 1$
$X: \quad$ anything
result: void

```
deriv_init_weights(D, params):
```

D: transmorphic
params: $\quad N \times 1$
result: void

```
deriv_init_h(D, h):
            D: transmorphic
            h: }1\timesn
    result: void
deriv_init_h(D):
    D: transmorphic
    result: }1\timesn
deriv_init_scale(D, scale):
    D: transmorphic
    scale: }\quad1\timesnp\mathrm{ (type d and v evaluator)
        nv\timesnp (type t evaluator)
    void: void
deriv_init_bounds(D, minmax):
            D: transmorphic
        minmax: }1\times
        result: void
deriv_init_bounds(D):
            D: transmorphic
    result: }1\times
deriv_init_min(D, min):
            D: transmorphic
            min: }1\times
    result: void
deriv_init_min(D):
            D: transmorphic
        result: }1\times
deriv(D, 0):
            D: transmorphic
        result: }1\times
        1\timesnv(type t evaluator)
deriv(D, 1):
    D: transmorphic
    result: }1\timesn
        nv\timesnp (type t evaluator)
deriv(D, 2):
    D: transmorphic
        result: }\quadnp\timesn
```

```
deriv_result_values(D):
    D: transmorphic
    result: }\quadN\times
        1\timesnv (type t evaluator)
        N\times1 (type v evaluator)
_deriv_result_values(D,v):
            D: transmorphic
            v: }\quadN\times
        1\timesnv (type t evaluator)
        N\times1 (type v evaluator)
    result: void
deriv_result_gradient(D):
            D: transmorphic
        result: }1\timesn
_deriv_result_gradient(D,g):
            D: transmorphic
            g: 1 
    result: void
deriv_result_scores(D):
            D: transmorphic
    result: }\quadN\timesn
_deriv_result_scores(D,S):
            D: transmorphic
            S: }\quadN\timesn
    result: void
deriv_result_Jacobian(D):
            D: transmorphic
    result: }\quadnv\timesn
_deriv_result_Jacobian(D,J):
            D: transmorphic
            J: }nv\timesn
    result: void
deriv_result_Hessian(D):
            D: transmorphic
    result: }np\timesn
_deriv_result_Hessian(D,H):
            D: transmorphic
            H: }np\timesn
    result: void
```

```
deriv_result_h( \(D\) ):
    D: transmorphic
    result: \(\quad 1 \times n p\)
deriv_result_scale(D):
    D: transmorphic
        result: \(\quad 1 \times n p\) (type d and v evaluator)
        \(n v \times n p\) (type t evaluator)
deriv_result_delta ( \(D\) ):
    D: transmorphic
    result: \(\quad 1 \times n p\) (type d and v evaluator)
        \(n v \times n p\) (type t evaluator)
```


## Diagnostics

All functions abort with error when used incorrectly.
deriv() aborts with error if it runs into numerical difficulties. _deriv() does not; it instead returns a nonzero error code.

## Methods and formulas

See sections 1.3.4 and 1.3.5 of Pitblado, Poi, and Gould (2024) for an overview of the methods and formulas deriv() uses to compute numerical derivatives.

Carl Gustav Jacob Jacobi (1804-1851) was born in Potsdam, Prussia, which is now Germany. Jacobi demonstrated great potential at a young age, meeting the qualifications to enter university at the age of 12 . He obtained his PhD from the University of Berlin in 1825. In 1829, Jacobi published a paper making significant contributions to elliptic functions, which were consequently named after him. He also published a paper on functional determinants, giving rise to the Jacobian matrix and Jacobian determinant. He made many contributions to number theory, including proofs of quadratic reciprocity and Fermat's two-square theorem. In addition to the functions and equations named after him, Jacobi was honored with a crater in his name. In 1833, the Royal Society elected Jacobi to fellowship, as did the Royal Society of Edinburgh in 1845.

Jacobi spent 15 years teaching at Königsberg University, where he also supervised the doctoral work of Ludwig Otto Hesse. Because of health complications, he spent some time in Italy and later relocated to Berlin, where he gave some lectures. However, after making some unfavorable political statements, he was denied the opportunity to work at the University of Berlin. This decision was later retracted, and he left his position at the University of Vienna to lecture at the University of Berlin. Jacobi died of smallpox in 1851.

## References

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## Also see

[M-5] Quadrature( ) - Numerical integration
[M-4] Mathematical - Important mathematical functions

## Title

[M-5] designmatrix() — Design matrices

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

designmatrix $(v)$ returns a rows $(v) \times \operatorname{colmax}(v)$ matrix with ones in the indicated columns and zero everywhere else.

## Syntax

real matrix designmatrix (real colvector $v$ )

## Remarks and examples

designmatrix $((1 \backslash 2 \backslash 3))$ is equal to $I(3)$, the $3 \times 3$ identity matrix.

## Conformability

## designmatrix(v):

```
            v:}\quadr\times
    result: }\quadr\times\operatorname{colmax}(v)\quad(0\times0\mathrm{ if }r=0
```


## Diagnostics

designmatrix $(v)$ aborts with error if any element of $v$ is $<1$.

## Also see

[M-4] Standard - Functions to create standard matrices

## Title

[M-5] $\operatorname{det}()$ - Determinant of matrix

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

$\operatorname{det}(A)$ returns the determinant of $A$.
dettriangular ( $A$ ) returns the determinant of $A$, treating $A$ as if it were triangular (even if it is not).

## Syntax

```
numeric scalar det(numeric matrix A)
numeric scalar dettriangular(numeric matrix A)
```


## Remarks and examples

Calculation of the determinant is made by obtaining the LU decomposition of $A$ and then calculating the determinant of $U$ :

$$
\begin{aligned}
\operatorname{det}(A) & =\operatorname{det}(P L U) \\
& =\operatorname{det}(P) \times \operatorname{det}(L) \times \operatorname{det}(U) \\
& = \pm 1 \times 1 \times \operatorname{det}(U) \\
& = \pm \operatorname{det}(U)
\end{aligned}
$$

Since $U$ is (upper) triangular, $\operatorname{det}(U)$ is simply the product of its diagonal elements. See [M-5] lud().

## Conformability

```
det(A), dettriangular(A):
            A: }n\times
    result: }\quad1\times
```


## Diagnostics

$\operatorname{det}(A)$ and dettriangular $(A)$ return 1 if $A$ is $0 \times 0$.
$\operatorname{det}(A)$ aborts with error if $A$ is not square and returns missing if $A$ contains missing values.
dettriangular ( $A$ ) aborts with error if $A$ is not square and returns missing if any element on the diagonal of $A$ is missing.

Both $\operatorname{det}(A)$ and dettriangular $(A)$ will return missing value if the determinant exceeds $8.99 \mathrm{e}+307$.

## Also see

[M-5] lud( ) - LU decomposition
[M-4] Matrix - Matrix functions

## Title

[M-5] _diag( ) - Replace diagonal of a matrix

```
Description Syntax Conformability Diagnostics Also see
```


## Description

_diag $(Z, v)$ replaces the diagonal of the matrix $Z$ with $v . Z$ need not be square.

1. If $v$ is a vector, the vector replaces the principal diagonal.
2. If $v$ is $1 \times 1$, each element of the principal diagonal is replaced with $v$.
3. If $v$ is a void vector $(1 \times 0$ or $0 \times 1), Z$ is left unchanged.

## Syntax

void _diag (numeric matrix $Z$, numeric vector $v$ )

## Conformability

_diag $(Z, v)$ :
input:

$$
\begin{array}{rlrl}
Z: & n \times m, n \leq m & \\
v: & & 1 \times 1,1 \times n, \quad \text { or } & n \times 1
\end{array}
$$

or
$Z: \quad n \times m, n>m$
$v: \quad 1 \times 1,1 \times m$, or $m \times 1$
output:

$$
Z: \quad n \times m
$$

## Diagnostics

_diag $(Z, v)$ aborts with error if $Z$ or $v$ is a view.

## Also see

[M-5] diag() - Create diagonal matrix
[M-4] Manipulation - Matrix manipulation

## Title

[M-5] diag() — Create diagonal matrix

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

diag() creates diagonal matrices.
$\operatorname{diag}(Z), Z$ a matrix, extracts the principal diagonal of $Z$ to create a new matrix. $Z$ must be square.
$\operatorname{diag}(z), z$ a vector, creates a new matrix with the elements of $z$ on its diagonal.

## Syntax

```
numeric matrix diag(numeric matrix Z)
numeric matrix diag(numeric vector z)
```


## Remarks and examples

Do not confuse diag() with its functional inverse, diagonal(); see [M-5] diagonal(). diag() creates a matrix from a vector (or matrix); diagonal () extracts the diagonal of a matrix into a vector.

Use of diag() should be avoided because it wastes memory. The colon operators will allow you to use vectors directly:

| Desired calculation | Equivalent |
| :--- | :--- |
| $\operatorname{diag}(v) * X$, |  |
| $v$ is a column | $v: * X$ |
| $v$ is a row | $v^{\prime}: * X$ |
| $v$ is a matrix | diagonal $(v): * X$ |
| $X * \operatorname{diag}(v)$ |  |
| $v$ is a column | $X: * v^{\prime}$ |
| $v$ is a row | $X: * v$ |
| $v$ is a matrix | $X: * \operatorname{diagonal}(v)^{\prime}$ |

In the above table, it is assumed that $v$ is real. If $v$ might be complex, the transpose operators that appear must be changed to transposeonly() calls, because we do not want the conjugate. For instance, $v^{\prime}: * X$ would become transposeonly $(v): * X$.

## Conformability

$\operatorname{diag}(Z):$

$$
\begin{aligned}
\text { Z: } & m \times n \\
\text { result: } & \min (m, n) \times \min (m, n)
\end{aligned}
$$

$\operatorname{diag}(z):$

$$
\begin{aligned}
z: & 1 \times n \\
\text { result: } & n \times n
\end{aligned} \quad \text { or } \quad n \times 1
$$

## Diagnostics

None.

## Also see

[M-5] _diag() - Replace diagonal of a matrix
[M-5] diagonal() - Extract diagonal into column vector
[M-5] isdiagonal() - Whether matrix is diagonal
[M-4] Manipulation - Matrix manipulation

## Title

[M-5] diagOcnt() — Count zeros on diagonal

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

$\operatorname{diag} 0 \operatorname{cnt}(X)$ returns the number of principal diagonal elements of $X$ that are 0.

## Syntax

real scalar diag0cnt (real matrix $X$ )

## Remarks and examples

diag0cnt() is often used after invsym() (see [M-5] invsym()) to count the number of columns omitted because of collinearity.

## Conformability

diag0cnt ( $X$ ):

$$
\begin{aligned}
X: & r \times c \\
\text { result: } & 1 \times 1
\end{aligned}
$$

## Diagnostics

diag0cnt $(X)$ returns 0 if $X$ is void.

## Also see

[M-5] invsym ( ) - Symmetric real matrix inversion
[M-4] Utility - Matrix utility functions

## Title

[M-5] diagonal() - Extract diagonal into column vector

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

diagonal ( $A$ ) extracts the diagonal of $A$ and returns it in a column vector.

## Syntax

numeric colvector diagonal (numeric matrix A)

## Remarks and examples

diagonal() may be used with nonsquare matrices.
Do not confuse diagonal() with its functional inverse, diag(); see [M-5] diag(). diagonal() extracts the diagonal of a matrix into a vector; $\operatorname{diag}()$ creates a diagonal matrix from a vector.

## Conformability

diagonal ( $A$ ):

$$
\begin{aligned}
A: & r \times c \\
\text { result: } & \min (r, c) \times 1
\end{aligned}
$$

## Diagnostics

None.

## Also see

[M-5] blockdiag( ) - Block-diagonal matrix
[M-5] diag() - Create diagonal matrix
[M-5] isdiagonal( ) - Whether matrix is diagonal
[M-4] Manipulation - Matrix manipulation

## Title

[M-5] dir() — File list

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

dir(dirname, filetype, pattern) returns a column vector containing the names of the files in dir that match pattern.
dir (dirname, filetype, pattern, prefix) does the same thing but allows you to specify whether you want a simple list of files (prefix $=0$ ) or a list of filenames prefixed with dirname (prefix $\neq 0$ ). dir (dirname, filetype, pattern) is equivalent to dir (dirname, filetype, pattern, 0).
pattern is interpreted by [M-5] strmatch().

## Syntax

```
string colvector dir(dirname, filetype, pattern)
string colvector dir(dirname, filetype, pattern, prefix)
```

where

$$
\begin{aligned}
\text { dirname: } & \text { string scalar containing directory name } \\
\text { filetype: } & \text { string scalar containing "files", "dirs", or "other" } \\
\text { pattern: } & \text { string scalar containing match pattern } \\
\text { prefix: } & \text { real scalar containing } 0 \text { or } 1
\end{aligned}
$$

## Remarks and examples

Examples:

```
dir(".", "dirs", "*")
```

returns a list of all directories in the current directory.
dir(".", "files", "*")
returns a list of all regular files in the current directory.
dir(".", "files", "*.sthlp")
returns a list of all $*$.sthlp files found in the current directory.

## Conformability

dir(dirname, filetype, pattern, prefix):
dirname: $1 \times 1$
filetype: $\quad 1 \times 1$
pattern: $\quad 1 \times 1$
prefix: $1 \times 1 \quad$ (optional)
result: $\quad k \times 1, \quad k$ number of files matching pattern

## Diagnostics

dir (dirname, filetype, pattern, prefix) returns $\mathrm{J}(0,1, " ")$ if

1. no files matching pattern are found,
2. directory dirname does not exist, or
3. filetype is misspecified (is not equal to "files", "dirs", or "others").
dirname may be specified with or without the directory separator on the end.
dirname $=" "$ is interpreted the same as dirname $=" . "$; the current directory is searched.

## Also see

[M-4] IO - I/O functions

## Title

[M-5] direxists( ) - Whether directory exists
Description Syntax Conformability Diagnostics Also see

## Description

direxists (dirname) returns 1 if dirname contains a valid path to a directory and returns 0 otherwise.

## Syntax

real scalar direxists (string scalar dirname)

## Conformability

```
direxists(dirname):
    dirname: }1\times
    result: }\quad1\times
```


## Diagnostics

None.

## Also see

[M-4] IO - I/O functions

## Title

[M-5] direxternal() — Obtain list of existing external globals

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

direxternal (pattern) returns a column vector containing the names matching pattern of the existing external globals. direxternal() returns $\mathrm{J}(0,1, " \mathrm{l})$ if there are no such globals.
pattern is interpreted by [M-5] strmatch().

## Syntax

```
string colvector direxternal(string scalar pattern)
```


## Remarks and examples

See [M-5] findexternal() for the definition of a global.
A list of all globals can be obtained by direxternal("*").

## Conformability

```
direxternal(pattern):
    pattern: }1\times
        result: }n\times
```


## Diagnostics

direxternal (pattern) returns $\mathrm{J}(0,1, " \mathrm{l})$ when there are no globals matching pattern.

## Also see

[M-5] findexternal() - Find, create, and remove external globals
[M-4] Programming - Programming functions

## Title

[M-5] display() — Display text interpreting SMCL

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

display ( $s$ ) displays the string or strings contained in $s$.
display ( $s$, asis) does the same thing but allows you to control how SMCL codes are treated. display $(s, 0)$ is equivalent to display $(s)$; any SMCL codes are honored.
display ( $s$, asis), asis $\neq 0$, displays the contents of $s$ exactly as they are. For instance, when asis $\neq 0$, "\{it\}" is just the string of characters $\{, i, t$, and $\}$ and those characters are displayed; \{it\} is not given the SMCL interpretation of enter italic mode.

## Syntax

```
void display(string colvector s)
void display(string colvector s, real scalar asis)
```


## Remarks and examples

When $s$ is a scalar, the differences between coding

$$
: \text { display }(s)
$$

and coding
: $s$
are

1. display ( $s$ ) will not indent $s$; $s$ by itself causes $s$ to be indented by two spaces.
2. display ( $s$ ) will honor any SMCL codes contained in $s ; s$ by itself is equivalent to display ( $s, 1$ ). For example,
```
: s = "this is an {it:example}"
: display(s)
this is an example
: s
    this is an {it:example}
```

3. When $s$ is a vector, display $(s)$ simply displays the lines, whereas $s$ by itself adorns the lines with row and column numbers:
```
: s = ("this is line 1" \ "this is line 2")
: display(s)
this is line 1
this is line 2
: s
this is line 1
this is line 2
```

Another alternative to display() is printf(); see [M-5] printf(). When $s$ is a scalar, display() and printf() do the same thing:

```
: display("this is an {it:example}")
this is an example
: printf("%s\n", "this is an {it:example}")
this is an example
```

printf(), however, will not allow $s$ to be nonscalar; it has other capabilities.

## Conformability

display (s, asis)

| $s:$ | $k \times 1$ |  |
| ---: | :--- | :--- |
| asis: | $1 \times 1$ | (optional) |
| result: | void |  |

## Diagnostics

None.

## Also see

[M-5] displayas( ) - Set display level
[M-5] displayflush() - Flush terminal-output buffer
[M-5] printf() - Format output
[M-4] IO - I/O functions

## Title

[M-5] displayas( ) - Set display level

Description Syntax Remarks and examples Conformability<br>Diagnostics Also see

## Description

displayas(level) sets whether and how subsequent output is to be displayed.

## Syntax

> void displayas (string scalar level)
where level may be

| level | Minimum abbreviation |
| :--- | :---: |
| "result" | "res" |
| "text" | "txt" |
| "error" | "err" |
| "input" | "inp" |

## Remarks and examples

If this function is never invoked, then the output level is result. Say that Mata was invoked in such a way that all output except error messages is being suppressed (for example, quietly was coded in front of the mata command or in front of the ado-file that called your Mata function). If output is being suppressed, then Mata output is being suppressed, including any output created by your program. Say that you reach a point in your program where you wish to output an error message. You coded

```
printf("{err:you made a mistake}\n")
```

Even though you coded the SMCL directive \{err:\}, the error message will still be suppressed. SMCL determines how something is rendered, not whether it is rendered. What you need to code is

```
displayas("err")
printf("{err:you made a mistake}\n")
```

Actually, you could code

```
displayas("err")
printf("you made a mistake\n")
```

because, in addition to setting the output level (telling Stata that all subsequent output is of the specified level), it also sets the current SMCL rendering to what is appropriate for that kind of output. Hence, if you coded

```
displayas("err")
printf("{res:you made a mistake}\n")
```

the text you made a mistake would appear in the style of results despite any quietlys attempting to suppress output. Coding the above is considered bad style.

## Conformability

```
displayas(level):
    level: }1\times
    result: void
```


## Diagnostics

displayas (level) aborts with error if level contains an inappropriate string.

## Also see

[M-5] display () - Display text interpreting SMCL
[M-5] printf() - Format output
[M-4] IO - I/O functions

## Title

[M-5] displayflush( ) - Flush terminal-output buffer
Description Syntax Remarks and examples Diagnostics Also see

## Description

To achieve better performance, Stata buffers terminal output, so, within a program, output may not appear when a display() or printf() command is executed. The output might be held in a buffer and displayed later.
displayflush() forces Stata to display all pending output at the terminal. displayflush() is rarely used.

## Syntax

void displayflush()

## Remarks and examples

See [M-5] printf() for an example of the use of displayflush().
Use of displayflush() slows execution. Use displayflush() only when it is important that output be displayed at the terminal now, such as when providing messages indicating what your program is doing.

## Diagnostics

None.

## Also see

[M-5] display () - Display text interpreting SMCL
[M-5] printf() - Format output
[M-4] IO - I/O functions

## Title

[M-5] Dmatrix() - Duplication matrix

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Reference | Also see |  |

## Description

$\operatorname{Dmatrix}(n)$ returns the $n^{2} \times n(n+1) / 2$ duplication matrix D for which $\mathrm{D} * \operatorname{vech}(X)=\operatorname{vec}(X)$, where $X$ is an arbitrary $n \times n$ symmetric matrix.

## Syntax

real matrix Dmatrix (real scalar $n$ )

## Remarks and examples

Duplication matrices are frequently used in computing derivatives of functions of symmetric matrices. Section 9.5 of Lütkepohl (1996) lists many useful properties of duplication matrices.

## Conformability

Dmatrix(n):
$\begin{aligned} n: & 1 \times 1 \\ \text { result: } & n^{2} \times n(n+1) / 2\end{aligned}$

## Diagnostics

Dmatrix ( $n$ ) aborts with error if $n$ is less than 0 or is missing. $n$ is interpreted as trunc ( $n$ ).

## Reference

Lütkepohl, H. 1996. Handbook of Matrices. New York: Wiley.

## Also see

[M-5] Kmatrix () - Commutation matrix
[M-5] Lmatrix ( ) - Elimination matrix
[M-5] vec() - Stack matrix columns
[M-4] Standard - Functions to create standard matrices

## Title

[M-5] _docx*() — Generate Office Open XML (.docx) file

| Description | Syntax | Remarks and examples | Diagnostics |
| :--- | :--- | :--- | :--- |
| References | Also see |  |  |

## Description

_docx*() provides a set of Mata functions to generate Office Open XML (.docx) files compatible with Microsoft Word 2007 and later.

## Syntax

Syntax is presented under the following headings:
Create and save .docx file
Add paragraph and text
Add image file
Add table
Edit table
Query routines

## Create and save .docx file

real scalar
_docx_new()

In the rest of the manual entry, $d h$ is the value returned by _docx_new().

```
real scalar__docx_save(dh, string scalar filename [, real scalar replace])
real scalar _docx_append(dh,string scalar filename)
real scalar _docx_close(dh)
void _docx_closeall()
```


## Add paragraph and text

```
real scalar _docx_paragraph_new(dh, string scalar s)
real scalar __docx_paragraph_new_styledtext(dh, string scalar s, style)
real scalar _docx_paragraph_add_text(dh, string scalar s
    [, real scalar nospace])
real scalar _docx_text_add_text(dh, string scalar s
    [, real scalar nospace])
```


## Add image file

$$
\begin{gathered}
\text { real scalar } \quad \text {-docx_image_add }(d h \text {, string scalar filepath } \\
[, \text { real scalar link }, c x, c y])
\end{gathered}
$$

## Add table

$$
\left.\begin{array}{cc}
\text { real scalar } & \text { _docx_new_table }(d h, \text { real scalar row, col }[\text {, noadd }]) \\
\text { real scalar } & \text { _docx_add_matrix }(d h, \text { string scalar name, fint }, \\
\text { real scalar scalar colnames, rownames }[\text {, noadd }])
\end{array} \quad \begin{array}{c}
\text { rdocx_add_mata }(d h, \text { string matrix } m, \text { string scalar fint } \\
\end{array} \quad[, \text { real scalar noadd }]\right) \text { ) }
$$

real scalar _docx_add_data(dh, real scalar varnames, obsno, real matrix $i$, rowvector $j$ [, real scalar noadd, scalar selectvar $]$ )
_docx_new_table(), _docx_add_matrix(), _docx_add_mata(), and _docx_add_data() return an integer ID tid of the new table for future use. In the rest of the manual entry, tid is used to denote the integer ID of the table returned by _docx_new_table(), _docx_add_matrix(), _docx_add_mata(), or _docx_add_data().

## Edit table

After a table is successfully created, that table identified by tid can be edited using the following functions:

| real scalar | _docx_table_add_row (dh, tid, real scalar i, count) |
| :---: | :---: |
| real scalar | _docx_table_del_row (dh, tid, real scalar i) |
| real scalar | _docx_table_add_cell (dh, tid, real scalar i, $j$ [, string scalar s]) |
| real scalar | _docx_table_del_cell (dh, tid, real scalar i, j) |
| real scalar | _docx_cell_set_colspan(dh, tid, real scalar i, j, count) |
| real scalar | _docx_cell_set_rowspan(dh, tid, real scalar i, j, count) |
| real scalar | _docx_cell_set_span (dh, tid, real scalar $i, j$, rowcount, colcount) |
| real scalar | ```_docx_table_mod_cell(dh, tid, real scalar i, j, string scalar s [, real scalar append ])``` |
| real scalar | _docx_table_mod_cell_table(dh, tid, real scalar $i, j$, append, src_tid) |
| real scalar | _docx_table_mod_cell_image ( $d h$, tid, real scalar $i, j$, string scalar filepath $[$, real scalar link, append, cx, cy]) |

## Query routines

```
real scalar _docx_query (real matrix doc_ids)
real scalar _docx_query_table(dh, tid)
real scalar _docx_table_query_row (dh, tid, real scalar \(i\) )
```


## Remarks and examples

The following sections describe the purpose, input parameters, and return codes of the Mata functions.
Remarks are presented under the following headings:

```
Detailed description
Error codes
Functions
    Create and save .docx file
    Add paragraph and text
    Add image
    Add table
    Edit table
    Query routines
Save document to disk file
Current paragraph and text
Supported image types
Linked and embedded images
Styles
Performance
Examples
    Create a .docx document in memory
    Add paragraphs and text
    Display data
    Display regression results
    Add an image
    Display nested table
    Add images to table cells
    Save the .docx document in memory to a disk file
```


## Detailed description

_docx_new () creates an empty .docx document in memory.
_docx_save (dh, filename [, replace]) saves the document identified by ID $d h$ to file filename on disk. The file filename is overwritten if replace is specified and is not 0 .
_docx_append ( $d h$, filename) appends the document identified by ID $d h$ to file filename on disk.
_docx_close $(d h)$ closes the document identified by ID $d h$ in memory.
_docx_closeall() closes all .docx documents in memory.
_docx_paragraph_new $(d h, s)$ creates a new paragraph with the content specified in string scalar $s$.
_docx_paragraph_new_styledtext ( $d h, s$, style) creates a new paragraph with the content specified in string scalar $s$. The text has the style specified in style. The styles can be "Title", "Heading1", "Heading2", etc. See https://www.stata.com/docx_styles.html for more discussion on styles.
_docx_paragraph_add_text (dh, $s$ [, nospace $]$ ) adds text $s$ to the current paragraph. If nospace is specified and is not 0 , the leading spaces in $s$ are trimmed; otherwise, the leading spaces in $s$ are preserved.
_docx_text_add_text (dh, $s$ [, nospace $]$ ) adds text $s$ to the current text. If nospace is specified and is not 0 , the leading spaces in $s$ are trimmed; otherwise, the leading spaces in $s$ are preserved.
_docx_image_add (dh, path $[, \operatorname{link}, c x, c y])$ adds an image file to the document. The filepath is the path to the image file.
_docx_new_table (dh, row, col [, noadd] ) creates an empty table of size row by col.
_docx_add_matrix (dh, name, fint, colnames, rownames [, noadd]) adds a matrix in a table to the document and returns the table ID tid for future use.
_docx_add_mata (dh, m, fmt [, noadd]) adds a Mata matrix in a table to the document and returns the table ID tid for future use.
_docx_add_data(dh, varnames, obsno, $i, j[$, noadd, selectvar $]$ ) adds the current Stata dataset in memory in a table to the document and returns the table ID tid for future use.
_docx_table_add_row ( $d h$, tid, $i$, count ) adds a row with count columns to the table ID tid right after the $i$ th row.
_docx_table_del_row $(d h, t i d, i)$ deletes the $i$ th row from the table.
_docx_table_add_cell $(d h, t i d, i, j[, s])$ adds a cell to the table ID tid right after the $j$ th column on the $i$ th row.
_docx_table_del_cell ( $d h, t i d, i, j$ ) deletes the cell of the table ID $t i d$ on the $i$ th row, $j$ th column.
_docx_cell_set_colspan (dh, tid, $i, j$, count) sets the cell of the $j$ th column on the $i$ th row to span horizontally count cells to the right.
_docx_cell_set_rowspan ( $d h$, tid, $i, j$, count $)$ sets the cell of the $j$ th column on the $i$ th row to span vertically count cells downward.
_docx_cell_set_span(dh, tid, $i, j$, rowcount, colcount) sets the cell of the $j$ th column on the $i$ th row to span vertically rowcount cells downward and span horizontally colcount cells to the right.
_docx_table_mod_cell (dh, tid, $i, j, s[$, append $]$ ) modifies the cell on the $i$ th row and $j$ th column with text $s$.
_docx_table_mod_cell_table ( $d h$, tid, $i, j$, append, src_tid) modifies the cell on the $i$ th row and $j$ th column with a table identified by ID src_tid.
_docx_table_mod_cell_image (dh, tid, $i, j$, filepath [, link, append, cx, cy]) modifies the cell on the $i$ th row and $j$ th column with an image. The filepath is the path to the image file.
_docx_query ( $d o c \_i d s$ ) returns the number of all documents in memory. It stores document IDs in doc_ids as a row vector.
_docx_query_table $(d h, t i d)$ returns the total number of rows of table ID tid in document ID $d h$. _docx_table_query_row $(d h, t i d, i)$ returns the number of columns of the $i$ th row of table ID tid in document ID $d h$.

## Error codes

Functions can only abort if one of the input parameters does not meet the specification; for example, a string scalar is used when a real scalar is required. Functions return a negative error code when there is an error. The codes specific to _docx_*() functions are the following:

Negative
code Meaning

- 16510 an error occurred; document is not changed
-16511 an error occurred; document is changed
-16512 an error occurred
- 16513 document ID out of range
-16514 document ID invalid
- 16515 table ID out of range
- 16516 table ID invalid
-16517 row number out of range
- 16518 column number out of range
-16519 no current paragraph
-16520 invalid property value
- 16521 too many open documents
-16522 last remaining row of the table cannot be deleted
-16523 last remaining column of the row cannot be deleted
- 16524 invalid image file format or image file too big
-16525 function is not supported on this platform
-16526 too many columns
- 16527 no current text

Any function that takes a document ID $d h$ as an argument may return error codes -16513 or -16514 .
Any function that takes a table ID tid as an argument may return error codes -16515 or -16516 .
Any function that takes a row number as an argument may return error code -16517 . Any function that takes a column number as an argument may return error code -16518 .

Error code -16511 means an error occurred during a batch of changes being applied to the document. For example, an error may occur when adding a matrix to the document. When this happens, the document is changed, but not all the entries of the matrix are added.

## Functions

## Create and save .docx file

_docx_new () creates an empty .docx document in memory. The function returns an integer ID $d h$ that identifies the document for future use. The function returns a negative error code if an error occurs. The function returns -16521 if there are too many open documents. If this happens, you may use _docx_close() or _docx_closeall() to close one or all documents in memory to fix the problem.
_docx_save (dh, string scalar filename [, real scalar replace])
saves the document identified by ID $d h$ to file filename on disk. The file filename is overwritten if replace is specified and is not 0 .

Besides the error codes -16513 and -16514 for invalid or out of range document ID $d h$, the function may return the following error codes if replace is not specified or if specified as 0 :

Code Meaning
-602 file already exists
-3602 invalid filename

The function may return the following error codes if replace is specified and is not 0 :

> Code Meaning
-3621 attempt to write read-only file
-3602 invalid filename
_docx_append (dh, string scalar filename)
appends the document identified by ID $d h$ to file filename on disk.
Besides the error codes -16513 and -16514 for invalid or out of range document ID $d h$, the function may return the following error codes:

> Code Meaning
-601 file cannot be found or read
-3621 attempt to write read-only file
-3602 invalid filename
_docx_close(dh)
closes the document identified by ID $d h$ in memory. The function returns error code -16513 if the ID $d h$ is out of range.
_docx_closeall()
closes all .docx documents in memory.

## Add paragraph and text

_docx_paragraph_new ( $d h$, string scalar $s$ )
creates a new paragraph with the content specified in string scalar $s$. The function returns 0 if it is successful or returns a negative error code if it fails.
_docx_paragraph_new_styledtext (dh, string scalar s, style)
creates a new paragraph with content string scalar $s$. The text has the style specified in style. The styles can be "Title", "Heading1", and "Heading2", etc. See https://www.stata.com/docx_styles.html for more discussion on styles.

After _docx_paragraph_new() and _docx_paragraph_new_styledtext(), the newly created paragraph becomes the current paragraph.
_docx_paragraph_add_text(dh, string scalar s [, real scalar nospace])
adds text $s$ to the current paragraph. If nospace is specified and is not 0 , the leading spaces in $s$ are trimmed; otherwise, the leading spaces in $s$ are preserved. The function returns 0 if it is successful and returns a negative error code if it fails. It may return -16519 if there is no current paragraph. This case usually happens if this function is called before a_docx_paragraph_new () or _docx_paragraph_new_styledtext() function.
_docx_text_add_text( $d h$, string scalar s [, real scalar nospace ])
adds text $s$ to the current text. If nospace is specified and is not 0 , the leading spaces in $s$ are trimmed; otherwise, the leading spaces in $s$ are preserved. The function returns 0 if it is successful and returns a negative error code if it fails. It may return -16527 if there is no current text. This case usually happens if this function is called before a _docx_paragraph_add_text () function.

This is a convenience routine so the newly added text can have the same styles as the current text.

## Add image

_docx_image_add(dh, string scalar filepath [, real scalar link, $c x, c y]$ )
adds an image file to the document. The filepath is the path to the image file. It can be either the full path or the relative path from the current working directory. If link is specified and is not 0 , the image file is linked; otherwise, the image file is embedded.

The width of the image is controlled by $c x$. The height of the image is controlled by $c y . c x$ and $c y$ are measured in twips. A twip is $1 / 20$ of a point, $1 / 1440$ of an inch, or approximately $1 / 567$ of a centimeter.

If $c x$ is not specified or is less than or equal to 0 , the default size, which is determined by the image information and the page width of the document, is used. If the $c x$ is larger than the page width of the document, the page width is used. Otherwise, the width is $c x$ in twips.

If $c y$ is not specified or is less than or equal to 0 , the height of the image is determined by the width and the aspect ratio of the image; otherwise, the added image has height $c y$ in twips.

The function returns 0 if it is successful and returns a negative error code if it fails. The function may return error code -601 if the image file specified by filepath cannot be found or read. The function may return error code -16524 if the type of the image file specified by filepath is not supported or the file is too big.

The function is not supported in Stata for Mac running on OS X 10.9 (Mavericks) or console Stata for Mac. The function returns error code -16525 if specified on the above platforms.

## Add table

_docx_new_table (dh, real scalar row, col [, noadd] )
creates an empty table of size row by col. If it is successful, the function returns the table ID tid , which is an integer greater than or equal to 0 for future use. The function returns a negative error code if it fails. If noadd is specified and is not 0 , the table is created but not added to the document. This is useful if the table is intended to be added to a cell of another table.

Microsoft Word 2007/2010 allows a maximum of 63 columns in a table. The function returns error code -16526 if col is greater than 63.
_docx_add_matrix(dh, string scalar name, fmt, real scalar colnames, rownames [, noadd]) adds a matrix in a table to the document and returns the table ID tid for future use. The elements of the matrix are formatted using fmt. If $f m t$ is not a valid Stata numeric format, $\% 12.0 \mathrm{~g}$ is used. If colnames is not 0 , the first row of the table is filled with matrix colnames. If rownames is not 0 , the first column of the table is filled with matrix rownames. If noadd is specified and is not 0 , the table is created but not added to the document. This is useful if the table is intended to be added to a cell of another table.

The function returns a negative error code if it fails. The function may return -111 if the matrix specified by name cannot be found. The function may return error code -16511 if the table has been added and an error occurred while filling the table. In this case, the document is changed but the operation is not entirely successful. The function returns error code -16526 if the number of columns of the matrix is greater than 63.
_docx_add_mata (dh, string matrix $m$, string scalar fmt [, real scalar noadd $]$ )
adds a Mata matrix in a table to the document and returns the table ID tid for future use. The elements of the Mata matrix are formatted using fint. If firt is not a valid Stata numeric format, $\% 12.0 \mathrm{~g}$ is used. If noadd is specified and is not 0 , the table is created but not added to the document. This is useful if the table is intended to be added to a cell of another table.

The function returns a negative error code if it fails. The function may return error code -16511 if the table has been added and an error occurred while filling the table. In this case, the document is changed, but the operation is not entirely successful. The function returns error code -16526 if the number of columns of the matrix is greater than 63.
_docx_add_data (dh, real scalar varnames, obsno, real matrix $i$, rowvector $j$ [, real scalar noadd, scalar selectvar])
adds the current Stata dataset in memory in a table to the document and returns the table ID tid for future use. If there is a value label attached to the variable, the variable is displayed according to the value label. Otherwise, the variable is displayed according to its format. $i, j$, and selectvar are specified in the same way as with st_data(). Factor variables and time-series-operated variables are not allowed. If varnames is not 0 , the first row of the table is filled with variable names. If obsno is not 0 , the first column of the table is filled with observation numbers. If noadd is specified and is not 0 , the table is created but not added to the document. This is useful if the table is intended to be added to a cell of another table.

The function returns a negative error code if it fails. The function may return error code -16511 if the table has been added and an error occurred while filling the table. In this case, the document is changed, but the operation is not entirely successful. The function outputs missing . or empty string $" "$ if $i$ or $j$ is out of range; it does not abort with an error. The functions returns error code -16526 if the number of variables is greater than 63.

## Edit table

_docx_table_add_row (dh, tid, real scalar i, count)
adds a row with count columns to the table ID tid right after the $i$ th row. The range of $i$ is from 0 to $r$, where $r$ is the number of rows of the table. Specifying $i$ as 0 adds a row before the first row, which is equivalent to adding a new first row; specifying $i$ as $r$ adds a row right after the last row, which is equivalent to adding a new last row. The function returns error code -16517 if $i$ is out of range.
_docx_table_del_row(dh, tid, real scalar i)
deletes the $i$ th row from the table. The range of $i$ is from 1 to $r$, where $r$ is the number of rows of the table. The function returns error code -16517 if $i$ is out of range. If the table has only one row, the function returns error code -16522 , and the row is not deleted. This is to ensure the document can be properly displayed in Microsoft Word.
_docx_table_add_cell (dh, tid, real scalar $i, j[$, string scalar $s]$ )
adds a cell to the table ID tid right after the $j$ th column on the $i$ th row. The range of $i$ is from 1 to $r$, where $r$ is the number of rows of the table. The range of $j$ is from 0 to $c$, where $c$ is the number of columns of the $i$ th row. Specifying $j$ as 0 adds a cell to the first column on the row; specifying $j$ as $c$ adds a cell to the last column on the row. The function returns error code -16517 if $i$ is out of range. The function returns error code -16518 if $j$ is out of range.
_docx_table_del_cell (dh, tid, real scalar $i, j$ )
deletes a cell from the table tid on the $i$ th row, $j$ th column. The range of $i$ is from 1 to $r$, where $r$ is the number of rows of the table. The range of $j$ is from 1 to $c$, where $c$ is the number of columns of the $i$ th row. The function returns error code -16517 if $i$ is out of range. The function returns error code -16518 if $j$ is out of range. If the row has only one column, the function returns error code -16523 , and the column is not deleted. This is to ensure the document can be properly displayed in Microsoft Word.
_docx_cell_set_colspan(dh, tid, real scalar $i, j$, count $)$
sets the cell of the $j$ th column on the $i$ th row to span horizontally count cells to the right. This is equivalent to merging count -1 cells right of the cell on the same row into that cell. If $j+$ count -1 is larger than $c$, where $c$ is the total number of columns of the $i$ th row, the span stops at the last column. The function returns error code -16517 if $i$ is out of range. The function returns error code -16518 if $j$ is out of range or if count is less than 1 .
_docx_cell_set_rowspan(dh, tid, real scalar $i, j$, count)
sets the cell of the $j$ th column on the $i$ th row to span vertically count cells downward. This is equivalent to merging count -1 cells below the cell on the same column into it. If $i+$ count -1 is larger than $r$ where $r$ is the total number of rows of the table, the span stops at the last row. The function returns error code -16517 if $i$ is out of range or if count is less than 1 . The function returns error code -16518 if $j$ is out of range.
_docx_cell_set_span(dh, tid, real scalar $i, j$, rowcount, colcount)
sets the cell of the $j$ th column on the $i$ th row to span vertically rowcount cells downward and span horizontally colcount cells to the right. The function returns error code -16517 if $i$ is out of range. The function returns error code -16518 if $j$ is out of range.
_docx_table_mod_cell (dh, tid, real scalar $i, j$, string scalar $s[$, real scalar append $]$ ) modifies the cell on the $i$ th row and $j$ th column with text $s$. If append is specified and is not 0 , text $s$ is appended to the current content of the cell; otherwise, text $s$ replaces the current content of the cell. The function returns 0 if it is successful and returns a negative error code if it fails.
_docx_table_mod_cell_table(dh, tid, real scalar $i, j$, append, src_tid) modifies the cell on the $i$ th row and $j$ th column with a table identified by ID src_tid. If append is not 0 , table src_tid is appended to the current content of the cell; otherwise, table src_tid replaces the current content of the cell. The function returns error code -16515 or -16516 if src_id is out of range or invalid.
_docx_table_mod_cell_image (dh, tid, real scalar $i, j$, string scalar filepath [, real scalar link, append, cx, cy])
modifies the cell on the $i$ th row and $j$ th column with an image. The filepath is the path to the image file. It can be either the full path or the relative path from the current working directory. If link is specified and is not 0 , the image file is linked; otherwise, the image file is embedded.
$c x$ and $c y$ specify the width and the height of the image. $c x$ and $c y$ are measured in twips. A twip is $1 / 20$ of a point, is $1 / 1440$ of an inch, or approximately $1 / 567$ of a centimeter.

If $c x$ is not specified or is less than or equal to 0 , the width of cell $(i, j)$ is used; otherwise, the image has width $c x$ in twips.

If $c y$ is not specified or is less than or equal to 0 , the height of the image is determined by the width and the aspect ratio of the image; otherwise, the image has height $c y$ in twips.

If append is not 0 , the image is appended to the current content of the cell; otherwise, the image replaces the current content of the cell.

The function returns error code -601 if the image file specified by filepath cannot be found or read. The function may return error code -16524 if the type of the image file specified by filepath is not supported or the file is too big.

The function is not supported on Stata for Mac running on OS X 10.9 (Mavericks) or console Stata for Mac. The function returns error code -16525 if specified on the above platforms.

## Query routines

_docx_query (real matrix doc_ids)
The function returns the number of all documents in memory. It stores document IDs in doc_ids as a row vector. If there is no document in memory, the function returns 0 and doc_ids is not changed.
_docx_query_table(dh, tid)
returns the total number of rows of table ID $t i d$ in document ID $d h$. The function returns a negative error code if either $d h$ or tid is invalid.
_docx_table_query_row(dh, tid, real scalar i)
returns the number of columns of the $i$ th row of table ID tid in document ID $d h$. The function returns a negative error code if either $d h$ or $t i d$ is invalid. The function returns a negative error code if $i$ is out of range.

## Save document to disk file

A document in memory can be saved and resaved. It stays in memory and can be modified as long as the document is not closed. We do not support loading and modifying existing Word documents from disk into memory at this time.

## Current paragraph and text

When a paragraph is added, it becomes the current paragraph. The subsequent _docx_paragraph_add_text () call will add text to the current paragraph. When _docx_paragraph_add_text () is called, the newly added text becomes the current text. Functions changing paragraph styles are applied to the current paragraph. Functions changing text styles are applied to the current text.

When the current paragraph and text change, there is no way to go back. We do not have functions to move around the document. The only exception to this is the table. The table is identified by its ID and can be accessed at any time.

A new paragraph is always created when you add a table or an image. The table is added to the new paragraph, and this paragraph becomes the current paragraph.

## Supported image types

Images of types .emf, .png, and .tiff are supported. Images of types .pdf, .ps, and .eps are not supported.

## Linked and embedded images

The image is linked into the document if the link parameter is specified and is not 0 in _docx_image_add() and _docx_table_mod_cell_image(); otherwise, the image is embedded.

If the image is embedded, it becomes a part of the document and has no further relationship with the original image on the disk. If the image is linked, only a link to the image file is inserted into the document. The image file must be present so that the Word document can display the image.

If the Word document is moved to a different machine, all embedded images will display fine; all linked images will require the image files to be moved to the same directory of the Word document on the new machine to be displayed correctly.

If the original image file on the disk is updated, the linked image in the Word document will reflect the change; the embedded image will not.

## Styles

A wide range of styles-for example, font, color, text size, table width, justification-is supported. For a list of functions related to styles, see https://www.stata.com/docx_styles.html.

## Performance

Creating a new document in a new session of Stata can cause some noticeable delay, usually several seconds.

## Examples

## Create a .docx document in memory

In the following example, we create a Microsoft Word document using Stata data, results from a Stata estimation command, and Stata graphs.

We create a new .docx document in memory by calling _docx_new().

```
mata:
dh = _docx_new()
end
```

It is good practice to check if $d h$ is negative, which means the document has not been successfully created.

## Add paragraphs and text

After the document is successfully created, we can add paragraphs and text to it. We start by adding a title, a subtitle, and a heading.

```
mata:
_docx_paragraph_new_styledtext(dh, "Sample Document", "Title")
_docx_paragraph_new_styledtext(dh, "by Stata", "Subtitle")
_docx_paragraph_new_styledtext(dh, "Add", "Heading1")
end
```

The document ID $d h$ is returned from previously calling _docx_new().
Each function returns a real scalar. A negative return code indicates the function failed with error. If you would like to suppress the display of the return code, simply put (void) in front of the function.

Now we add a regular paragraph and some text to the document.

```
mata:
_docx_paragraph_new(dh, "Use auto dataset. ")
_docx_paragraph_add_text(dh, "Use -regress- with ")
_docx_paragraph_add_text(dh, "variables -mpg price foreign-.")
end
```

The function _docx_paragraph_add_text() can be used to break long sentences into pieces.

## Display data

We use auto.dta in the rest of the examples.

```
. sysuse auto
```

We may use _docx_add_data() to display observations $1-10$ of variables mpg, price, and foreign to the document as a table.

```
mata:
_docx_add_data(dh, 1, 1, (1,10), ("mpg", "price", "foreign"))
end
```

The table's first row contains variable names, and the first column contains observation numbers.

## Display regression results

We run a regression:

| Source | SS | df | MS | Number of obs |  |  | 74 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Model | 960.866305 | 2 | 480.433152 |  |  |  | 0.0000 |
| Residual | 1482.59315 | 71 | 20.8815937 |  |  |  | 0.3932 |
|  |  |  |  |  | squared |  | 0.3761 |
| Total | 2443.45946 | 73 | 33.4720474 |  |  |  | 4.5696 |
| mpg | Coefficient | Std. err. | t | $P>\|t\|$ | [95\% con | f. | interval] |
| price | -. 000959 | . 0001815 | -5.28 | 0.000 | -. 001321 |  | -. 000597 |
| foreign | 5.245271 | 1.163592 | 4.51 | 0.000 | 2.925135 |  | 7.565407 |
| _cons | 25.65058 | 1.271581 | 20.17 | 0.000 | 23.11512 |  | 28.18605 |

Now, we want to replicate the output in the document as two tables.

First, we replicate the header. We add an empty $6 \times 2$ table by using _docx_new_table(), then modify each cell of the table by using _docx_table_mod_cell() with the stored results in e() to replicate the above output. Note that Prob $>$ F is not stored but computed by

```
Ftail(e(df_m), e(df_r), e(F))
```

Also note the use of sprintf() to format the numeric values to string.

```
mata:
tid = _docx_new_table(dh, 6, 2)
_docx_table_mod_cell(dh, tid, 1, 1, "Number of obs")
result = sprintf("%g", st_numscalar("e(N)"))
_docx_table_mod_cell(dh, tid, 1, 2, result)
result = sprintf("F(%g, %g)",
    st_numscalar("e(df_m)"),
    st_numscalar("e(df_r)"))
_docx_table_mod_cell(dh, tid, 2, 1, result)
result = sprintf("%8.2g", st_numscalar("e(F)"))
_docx_table_mod_cell(dh, tid, 2, 2, result)
_docx_table_mod_cell(dh, tid, 3, 1, "Prob > F")
prob = Ftail(st_numscalar("e(df_m)"),
                            st_numscalar("e(df_r)"),
                            st_numscalar("e(F)"))
result = sprintf("%10.4g", prob)
_docx_table_mod_cell(dh, tid, 3, 2, result)
_docx_table_mod_cell(dh, tid, 4, 1, "R-squared")
result = sprintf("%10.4g", st_numscalar("e(r2)"))
_docx_table_mod_cell(dh, tid, 4, 2,result)
_docx_table_mod_cell(dh, tid, 5, 1, "Adj R-squared")
result = sprintf("%10.4g", st_numscalar("e(r2_a)"))
_docx_table_mod_cell(dh, tid, 5, 2, result)
_docx_table_mod_cell(dh, tid, 6, 1, "Root MSE")
result = sprintf("%10.4g", st_numscalar("e(rmse)"))
_docx_table_mod_cell(dh, tid, 6, 2, result)
end
```

To replicate the regression table, we store the numeric values in $r$ (table). But $r$ (table) is in the transposed form and contains extra rows, and all row and column names are not what we want. We extract the stored results from $r$ (table) by typing

```
mat define r_table = r(table)'
mat r_table = r_table[1..3, 1..6]
```

Then we add the extracted matrix r_table to the document by using _docx_add_matrix:

```
mata:
tid = _docx_add_matrix(dh, "r_table", "%10.0g", 1, 1)
end
```

Notice that we are including the row and column names although they are not what we want. We modify them by using _docx_table_mod_cell():

```
mata:
_docx_table_mod_cell(dh, tid, 1, 1, "mpg")
_docx_table_mod_cell(dh, tid, 1, 2, "Coefficient")
_docx_table_mod_cell(dh, tid, 1, 3, "Std. err.")
_docx_table_mod_cell(dh, tid, 1, 4, "t")
_docx_table_mod_cell(dh, tid, 1, 5, "P>|t|")
_docx_table_mod_cell(dh, tid, 1, 6, "[95% conf. interval]")
end
```

We set the last column of the first row in the regression table to have a column span of 2 to match the Stata output by typing

```
mata:
_docx_cell_set_colspan(dh, tid, 1, 6, 2)
end
```


## Add an image

To add a graph to the document, we first need to export the Stata graph to an image file of type .emf, .png, or .tif.
. scatter mpg price
. graph export auto.png
Then we can add the image to the document by using _docx_image_add():

```
mata:
_docx_image_add(dh, "auto.png")
end
```


## Display nested table

If we want to output something like the table below to the document,

|  | mpg |
| :--- | :--- |
| price | -0.001 |
|  | $(5.28)^{* *}$ |
| foreign | 5.245 |
|  | $(4.51)^{* *}$ |
| _cons | 25.651 |
|  | $(20.17)^{* *}$ |
| $R^{2}$ | 0.39 |
|  | 74 |
| $* p<0.05 ;$ | $* * p<0.01$ |

we can either create a $10 \times 2$ table and fill in the content or build it in pieces and combine the pieces. Notice that the middle part of the table for each variable has a similar pattern. First, we run the regression and get the saved table.

Then in Mata, we can build a $2 \times 2$ table for each variable by coding

```
mata:
mr_table = st_matrix("r(table)")
colnames = st_matrixcolstripe("r(table)")
tids = J(1, cols(mr_table), .)
for(i=1; i<=cols(mr_table); i++) {
    tids[i] = _docx_new_table(dh, 2, 2, 1)
    _docx_table_mod_cell(dh, tids[i], 1, 1, colnames[i, 2])
    output = sprintf("%10.0g", mr_table[1, i])
    _docx_table_mod_cell(dh, tids[i], 1, 2, output)
    if(mr_table[1, i]<0) {
output = sprintf("(%10.0g)", mr_table[3, i])
    }
    else {
        output = sprintf("%10.0g", mr_table[3, i])
    }
    if(mr_table[4, i]<0.05) {
    output = output +"*"
    }
    if(mr_table[4, i]<0.01) {
        output = output +"*"
    }
    _docx_table_mod_cell(dh, tids[i], 2, 2, output)
    _docx_cell_set_rowspan(dh, tids[i], 1, 1, 2)
}
end
```

Now we can combine them with the header and bottom three rows.

```
mata:
tid = _docx_new_table(dh, cols(mr_table)+4, 2)
_docx_table_mod_cell(dh, tid, 1, 2, "mpg")
for(i=2; i<=cols(mr_table)+1; i++) {
    _docx_cell_set_colspan(dh, tid, i, 1, 2)
    _docx_table_mod_cell_table(dh, tid, i, 1,
        0, tids[i-1])
}
_docx_table_mod_cell(dh, tid, cols(mr_table)+2, 1, "R2")
output = sprintf("%10.4g", st_numscalar("e(r2)"))
_docx_table_mod_cell(dh, tid, cols(mr_table)+2, 2, output)
_docx_table_mod_cell(dh, tid, cols(mr_table)+3, 1, "N")
output = sprintf("%10.4g", st_numscalar("e(N)"))
_docx_table_mod_cell(dh, tid, cols(mr_table)+3, 2, output)
_docx_table_mod_cell(dh, tid, cols(mr_table)+4, 1,
    "* p<0.05; ** p<0.01")
_docx_cell_set_colspan(dh, tid, cols(mr_table)+4, 1, 2)
end
```


## Add images to table cells

We can also add an image to a table cell. First, we create the images:
. histogram price, title("Prices")

- graph export prices.png
. histogram mpg, title("MPG")
- graph export mpg.png

We can add auto0.png and auto1.png to different cells of a table by using _docx_table_mod_cell_image().

```
mata:
tid = _docx_new_table(dh, 1, 2)
_docx_table_mod_cell_image(dh, tid, 1, 1, "prices.png")
_docx_table_mod_cell_image(dh, tid, 1, 2, "mpg.png")
end
```


## Save the .docx document in memory to a disk file

We use _docx_save() to save the document to a disk file:

```
mata:
res = _docx_save(dh, "example.docx")
end
```

Notice that we did not specify the third parameter replace; hence, the function may fail if example.docx already exists in the current working directory. It is always good practice to check the return code of _docx_save().

## Diagnostics

See Remarks and examples.

## References

Gallup, J. L. 2012. A programmer's command to build formatted statistical tables. Stata Journal 12: 655-673.
Lo Magno, G. L. 2013. Sar: Automatic generation of statistical reports using Stata and Microsoft Word for Windows. Stata Journal 13: 39-64.

Quintó, L. 2012. HTML output in Stata. Stata Journal 12: 702-717.

## Also see

[M-5] Pdf*() - Create a PDF file
[M-5] xl() - Excel file I/O class
[M-4] IO - I/O functions
[RPT] putdocx intro - Introduction to generating Office Open XML (.docx) files
[RPT] putexcel - Export results to an Excel file
[RPT] putpdf intro - Introduction to generating PDF files

## Title

[M-5] dsign( ) - FORTRAN-like DSIGN( ) function
Description Syntax Conformability Diagnostics Also see

## Description

dsign $(a, b)$ returns $a$ with the sign of $b$, defined as $|a|$ if $b \geq 0$ and $-|a|$ otherwise.
This function is useful when translating FORTRAN programs.
The in-line construction

$$
(b>=0 \text { ? } \operatorname{abs}(a):-\operatorname{abs}(a))
$$

is clearer. Also, differentiate carefully between what dsign() returns (equivalent to the above construction) and signum $(b) * \operatorname{abs}(a)$, which is almost equivalent but returns 0 when $b$ is 0 rather than abs (a). (Message: dsign() is not one of our favorite functions.)

## Syntax

real scalar dsign(real scalar a, real scalar b)

## Conformability

dsign $(a, b)$ :

| $a:$ | $1 \times 1$ |
| ---: | :--- |
| $b:$ | $1 \times 1$ |
| result: | $1 \times 1$ |

## Diagnostics

dsign(., b) returns . for all $b$.
$\operatorname{dsign}(a,$.$) returns abs (a)$ for all $a$.

## Also see

[M-5] $\operatorname{sign}()$ - Sign and complex quadrant functions
[M-4] Scalar - Scalar mathematical functions

## Title

[M-5] e() - Unit vectors

```
Description Syntax Conformability Diagnostics Also see
```


## Description

$\mathrm{e}(i, n)$ returns a $1 \times n$ unit vector, a vector with all elements equal to zero except for the $i$ th, which is set to one.

## Syntax

real rowvector $\mathrm{e}($ real scalar $i$, real scalar $n)$

## Conformability

$\mathrm{e}(i, n)$ :

| $i:$ | $1 \times 1$ |
| ---: | :--- |
| $n:$ | $1 \times 1$ |
| result $:$ | $1 \times n$ |

## Diagnostics

$\mathrm{e}(i, n)$ aborts with error if $n<1$ or if $i<1$ or if $i>n$. Arguments $i$ and $n$ are interpreted as trunc ( $i$ ) and trunc ( $n$ ).

## Also see

[M-4] Standard - Functions to create standard matrices

## Title

[M-5] editmissing() - Edit matrix for missing values

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

editmissing $(A, v)$ returns $A$ with any missing values changed to $v$.
_editmissing $(A, v)$ replaces all missing values in $A$ with $v$.

## Syntax

```
numeric matrix editmissing(numeric matrix A, numeric scalar v)
void
_editmissing(numeric matrix a, numeric scalar v)
```


## Remarks and examples

editmissing() and _editmissing() are very fast.
If you want to change nonmissing values to other values, including missing, see [M-5] editvalue().

## Conformability

editmissing $(A, v)$ :

| $A:$ | $r \times c$ |
| ---: | :--- |
| $v:$ | $1 \times 1$ |
| result: | $r \times c$ |

input:

| A: |  | $r \times c$ |
| ---: | :--- | ---: |
| $v:$ |  | $1 \times 1$ |

output:
A: $\quad r \times c$

## Diagnostics

editmissing $(A, v)$ and _editmissing $(A, v)$ change all missing elements to $v$, including not only . but also .a, .b,..., .z.

## Also see

[M-5] editvalue() - Edit (change) values in matrix
[M-4] Manipulation - Matrix manipulation

## Title

[M-5] edittoint( ) - Edit matrix for roundoff error (integers)

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

These edit functions set elements of matrices to integers that are close to integers. edittoint ( $Z, a m t$ ) and _edittoint ( $Z, a m t$ ) set

$$
Z_{i j}=\operatorname{round}\left(Z_{i j}\right) \quad \text { if }\left|Z_{i j}-\operatorname{round}\left(Z_{i j}\right)\right| \leq \mid \text { tol } \mid
$$

for $Z$ real and set

$$
\begin{array}{ll}
\operatorname{Re}\left(Z_{i j}\right)=\operatorname{round}\left(\operatorname{Re}\left(Z_{i j}\right)\right) & \text { if }\left|\operatorname{Re}\left(Z_{i j}\right)-\operatorname{round}\left(\operatorname{Re}\left(Z_{i j}\right)\right)\right| \leq|t o l| \\
\operatorname{Im}\left(Z_{i j}\right)=\operatorname{round}\left(\operatorname{Im}\left(Z_{i j}\right)\right) & \text { if }\left|\operatorname{Im}\left(Z_{i j}\right)-\operatorname{round}\left(\operatorname{Im}\left(Z_{i j}\right)\right)\right| \leq|t o l|
\end{array}
$$

for $Z$ complex, where in both cases

$$
t o l=\operatorname{abs}(a m t) * \operatorname{epsilon}(\operatorname{sum}(\operatorname{abs}(Z)) /(\operatorname{rows}(Z) * \operatorname{cols}(Z)))
$$

edittoint() leaves $Z$ unchanged and returns the edited matrix. _edittoint() edits $Z$ in place. edittointtol $(Z, t o l)$ and _edittointtol $(Z, t o l)$ do the same thing, except that tol is specified directly.

## Syntax

| numeric matrix | edittoint(numeric matrix $Z$, real scalar amt) |
| :--- | :--- |
| void | _edittoint(numeric matrix $Z$, real scalar amt) |
| numeric matrix | edittointtol (numeric matrix $Z$, real scalar tol) |
| void | _edittointtol (numeric matrix $Z$, real scalar tol) |

## Remarks and examples

These functions mirror the edittozero() functions documented in [M-5] edittozero(), except that, rather than solely resetting to zero values close to zero, they reset to integer values close to integers.

See [M-5] edittozero(). Whereas use of the functions documented there is recommended, use of the functions documented here generally is not. Although zeros commonly arise in real problems so that there is reason to suspect small numbers would be zero but for roundoff error, integers arise more rarely.

If you have reason to believe that integer values are likely, then by all means use these functions.

## Conformability

```
edittoint(Z, amt):
                Z: r}\quadr\times
            amt: }1\times
result: }\quadr\times
_edittoint(Z,amt):
input:
\[
\begin{aligned}
Z: & r \times c \\
a m t: & 1 \times 1
\end{aligned}
\]
```

output:

$$
Z: \quad r \times c
$$

edittointtol(Z, tol):

$$
\begin{aligned}
Z: & r \times c \\
\text { tol: } & 1 \times 1 \\
\text { result: } & r \times c
\end{aligned}
$$

_edittointtol(Z, tol):
input:

$$
\begin{aligned}
Z: & & r \times c \\
\text { tol: } & & 1 \times 1
\end{aligned}
$$

output:

$$
Z: \quad r \times c
$$

## Diagnostics

None.

## Also see

[M-5] edittozero( ) - Edit matrix for roundoff error (zeros)
[M-4] Manipulation - Matrix manipulation

## Title

[M-5] edittozero() — Edit matrix for roundoff error (zeros)

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

These edit functions set elements of matrices to zero that are close to zero. edittozero ( $Z$, amt) and _edittozero ( $Z$, amt) set

$$
Z_{i j}=0 \quad \text { if }\left|Z_{i j}\right| \leq|t o l|
$$

for $Z$ real and set

$$
\begin{array}{ll}
\operatorname{Re}\left(Z_{i j}\right)=0 & \text { if }\left|\operatorname{Re}\left(Z_{i j}\right)\right| \leq|t o l| \\
\operatorname{Im}\left(Z_{i j}\right)=0 & \text { if }\left|\operatorname{Im}\left(Z_{i j}\right)\right| \leq|t o l|
\end{array}
$$

for $Z$ complex, where in both cases

$$
t o l=\operatorname{abs}(a m t) * \operatorname{epsilon}(\operatorname{sum}(\operatorname{abs}(Z)) /(\operatorname{rows}(Z) * \operatorname{cols}(Z)))
$$

edittozero() leaves $Z$ unchanged and returns the edited matrix. _edittozero() edits $Z$ in place. edittozerotol $(Z, t o l)$ and _edittozerotol $(Z, t o l)$ do the same thing, except that tol is specified directly.

## Syntax

numeric matrix edittozero (numeric matrix $Z$, real scalar amt)
void _edittozero (numeric matrix $Z$, real scalar amt)
numeric matrix edittozerotol(numeric matrix $Z$, real scalar tol)
void _edittozerotol(numeric matrix $Z$, real scalar tol)

## Remarks and examples

Remarks are presented under the following headings:

Background<br>Treatment of complex values Recommendations

## Background

Numerical roundoff error leads to, among other things, numbers that should be zero being small but not zero, and so it is sometimes desirable to reset those small numbers to zero.

The problem is in identifying those small numbers. Is $1 \mathrm{e}-14$ small? Is 10,000 ? The answer is that, given some matrix, $1 \mathrm{e}-14$ might not be small because most of the values in the matrix are around $1 \mathrm{e}-14$, and the small values are $1 \mathrm{e}-28$, and given some other matrix, 10,000 might indeed be small because most of the elements are around $1 \mathrm{e}+18$.
edittozero() makes an attempt to determine what is small. edittozerotol() leaves that determination to you. In edittozerotol $(Z, t o l)$, you specify tol and elements for which $\left|Z_{i j}\right| \leq t o l$ are set to zero.

Using edittozero ( $Z$, amt), however, you specify amt and then tol is calculated for you based on the size of the elements in $Z$ and $a m t$, using the formula

$$
\text { tol } \left.=a m t * \text { epsilon (average value of }\left|Z_{i j}\right|\right)
$$

epsilon() refers to machine precision, and epsilon $(x)$ is the function that returns machine precision in units of $x$ :

$$
\operatorname{epsilon}(x)=|x| * e p s i l o n(1)
$$

where epsilon(1) is approximately $2.22 \mathrm{e}-16$ on most computers; see [M-5] epsilon().

## Treatment of complex values

The formula

$$
\text { tol } \left.=a m t * \text { epsilon(average value of }\left|Z_{i j}\right|\right)
$$

is used for both real and complex values. For complex, $\left|Z_{i j}\right|$ refers to the modulus (length) of the complex element.

However, rather than applying the reset rule

$$
Z_{i j}=0 \quad \text { if }\left|Z_{i j}\right| \leq|t o l|
$$

as is done when $Z$ is real, the reset rules are

$$
\begin{array}{ll}
\operatorname{Re}\left(Z_{i j}\right)=0 & \text { if }\left|\operatorname{Re}\left(Z_{i j}\right)\right| \leq|t o l| \\
\operatorname{Im}\left(Z_{i j}\right)=0 & \text { if }\left|\operatorname{Im}\left(Z_{i j}\right)\right| \leq|t o l|
\end{array}
$$

The first rule, applied even for complex, may seem more appealing, but the use of the second has the advantage of mapping numbers close to being purely real or purely imaginary to purely real or purely imaginary results.

## Recommendations

1. Minimal editing is performed by edittozero $(Z, 1)$. Values less than $2.22 \mathrm{e}-16$ times the average would be set to zero.
2. It is often reasonable to code edittozero ( $Z, 1000$ ), which sets to zero values less than $2.22 \mathrm{e}-13$ times the average.
3. For a given calculation, the amount of roundoff error that arises with complex matrices (matrices with nonzero imaginary part) is greater than the amount that arises with real matrices (matrices with zero imaginary part even if stored as complex). That is because, in addition to all the usual sources of roundoff error, multiplication of complex values involves the addition operator, which introduces additional roundoff error. Hence, whatever is the appropriate value of amt or tol with real matrices, it is larger for complex matrices.

## Conformability

```
edittozero(Z, amt):
                    Z: r
            amt: }\quad1\times
            result: }\quadr\times
_edittozero(Z, amt):
```

input:

$$
\begin{aligned}
Z: & r \times c \\
\text { amt: } & 1 \times 1
\end{aligned}
$$

output:

$$
Z: \quad r \times c
$$

edittozerotol(Z, tol):

$$
\begin{aligned}
& Z: \\
& \text { tol: } \\
& \text { result: } \\
& r \times 1 \times c
\end{aligned}
$$

_edittozerotol (Z, tol):
input:

$$
\begin{aligned}
Z: & & r \times c \\
\text { tol: } & & 1 \times 1
\end{aligned}
$$

output:

$$
Z: \quad r \times c
$$

## Diagnostics

edittozero ( $Z, a m t$ ) and _edittozero ( $Z, a m t$ ) leave scalars unchanged because they base their calculation of the likely roundoff error on the average value of $\left|Z_{i j}\right|$.

## Also see

[M-5] edittoint() - Edit matrix for roundoff error (integers)
[M-4] Manipulation - Matrix manipulation

## Title

[M-5] editvalue() - Edit (change) values in matrix

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

editvalue ( $A$, from, to) returns $A$ with all elements equal to from changed to to.
_editvalue ( $A$, from, to) does the same thing but modifies $A$ itself.

## Syntax

matrix editvalue(matrix A, scalar from, scalar to)
void $\quad$ _editvalue(matrix A, scalar from, scalar to)
where $A$, from, and to may be real, complex, or string.

## Remarks and examples

editvalue() and _editvalue() are fast.
If you wish to change missing values to nonmissing values, it is better to use [M-5] editmissing(). editvalue ( $A, ., 1$ ) would change all . missing values to 1 but leave .a, $b, \ldots, . z$ unchanged. editmissing $(A, 1)$ would change all missing values to 1 .

## Conformability

editvalue( $A$, from, to):

| $A:$ |  | $r \times c$ |
| ---: | :--- | ---: | :--- |
| from: |  | $1 \times 1$ |
| to: |  | $1 \times 1$ |
| result: |  | $r \times c$ |

_editvalue ( $A$, from, to):
input:

$$
\begin{aligned}
& A: \\
& \text { from: } \\
& \text { to: } \\
& 1 \times c \\
& 1 \times 1
\end{aligned}
$$

output:
A: $\quad r \times c$

## Diagnostics

editvalue ( $A$, from, to) returns a matrix of the same type as $A$.
editvalue ( $A$, from, to) and _editvalue ( $A$, from, to) abort with error if from and to are incompatible with $A$. That is, if $A$ is real, to and from must be real. If $A$ is complex, to and from must each be either real or complex. If $A$ is string, to and from must be string.
_editvalue ( $A$, from, to) aborts with error if $A$ is a view.

## Also see

[M-5] editmissing() — Edit matrix for missing values
[M-4] Manipulation - Matrix manipulation

## Title

[M-5] eigensystem() — Eigenvectors and eigenvalues

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | References | Also see |  |

## Description

These routines calculate eigenvectors and eigenvalues of square matrix $A$.
eigensystem ( $A, X, L$, rcond, nobalance) calculates eigenvectors and eigenvalues of a general, real or complex, square matrix $A$. Eigenvectors are returned in $X$ and eigenvalues in $L$. The remaining arguments are optional:

1. If rcond is not specified, then reciprocal condition numbers are not returned in rcond.

If $r$ cond is specified and contains a value other than 0 or missing—rcond $=1$ is suggested-in rcond will be placed a vector of the reciprocals of the condition numbers for the eigenvalues. Each element of the new rcond measures the accuracy to which the corresponding eigenvalue has been calculated; large numbers (numbers close to 1 ) are better and small numbers (numbers close to 0) indicate inaccuracy; see Eigenvalue condition below.
2. If nobalance is not specified, balancing is performed to obtain more accurate results.

If nobalance is specified and is not zero nor missing, balancing is not used. Results are calculated more quickly, but perhaps a little less accurately; see Balancing below.
lefteigensystem( $A, X, L$, rcond, nobalance) mirrors eigensystem(), the difference being that lefteigensystem() solves for left eigenvectors solving $X A=\operatorname{diag}(L) * X$ instead of right eigenvectors solving $A X=X * \operatorname{diag}(L)$.
eigenvalues ( $A$, rcond, nobalance) returns the eigenvalues of square matrix $A$; the eigenvectors are not calculated. Arguments rcond and nobalance are optional.
symeigensystem $(A, X, L)$ and symeigenvalues $(A)$ mirror eigensystem() and eigenvalues(), the difference being that $A$ is assumed to be symmetric (Hermitian). The eigenvalues returned are real. (Arguments rcond and nobalance are not allowed; rcond because symmetric matrices are inherently well conditioned; nobalance because it is unnecessary.)

The underscore routines mirror the routines of the same name without underscores, the difference being that $A$ is damaged during the calculation and so the underscore routines use less memory.
_eigen_la() is the interface to the Netlib LAPACK routines used to implement the above functions.
_symeigen_la() is the interface to the LAPACK routines used to implement the above functions. It uses MKL LAPACK by default.

Direct use of _eigen_la() and _symeigen_la() is not recommended.
_eigengelapacke() is the interface to the MKL LAPACK routines that are used by default to compute the eigenvalues, eigenvectors, and condition numbers.

## Syntax

void eigensystem ( $A, X, L[$, rcond $[$, nobalance $]])$
void lefteigensystem ( $A, X, L[$, rcond [, nobalance $]]$ )
complex rowvector
eigenvalues $(A \quad[$, rcond $[$, nobalance $]])$
void
symeigensystem $(A, X, L)$
real rowvector
symeigenvalues ( $A$ )
void
_eigensystem (A, X, L [, rcond $[$, nobalance $]])$
void
complex rowvector
void
_lefteigensystem $(A, X, L[$, rcond $[$, nobalance $]])$
_eigenvalues $(A \quad[$, rcond $[$, nobalance $]])$
real rowvector
_symeigensystem ( $A, X, L$ )
_symeigenvalues ( $A$ )
where inputs are

| A: | numeric matrix |
| ---: | :--- |
| rcond: | real scalar (whether rcond desired) |
| nobalance: | real scalar (whether to suppress balancing) |

and outputs are
$X$ : numeric matrix of eigenvectors
$L$ : numeric vector of eigenvalues
rcond: real vector of reciprocal condition numbers

The columns of $X$ will contain the eigenvectors except when using _lefteigensystem(), in which case the rows of $X$ contain the eigenvectors.

The following routines are used in implementing the above routines:
real scalar _eigen_la(real scalar todo, numeric matrix $A, X, L$, real scalar rcond, real scalar nobalance)
real scalar _symeigen_la(real scalar todo, numeric matrix $A, X, L$ )
The following routine is an interface to the LAPACK routines that compute the eigenvalues and eigenvectors:
real scalar _eigengelapacke (real scalar todo, numeric matrix $A, X, L$, real scalar rcond, real scalar balancing)

## Remarks and examples

Remarks are presented under the following headings:

```
Eigenvalues and eigenvectors
Left eigenvectors
Symmetric eigensystems
Normalization and order
Eigenvalue condition
Balancing
eigensystem() and eigenvalues()
lefteigensystem()
symeigensystem() and symeigenvalues()
```


## Eigenvalues and eigenvectors

A scalar $\lambda$ is said to be an eigenvalue of square matrix $A$ : $n \times n$ if there is a nonzero column vector $x: n \times 1$ (called the eigenvector) such that

$$
\begin{equation*}
A x=\lambda x \tag{1}
\end{equation*}
$$

(1) can also be written as

$$
(A-\lambda I) x=0
$$

where $I$ is the $n \times n$ identity matrix. A nontrivial solution to this system of $n$ linear homogeneous equations exists if and only if

$$
\begin{equation*}
\operatorname{det}(A-\lambda I)=0 \tag{2}
\end{equation*}
$$

This $n$th degree polynomial in $\lambda$ is called the characteristic polynomial or characteristic equation of $A$, and the eigenvalues $\lambda$ are its roots, also known as the characteristic roots.

There are, in fact, $n$ solutions ( $\lambda_{i}, x_{i}$ ) that satisfy (1)—although some can be repeated-and we can compactly write the full set of solutions as

$$
\begin{equation*}
A X=X * \operatorname{diag}(L) \tag{3}
\end{equation*}
$$

where

$$
\begin{array}{ll}
X=\left(x_{1}, x_{2}, \ldots\right) & (X: n \times n) \\
L & =\left(\lambda_{1}, \lambda_{2}, \ldots\right) \\
(L: 1 \times n)
\end{array}
$$

For instance,

```
A = (1, 2\ 9,4)
X = .
L = .
eigensystem(A, X, L)
X
1 2
-.316227766 
L
    1 2
1
7 -2
```

The first eigenvalue is 7 , and the corresponding eigenvector is $(-0.316 \backslash-0.949)$. The second eigenvalue is -2 , and the corresponding eigenvector is $(-0.555 \backslash 0.832)$.

In general, eigenvalues and vectors can be complex even if $A$ is real.

## Left eigenvectors

What we have defined above is properly known as the right-eigensystem problem:

$$
\begin{equation*}
A x=\lambda x \tag{1}
\end{equation*}
$$

In the above, $x$ is a column vector. The left-eigensystem problem is to find the row vector $x$ satisfying

$$
x A=\lambda x
$$

The eigenvalue $\lambda$ is the same in (1) and ( $1^{\prime}$ ), but $x$ can differ.
The $n$ solutions ( $\lambda_{i}, x_{i}$ ) that satisfy ( $1^{\prime}$ ) can be compactly written as

$$
X A=\operatorname{diag}(L) * X
$$

where

$$
\begin{aligned}
& X=\left[\begin{array}{c}
x_{1} \\
x_{2} \\
\vdots \\
x_{n}
\end{array}\right] \quad L=\left[\begin{array}{c}
\lambda_{1} \\
\lambda_{2} \\
\vdots \\
\lambda_{n}
\end{array}\right] \\
& n \times n \quad n \times 1
\end{aligned}
$$

For instance,
$\mathrm{A}=(1,2 \backslash 9,4)$
X = .
: L = .
: lefteigensystem(A, X, L)
: X
$1 \quad 2$
$-.832050294-.554700196$
-.948683298 . 316227766
L
1

| 1 | 7 |
| :--- | ---: |
| 2 | -2 |

The first eigenvalue is 7 , and the corresponding eigenvector is $(-0.832,-0.555)$. The second eigenvalue is -2 , and the corresponding eigenvector is $(-0.949,0.316)$.

The eigenvalues are the same as in the previous example; the eigenvectors are different.

## Symmetric eigensystems

Below we use the term symmetric to encompass Hermitian matrices, even when we do not emphasize the fact.

Eigensystems of symmetric matrices are conceptually no different from general eigensystems, but symmetry introduces certain simplifications:

1. The eigenvalues associated with symmetric matrices are real, whereas those associated with general matrices may be real or complex.
2. The eigenvectors associated with symmetric matrices-which may be real or complex—are orthogonal.
3. The left and right eigenvectors of symmetric matrices are transposes of each other.
4. The eigenvectors and eigenvalues of symmetric matrices are more easily, and more accurately, computed.

For item 3, let us begin with the right-eigensystem problem:

$$
A X=X * \operatorname{diag}(L)
$$

Taking the transpose of both sides results in

$$
X^{\prime} A=\operatorname{diag}(L) * X^{\prime}
$$

because $A=A^{\prime}$ if $A$ is symmetric (Hermitian).
symeigensystem $(A, X, L)$ calculates right eigenvectors. To obtain the left eigenvectors, you simply transpose $X$.

## Normalization and order

If $x$ is a solution to

$$
A x=\lambda x
$$

then so is $c x, c: 1 \times 1, c \neq 0$.
The eigenvectors returned by the above routines are scaled to have length (norm) 1 .
The eigenvalues are combined and returned in a vector $(L)$ and the eigenvectors in a matrix $(X)$. The eigenvalues are ordered from largest to smallest in absolute value (or, if the eigenvalues are complex, by modulus). The eigenvectors are ordered to correspond to the eigenvalues.

## Eigenvalue condition

Optional argument rcond may be specified as a value other than 0 or missing—rcond $=1$ is suggestedand then roond will be filled in with a vector containing the reciprocals of the condition numbers for the eigenvalues. Each element of rcond measures the accuracy with which the corresponding eigenvalue has been calculated; large numbers (numbers close to 1 ) are better and small numbers (numbers close to 0 ) indicate inaccuracy.

The reciprocal condition number is calculated as abs $(y * x)$, where $y: 1 \times n$ is the left eigenvector and $x: n \times 1$ is the corresponding right eigenvector. Since $y$ and $x$ each have norm $1, \operatorname{abs}(y * x)$ $=\operatorname{abs}(\cos ($ theta $))$, where theta is the angle made by the vectors. Thus $0 \leq \operatorname{abs}(y * x) \leq 1$. For symmetric matrices, $y^{*} x$ will equal 1. It can be proved that abs $(y * x)$ is the reciprocal of the condition number for a simple eigenvalue, and so it turns out that the sensitivity of the eigenvalue to a perturbation is a function of how far the matrix is from symmetric on this scale.

Requesting that rcond be calculated increases the amount of computation considerably.

## Balancing

By default, balancing is performed for general matrices. Optional argument nobalance allows you to suppress balancing.

Balancing is related to row-and-column equilibration; see [M-5] _equilre( ). Here, however, a diagonal matrix $D$ is found such that $D A D^{-1}$ is better balanced, the eigenvectors and eigenvalues for $D A D^{-1}$ are extracted, and then the eigenvectors and eigenvalues are adjusted by $D$ so that they reflect those for the original $A$ matrix.

There is no gain from these machinations when $A$ is symmetric, so the symmetric routines do not have a nobalance argument.

Some additional balancing modes are available with _eigengelapacke(). It allows for permuting the rows and columns of the matrix in addition to scaling by a diagonal matrix $D$.

In _eigengelapacke (..., real scalar balancing), if balancing is 0 , then no balancing is performed; if balancing is 1 , only scaling with a diagonal matrix $D$ is performed; if balancing is 2 , only permutation of the rows and columns is performed; in all other cases, both scaling and permutation are performed.

## eigensystem() and eigenvalues()

1. Use $L=$ eigenvalues $(A)$ and eigensystem $(A, X, L)$ for general matrices $A$.
2. Use $L=$ eigenvalues ( $A$ ) when you do not need the eigenvectors; it will save both time and memory.
3. The eigenvalues returned by $L=$ eigenvalues ( $A$ ) and by eigensystem $(A, X, L$ ) are of storage type complex even if the eigenvalues are real (that is, even if $\operatorname{Im}(L)==0$ ). If the eigenvalues are known to be real, you can save computer memory by subsequently coding

$$
\mathrm{L}=\operatorname{Re}(\mathrm{L})
$$

If you wish to test whether the eigenvalues are real, examine mreldifre $(L)$; see [M-5] reldif().
4. The eigenvectors returned by eigensystem $(A, X, L)$ are of storage type complex even if the eigenvectors are real (that is, even if $\operatorname{Im}(X)==0$ ). If the eigenvectors are known to be real, you can save computer memory by subsequently coding

$$
X=\operatorname{Re}(X)
$$

If you wish to test whether the eigenvectors are real, examine mreldifre $(X)$; see [M-5] reldif().
5. If you are using eigensystem $(A, X, L)$ interactively (outside a program), $X$ and $L$ must be predefined. Type

```
: eigensystem(A, X=., L=.)
```


## lefteigensystem()

What was just said about eigensystem() applies equally well to lefteigensystem().
If you need only the eigenvalues, use $L=$ eigenvalues $(A)$. The eigenvalues are the same for both left and right systems.

## symeigensystem( ) and symeigenvalues( )

1. Use $L=$ symeigenvalues $(A)$ and symeigensystem $(A, X, L)$ for symmetric or Hermitian matrices $A$.
2. Use $L=$ symeigenvalues $(A)$ when you do not need the eigenvectors; it will save both time and memory.
3. The eigenvalues returned by $L=$ symeigenvalues ( $A$ ) and by symeigensystem ( $A, X$, $L$ ) are of storage type real. Eigenvalues of symmetric and Hermitian matrices are always real.
4. The eigenvectors returned by symeigensystem $(A, X, L)$ are of storage type real when $A$ is of storage type real and of storage type complex when $A$ is of storage type complex.
5. If you are using symeigensystem ( $A, X, L$ ) interactively (outside a program), $X$ and $L$ must be predefined. Type
```
: symeigensystem(A, X=., L=.)
```


## Conformability

eigensystem( $A, X, L$, rcond, nobalance):
input:

| A: | $n \times n$ |  |
| ---: | :--- | :--- |
| rcond: | $1 \times 1$ | (optional, specify as 1 to obtain rcond) |
| nobalance: | $1 \times 1$ | (optional, specify as 1 to prevent balancing) |

output:

| X: | $n \times n$ | (columns contain eigenvectors) |
| ---: | :--- | :--- |
| $L:$ | $1 \times n$ |  |
| rcond: | $1 \times n$ | (optional) |
| result: | void |  |

lefteigensystem( $A, X, L$, rcond, nobalance):
input:

| A: | $n \times n$ |  |
| ---: | :--- | :--- |
| rcond: | $1 \times 1$ | (optional, specify as 1 to obtain rcond) |
| nobalance: | $1 \times 1$ | (optional, specify as 1 to prevent balancing) |

output:

```
            \(X: \quad n \times n \quad\) (rows contain eigenvectors)
            \(L: \quad n \times 1\)
            rcond: \(\quad n \times 1\) (optional)
            result: void
```

eigenvalues ( $A$, rcond, nobalance):
input:
A: $\quad n \times n$
rcond: $\quad 1 \times 1$ (optional, specify as 1 to obtain rcond)
nobalance: $1 \times 1$ (optional, specify as 1 to prevent balancing)
output:
rcond: $\quad 1 \times n \quad$ (optional)
result: $\quad 1 \times n \quad$ (contains eigenvalues)
symeigensystem ( $A, X, L$ ):
input:
A: $\quad n \times n$
output:
$X: \quad n \times n \quad$ (columns contain eigenvectors)
L: $\quad 1 \times n$
result: void
symeigenvalues ( $A$ ):
input:

$$
A: \quad n \times n
$$

output:
result: $\quad 1 \times n \quad$ (contains eigenvalues)
_eigensystem( $A, X, L$, rcond, nobalance):
input:

| $A:$ | $n \times n$ |  |
| ---: | :--- | :--- |
| rcond: | $1 \times 1$ | (optional, specify as 1 to obtain rcond) |
| nobalance: | $1 \times 1$ | (optional, specify as 1 to prevent balancing) |

output:
A: $\quad 0 \times 0$
$X: \quad n \times n \quad$ (columns contain eigenvectors)
$L: \quad 1 \times n$
rcond: $\quad 1 \times n \quad$ (optional)
result: void
_lefteigensystem ( $A, X, L$, rcond, nobalance) :
input:

| A: | $n \times n$ |  |
| ---: | :--- | :--- |
| rcond: | $1 \times 1$ | (optional, specify as 1 to obtain rcond) |
| nobalance: | $1 \times 1$ | (optional, specify as 1 to prevent balancing) |

output:
A: $\quad 0 \times 0$
$X: \quad n \times n \quad$ (rows contain eigenvectors)
L: $\quad n \times 1$
rcond: $\quad n \times 1 \quad$ (optional)
result: void
_eigenvalues ( $A$, rcond, nobalance):
input:

$$
\begin{array}{rll}
\text { A: } & n \times n & \\
\text { rcond: } & 1 \times 1 & \text { (optional, specify as } 1 \text { to obtain } \text { rcond) } \\
\text { nobalance: } & 1 \times 1 & \text { (optional, specify as } 1 \text { to prevent balancing) }
\end{array}
$$

output:

| A: | $0 \times 0$ |  |
| ---: | :--- | :--- |
| rcond: | $1 \times n$ | (optional) |
| result: | $1 \times n$ | (contains eigenvalues) |

_symeigensystem ( $A, X, L$ ):
input:
A: $\quad n \times n$
output:
A: $\quad 0 \times 0$
$X: \quad n \times n \quad$ (columns contain eigenvectors)
L: $\quad 1 \times n$
result: void
_symeigenvalues $(A)$ :
input:
A: $\quad n \times n$
output:

$$
\begin{array}{rll}
A: & 0 \times 0 & \\
\text { result: } & 1 \times n \quad \text { (contains eigenvalues) }
\end{array}
$$

_eigen_la(todo, $A, X, L$, rcond, nobalance):
input:

$$
\begin{array}{rr}
\text { todo: } & 1 \times 1 \\
A: & n \times n \\
r c o n d: & 1 \times 1 \\
\text { nobalance: } & 1 \times 1
\end{array}
$$

output:

$$
\begin{array}{rllll}
A: & 0 \times 0 & & & \\
X: & n \times n & & & \\
L: & 1 \times n & \text { or } & n \times 1 & \\
\text { rcond: } & 1 \times n & \text { or } & n \times 1 & \text { (optional) } \\
\text { result: } & 1 \times 1 & \text { (return code) } &
\end{array}
$$

_symeigen_la(todo, $A, X, L)$ :
input:

$$
\text { todo: } \quad 1 \times 1
$$

A: $\quad n \times n$
output:
A: $\quad 0 \times 0$
$X: \quad n \times n$
L: $\quad 1 \times n$
result: $1 \times 1$ (return code)
_eigengelapacke(todo, $A, X, L$, rcond, balancing):
input:

| todo: | $1 \times 1$ |
| ---: | ---: |
| $A:$ | $n \times n$ |
| rcond: | $1 \times 1$ |
| balancing: |  |
|  |  |

output:

$$
\begin{array}{rllll}
A: & 0 \times 0 & & & \\
X: & n \times n & & & \\
L: & 1 \times n & \text { or } & n \times 1 & \\
\text { rcond: } & 1 \times n & \text { or } & n \times 1 & \text { (optional) } \\
\text { result: } & 1 \times 1 & (\text { return code) } &
\end{array}
$$

## Diagnostics

All functions return missing-value results if $A$ has missing values.
symeigensystem(), symeigenvalues(), _symeigensystem(), and _symeigenvalues() use the lower triangle of $A$ without checking for symmetry. When $A$ is complex, only the real part of the diagonal is used.
_eigengelapacke() aborts with an error message when MKL is not available or is set to not use MKL.

## References

Gould, W. W. 2011a. Understanding matrices intuitively, part 1. The Stata Blog: Not Elsewhere Classified. http://blog.stata.com/2011/03/03/understanding-matrices-intuitively-part-1/.
_-. 2011b. Understanding matrices intuitively, part 2, eigenvalues and eigenvectors. The Stata Blog: Not Elsewhere Classified. http://blog.stata.com/2011/03/09/understanding-matrices-intuitively-part-2/.

## Also see

[M-5] matexpsym() - Exponentiation and logarithms of symmetric matrices
[M-5] matpowersym() - Powers of a symmetric matrix
[M-4] Matrix - Matrix functions

## Title

[M-5] eigensystemselect() — Compute selected eigenvectors and eigenvalues

Description Syntax Remarks and examples Conformability

## Description

eigensystemselectr ( $A$, range, $X, L$ ) computes selected right eigenvectors of a square, numeric matrix $A$ along with their corresponding eigenvalues. Only the eigenvectors corresponding to selected eigenvalues are computed. Eigenvalues that lie in a range are selected. The selected eigenvectors are returned in $X$, and their corresponding eigenvalues are returned in $L$.
range is a vector of length 2 . All eigenvalues with absolute value in the half-open interval (range [1], range [2]] are selected.
lefteigensystemselectr ( $A$, range, $X, L$ ) mirrors eigensystemselectr(), the difference being that it computes selected left eigenvectors instead of selected right eigenvectors.
eigensystemselecti ( $A$, index $, X, L$ ) computes selected right eigenvectors of a square, numeric matrix, $A$, along with their corresponding eigenvalues. Only the eigenvectors corresponding to selected eigenvalues are computed. Eigenvalues are selected by an index. The selected eigenvectors are returned in $X$, and the selected eigenvalues are returned in $L$.
index is a vector of length 2 . The eigenvalues are sorted by their absolute values, in descending order. The eigenvalues whose rank is index [1] through index [2], inclusive, are selected.
lefteigensystemselecti( $A$, index, $X, L$ ) mirrors eigensystemselecti(), the difference being that it computes selected left eigenvectors instead of selected right eigenvectors.
eigensystemselectf $(A, f, X, L)$ computes selected right eigenvectors of a square, numeric matrix, $A$, along with their corresponding eigenvalues. Only the eigenvectors corresponding to selected eigenvalues are computed. Eigenvalues are selected by a user-written function described below. The selected eigenvectors are returned in $X$, and the selected eigenvalues are returned in $L$.
lefteigensystemselectf ( $A, f, X, L$ ) mirrors eigensystemselectf(), the difference being that it computes selected left eigenvectors instead of selected right eigenvectors.
symeigensystemselectr ( $A$, range, $X, L$ ) computes selected eigenvectors of a symmetric (Hermitian) matrix, $A$, along with their corresponding eigenvalues. Only the eigenvectors corresponding to selected eigenvalues are computed. Eigenvalues that lie in a range are selected. The selected eigenvectors are returned in $X$, and their corresponding eigenvalues are returned in $L$.
symeigensystemselecti ( $A$, index $, X, L$ ) computes selected eigenvectors of a symmetric (Hermitian) matrix, $A$, along with their corresponding eigenvalues. Only the eigenvectors corresponding to selected eigenvalues are computed. Eigenvalues are selected by an index. The selected eigenvectors are returned in $X$, and the selected eigenvalues are returned in $L$.
_eigenselectr_la(), _eigenselecti_la(), _eigenselectf_la(), _eigenselect_la(), and _symeigenselect_la() are the interfaces into the [M-1] LAPACK routines used to implement the above functions. Their direct use is not recommended.

## Syntax

| void | eigensystemselectr $(A$, range, $X, L)$ |
| :--- | ---: |
| void | lefteigensystemselectr $(A$, range, $X, L)$ |
| void | eigensystemselecti $(A$, index, $X, L)$ |
| void | lefteigensystemselecti $(A$, index, $X, L)$ |
| void | eigensystemselectf $(A, f, X, L)$ |
| void | lefteigensystemselectf $(A, f, X, L)$ |
| void | symeigensystemselectr $(A$, range, $X, L)$ |
| void | symeigensystemselecti $(A$, index, $X, L)$ |

where inputs are
A: numeric matrix
range: real vector (range of eigenvalues to be selected)
index: real vector (indices of eigenvalues to be selected)
$f$ : pointer scalar (points to a function used to select eigenvalue)
and outputs are
$X$ : numeric matrix of eigenvectors
$L$ : numeric vector of eigenvalues
The following routines are used in implementing the above routines:

> void _eigenselecti_la(numeric matrix $A, X L, X R, L$, string scalar side, real vector index)
> void _eigenselectr_la(numeric matrix $A, X L, X R, L$, string scalar side, real vector range)
> void _eigenselectf_la(numeric matrix $A, X L, X R, L$, string scalar side, pointer scalar $f$ )
real scalar _eigenselect_la(numeric matrix $A, X L, X R, L$, select, string scalar side, real scalar noflopin)
real scalar _symeigenselect_la(numeric matrix $A, X, L$, ifail, real scalar type, lower, upper, abstol)

## Remarks and examples

Remarks are presented under the following headings:

Introduction<br>Range selection<br>Index selection<br>Criterion selection<br>Other functions

## Introduction

These routines compute subsets of the available eigenvectors. This computation can be much faster than computing all the eigenvectors. (See [M-5] eigensystem() for routines to compute all the eigenvectors and an introduction to the eigensystem problem.)

There are three methods for selecting which eigenvectors to compute; all of them are based on the corresponding eigenvalues. First, we can select only those eigenvectors whose eigenvalues have absolute values that fall in a half-open interval. Second, we can select only those eigenvectors whose eigenvalues have certain indices, after sorting the eigenvalues by their absolute values in descending order. Third, we can select only those eigenvectors whose eigenvalues meet a criterion encoded in a function.

Below we illustrate each of these methods. For comparison purposes, we begin by computing all the eigenvectors of the matrix
: A

|  | 1 | 2 | 3 | 4 |
| :--- | ---: | ---: | ---: | ---: |
| 1 | .31 | .69 | .13 | .56 |
| 2 | .31 | .5 | .72 | .42 |
| 3 | .68 | .37 | .71 | .8 |
| 4 | .09 | .16 | .83 | .9 |
|  |  |  |  |  |

We perform the computation with eigensystem():

```
: eigensystem(A, X=., L=.)
```

The absolute values of the eigenvalues are

```
: abs(L)
```

$1 \quad 2$
$3 \quad 4$


The corresponding eigenvectors are


## Range selection

In applications, an eigenvalue whose absolute value is greater than 1 frequently corresponds to an explosive solution, whereas an eigenvalue whose absolute value is less than 1 corresponds to a stable solution. We frequently want to use only the eigenvectors from the stable solutions, which we can do using eigensystemselectr(). We begin by specifying

```
: range = (-1, .999999999)
```

which starts from -1 to include 0 and stops at .999999999 to exclude 1. (The half-open interval in range is open on the left and closed on the right.)

Using this range in eigensystemselectr() requests each eigenvector for which the absolute value of the corresponding eigenvalue falls in the interval ( $-1, .999999999$ ]. For the example at hand, we have
eigensystemselectr(A, range, X=., L=.)
eigensystemselectr(A, range, X=., L=.)
X
1 2
23

$$
\begin{array}{rrr}
\hline-.442004357 & .201218963-.875384534 i & .201218963+.875384534 \mathrm{i} \\
-.668468693 & .136296114+.431873675 \mathrm{i} & .136296114-.431873675 \mathrm{i} \\
-.215927364 & -.706872994-.022093594 \mathrm{i} & -.706872994+.022093594 \mathrm{i} \\
.750771548 & .471845361+.218651289 \mathrm{i} & .471845361-.218651289 \mathrm{i}
\end{array}
$$

L
$1 \quad 2$
2 3

abs (L)
1


The above output illustrates that eigensystemselectr() has not included the results for the eigenvalue whose absolute value is greater than 1 , as desired.

## Index selection

In many statistical applications, an eigenvalue measures the importance of an eigenvector factor. In these applications, we want only to compute several of the largest eigenvectors. Here we use eigensystemselecti () to compute the eigenvectors corresponding to the two largest eigenvalues:

```
: index = (1, 2)
: eigensystemselecti(A, index, X=., L=.)
: L
    1 2
1
    2.10742167 . }4658398
```

$$
\begin{array}{ll}
1 & 2
\end{array}
$$

```
. 385302069 -. 442004357
.477773165 -. }66846869
. }604617181-.21592736
. }50765459 .75077154
```


## Criterion selection

In some applications, we want to compute only those eigenvectors whose corresponding eigenvalues satisfy a more complicated criterion. We can use eigensystemselectf() to solve these problems.

We must pass eigensystemselectf() a pointer to a function that implements our criterion. The function must accept a complex scalar argument so that it can receive an eigenvalue, and it must return the real value 0 to indicate rejection and a nonzero real value to indicate selection.

In the example below, we consider the common criterion of whether the eigenvalue is real. We want only to compute the eigenvectors corresponding to real eigenvalues. After deciding that anything smaller than $1 \mathrm{e}-15$ is zero, we define our function to be

```
real scalar onlyreal(complex scalar ev)
{
return( (abs(Im(ev))<1e-15))
}
```

We compute only the eigenvectors corresponding to the real eigenvalues by typing

```
: eigensystemselectf(A, &onlyreal(), X=., L=.)
```

The eigenvalues that satisfy this criterion and their corresponding eigenvectors are

```
: L
    1 2
    2.10742167 . }4658398
X
    1 2
    .385302069 
```


## Other functions

lefteigensystemselectr() and symeigensystemselectr() use a range like eigensystemselectr().
lefteigensystemselecti() and symeigensystemselecti() use an index like eigensystemselecti().
lefteigensystemselectf() uses a pointer to a function like eigensystemselectf().

## Conformability

eigensystemselectr ( $A$, range, $X, L$ ):
input:
A: $\quad n \times n$
range: $\quad 1 \times 2$ or $2 \times 1$
output:

$$
\begin{array}{ll}
X: & \\
L \times m \\
L: & \\
1 \times m
\end{array}
$$

lefteigensystemselectr ( $A$, range, $X, L$ ):
input:

$$
\begin{aligned}
A: & n \times n \\
\text { range: } & 1 \times 2 \text { or } 2 \times 1
\end{aligned}
$$

output:

$$
\begin{array}{cc}
X: & m \times n \\
L: & 1 \times m
\end{array}
$$

eigensystemselecti( $A$, index, $X, L$ ):
input:
A: $\quad n \times n$
index: $\quad 1 \times 2$ or $2 \times 1$
output:
$X: \quad n \times m$
$L: \quad 1 \times m$
lefteigensystemselecti ( $A$, index, $X, L$ ):
input:
A: $\quad n \times n$
index: $\quad 1 \times 2$ or $2 \times 1$
output:
$X: \quad m \times n$
$L: \quad 1 \times m$
eigensystemselectf ( $A, f, X, L$ ):
input:
A: $\quad n \times n$
$f: \quad 1 \times 1$
output:
$\begin{aligned} \text { X: } & \\ L: & n \times m \\ & 1 \times m\end{aligned}$
$L: \quad 1 \times m$
lefteigensystemselectf $(A, f, X, L)$ :
input:

$$
\begin{array}{cc}
A: & \\
f: & \\
f: & 1 \times 1
\end{array}
$$

output:
$X: \quad m \times n$
$L: \quad 1 \times m$
symeigensystemselectr ( $A$, range, $X, L$ ):
input:
A: $\quad n \times n$

$$
\text { range: } \quad 1 \times 2 \text { or } 2 \times 1
$$

output:
$X: \quad n \times m$
$L: \quad 1 \times m$
symeigensystemselecti ( $A$, index, $X, L$ ):
input:
A: $\quad n \times n$
index: $\quad 1 \times 2$ or $2 \times 1$
output:
$X: \quad n \times m$
$L: \quad 1 \times m$

## Diagnostics

All functions return missing-value results if $A$ has missing values.
symeigensystemselectr() and symeigensystemselecti() use the lower triangle of $A$ without checking for symmetry. When $A$ is complex, only the real part of the diagonal is used.

If the $i$ th eigenvector failed to converge, symeigensystemselectr () and symeigensystemselecti() insert a vector of missing values into the $i$ th column of the returned eigenvector matrix.

## Also see

[M-1] LAPACK - Linear algebra package (LAPACK) routines
[M-5] eigensystem( ) - Eigenvectors and eigenvalues
[M-5] matexpsym( ) - Exponentiation and logarithms of symmetric matrices
[M-5] matpowersym() - Powers of a symmetric matrix
[M-4] Matrix - Matrix functions

## Title

[M-5] eltype() - Element type, organizational type, and type name of object

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

eltype() returns the current eltype of the argument.
orgtype () returns the current orgtype of the argument.
classname() returns the name of the class for a Mata class scalar.
structname() returns the name of the struct for a Mata struct scalar.
See [M-6] Glossary for a definition of eltype and orgtype.

## Syntax

```
string scalar eltype(X)
string scalar orgtype(X)
string scalar classname(X)
string scalar structname(X)
```


## Remarks and examples

If $X$ is a matrix (syntax 1 ), returned is

| eltype $(X)$ | orgtype $(X)$ |
| :--- | :--- |
| real | scalar |
| complex | rowvector |
| string | colvector |
| pointer | matrix |
| struct |  |
| class |  |

The returned value reflects the current contents of $X$. That is, $X$ might be declared a transmorphic matrix, but at any instant, it contains something, and if that something were 5 , returned would be "real" and "scalar".

For orgtype(), returned is "scalar" if the object is currently $1 \times 1$; "rowvector" if it is $1 \times$ $k, k \neq 1$; "colvector" if it is $k \times 1, k \neq 1$; and "matrix" otherwise (it is $r \times c, r \neq 1, c \neq 1$ ).
$X$ can be a function (syntax 2). Returned is

| eltype(*(\&func())) | orgtype(*(\&func())) |
| :--- | :--- |
| transmorphic | matrix |
| numeric | vector |
| real | rowvector |
| complex | colvector |
| string | scalar |
| pointer | void |
| struct |  |
| structdef |  |
| class |  |
| classdef |  |

These types are obtained from the declaration of the function.
Aside: struct and structdef have to do with structures; see [M-2] struct. structdef indicates that the function not only returns a structure but is the routine that defines the structure as well. class and classdef have to do with Mata classes; see [M-2] class. classdef indicates the function not only returns a class but is the routine that defines the class as well.
classname() returns the name " $c A$ " if the object is a class cA scalar. The function returns " " if the object has element type other than class or has organizational type other than scalar.
structname() returns the name "sA" if the object is a struct sA scalar. The function returns " " if the object has element type other than struct or has organizational type other than scalar.

## Conformability

```
eltype(X), orgtype(X), classname(X), structname(X):
    X: r
    result: }1\times
```


## Diagnostics

None.

## Also see

[M-5] isreal() - Storage type of matrix
[M-5] isview () - Whether matrix is view
[M-4] Utility - Matrix utility functions

## Title

[M-5] epsilon( ) - Unit roundoff error (machine precision)

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

epsilon ( $x$ ) returns the unit roundoff error in quantities of size abs $(x)$.

## Syntax

real scalar epsilon(real scalar $x$ )

## Remarks and examples

On all computers on which Stata and Mata are currently implemented-which are computers following IEEE standards-epsilon(1) is $1.0 \mathrm{X}-34$, or about $2.22045 \mathrm{e}-16$. This is the smallest amount by which a real number can differ from 1.
epsilon $(x)$ is abs $(x) * e p s i l o n(1)$. This is an approximation of the smallest amount by which a real number can differ from $x$. The approximation is exact at integer powers of 2.

## Conformability

epsilon( $x$ ):

| $x:$ | $1 \times 1$ |
| ---: | :--- |
| result: | $1 \times 1$ |

## Diagnostics

epsilon(x) returns. if $x$ is missing.

## Also see

[M-5] edittozero() - Edit matrix for roundoff error (zeros)
[M-5] mindouble( ) - Minimum and maximum nonmissing value
[M-4] Utility - Matrix utility functions

## Title

[M-5] _equilrc() — Row and column equilibration

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

_equilrc $(A, r, c)$ performs row and column equilibration (balancing) on matrix $A$, returning the equilibrated matrix in $A$, the row-scaling factors in $r$, and the column-scaling factors in $c$.
_equilr $(A, r)$ performs row equilibration on matrix $A$, returning the row-equilibrated matrix in $A$ and the row-scaling factors in $r$.
_equilc $(A, c)$ performs column equilibration on matrix $A$, returning the column-equilibrated matrix in $A$ and the column-scaling factors in $c$.
_perhapsequilrc $(A, r, c)$ performs row and/or column equilibration on matrix $A$ —as is necessary and which decision is made by _perhapsequilrc()—returning the equilibrated matrix in $A$, the row-scaling factors in $r$, the column-scaling factors in $c$, and returning 0 (no equilibration performed), 1 (row equilibration performed), 2 (column equilibration performed), or 3 (row and column equilibration performed).
_perhapsequilr $(A, r)$ performs row equilibration on matrix $A$-if necessary and which decision is made by _perhapsequilr()—returning the equilibrated matrix in $A$, the row-scaling factors in $r$, and returning 0 (no equilibration performed) or 1 (row equilibration performed).
_perhapsequilc $(A, c)$ performs column equilibration on matrix $A$-if necessary and which decision is made by _perhapsequilc()—returning the equilibrated matrix in $A$, the column-scaling factors in $c$, and returning 0 (no equilibration performed) or 1 (column equilibration performed).
rowscalefactors $(A)$ returns the row-scaling factors of $A$.
colscalefactors ( $A$ ) returns the column-scaling factors of $A$.

## Syntax

| void | _equilrc (numeric matrix $A, r, c$ ) |
| :---: | :---: |
| void | _equilr (numeric matrix $A, r$ ) |
| void | _equilc(numeric matrix $A, c$ ) |
| real scalar | _perhapsequilrc (numeric matrix $A, r, c$ ) |
| real scalar | _perhapsequilr (numeric matrix $A, r$ ) |
| real scalar | _perhapsequilc (numeric matrix $A, c$ ) |
| real colvector | rowscalefactors (numeric matrix $A$ ) |
| real rowvector | colscalefactors (numeric matrix $A$ ) |

The types of $r$ and $c$ are irrelevant because they are overwritten.

## Remarks and examples

Remarks are presented under the following headings:

```
Introduction
Is equilibration necessary?
The _equil*() family of functions
The _perhapsequil*() family of functions
rowscalefactors() and colscalefactors()
```


## Introduction

Equilibration (also known as balancing) takes a matrix with poorly scaled rows and columns, such as
$1 \quad 2$

```
1.00000e+10 5.00000e+10
2.00000e-10 8.00000e-10
```

and produces a related matrix, such as

|  | 1 | 2 |
| :--- | ---: | :--- |
| 1 | .2 | 1 |
| 2 | .25 | 1 |
|  |  |  |

that will yield improved accuracy in, for instance, the solution to linear systems. The improved matrix above has been row equilibrated. All we did was find the maximum of each row of the original and then divide the row by its maximum. If we were to take the result and repeat the process on the columns-divide each column by the column's maximum-we would obtain

|  | 1 | 2 |
| :---: | :---: | :---: |
| 1 | . 8 | 1 |
| 2 | 1 | 1 |

which is the row-and-column equilibrated form of the original matrix.
In terms of matrix notation, equilibration can be thought about in terms of multiplication by diagonal matrices. The row-equilibrated form of $A$ is $R A$, where $R$ contains the reciprocals of the row maximums on its diagonal. The column-equilibrated form of $A$ is $A C$, where $C$ contains the reciprocals of the column maximums on its diagonal. The row-and-column equilibrated form of $A$ is $R A C$, where $R$ contains the reciprocals of the row maximums of $A$ on its diagonal, and $C$ contains the reciprocals of the column maximums of $R A$ on its diagonal.

Say we wished to find the solution $x$ to

$$
A x=b
$$

We could compute the solution by solving for $y$ in the equilibrated system

$$
(R A C) y=R b
$$

and then setting

$$
x=C y
$$

Thus routines that perform equilibration need to return to you, in some fashion, $R$ and $C$. The routines here do that by returning $r$ and $c$, the reciprocals of the maximums in vector form. You could obtain $R$ and $C$ from them by coding

$$
\begin{aligned}
& R=\operatorname{diag}(r) \\
& C=\operatorname{diag}(c)
\end{aligned}
$$

but that is not in general necessary, and it is wasteful of memory. In code, you will need to multiply by $R$ and $C$, and you can do that using the $: *$ operator with $r$ and $c$ :

$$
\begin{aligned}
R A & \leftrightarrow r: * A \\
A C & \leftrightarrow A: * c \\
R A C & \leftrightarrow r: * A: * c
\end{aligned}
$$

## Is equilibration necessary?

Equilibration is not a panacea. Equilibration can reduce the condition number of some matrices and thereby improve the accuracy of the solution to linear systems, but equilibration is not guaranteed to reduce the condition number, and counterexamples exist in which equilibration actually decreases the accuracy of the solution. That said, you have to look long and hard to find such examples.

Equilibration is not especially computationally expensive, but neither is it cheap, especially when you consider the extra computational costs of using the equilibrated matrices. In statistical contexts, equilibration may buy you little because matrices are already nearly equilibrated. Data analysts know variables should be on roughly the same scale, and observations are assumed to be draws from an underlying distribution. The computational cost of equilibration is probably better spent somewhere else. For instance, consider obtaining regression estimates from $X^{\prime} X$ and $X^{\prime} y$. The gain from equilibrating $X^{\prime} X$ and $X^{\prime} y$, or even from equilibrating the original $X$ matrix, is nowhere near that from the gain to be had in removal of the means before $X^{\prime} X$ and $X^{\prime} y$ are formed.

In the example in the previous section, we showed you a matrix that assuredly benefited from equilibration. Even so, after row equilibration, column equilibration was unnecessary. It is often the case that solely row or column equilibration is sufficient, and in those cases, although the extra equilibration will do no numerical harm, it will burn computer cycles. And, as we have already argued, some matrices do not need equilibration at all.

Thus programmers who want to use equilibration and obtain the best speed possible examine the matrix and on that basis perform (1) no equilibration, (2) row equilibration, (3) column equilibration, or (4) both. They then write four branches in their subsequent code to handle each case efficiently.

In terms of determining whether equilibration is necessary, measures of the row and column condition can be obtained from $\min (r) / \max (r)$ and $\min (c) / \max (c)$, where $r$ and $c$ are the scaling factors (vectors of the reciprocals of the row and column maximums). If those measures are near 1 (LAPACK uses $\geq .1$ ), then equilibration can be skipped.

There is also the issue of the overall scale of the matrix. In theory, the overall scale should not matter, but many packages set tolerances in absolute rather than relative terms, and so overall scale does matter. In most of Mata's other functions, relative scales are used, but provisions are made so that you can specify tolerances in relative or absolute terms. In any case, LAPACK uses the rule that equilibration is necessary if the matrix is too small (its maximum value is less than epsilon(100), approximately $2.22045 \mathrm{e}-14$ ) or too large (greater than $1 / \mathrm{epsilon}(100)$, approximately $4.504 \mathrm{e}+13$ ).

To summarize,

1. In statistical applications, we believe that equilibration burns too much computer time and adds too much code complexity for the extra accuracy it delivers. This is a judgment call, and we would probably recommend the use of equilibration were it computationally free.
2. If you are going to use equilibration, there is nothing numerically wrong with simply equilibrating matrices in all cases, including those in which equilibration is not necessary. The advantages of this is that you will gain the precision to be had from equilibration while still keeping your code reasonably simple.
3. If you wish to minimize execution time, then you want to perform the minimum amount of equilibration possible and write code to deal efficiently with each case: 1) no equilibration, 2) row equilibration, 3) column equilibration, and 4) row and column equilibration. The defaults used by LAPACK and incorporated in Mata's _perhapsequil*() routines are
a. Perform row equilibration if $\min (r) / \max (r)<.1$, or if $\min (\operatorname{abs}(A))<\operatorname{epsilon}(100)$, or if $\min (\operatorname{abs}(A))>1 /$ epsilon(100).
b. After performing row equilibration, perform column equilibration if $\min (c) / \max (c)$ $<.1$, where $c$ is calculated on $r: * A$, the row-equilibrated $A$, if row equilibration was performed.

## The _equil ${ }^{*}()$ family of functions

The _equil*() family of functions performs equilibration as follows:
_equilrc $(A, r, c)$ performs row equilibration followed by column equilibration; it returns in $r$ and in $c$ the row- and column-scaling factors, and it modifies $A$ to be the fully equilibrated matrix $r: * A: * c$.
_equilr $(A, r)$ performs row equilibration only; it returns in $r$ the row-scaling factors, and it modifies $A$ to be the row-equilibrated matrix $r: * A$.
_equilc $(A, c)$ performs column equilibration only; it returns in $c$ the row-scaling factors, and it modifies $A$ to be the row-equilibrated matrix $A: * c$.

Here is code to solve $A x=b$ using the fully equilibrated form, which damages $A$ in the process:

```
_equilrc(A, r, c)
x = c:*lusolve(A, r:*b)
```


## The _perhapsequil*() family of functions

The _perhapsequil*() family of functions mirrors _equil*(), except that these functions apply the rules mentioned above for whether equilibration is necessary.

Here is code to solve $A x=b$, which may damage $A$ in the process:

```
result = _perhapsequilrc(A, r, c)
if (result==0) x = lusolve(A, b)
else if (result==1) x = lusolve(A, r:*b)
else if (result==2) x = c:*lusolve(A, b)
else if (result==3) x = c:*lusolve(A, r:*b)
```

As a matter of fact, the _perhapsequil*() family returns a vector of 1 s when equilibration is not performed, so you could code

$$
\begin{aligned}
& \text { (void) _perhapsequilrc(A, r, c) } \\
& x=c: * \operatorname{lusolve}(A, r: * b)
\end{aligned}
$$

but that would be inefficient.

## rowscalefactors( ) and colscalefactors( )

rowscalefactors ( $A$ ) and colscalefactors $(A)$ return the scale factors (reciprocals of row and column maximums) to perform row and column equilibration. These functions are used by the other functions above and are provided for those who wish to write their own equilibration routines.

## Conformability

_equilrc $(A, r, c)$ :
input:
A: $\quad m \times n$
output:
A: $\quad m \times n$
$r: \quad m \times 1$
$c: \quad 1 \times n$
_equilr $(A, r)$ :
input:
A: $\quad m \times n$
output:
A: $\quad m \times n$
$r: \quad m \times 1$
_equilc $(A, c)$ :
input:
A: $\quad m \times n$
output:
A: $\quad m \times n$
$c: \quad 1 \times n$
_perhapsequilrc $(A, r, c)$ :
input:
A: $\quad m \times n$
output:
A: $\quad m \times n \quad($ unmodified if result $=0)$
$r: \quad m \times 1$
$c: \quad 1 \times n$
result: $\quad 1 \times 1$
_perhapsequilr $(A, r)$ :
input:
A: $\quad m \times n$
output:

$$
\begin{aligned}
A: & m \times n \quad(\text { unmodified if result }=0) \\
r: & m \times 1 \\
\text { result: } & 1 \times 1
\end{aligned}
$$

_perhapsequilc $(A, c)$ :
input:
A: $\quad m \times n$
output:
A: $\quad m \times n \quad($ unmodified if result $=0)$
$c: \quad 1 \times n$
result: $\quad 1 \times 1$
rowscalefactors ( $A$ ):
A: $\quad m \times n$
result: $\quad m \times 1$
colscalefactors ( $A$ ):
$\begin{aligned} \text { A: } & m \times n \\ \text { result: } & 1 \times n\end{aligned}$

## Diagnostics

Scale factors used and returned by all functions are calculated by rowscalefactors $(A)$ and colscalefactors $(A)$. The functions are defined as $1: /$ rowmaxabs $(A)$ and $1: / \operatorname{colmaxabs}(A)$, with missing values changed to 1 . Thus rows or columns that contain missing or are entirely zero are defined to have scale factors of 1 .

Equilibration functions do not equilibrate rows or columns that contain missing or all zeros.
The _equil* () functions convert $A$ to an array if $A$ was a view. The Stata dataset is not changed.
The _perhapsequil*() functions convert $A$ to an array if $A$ was a view and returned is nonzero. The Stata dataset is not changed.

## Also see

[M-4] Matrix - Matrix functions

## Title

[M-5] error() — Issue error message

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

error ( $r c$ ) displays the standard Stata error message associated with return code $r c$ and returns $r c$; see [P] error for a listing of return codes. error() does not abort execution; standard usage is exit(error (rc)).
_error () aborts execution and produces a traceback log.
_error (errnum) produces a traceback log with standard Mata error message errnum; see [M-2] Errors for a listing of the standard Mata error codes.
_error (errtxt) produces a traceback log with error number 3498 and custom text errtxt.
_error (errnum, errtxt) produces a traceback log with error number errnum and custom text errtxt. If errtxt is specified, it should contain straight text; SMCL codes are not interpreted.

## Syntax

```
real scalar error(real scalar rc)
void _error(real scalar errnum)
void _error(string scalar errtxt)
void _error(real scalar errnum, string scalar errtxt)
```


## Remarks and examples

Remarks are presented under the following headings:

```
Use of _error()
Use of error()
```


## Use of _error()

_error () aborts execution and produces a traceback log:

```
: myfunction(A,B)
    mysub(): 3200 conformability error
    myfunction(): - function returned error
    <istmt>: - function returned error
r(3200);
```

The above output was created because function mysub() contained the line

```
_error(3200)
```

and 3200 is the error number associated with the standard message "conformability error"; see [M-2] Errors. Possibly, the code read

```
if (rows(A)!=rows(B) | cols(A)!=cols(B)) {
    _error(3200)
}
```

Another kind of mistake might produce

```
: myfunction(A,B)
    mysub(): 3498 zeros on diagonal not allowed
    myfunction(): - function returned error
    <istmt>: - function returned error
r(3498);
```

and that could be produced by the code

```
if (diag0cnt(A)>0) {
    _error("zeros on diagonal not allowed")
}
```

If we wanted to produce the same text but change the error number to 3300 , we could have coded

```
if (diag0cnt(A)>0) {
    _error(3300, "zeros on diagonal not allowed")
}
```

Coding _error () is not always necessary. In our conformability-error example, imagine that more of the code read

```
if (rows(A)!=rows(B) | cols(A)!=cols(B)) {
    _error(3200)
}
C = A + B
```

If we simplified the code to read

$$
C=A+B
$$

the conformability error would still be detected because + requires p-conformability:

```
: myfunction(A,B)
    +: 3200 conformability error
    mysub(): - conformability error
    myfunction(): - function returned error
    <istmt>: - function returned error
r(3200);
```

Sometimes, however, you must detect the error yourself. For instance,

```
if (rows(A)!=rows(B) | cols(A)!=cols(B) | rows(A)!=2*cols(A)) {
    _error(3200)
}
C = A + B
```

We assume we have some good reason to require that A has twice as many rows as columns. + , however, will not require that, and perhaps no other calculation we will make will require that, either. Or perhaps it will be subsequently detected, but in a way that leads to a confusing error message for the caller.

## Use of error()

error ( $r c$ ) does not cause the program to terminate. Standard usage is

```
exit(error(rc))
```

such as

```
exit(error(503))
```

In any case, error () does not produce a traceback log:

```
: myfunction(A,B)
conformability error
r(503);
```

error() is intended to be used in functions that are subroutines of ado-files:


This way, when the example command is used incorrectly, the user will see

```
. example ...
conformability error
r(503);
```

rather than the traceback $\log$ that would have been produced had we omitted the test and exit(error(503)):

```
. example ...
    +: 3200 conformability error
    myfunction(): - function returned error
    <istmt>: - function returned error
r(3200);
```


## Conformability

```
error(rc):
```

| rc: | $1 \times 1$ |
| ---: | :--- |
| result: | $1 \times 1$ |
| _error (errnum): |  |
| errnum: | $1 \times 1$ |
| result: | void |
| _error (errtxt): |  |
| errtxt: | $1 \times 1$ |
| result: | void |
| _error (errnum, errtxt): |  |
| errnum: | $1 \times 1$ |
| errtxt: | $1 \times 1$ |
| result: | void |

## Diagnostics

error $(r c)$ does not abort execution; code exit (error $(r c)$ ) if that is your desire; see [M-5] exit().
The code error $(r c)$ returns can differ from $r c$ if $r c$ is not a standard code or if there is a better code associated with it.
error ( $r c$ ) with $r c=0$ produces no output and returns 0 .
_error (errnum), _error (errtxt), and _error (errnum, errtxt) always abort with error. _error () will abort with error because you called it wrong if you specify an errnum less than 1 or greater than $2,147,483,647$ or if you specify an errtxt longer than 100 characters. If you specify an errnum that is not a standard code, the text of the error messages will read "Stata returned error".

## Also see

[M-2] Errors - Error codes
[M-5] errprintf() - Format output and display as error message
[M-5] exit() - Terminate execution
[M-4] Programming — Programming functions

## Title

[M-5] errprintf( ) - Format output and display as error message

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

errprintf() is a convenience tool for displaying an error message.
errprintf (...) is equivalent to printf (...) except that it executes displayas("error") before the printf() is executed; see $[\mathrm{M}-5]$ printf() and [M-5] displayas().

## Syntax

```
void errprintf(string scalar fmt, r},\mp@subsup{r}{1}{},\mp@subsup{r}{2}{},\ldots,\mp@subsup{r}{N}{}
```


## Remarks and examples

You have written a program. At one point in the code, you have variable fn that should contain the name of an existing file:

```
if (!fileexists(fn)) {
    // you wish to display the error message
    // file ____ not found
    exit(601)
}
```

One solution is

```
if (!fileexists(fn)) {
    displayas("error")
    printf("file %s not found\n", fn)
    exit(601)
}
```

Equivalent is

```
if (!fileexists(fn)) {
    errprintf("file %s not found\n", fn)
    exit(601)
}
```

It is important that you either displayas("error") before using printf() or that you use errprintf(), to ensure that your error message is displayed (is not suppressed by a quietly) and that it is displayed in red; see [M-5] displayas().

## Conformability

$$
\begin{array}{cc}
\text { errprintf }(f m t, & \left.r_{1}, r_{2}, \ldots, r_{N}\right) \\
\text { fmt: } & 1 \times 1 \\
r_{1}: & 1 \times 1 \\
r_{2}: & 1 \times 1 \\
\cdots & \\
r_{N}: & 1 \times 1 \\
\text { result: } & \text { void }
\end{array}
$$

## Diagnostics

errprintf () aborts with error if a \%fmt is misspecified, if a numeric \%fmt corresponds to a string result or a string $\% f m t$ corresponds to a numeric result, or there are too few or too many $\%$ fimts in $f m t$ relative to the number of results specified.

## Also see

[M-5] displayas( ) - Set display level
[M-5] error( ) - Issue error message
[M-5] printf() - Format output
[M-4] IO - I/O functions

## Title

[M-5] exit( ) - Terminate execution

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

exit (rc) terminates execution and sets the overall return code to $r c$.
exit() with no argument specified is equivalent to exit(0).

## Syntax

```
exit(real scalar rc)
exit()
```


## Remarks and examples

Do not confuse exit () and return. return stops execution of the current function and returns to the caller, whereupon execution continues. exit() terminates execution. For instance, consider

```
function first()
{
        "begin execution"
        second()
        "this message will never be seen"
}
function second()
{
    "hello from second()"
    exit(0)
}
```

The result of running this would be

```
: first()
begin execution
hello from second()
```

If we changed the exit(0) to be exit (198) in second(), the result would be

```
: first()
    begin execution
    hello from second()
r(198);
```

No error message is presented. If you want to present an error message and exit, you should code exit(error(198)); see [M-5] error().

## Conformability

```
exit(rc):
    rc:1\times1 (optional)
```


## Diagnostics

exit( $r c$ ) and exit() do not return.

## Also see

[M-5] error( ) - Issue error message
[M-4] Programming — Programming functions

## Title

[M-5] exp( ) - Exponentiation and logarithms
Description Syntax Conformability Diagnostics Also see

## Description

$\exp (Z)$ returns the elementwise exponentiation of $Z$. $\exp ()$ returns real if $Z$ is real and complex if $Z$ is complex.
$\ln (Z)$ and $\log (Z)$ return the elementwise natural logarithm of $Z$. The functions are synonyms. $\ln ()$ and $\log ()$ return real if $Z$ is real and complex if $Z$ is complex.
$\ln (x), x$ real, returns the natural logarithm of $x$ or returns missing (.) if $x \leq 0$.
$\ln (z), z$ complex, returns the complex natural logarithm of $z \cdot \operatorname{Im}(\ln ())$ is chosen to be in the interval ( $-p i, p i]$.
$\log 10(Z)$ returns the elementwise $\log$ base 10 of $Z . \log 10()$ returns real if $Z$ is real and complex if $Z$ is complex. $\log 10(Z)$ is defined mathematically and operationally as $\ln (Z) / \ln (10)$.
$\operatorname{expm} 1(Z)$ returns $\exp (z)-1$ for every element $z$ of real matrix $Z$. $\operatorname{expm} 1(z)$ is more accurate than $\exp (z)-1$ for small values of $|z|$.
$\ln 1 \mathrm{p}(Z)$ and $\log 1 \mathrm{p}(Z)$ return $\log (1+z)$ for every element $z$ of real matrix $Z$. The functions are synonyms. $\ln 1 \mathrm{p}(z)$ is more accurate than $\ln (1+z)$ for small values of $|z|$.
$\ln 1 \mathrm{~m}(Z)$ and $\log 1 \mathrm{~m}(Z)$ return $\log (1-z)$ for every element $z$ of real matrix $Z$. The functions are synonyms. $\ln 1 \mathrm{~m}(z)$ is more accurate than $\ln (1-z)$ for small values of $|z|$.

## Syntax

| numeric matrix | $\exp ($ numeric matrix $Z)$ |
| :--- | :--- |
| numeric matrix | $\ln ($ numeric matrix $Z)$ |
| numeric matrix | $\log ($ numeric matrix $Z)$ |
| numeric matrix | $\log 10$ (numeric matrix $Z$ ) |
| numeric matrix | $\operatorname{expm} 1$ (numeric matrix $Z$ ) |
| numeric matrix | $\ln 1 \mathrm{p}($ numeric matrix $Z)$ |
| numeric matrix | $\log 1 \mathrm{p}($ numeric matrix $Z)$ |
| numeric matrix | $\ln 1 \mathrm{~m}($ numeric matrix $Z)$ |
| numeric matrix | $\log 1 \mathrm{~m}($ numeric matrix $Z)$ |

## Conformability

$\exp (Z), \ln (Z), \log (Z), \log 10(Z), \operatorname{expm} 1(Z), \ln 1 \mathrm{p}(Z), \log 1 \mathrm{p}(Z), \ln 1 \mathrm{~m}(Z), \log 1 \mathrm{~m}(Z):$
$Z: \quad r \times c$
result: $\quad r \times c$

## Diagnostics

$\exp (Z)$ returns missing when $\operatorname{Re}(Z)>709$.
$\ln (Z), \log (Z)$, and $\log 10(Z)$ return missing when $Z$ is real and $Z \leq 0$. In addition, the functions return missing (.) for real arguments when the result would be complex. For instance, $\ln (-1)=$. , whereas $\ln (-1+0 i)=3.14159265 i$.
expm1 $(Z)$ returns missing when $Z>709$.
$\ln 1 \mathrm{p}(z)$ and $\log 1 \mathrm{p}(z)$ return missing when $1+z \leq 0$.
$\ln 1 \mathrm{~m}(z)$ and $\log 1 \mathrm{~m}(z)$ return missing when $1-z \leq 0$.

## Also see

[M-4] Scalar - Scalar mathematical functions

## Title

[M-5] factorial() — Factorial and gamma function
Description Syntax Conformability Diagnostics Also see

## Description

factorial ( $R$ ) returns the elementwise factorial of $R$.
$\operatorname{lnfactorial}(R)$ returns the elementwise $\ln (f \operatorname{actorial}(R))$, calculated differently. Very large values of $R$ may be evaluated.

Ingamma $(Z)$, for $Z$ real, returns the elementwise real result $\ln (\operatorname{abs}(\operatorname{gamma}(Z)))$, but calculated differently. Ingamma $(Z)$, for $Z$ complex, returns the elementwise $\ln$ (gamma $(Z)$ ), calculated differently. Thus, Ingamma $(-2.5)=-0.056244$, whereas Ingamma $(-2.5+0 \mathrm{i})=-0.056244+3.1416 \mathrm{i}$. In both cases, very large values of $Z$ may be evaluated.
gamma $(Z)$ returns $\exp (\operatorname{lngamma}(Z))$ for complex arguments and $\operatorname{Re}(\exp (\operatorname{lngamma}(C(Z)))$ ) for real arguments. Thus gamma() can correctly calculate, say, gamma ( -2.5 ) even for real arguments. digamma ( $R$ ) returns the derivative of Ingamma() for $R>0$, sometimes called the psi function. digamma() requires a real argument.
trigamma( $R$ ) returns the second derivative of Ingamma() for $R>0$. trigamma() requires a real argument.

## Syntax

| real matrix | factorial (real matrix $R$ ) |
| :--- | :--- |
| real matrix | lnfactorial (real matrix $R$ ) |
| numeric matrix | lngamma (numeric matrix $Z$ ) |
| numeric matrix | gamma(numeric matrix $Z$ ) |
| real matrix | digamma (real matrix $R$ ) |
| real matrix | trigamma (real matrix $R)$ |

## Conformability

All functions return a matrix of the same dimension as input, containing element-by-element calculated results.

## Diagnostics

factorial() returns missing for noninteger arguments, negative arguments, and arguments $>167$.
lnfactorial() returns missing for noninteger arguments, negative arguments, and arguments $>$ $1 \mathrm{e}+305$.

Ingamma() returns missing for 0 , negative integer arguments, negative arguments $\leq-2,147,483,648$, and arguments $>1 \mathrm{e}+305$.
gamma() returns missing for real arguments $>171$ and for negative integer arguments.
digamma() returns missing for 0 and negative integer arguments and for arguments $<-10,000,000$.
trigamma() returns missing for 0 and negative integer arguments and for arguments $<-10,000,000$.

## Also see

[M-4] Scalar - Scalar mathematical functions
[M-4] Statistical - Statistical functions

## Title

[M-5] favorspeed() - Whether speed or space is to be favored

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

favorspeed() returns 1 if the user has mata set matafavor speed and 0 if the user has mata set matafavor space or has not set matafavor at all; see [M-3] mata set.

## Syntax

```
real scalar favorspeed()
```


## Remarks and examples

Sometimes in programming you can choose between writing code that runs faster but consumes more memory or writing code that conserves memory at the cost of execution speed. favorspeed() tells you the user's preference:

```
if (favorspeed()) {
    /* code structured for speed over memory */
}
else {
    /* code structured for memory over speed */
}
```


## Conformability

favorspeed():
result: $\quad 1 \times 1$

## Diagnostics

None.

## Also see

[M-3] mata set - Set and display Mata system parameters
[M-4] Programming - Programming functions

## Title

[M-5] ferrortext( ) - Text and return code of file error code

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

ferrortext (ec) returns the text associated with a file error code returned by, for instance, _fopen(), _fwrite(), fstatus(), or any other file-processing functions that return an error code. See [M-5] fopen().
freturncode (ec) returns the Stata return code associated with the file error code.

## Syntax

```
string scalar ferrortext(real scalar ec)
real scalar freturncode(real scalar ec)
```


## Remarks and examples

Most file-processing functions abort with error if something goes wrong. You attempt to read a nonexisting file, or attempt to write a read-only file, and the file functions you use to do that, usually documented in [M-5] fopen(), abort with error. Abort with error means not only that the file function you called stops when the error arises but also that the calling function is aborted. The user is presented with a traceback log documenting what went wrong.

Sometimes you will want to write code to handle such errors for itself. For instance, you are writing a subroutine for a command to be used in Stata and, if the file does not exist, you want the subroutine to present an informative error message and exit without a traceback $\log$ but with a nonzero return code. Or in another application, if the file does not exist, that is not an error at all; your code will create the file.

Most file-processing functions have a corresponding underscore function that, rather than aborting, returns an error code when things go wrong. fopen() opens a file or aborts with error. $\quad$ fopen() opens a file or returns an error code. The error code is sufficient for your program to take the appropriate action. One uses the underscore functions when the calling program will deal with any errors that might arise.

Let's take the example of simply avoiding traceback logs. If you code

```
fh = fopen(filename, "r")
```

and the file does not exist, execution aborts and you see a traceback log. If you code

```
if ((fh = _fopen(filename, "r"))<0) {
    errprintf("%s\n", ferrortext(fh))
    exit(freturncode(fh))
}
```

execution still stops if the file does not exist, but this time, it stops because you coded exit (). You still see an error message, but this time, you see the message because you coded errprintf (). No traceback $\log$ is produced because you did not insert code to produce one. You could have coded _exit() if you wanted one.

The file error codes and the messages associated with them are

| Negative <br> code | Meaning |
| ---: | :--- |
| 0 | all is well |
| -1 | end of file (this code is usually not an error) |
| -601 | file not found |
| -602 | file already exists |
| -603 | file could not be opened |
| -608 | file is read-only |
| -610 | file not Stata format |
| -612 | unexpected end of file |
| -630 | web files not supported by this version of Stata |
| -631 | host not found |
| -632 | web file not allowed in this context |
| -633 | may not write to web file |
| -639 | file transmission error-checksums do not match |
| -660 | proxy host not found |
| -662 | proxy server refused request to send |
| -663 | remote connection to proxy failed |
| -665 | could not set socket nonblocking |
| -667 | wrong version of winsock.dll |
| -668 | could not find valid winsock. dll or astsys0.lib |
| -669 | invalid URL |
| -670 | invalid network port number |
| -671 | unknown network protocol |
| -672 | server refused to send file |
| -673 | authorization required by server |
| -674 | unexpected response from server |
| -675 | server reported error |
| -676 | server refused request to send |
| -677 | remote connection failed-see r (677) for troubleshooting |
| -678 | could not open local network socket |
| -679 | unexpected web error |

$$
\begin{array}{ll}
-680 & \text { could not find valid odbc32.dll } \\
-681 & \text { too many open files } \\
-682 & \text { could not connect to ODBC data source name } \\
-683 & \text { could not fetch variable in ODBC table } \\
-684 & \text { could not find valid dlxabi32.dll } \\
-691 & \text { I/O error } \\
-699 & \text { insufficient disk space } \\
-3601 & \text { invalid file handle } \\
-3602 & \text { invalid filename } \\
-3611 & \text { too many open files } \\
-3621 & \text { attempt to write read-only file } \\
-3622 & \text { attempt to read write-only file } \\
-3623 & \text { attempt to seek append-only file } \\
-3698 & \text { file seek error }
\end{array}
$$

File error codes are usually negative, but neither ferrortext(ec) nor freturncode(ec) cares whether $e c$ is of the positive or negative variety.

## Conformability

```
ferrortext(ec), freturncode(ec):
    ec: 1 }\times
    result: 1 }\times
```


## Diagnostics

ferrortext (ec) and freturncode(ec) treat $e c=-1$ (end of file) the same as $e c=612$ (unexpected end of file). Code -1 usually is not an error; it just denotes end of file. It is assumed that you will not call ferrortext() and freturncode() in such cases. If you do call the functions here, it is assumed that you mean that the end of file was unexpected.

## Also see

[M-5] fopen() - File I/O
[M-4] IO - I/O functions

## Title

[M-5] fft() — Fourier transform

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

$H=f f t(h)$ and $h=\operatorname{invfft}(H)$ calculate the Fourier transform and inverse Fourier transform. The length of $H$ and $h$ must be a power of 2 .
_fft (h) and _invfft $(H)$ do the same thing, but they perform the calculation in place, replacing the contents of $h$ and $H$.
convolve $(r, s)$ returns the convolution of the signal $s$ with the response function $r$. deconvolve ( $r$, $s m$ ) deconvolves the smeared signal $s m$ with the response function $r$ and is thus the inverse of convolve().
$\operatorname{Corr}(g, h, k)$ returns a $2 k+1$ element vector containing the correlations of $g$ and $h$ for lags and leads as large as $k$.
ftperiodogram $(H)$ returns a real vector containing the one-sided periodogram of $H$.
ftpad ( $h$ ) returns $h$ padded with 0 s to have a length that is a power of 2 .
ftwrap $(r, n)$ converts the symmetrically stored response function $r$ into wraparound format of length $n, n \geq \operatorname{rows}(r) * \operatorname{cols}(r)$ and rows $(r) * \operatorname{cols}(r)$ odd.
ftunwrap $(H)$ unwraps frequency-wraparound order as returned by fft(). You may find this useful when graphing or listing results, but it is otherwise unnecessary.
ftretime ( $r, s$ ) retimes the signal $s$ to be on the same time scale as convolve $(r, s)$. This is useful in graphing data and listing results but is otherwise not required.
ftfreqs ( $H$, delta) returns a vector containing the frequencies associated with the elements of $H$; delta is the sampling interval and is often specified as 1.

## Syntax

```
complex vector fft(numeric vector h)
numeric vector invfft(numeric vector H)
void _fft(complex vector h)
void _invfft(complex vector H)
numeric vector convolve(numeric vector r, numeric vector s)
numeric vector deconvolve(numeric vector r, numeric vector sm)
numeric vector Corr(numeric vector g, numeric vector h, real scalar k)
real vector ftperiodogram(numeric vector H)
numeric vector ftpad(numeric vector h)
numeric vector ftwrap(numeric vector r, real scalar n)
numeric vector ftunwrap(numeric vector H)
numeric vector ftretime(numeric vector r, numeric vector s)
real vector ftfreqs(numeric vector H, real scalar delta)
```


## Remarks and examples

Remarks are presented under the following headings:
Definitions, notation, and conventions
Fourier transform
Convolution and deconvolution
Correlation
Utility routines
Warnings

## Definitions, notation, and conventions

A signal $h$ is a row or column vector containing real or complex elements. The length of the signal is defined as the number of elements of the vector. It is occasionally necessary to pad a signal to a given length. This is done by forming a new vector equal to the original and with zeros added to the end.

The Fourier transform of a signal $h$, typically denoted by capital letter $H$ of $h$, is stored in frequencywraparound order. That is, if there are $n$ elements in $H$ :

```
H[1] frequency 0
H[2] frequency 1
H[3] frequency 2
H[n/2] frequency n/2-1
H[n/2+1] frequency }n/2(-n/2,\mathrm{ aliased)
H[n/2+2] frequency -(n/2-1)
H[n-1] frequency -2
H[n] frequency -1
```

All routines expect and use this order, but see ftunwrap() below.
A response function $r$ is a row or column vector containing $m=2 k+1$ real or complex elements. $m$ is called the duration of the response function. Response functions are generally stored symmetrically, although the response function itself need not be symmetric. The response vector contains

```
r[1] response at lag - }
r[2] response at lag -k+1
r[k] response at lag -1
r[k+1] contemporaneous response
r[k+2] response at lead 1
r[k+3] response at lead 2
r[2k+1] response at lead k
```

Response functions always have odd lengths. Response vectors are never padded.
You may occasionally find it convenient to store a response vector in "wraparound" order (similar to frequency-wraparound order), although none of the routines here require this. In wraparound order:

```
wrap[1] contemporaneous response
wrap[2] response at lead 1
wrap[3] response at lead 2
wrap[k+1] response at lead k
wrap[k+2] response at lag -k
wrap[k+3] response at lag -k+1
wrap[2k+1] response at lag -1
```

Response vectors stored in wraparound order may be internally padded (as opposed to merely padded) to a given length by the insertion of zeros between wrap $[k+1]$ and wrap $[k+2]$.

## Fourier transform

fft ( $h$ ) returns the discrete Fourier transform of $h . h$ may be either real or complex, but its length must be a power of 2 , so one typically codes $f f t(f t p a d(h))$; see $f t p a d()$, below. The returned result is p-conformable with $h$. The calculation is performed by _fft().
invfft $(H)$ returns the discrete inverse Fourier transform of $H . H$ may be either real or complex, but its length must be a power of 2 . The returned result is p-conformable with $H$. The calculation is performed by _invfft().
invfft $(H)$ may return a real or complex. This should be viewed as a feature, but if you wish to ensure the complex interpretation, code C(invfft $(H)$ ).
_fft (h) is the built-in procedure that performs the fast Fourier transform in place. $h$ must be complex, and its length must be a power of 2 .
_invfft $(H)$ is the built-in procedure that performs the inverse fast Fourier transform in place. $H$ must be complex, and its length must be a power of 2 .

## Convolution and deconvolution

convolve $(r, s)$ returns the convolution of the signal $s$ with the response function $r$. Calculation is performed by taking the $f f t()$ of the elements, multiplying, and using invfft() to transform the results back. Nevertheless, it is not necessary that the length of $s$ be a power of 2 . convolve() handles all paddings necessary, including paddings to $s$ required to prevent the result from being contaminated by erroneous wrapping around of $s$. Although one thinks of the convolution operator as being commutative, convolve() is not commutative since required zero-padding of the response and signal differ.

If $n$ is the length of the signal and $2 k+1$ is the length of the response function, the returned result has length $n+2 k$. The first $k$ elements are the convoluted signal before the true signal begins, and the last $k$ elements are the convoluted signal after the true signal ends. See ftretime(), below. In any case, you may be interested only in the elements convolve() [|k+1\n-k|], the part contemporaneous with $s$.

The returned vector is a row vector if $s$ is a row vector and a column vector otherwise. The result is guaranteed to be real if both $r$ and $s$ are real; the result may be complex or real, otherwise.

It is not required that the response function be shorter than the signal, although this will typically be the case.
deconvolve ( $r, s m$ ) deconvolves the smeared signal $s m$ with the response function $r$ and is thus the inverse of convolve(). In particular,

$$
\text { deconvolve }(r, \operatorname{convolve}(r, s))=s \quad \text { (up to roundoff error) }
$$

Everything said about convolve() applies equally to deconvolve().

## Correlation

Here we refer to correlation in the signal-processing sense, not the statistical sense.
$\operatorname{Corr}(g, h, k)$ returns a $2 k+1$ element vector containing the correlations of $g$ and $h$ for lags and leads as large as $k$. For instance, $\operatorname{Corr}(g, h, 2)$ returns a five-element vector, the first element of which contains the correlation for lag 2 , the second element lag 1 , the third (middle) element the contemporaneous correlation, the fourth element lead 1, and the fifth element lead 2. $k$ must be greater than or equal to 1 . The returned vector is a row or column vector depending on whether $g$ is a row or column vector. $g$ and $h$ must have the same number of elements but need not be p-conformable.

The result is obtained by padding with zeros to avoid contamination, taking the Fourier transform, multiplying $G \times \operatorname{conj}(H)$, and rearranging the inverse transformed result. Nevertheless, it is not required that the number of elements of $g$ and $h$ be powers of 2 because the program pads internally.

## Utility routines

ftpad ( $h$ ) returns $h$ padded with 0 s to have a length that is a power of 2 . For instance,

```
: h = (1,2,3,4,5)
: ftpad(h)
\begin{tabular}{l|llllllll|} 
& 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
\cline { 2 - 7 } & 1 & 2 & 3 & 4 & 5 & 0 & 0 & 0 \\
\hline
\end{tabular}
```

If $h$ is a row vector, a row vector is returned. If $h$ is a column vector, a column vector is returned.
ftwrap $(r, n)$ converts the symmetrically stored response function $r$ into wraparound format of length $n, n \geq \operatorname{rows}(r) * \operatorname{cols}(r)$ and rows $(r) * \operatorname{cols}(r)$ odd. A symmetrically stored response function is a vector of odd length, for example:

$$
(.1, .5,1, .2, .4)
$$

The middle element of the vector represents the response function at lag 0. Elements before the middle represent lags while elements after the middle represent leads. Here .1 is the response for lag 2 and .5 for lag 1,1 the contemporaneous response, .2 the response for lead 1 , and .4 the response for lead 2. The wraparound format of a response function records the response at times 0,1 , and so on in the first positions, has some number of zeros, and then records the most negative time value of the response function, and so on.

For instance,

```
: r
```



```
: ftwrap(r, 5)
    1
: ftwrap(r, 6)
    1
    1
    \begin{array} { | l l l l l l } { \hline 1 } & { . 2 } & { . 4 } & { 0 } & { . 1 } & { . 5 } \\ { \hline } \end{array}
: ftwrap(r, 8)
\begin{tabular}{|cccccccc|}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
\hline 1 & .2 & .4 & 0 & 0 & 0 & .1 & .5 \\
\hline
\end{tabular}
```

ftunwrap ( $H$ ) unwraps frequency-wraparound order such as returned by fft(). You may find this useful when graphing or listing results, but it is otherwise unnecessary. Frequency-unwrapped order is defined as

```
unwrap[1] frequency -(n/2) +1
unwrap[2] frequency }-(n/2)+
unwrap[n/2-1] frequency -1
unwrap[n/2] frequency 0
unwrap[n/2+1] frequency 1
unwrap[n-1] frequency n/2 - 1
unwrap[n] frequency }n/
```

Here we assume that $n$ is even, as will usually be true. The aliased (highest) frequency is assigned the positive sign.

Also see ftperiodogram(), below.
ftretime ( $r, s$ ) retimes the signal $s$ to be on the same time scale as convolve ( $r, s$ ). This is useful in graphing and listing results but is otherwise not required. ftretime () uses only the length of $r$, and not its contents, to perform the retiming. If the response vector is of length $2 k+1$, a vector containing $k$ zeros, $s$, and $k$ more zeros is returned. Thus the result of ftretime $(r, s)$ is p-conformable with convolve ( $r, s$ ).
ftfreqs $(H$, delta) returns a p-conformable-with- $H$ vector containing the frequencies associated with the elements of $H$. delta is the sampling interval and is often specified as 1 .
ftperiodogram $(H)$ returns a real vector of length $n / 2$ containing the one-sided periodogram of $H$ (length $n$ ), calculated as

$$
|H(f)|^{2}+|H(-f)|^{2}
$$

excluding frequency 0 . Thus ftperiodogram $(H)$ [1] corresponds to frequency $1(-1)$, ftperiodogram $(H)[2]$ to frequency $2(-2)$, and so on.

## Warnings

invfft $(H)$ will cast the result down to real if possible. Code $C(\operatorname{invfft}(H))$ if you want to be assured of the result being stored as complex.
convolve ( $r, s$ ) is not the same as convolve ( $s, r$ ).
convolve ( $r, s$ ) will cast the result down to real if possible. Code C ( $\operatorname{convolve}(r, s)$ ) if you want to be assured of the result being stored as complex.

For convolve $(r, s)$, the response function $r$ must have odd length.

## Conformability

fft (h):

$$
\begin{array}{rlll}
h: & 1 \times n & \text { or } & n \times 1, \\
\text { result: } & 1 \times n \text { a power of } 2 \\
& \text { or } & n \times 1
\end{array}
$$

invfft ( $H$ ):

$$
\begin{array}{rlll}
H: & 1 \times n & \text { or } & n \times 1, \\
\text { result: } & 1 \times n \text { a power of } 2 \\
& \text { or } & n \times 1
\end{array}
$$

_fft (h):

| $h:$ | $1 \times n$ |
| ---: | :--- |
| result: | void |

_invfft $(H)$ :

| $H:$ | $1 \times n$ |
| ---: | :--- |
| result: | void |

convolve ( $r, s$ ):

$$
\begin{array}{rllrl}
r: & 1 \times n & \text { or } & n \times 1, & n>0, n \text { odd } \\
s: & 1 \times 2 k+1 & \text { or } & 2 k+1 \times 1, & \text { i.e., } s \text { of odd length } \\
\text { result: } & 1 \times 2 k+n & \text { or } & 2 k+n \times 1 &
\end{array}
$$

deconvolve ( $r, s m$ ):

$$
\begin{array}{rllrr}
r: & 1 \times n & \text { or } & n \times 1, \quad n>0, n \text { odd } \\
\text { sm: } & 1 \times 2 k+n & \text { or } & 2 k+n \times 1 & \\
\text { result: } & 1 \times 2 k+1 & \text { or } & 2 k+1 \times 1 &
\end{array}
$$

$\operatorname{Corr}(g, h, k):$

| $g:$ | $1 \times n$ | or | $n \times 1$, | $n>0$ |
| ---: | :--- | :--- | ---: | :--- |
| $h:$ | $1 \times n$ | or | $n \times 1$ |  |
| $k:$ | $1 \times 1$ | or | $1 \times 1$, | $k>0$ |
| result: | $1 \times 2 k+1$ | or | $2 k+1 \times 1$ |  |

ftperiodogram ( $H$ ):

| $H:$ | $1 \times n$ | or | $n \times 1$, | $n$ even |
| ---: | :--- | :--- | ---: | :--- |
| result: | $n / 2 \times 1$ | or | $1 \times n / 2$ |  |

ftpad (h):

| $h:$ | $1 \times n$ | or | $n \times 1$ |
| ---: | :---: | :---: | :---: | :---: |
| result: | $1 \times N$ | or | $N \times 1, \quad N=n$ rounded up to power of 2 |

ftwrap $(r, n)$ :

| $r:$ | $1 \times m$ | or | $m \times 1$, | $m>0, m$ odd |
| ---: | :--- | :--- | :---: | :--- |
| $n:$ | $1 \times 1$ | or | $1 \times 1$, | $n \geq m$ |
| result $:$ | $1 \times n$ | or | $n \times 1$ |  |

ftunwrap ( $H$ ):

| $H:$ | $1 \times n$ | or | $n \times 1$ |
| ---: | :--- | :--- | :--- |
| result: | $1 \times n$ | or | $n \times 1$ |

ftretime ( $r, s$ ):

| $r:$ | $1 \times n$ | or | $n \times 1$, | $n>0, n$ odd |
| ---: | :--- | :--- | :--- | :--- |
| $s:$ | $1 \times 2 k+1$ | or | $2 k+1 \times 1$, | i.e., $s$ of odd length |
| result: | $1 \times 2 k+n$ | or | $2 k+n \times 1$ |  |

ftfreqs ( $H$, delta) :

| $H:$ | $1 \times n$ | or | $n \times 1, n$ even |  |
| ---: | :--- | :--- | :--- | :--- |
| delta: | $1 \times 1$ |  |  |  |
| result: | $1 \times n$ | or | $n \times 1$ |  |

## Diagnostics

All functions abort with error if the conformability requirements are not met. This is always true, of course, but pay particular attention to the requirements outlined under Conformability directly above.
fft( $h$ ), _fft( $h$ ), invfft( $H$ ), _invfft $(H)$, convolve $(r, s)$, deconvolve( $r, s m$ ), and $\operatorname{Corr}(g, h, k)$ return missing results if any argument contains missing values.
ftwrap $(r, n)$ aborts with error if $n$ contains missing value.

## Also see

[M-4] Mathematical - Important mathematical functions

## Title

[M-5] fileexists( ) - Whether file exists

Description Syntax Conformability Diagnostics Also see

## Description

fileexists ( $f n$ ) returns 1 if file $f n$ exists and is readable and returns 0 otherwise.

## Syntax

real scalar fileexists(string scalar fn)

## Conformability

```
fileexists(fn):
        fn: 
    result: }1\times
```


## Diagnostics

None.

## Also see

[M-4] IO $-\mathrm{I} / \mathrm{O}$ functions

## Title

[M-5] _fillmissing() — Fill matrix with missing values

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

_fillmissing (transmorphic matrix $A$ ) changes the contents of $A$ to missing values.

## Syntax

void _fillmissing(transmorphic matrix A)

## Remarks and examples

The definition of missing depends on the storage type of $A$ :

| Storage type | Contents |
| :--- | :--- |
| real | $\dot{C}()$. |
| complex | "" |
| string | NULL |
| pointer |  |

## Conformability

_fillmissing $(A)$ :
input:
A: $\quad r \times c$
output:
A: $\quad r \times c$

## Diagnostics

None.

## Also see

[M-4] Manipulation - Matrix manipulation

## Title

[M-5] findexternal( ) - Find, create, and remove external globals

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

findexternal (name) returns a pointer (see [M-2] pointers) to the external global matrix, vector, or scalar whose name is specified by name, or to the external global function if the contents of name end in (). findexternal() returns NULL if the external global is not found.
crexternal (name) creates a new external global $0 \times 0$ real matrix with the specified name and returns a pointer to it; it returns NULL if an external global of that name already exists.
rmexternal (name) removes (deletes) the specified external global or does nothing if no such external global exists.
nameexternal ( $p$ ) returns the name of $* p$.

## Syntax

| pointer () | scalar | findexternal(string scalar name) |
| :--- | :--- | :--- |
| pointer () | scalar | crexternal(string scalar name) |
| void |  | rmexternal(string scalar name) |
| string | scalar | nameexternal(pointer() scalar p) |

## Remarks and examples

Remarks are presented under the following headings:

Also see Linking to external globals in [M-2] Declarations.

## Definition of a global

When you use Mata interactively, any variables you create are known, equivalently, as externals, globals, or external globals.

```
: myvar = x
```

Such variables can be used by subsequent functions that you run, and there are two ways that can happen:

```
function example1(...)
{
    external real myvar
    ... myvar ...
}
```

and

```
function example2(...)
{
    pointer(real) p
    p = findexternal("myvar")
    ... *p ...
}
```

Using the first method, you must know the name of the global at the time you write the source code, and when you run your program, if the global does not exist, it will refuse to run (abort with myvar not found). With the second method, the name of the global can be specified at run time and what is to happen when the global is not found is up to you.

In the second example, although we declared $p$ as a pointer to a real, myvar will not be required to contain a real. After $p=$ findexternal ("myvar"), if $p!=$ NULL, $p$ will point to whatever myvar contains, whether it be real, complex, string, or another pointer. (You can diagnose the contents of *p using eltype (*p) and orgtype (*p) ; see [M-5] eltype().)

## Use of globals

Globals are useful when a function must remember something from one call to the next:

```
function example3(real scalar x)
{
    pointer() scalar p
    if ( (p = findexternal("myprivatevar")) == NULL) {
        printf("you haven't called me previously")
        p = crexternal("myprivatevar")
    }
    else {
        printf("last time, you said "%g", *p)
    }
    *p = x
}
: example3(2)
you haven't called me previously
: example3(31)
last time, you said 2
: example3(9)
last time, you said 31
```

Note our use of the name myprivatevar. It actually is not a private variable; it is global, and you would see the variable listed if you described the contents of Mata's memory. Because global variables are so exposed, it is best that you give them long and unlikely names.

In general, programs do not need global variables. The exception is when a program must remember something from one invocation to the next, and especially if that something must be remembered from one invocation of Mata to the next.

When you do need globals, you probably will have more than one thing you will need to recall. There are two ways to proceed. One way is simply to create separate global variables for each thing you need to remember. The other way is to create one global pointer vector and store everything in that. In the following example, we remember one scalar and one matrix:

```
function example4()
{
    pointer(pointer() vector) scalar p
    scalar s
    real matrix X
    pointer() scalar ps, pX
    if ( (p = findexternal("mycollection")) == NULL) {
        ... calculate scalar s and X from nothing ...
        ... and save them:
        p = crexternal("mycollection")
        *p = (&s, &X)
    }
    else {
        ps = (*p)[1]
        pX = (*p)[2]
        ... calculate using *ps and *pX ...
    }
}
```

In the above example, even though crexternal() created a $0 \times 0$ real global, we morphed it into a $1 \times 2$ pointer vector:

```
p = crexternal("mycollection") *p is 0 }\times0\mathrm{ real
*p = (&s, &X) *p is 1 }\times2\mathrm{ vector
```

just as we could with any nonpointer object.
In the else part of our program, where we use the previous values, we do not use variables $s$ and $X$, but ps and pX . Actually, we did not really need them, we could just as well have used $*((* \mathrm{p})[1])$ and $*((* \mathrm{p})[2])$, but the code is more easily understood by introducing $* \mathrm{ps}$ and $* \mathrm{pX}$.

Actually, we could have used the variables s and X by changing the else part of our program to read

```
else {
    s = *(*p) [1]
    X = *(*p) [2]
    ... calculate using s and X ...
    *p = (&s, &X) 
}
```

Doing that is inefficient because $s$ and $X$ contain copies of the global values. Obviously, the amount of inefficiency depends on the sizes of the elements being copied. For s, there is really no inefficiency at all because $s$ is just a scalar. For X , the amount of inefficiency depends on the dimensions of X . Making a copy of a small X matrix would introduce just a little inefficiency.

The best balance between efficiency and readability is achieved by introducing a subroutine:

```
function example5()
{
    pointer(pointer() vector) scalar p
    scalar s
    real matrix X
    if ( (p = findexternal("mycollection")) == NULL) {
        example5_sub(1, s=., X=J (0,0,.))
        p = crexternal("mycollection")
        *p = (&s, &X)
    }
    else {
        example5_sub(0, (*p)[1], (*p)[2])
    }
}
function example5_sub(scalar firstcall, scalar x, matrix X)
{
}
```

The last two lines in the not-found case

```
p = crexternal("mycollection")
*p = (&s, &X)
```

could also be coded

```
*crexternal("mycollection") = (&s, &X)
```


## Conformability

```
findexternal(name), crexternal(name):
    name: }1\times
    result: }1\times
rmexternal(name):
    name: 
    result: void
nameexternal(p):
    p: 
```


## Diagnostics

findexternal(name), crexternal(name), and rmexternal(name) abort with error if name contains an invalid name.
findexternal (name) returns NULL if name does not exist.
crexternal (name) returns NULL if name already exists.
nameexternal ( $p$ ) returns "" if $p=$ NULL. Also, nameexternal() may be used not just with pointers to globals but pointers to locals as well. For example, you can code nameexternal (\&myx), where myx is declared in the same program or a calling program. nameexternal() will usually return the expected local name, such as "myx". In such cases, however, it is also possible that "" will be returned. That can occur because, in the compilation/optimization process, the identity of local variables can be lost.

## Also see

[M-5] valofexternal() - Obtain value of external global
[M-4] Programming - Programming functions

## Title

[M-5] findfile() — Find file

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

findfile ( $f n$, dirlist) looks for file $f n$ along the semicolon-separated list of directories dirlist and returns the fully qualified path and filename if $f n$ is found. findfile() returns " " if the file is not found.
findfile(fn) is equivalent to findfile(fn, c("adopath")). findfile() with one argument looks along the official Stata ado-path for file $f n$.

## Syntax

string scalar findfile (string scalar fn, string scalar dirlist)
string scalar findfile(string scalar fn)

## Remarks and examples

For instance,

```
findfile("kappa.ado")
```

might return C: \Program Files $\backslash$ Stata18\ado\base $\backslash k \backslash k a p p a . a d o . ~$

## Conformability

```
findfile(fn, dirlist):
        fn: 
    dirlist: }1\times1\quad\mathrm{ (optional)
    result: }1\times
```


## Diagnostics

findfile(fn, dirlist) and findfile(fn) return "" if the file is not found. If the file is found, the returned fully qualified path and filename is guaranteed to exist and be readable at the instant findfile() concluded.

## Also see

[M-4] IO - I/O functions

## Title

[M-5] floatround() - Round to float precision

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

floatround ( $x$ ) returns $x$ rounded to IEEE 4-byte real (float) precision. floatround() is the element-by-element equivalent of Stata's float() function. The Mata function could not be named float() because the word float is reserved in Mata.

## Syntax

```
real matrix floatround(real matrix x)
```


## Remarks and examples

```
: printf(" %21x\n", .1)
    +1.999999999999aX-004
: printf(" %21x\n", floatround(.1))
    +1.99999a0000000X-004
```


## Conformability

floatround $(x)$ :

| $x:$ | $r \times c$ |
| ---: | :--- |
| result: | $r \times c$ |

## Diagnostics

floatround ( $x$ ) returns missing (.) if $x<-1 . \mathrm{fffffe} \mathrm{X}+7 \mathrm{e}$ (approximately $-1.70141173319 \mathrm{e}+38$ ) or $x>1$.fffffeX+7e (approximately $1.70141173319 \mathrm{e}+38$ ).

In contrast with most functions, floatround $(x)$ returns the same kind of missing value as $x$ if $x$ contains missing; . if $x==$., a if $x==$.a, .b if $x==. \mathrm{b}, \ldots$, and . z if $x==$. z .

## Also see

[M-4] Utility - Matrix utility functions

## Title

[M-5] fmtwidth() — Width of \%fmt

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

fmtwidth $(f)$ returns the width of the $\% f m t$ contained in $f$.

## Syntax

real matrix fmtwidth (string matrix $f$ )

## Remarks and examples

fmtwidth("\%9.2f") returns 9.
fmtwidth("\%20s") returns 20.
fmtwidth("\%tc") returns 18.
fmtwidth("\%tcDay_Mon_DD_hh:mm:ss_!C!D!T_CCYY") returns 28.
fmtwidth("not a format") returns . (missing).

## Conformability

```
fmtwidth(f):
    f: r
    result: }\quadr\times
```


## Diagnostics

fmtwidth $(f)$ returns . (missing) when $f$ does not contain a valid Stata format.

## Also see

[M-5] strlen( ) - Length of string in bytes
[M-4] String - String manipulation functions

## Title

[M-5] fopen() — File I/O

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

These functions read and write files. First, open the file and get back a file handle (fh). The file handle, which is nothing more than an integer, is how you refer to the file in the calls to other file I/O functions. When you are finished, close the file.

Most file I/O functions come in two varieties: without and with an underscore in front of the name, such as fopen() and _fopen(), and fwrite() and _fwrite().

In functions without a leading underscore, errors cause execution to be aborted. For instance, you attempt to open a file for read and the file does not exist. Execution stops. Or, having successfully opened a file, you attempt to write into it and the disk is full. Execution stops. When execution stops, the appropriate error message is presented.

In functions with the leading underscore, execution continues and no error message is displayed; it is your responsibility (1) to verify that things went well and (2) to take the appropriate action if they did not. Concerning (1), some underscore functions return a status value; others require that you call fstatus () to obtain the status information.

You can mix and match use of underscore and nonunderscore functions, using, say, _fopen() to open a file and fread() to read it, or fopen() to open and fiwrite() to write.

## Syntax

```
real scalar fopen(string scalar fn, mode)
real scalar fopen(string scalar fn, mode, public)
real scalar _fopen(string scalar fn, mode)
real scalar _fopen(string scalar fn, mode, public)
```

where
mode: string scalar containing "r", "w", "rw", or "a"
public: optional real scalar containing zero or nonzero

In what follows, $f$ f is the value returned by fopen() or _fopen():

| void | fclose $(f h)$ |
| :--- | :--- |
| real scalar | _fclose $(f h)$ |
|  |  |
| string scalar | fget $(f h)$ |
| string scalar | _fget $(f h)$ |
| string scalar | fgetnl $(f h)$ |
| string scalar | _fgetnl $(f h)$ |

string scalar fread (fh, real scalar len)
string scalar _fread $(f h$, real scalar len)
void fput (fh, string scalar $s$ )
real scalar _fput(fh, string scalar s)
void $\quad$ fwrite (fh, string scalar $s$ )
real scalar _fwrite (fh, string scalar s)
matrix fgetmatrix (fh[, real scalar isstrict $]$ )
matrix _fgetmatrix (fh[, real scalar isstrict $]$ )
void fputmatrix ( $f$ f, transmorphic matrix $X$ )
real scalar _fputmatrix (fh, transmorphic matrix $X$ )
real scalar fstatus( $f$ )
real scalar ftell(fh)
real scalar _ftell(fh)
void fseek(fh, real scalar offset, real scalar whence)
real scalar _fseek(fh, real scalar offset, real scalar whence)
(whence is coded $-1,0$, or 1 , meaning from start of file, from current position, or from end of file; offset is signed: positive values mean after whence and negative values mean before)
void ftruncate (fh)
real scalar _ftruncate(fh)

## Remarks and examples

Remarks are presented under the following headings:

```
Opening and closing files
Reading from a file
Writing to a file
Reading and writing in the same file
Reading and writing matrices
Repositioning in a file
Truncating a file
Error codes
```


## Opening and closing files

Functions
fopen(string scalar fn, string scalar mode)
_fopen (string scalar fn, string scalar mode)
fopen(string scalar fn, string scalar mode, real scalar public)
_fopen(string scalar fn, string scalar mode, real scalar public)
open a file. The file may be on a local disk, a network disk, or even on the web (such as https://www.stata.com/index.html). fn specifies the filename, and mode specifies how the file is to opened:
mode Meaning
" r " Open for reading; file must exist and be readable.
File may be "https://..." file.
File will be positioned at the beginning.
"w" Open for writing; file must not exist and the directory be writable.
File may not be "https://..." file.
File will be positioned at the beginning.
"rw" Open for reading and writing; file must either exist and be writable or not exist and directory be writable.
File may not be "https://..." file.
File will be positioned at the beginning (new file) or at the end (existing file).
"a" Open for appending; file must either exist and be writable or not exist and directory be writable.
File may not be "https://..." file.
File will be positioned at the end.
Other values for mode cause fopen() and _fopen() to abort with an invalid-mode error.
Optional third argument public specifies whether the file, if it is being created, should be given permissions so that everyone can read it, or if it instead should be given the normal permissions. Not specifying public, or specifying public as 0, gives the file the normal permissions. Specifying public as nonzero makes the file publicly readable. public is relevant only when the file is being created, that is, is being opened "w", or being opened "rw" and not previously existing.
fopen() returns a file handle; the file is opened or execution is aborted.
_fopen() returns a file handle or returns a negative number. If a negative number is returned, the file is not open, and the number indicates the reason. For _fopen(), there are a few likely possibilities

| Negative <br> value | Meaning |
| :--- | :--- |
| -601 | file not found |
| -602 | file already exists |
| -603 | file could not be opened |
| -608 | file is read-only |
| -691 | I/O error |

and there are many other possibilities. For instance, perhaps you attempted to open a file on the web (say, http://www.newurl.org/upinfo.doc) and the URL was not found, or the server refused to send back the file, etc. See Error codes below for a complete list of codes.

After opening the file, you use the other file I/O commands to read and write it, and then you close the file with fclose() or _fclose(). fclose() returns nothing; if the file cannot be closed, execution is aborted. _fclose returns 0 if successful, or a negative number otherwise. For $\quad$ fclose(), the likely possibilities are

| Negative <br> value | Meaning |
| :--- | :--- |
| -691 | filesystem I/O error |

## Reading from a file

You may read from a file opened " r " or "rw". The commands to read are

```
fget(fh)
fgetnl(fh)
fread(fh, real scalar len)
```

and, of course,

```
_fget(fh)
_fgetnl(fh)
_fread(fh, real scalar len)
```

All functions, with or without an underscore, require a file handle be specified, and all the functions return a string scalar or they return $J(0,0, " ")$, a $0 \times 0$ string matrix. They return $J(0,0, " ")$ on end of file and, for the underscore functions, when the read was not successful for other reasons. When using the underscore functions, you use fstatus () to obtain the status code; see Error codes below. The underscore read functions are rarely used because the only reason a read can fail is I/O
error, and there is not much that can be done about that except abort, which is exactly what the nonunderscore functions do.
fget ( fh ) is for reading text files; the next line from the file is returned, without end-of-line characters. (If the line is longer then 32,768 characters, the first 32,768 characters are returned.)
fgetnl( fh ) is much the same as fget(), except that the new-line characters are not removed from the returned result. (If the line is longer then 32,768 characters, the first 32,768 characters are returned.)
fread ( $f$ h, len) is usually used for reading binary files and returns the next len characters (bytes) from the file or, if there are fewer than that remaining to be read, however many remain. (len may not exceed $9,007,199,254,740,991$ [sic] on 64-bit computers; memory shortage for storing the result will arise long before this limit is reached on most computers.)

The following code reads and displays a file:

```
fh = fopen(filename, "r")
while ((line=fget(fh))!=J(0,0,"")) {
    printf("%s\n", line)
}
fclose(fh)
```


## Writing to a file

You may write to a file opened "w", "rw", or "a". The functions are

```
fput(fh, string scalar s)
fwrite(fh, string scalar s)
```

and, of course,

```
_fput(fh, string scalar s)
_fwrite(fh, string scalar s)
```

$f h$ specifies the file handle, and $s$ specifies the string to be written. fput() writes $s$ followed by the new-line characters appropriate for your operating system. fwrite() writes $s$ alone.
fput() and fwrite() return nothing; _fput() and _fwrite() return a real scalar equal to 0 if all went well or a negative error code; see Error codes below.

The following code copies text from one file to another:

```
fh_in = fopen(inputname, "r")
fh_out = fopen(outputname, "w")
while ((line=fget(fh_in))!=J(0,0,"")) {
        fput(fh_out, line)
}
fclose(fh_out)
fclose(fh_in)
```

The following code reads a file (binary or text) and changes every occurrence of "a" to "b":

```
fh_in = fopen(inputname, "r")
fh_out = fopen(outputname, "w")
while ((c=fread(fh_in, 1))!=J(0,0,"")) {
    fwrite(fh_out, (c=="a" ? "b" : c))
}
fclose(fh_out)
fclose(fh_in)
```


## Reading and writing in the same file

You may read and write from a file opened "rw", using any of the read or write functions described above. When reading and writing in the same file, one often uses file repositioning functions, too; see Repositioning in a file below.

## Reading and writing matrices

## Functions

fputmatrix (fh, transmorphic matrix $X$ )
and

$$
\text { _fputmatrix }(f h, \text { transmorphic matrix } X)
$$

will write a matrix to a file. In the usual fashion, fputmatrix () returns nothing (it aborts if there is an I/O error) and _fputmatrix() returns a scalar equal to 0 if all went well and a negative error code otherwise.

Functions

$$
\text { fgetmatrix }(f h[, \text { real scalar isstrict }])
$$

and

$$
\text { _fgetmatrix }(f h[, \text { real scalar isstrict }])
$$

will read a matrix written by fputmatrix() or _fputmatrix(). Both functions return the matrix read or return $J(0,0,$.$) on end of file (both functions) or error (_fgetmatrix() only). Because$ $J(0,0,$.$) could be the matrix that was written, distinguishing between that and end of file requires$ subsequent use of fstatus(). fstatus() will return 0 if fgetmatrix() or _fgetmatrix() returned a written matrix, -1 if end of file, or (after _fgetmatrix()) a negative error code.

For a Mata struct or class matrix, a matrix according to the current definition will be created, and the saved matrix will be used to initialize the new matrix based on member-name matching.

Optional argument isstrict affects the behavior of the functions if the matrix being read is a Mata struct or class matrix. When the argument is set and not zero, the current struct or class definition in memory will be checked against the saved matrix to ensure that all variable names, variable eltypes, and variable orgtypes match each other.
fputmatrix() writes matrices in a compact, efficient, and portable format; a matrix written on a Windows computer can be read back on a Mac or Unix computer and vice versa.

The following code creates a file containing three matrices,

```
fh = fopen(filename, "w")
fputmatrix(fh, a)
fputmatrix(fh, b)
fputmatrix(fh, c)
fclose(fh)
```

and the following code reads them back:

```
fh = fopen(filename, "r")
a = fgetmatrix(fh)
b = fgetmatrix(fh)
c = fgetmatrix(fh)
fclose(fh)
```

The following code saves a Mata class scalar to file,

```
class sA {
    real scalar r
    string scalar s
    static scalar sr
    void new()
}
a = sA()
a.r = 1
a.s = "sA instance"
fh = fopen(filename, "w")
fputmatrix(fh, a)
fclose(fh)
```

and the following code reads the class matrix back:

```
fh = fopen(filename, "r")
a = fgetmatrix(fh)
fclose(fh)
```

The contents of a depends on the current definition of class sA in memory. If the definition does not change, a.r will be 1 and a.s will be "sA instance". Note: Only regular variables are saved and read back; static variables and functions are not saved. Also, the new () function will not be called in the created class scalar. If the class definition has been changed,

```
class sA {
    real scalar r
    real scalar b
    static scalar sr
    void new()
}
```

the function will not read the matrix if optional argument isstrict is specified and not zero. Otherwise, a class sA scalar a according to the current definition will be created. (Note: new () will not be called.) Member variables with matching names and compatible eltypes and orgtypes will be initialized using the values in the saved matrix. In this example, a.r will be 1 , and $\mathrm{a} . \mathrm{b}$ will be missing because b is not a member in the class sA definition when it was saved.

- Technical note

You may even write pointer matrices

```
mats = (&a, &b, &c, NULL)
fh = fopen(filename,"w")
fputmatrix(fh, mats)
fclose(fh)
```

and read them back:

```
fh = fopen(filename, "r")
mats = fgetmatrix(fh)
fclose(fh)
```

When writing pointer matrices, fputmatrix() writes NULL for any pointer-to-function value. It is also recommended that you do not write self-referential matrices (matrices that point to themselves, either directly or indirectly), although the elements of the matrix may be cross linked and even recursive themselves. If you are writing pointer matrix p , no element of $\mathrm{p}, * \mathrm{p}, * * \mathrm{p}$, etc., should contain \&p. That one address cannot be preserved because in the assignment associated with reading back the matrix (the "result $=$ " part of result $=$ fgetmatrix $(f h)$ ), a new matrix with a different address is associated with the contents.

## Repositioning in a file

The function

```
ftell(fh)
```

returns a real scalar reporting where you are in a file, and function
fseek (fh, real scalar offset, real scalar whence)
changes where you are in the file to be offset bytes from the beginning of the file (whence $=-1$ ), offset bytes from the current position (whence $=0$ ), or offset bytes from the end of the file (whence $=1$ ).

Functions _ftell() and _fseek() do the same thing as ftell() and fseek(), the difference being that, rather than aborting on error, the underscore functions return negative error codes. $f$ ftell () is pretty well useless as the only error that can arise is I/O error, and what else are you going to do other than abort? _fseek(), however, has a use, because it allows you to try out a repositioning and check whether it was successful. With fseek(), if the repositioning is not successful, execution is aborted.

Say you open a file for read:

```
fh = fopen(filename, "r")
```

After opening the file in mode $r$, you are positioned at the beginning of the file or, in the jargon of file processing, at position 0 . Now say that you read 10 bytes from the file:

```
part1 = fread(fh, 10)
```

Assuming that was successful, you are now at position 10 of the file. Say that you next read a line from the file

$$
\text { line }=\text { fget }(f h)
$$

and assume that fget() returns "abc". You are now at position 14 or 15. (No, not 13: fget () read the line and the new-line characters and returned the line. abc was followed by carriage return and line feed (two characters) if the file was written under Windows and by a carriage return or line feed alone (one character) if the file was written under Mac or Unix).
ftell ( $f h$ ) and _ftell ( $f h$ ) tell you where you are in the file. Coding

```
pos = ftell(fh)
```

would store 14 or 15 in pos. Later in your code, after reading more of the file, you could return to this position by coding

```
fseek(fh, pos, -1)
```

You could return to the beginning of the file by coding

$$
\text { fseek }(\mathrm{fh}, 0,-1)
$$

and you could move to the end of the file by coding

```
fseek(fh, 0, 1)
```

_fseek $(f h, 0,0)$ is equivalent to $\mathrm{ftell}(f h)$.
Repositioning functions cannot be used when the file has been opened "a".

## Truncating a file

Truncation refers to making a longer file shorter. If a file was opened "w" or "rw", you may truncate it at its current position by using

```
ftruncate(fh)
```

or

```
_ftruncate(fh)
```

ftruncate() returns nothing; _ftruncate() returns 0 on success and otherwise returns a negative error code.

The following code shortens a file to its first 100 bytes:

```
fh = fopen(filename, "rw")
fseek(fh, 100, -1)
ftruncate(fh)
fclose(fh)
```


## Error codes

If you use the underscore I/O functions, if there is an error, they will return a negative code. Those codes are

| Negative code | Meaning |
| ---: | :--- |
| 0 | all is well |
| -1 | end of file |
| -601 | connection timed out |
| -602 | file not found |
| -603 | file already exists |
| -608 | file is read-only opened |
| -610 | file format error |
| -612 | unexpected end of file |
| -630 | web files not supported in this version of Stata |
| -631 | host not found |
| -632 | web file not allowed in this context |
| -633 | may not write to web file |
| -660 | proxy host not found |
| -661 | host or file not found |
| -662 | proxy server refused request to send |
| -663 | remote connection to proxy failed |
| -665 | could not set socket nonblocking |
| -669 | invalid URL |
| -670 | invalid network port number |
| -671 | unknown network protocol |
| -672 | server refused to send file |
| -673 | authorization required by server |
| -674 | unexpected response from server |
| -675 | server reported server error |
| -676 | server refused request to send |
| -677 | remote connection failed |
| -678 | could not open local network socket |
| -679 | unexpected web error |
| -691 | I/O error |
| -699 | insufficient disk space |
| -3601 | invalid file handle |
| -3602 | invalid filename |
| -3603 | invalid file mode |
| -3611 | too many open files |
| -3621 | attempt to write read-only file |
| -3622 | attempt to read write-only file |
| -3623 | attempt to seek append-only file |
| file seek error |  |

Other codes in the -600 to -699 range are possible. The codes in this range correspond to the negative of the corresponding Stata return code; see $[\mathrm{P}]$ error.

Underscore functions that return a real scalar will return one of these codes if there is an error.
If an underscore function does not return a real scalar, then you obtain the outcome status using fstatus(). For instance, the read-string functions return a string scalar or $J(0,0, " ")$ on end of file. The underscore variants do the same, and they also return $\mathrm{J}(0,0, " ")$ on error, meaning error looks like end of file. You can determine the error code using the function

## fstatus(fh)

fstatus() returns 0 (no previous error) or one of the negative codes above.
fstatus() may be used after any underscore I/O command to obtain the current error status.
fstatus() may also be used after the nonunderscore I/O commands; then fstatus() will return -1 or 0 because all other problems would have stopped execution.

## Conformability

fopen( $f n$, mode, public), _fopen(fn, mode, public):

| fn: | $1 \times 1$ |  |
| ---: | :--- | :--- | :--- |
| mode: | $1 \times 1$ |  |
| public: | $1 \times 1$ | (optional) |
| result: | $1 \times 1$ |  |

## fclose(fh):

$$
\begin{aligned}
\text { fh: } & 1 \times 1 \\
\text { result: } & \text { void }
\end{aligned}
$$

_fclose (fh):

| fh: |  | $1 \times 1$ |
| ---: | :--- | ---: |
| result: |  | $1 \times 1$ |

fget (fh), _fget (fh), fgetnl(fh), _fgetnl(fh):
$f h: \quad 1 \times 1$
result: $1 \times 1$ or $0 \times 0$ if end of file
fread (fh, len), _fread (fh, len):
fh: $\quad 1 \times 1$
len: $\quad 1 \times 1$
result: $1 \times 1$ or $0 \times 0$ if end of file
fput ( $f$ fh, $s$ ), fwrite ( $f$ h, $s$ ):
fh: $\quad 1 \times 1$
$s: \quad 1 \times 1$
result: void
_fput ( $f$ h, $s$ ), _fwrite $(f h, s)$ :
fh: $\quad 1 \times 1$
$s: \quad 1 \times 1$
result: $\quad 1 \times 1$
fgetmatrix ( $f$ h), _fgetmatrix $(f h)$ :
fh: $\quad 1 \times 1$
result: $\quad r \times c$ or $0 \times 0$ if end of file

```
fputmatrix(fh, X):
    fh: 
    X: }\quadr\times
result: void
_fputmatrix(fh, X):
    fh: 
    result: }1\times
fstatus(fh):
\begin{tabular}{rlr} 
fh: & & \(1 \times 1\) \\
result: & & \(1 \times 1\)
\end{tabular}
ftell(fh), _ftell(fh):
            fh: }\quad1\times
        result: }\quad1\times
fseek(fh, offset, whence):
            fh: 
        offset: }\quad1\times
        whence: }1\times
        result: void
_fseek(fh, offset, whence):
\begin{tabular}{rr} 
fh: & \(1 \times 1\) \\
offset: & \(1 \times 1\) \\
whence: & \(1 \times 1\) \\
result: & \(1 \times 1\)
\end{tabular}
ftruncate(fh):
\begin{tabular}{rl} 
fh: & \\
result: & \\
void
\end{tabular}
_ftruncate(fh):
\begin{tabular}{rl} 
fh: & \(1 \times 1\) \\
result: & \(1 \times 1\)
\end{tabular}
```


## Diagnostics

fopen ( $f n$, mode) aborts with error if mode is invalid or if $f n$ cannot be opened or if an attempt is made to open too many files simultaneously.
_fopen ( $f n$, mode) aborts with error if mode is invalid or if an attempt is made to open too many files simultaneously. _fopen() returns the appropriate negative error code if fn cannot be opened.

All remaining I/O functions-even functions with leading underscore-abort with error if $f h$ is not a handle to a currently open file.

Also, the functions that do not begin with an underscore abort with error when a file was opened read-only and a request requiring write access is made, when a file is opened write-only and a request requiring read access is made, etc. See Error codes above; all problems except code -1 (end of file) cause the nonunderscore functions to abort with error.

Finally, the following functions will also abort with error for the following specific reasons:
fseek(fh, offset, whence) and _fseek(fh, offset, whence) abort with error if offset is outside the range $\pm 9,007,199,254,740,991$ on 64 -bit computers or if whence is not $-1,0$, or 1 .

## Also see

[M-5] bufio() — Buffered (binary) I/O
[M-5] cat() - Load file into string matrix sprintf() in
[M-5] printf() - Format output
[M-4] IO - I/O functions

## Title

[M-5] fullsvd() — Full singular value decomposition

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

fullsvd $(A, U, s, V t)$ calculates the singular value decomposition of $m \times n$ matrix $A$, returning the result in $U, s$, and $V t$. Singular values in $s$ are sorted from largest to smallest.
fullsdiag ( $s, k$ ) converts column vector $s$ returned by fullsvd () into matrix $S$. In all cases, the appropriate call for this function is

$$
S=\operatorname{full} \operatorname{sdiag}(s, \operatorname{rows}(A)-\operatorname{cols}(A))
$$

_fullsvd $(A, U, s, V t)$ does the same as fullsvd(), except that, in the process, it destroys $A$. Use of _fullsvd() in place of fullsvd() conserves memory.
_svd_la() is the interface to the LAPACK SVD routines and is used in the implementation of the previous functions. There is no reason you should want to use it. _svd_la() is similar to _fullsvd(). It differs in that it returns a real scalar equal to 1 if the numerical routines fail to converge, and it returns 0 otherwise. The previous SVD routines set $s$ to contain missing values in this unlikely case.

## Syntax

| void | fullsvd(numeric matrix $A, U, s, V t)$ |
| :--- | :--- |
| numeric matrix | fullsdiag(numeric colvector $s$, real scalar $k$ ) |
| void | _fullsvd(numeric matrix $A, U, s, V t)$ |
| real scalar | _svd_la(numeric matrix $A, U, s, V t)$ |

## Remarks and examples

Remarks are presented under the following headings:

Introduction<br>Relationship between the full and thin SVDs<br>The contents of $s$<br>Possibility of convergence problems

Documented here is the full SVD, appropriate in all cases, but of interest mainly when $A$ : $m \times n$, $m<n$. There is a thin SVD that conserves memory when $m \geq n$; see $[\mathrm{M}-5] \operatorname{svd}()$. The relationship between the two is discussed in Relationship between the full and thin SVDs below.

## Introduction

The SVD is used to compute accurate solutions to linear systems and least-squares problems, to compute the 2 -norm, and to determine the numerical rank of a matrix.

The singular value decomposition (SVD) of $A: m \times n$ is given by

$$
A=U S V^{\prime}
$$

where

$$
\begin{array}{cl}
U: & m \times m \text { and orthogonal (unitary) } \\
S: & m \times n \text { and diagonal } \\
V: & n \times n \text { and orthogonal (unitary) }
\end{array}
$$

When $A$ is complex, the transpose operator ' is understood to mean the conjugate transpose operator.
Diagonal matrix $S$ contains the singular values and those singular values are real even when $A$ is complex. It is usual (but not required) that $S$ is arranged so that the largest singular value appears first, then the next largest, and so on. The SVD routines documented here do this.

The full SVD routines return $U$ and $V t=V^{\prime} . S$ is returned as a column vector $s$, and $S$ can be obtained by

$$
S=\text { fullsdiag }(s, \operatorname{rows}(A)-\operatorname{cols}(A))
$$

so we will write the SVD as

$$
A=U * \operatorname{full} \operatorname{sdiag}(s, \operatorname{rows}(A)-\operatorname{cols}(A)) * V t
$$

Function fullsvd $(A, U, s, V t)$ returns the $U, s$, and $V t$ corresponding to $A$.

## Relationship between the full and thin SVDs

A popular variant of the SVD is known as the thin SVD and is suitable for use when $m \geq n$. Both SVDs have the same formula,

$$
A=U S V^{\prime}
$$

but $U$ and $S$ have reduced dimensions in the thin version:

| Matrix | Full SVD | Thin SVD |
| :--- | :---: | :---: |
| $U:$ | $m \times m$ | $m \times n$ |
| $S:$ | $m \times n$ | $n \times n$ |
| $V:$ | $n \times n$ | $n \times n$ |

When $m=n$, the two variants are identical.
The thin SVD is of use when $m>n$, because then only the first $n$ diagonal elements of $S$ are nonzero, and therefore only the first $n$ columns of $U$ are relevant in $A=U S V^{\prime}$. There are considerable memory savings to be had in calculating the thin SVD when $m \gg n$.

As a result, many people call the thin SVD the SVD and ignore the full SVD altogether. If the matrices you deal with have $m \geq n$, you will want to do the same. To obtain the thin SVD, see $[M-5] \operatorname{svd}()$.

Regardless of the dimension of your matrix, you may wish to obtain only the singular values. In this case, see $\operatorname{svdsv}()$ documented in $[M-5] \operatorname{svd}()$. That function is appropriate in all cases.

## The contents of $s$

Given $A: m \times n$, the singular values are returned in $s: \min (m, n) \times 1$.
Let's consider the $m=n$ case first. $A$ is $m \times m$ and the $m$ singular values are returned in $s$, an $m \times$ 1 column vector. If $A$ were $3 \times 3$, perhaps we would get back
s
1

```
13.47
    5.8
    2.63
```

If we needed it, we could obtain $S$ from $s$ simply by creating a diagonal matrix from $s$

```
: S = diag(s)
: S
[symmetric]
\begin{tabular}{l|rrr|}
\multicolumn{1}{c}{} & 1 & 2 & \multicolumn{1}{c}{3} \\
\cline { 2 - 4 } 1 & 13.47 & & \\
2 & 0 & 5.8 & \\
3 & 0 & 0 & 2.63 \\
\cline { 2 - 4 } & & &
\end{tabular}
```

although the official way we are supposed to do this is

```
: S = fullsdiag(s, rows(A)-cols(A))
```

and that will return the same result.
Now let's consider $m<n$. Let's pretend that $A$ is $3 \times 4$. The singular values will be returned in 3 $\times 1$ vector $s$. For instance, $s$ might still contain
: s
1
13.47
5.8
2.63

The $S$ matrix here needs to be $3 \times 4$, and fullsdiag() will form it:
: fullsdiag(s, rows(A)-cols(A))

|  |  |  |  |  |
| :--- | ---: | ---: | ---: | :--- |
|  | 1 | 2 | 3 | 4 |
| 2 | 13.47 | 0 | 0 | 0 |
| 3 | 0 | 5.8 | 0 | 0 |
|  | 0 | 0 | 2.63 | 0 |

The final case is $m>n$. We will pretend that $A$ is $4 \times 3$. The $s$ vector we get back will look the same
: s
1
13.47
5.8
2.63
but this time, we need a $4 \times 3$ rather than a $3 \times 4$ matrix formed from it.

```
: fullsdiag(s, rows(A)-cols(A))
```

|  | 13.47 | 0 | 0 |
| :--- | ---: | ---: | ---: |
| 2 | 0 | 5.8 | 0 |
| 3 | 0 | 0 | 2.63 |
| 4 | 0 | 0 | 0 |
|  |  |  |  |

## Possibility of convergence problems

See Possibility of convergence problems in [M-5] svd(); what is said there applies equally here.

## Conformability

## fullsvd $(A, U, s, V t)$ :

input:

$$
A: \quad m \times n
$$

output:

$$
\begin{aligned}
U: & m \times m \\
s: & \min (m, n) \times 1 \\
V t: & n \times n \\
\text { result }: & \text { void }
\end{aligned}
$$

fullsdiag $(s, k)$ :
input:

$$
\begin{array}{ll}
s: & \\
k: & \\
k: & 1 \times 1 \\
\hline
\end{array}
$$

output:

$$
\text { result: } \quad r+k \times r, \quad \text { if } \mathrm{k} \geq 0
$$

$$
r \times r-k, \quad \text { otherwise }
$$

_fullsvd $(A, U, s, V t)$ :
input:
A: $\quad m \times n$
output:
A: $\quad 0 \times 0$
$U: \quad m \times m$
$s: \quad \min (m, n) \times 1$
Vt: $\quad n \times n$
result: void
_svd_la $(A, U, s, V t)$ :
input:
A: $\quad m \times n$
output:

$$
\begin{aligned}
A: & m \times n, \quad \text { but contents changed } \\
U: & m \times m \\
s: & \min (m, n) \times 1 \\
V t: & n \times n \\
\text { result: } & 1 \times 1
\end{aligned}
$$

## Diagnostics

fullsvd $(A, U, s, V t)$ and _fullsvd $(A, s, V t)$ return missing results if $A$ contains missing. In all other cases, the routines should work, but there is the unlikely possibility of convergence problems, in which case missing results will also be returned; see Possibility of convergence problems in [M-5] svd( ).
_fullsvd() aborts with error if $A$ is a view.
Direct use of _svd_la() is not recommended.

## Also see

[M-5] norm() - Matrix and vector norms
[M-5] pinv() - Moore-Penrose pseudoinverse
[M-5] rank( ) - Rank of matrix
[M-5] svd() - Singular value decomposition
[M-5] svsolve( ) - Solve AX=B for X using singular value decomposition
[M-4] Matrix - Matrix functions

## Title

[M-5] geigensystem() — Generalized eigenvectors and eigenvalues

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | References | Also see |  |

## Description

geigensystem $(A, B, X, w, b)$ computes generalized eigenvectors of two general, real or complex, square matrices, $A$ and $B$, along with their corresponding generalized eigenvalues.

- $A$ and $B$ are two general, real or complex, square matrices with the same dimensions.
- $X$ contains generalized eigenvectors.
- $w$ contains numerators of generalized eigenvalues.
- $b$ contains denominators of generalized eigenvalues.
leftgeigensystem ( $A, B, X, w, b$ ) mirrors geigensystem(), the difference being that leftgeigensystem() computes left, generalized eigenvectors.
geigensystemselectr $(A, B$, range, $X, w, b)$ computes selected generalized eigenvectors of two general, real or complex, square matrices, $A$ and $B$, along with their corresponding generalized eigenvalues. Only the generalized eigenvectors corresponding to selected generalized eigenvalues are computed. Generalized eigenvalues that lie in a range are selected. The selected generalized eigenvectors are returned in $X$, and their corresponding generalized eigenvalues are returned in $(w, b)$.
range is a vector of length 2 . All finite, generalized eigenvalues with absolute value in the half-open interval (range [1], range [2]] are selected.
leftgeigensystemselectr ( $A, B$, range, $X, w, b$ ) mirrors geigensystemselectr () , the difference being that leftgeigensystemr() computes left, generalized eigenvectors.
geigensystemselecti $(A, B$, index $, X, w, b)$ computes selected right, generalized eigenvectors of two general, real or complex, square matrices, $A$ and $B$, along with their corresponding generalized eigenvalues. Only the generalized eigenvectors corresponding to selected generalized eigenvalues are computed. Generalized eigenvalues are selected by an index. The selected generalized eigenvectors are returned in $X$, and the selected generalized eigenvalues are returned in $(w, b)$.

The finite, generalized eigenvalues are sorted by their absolute values, in descending order, followed by the infinite, generalized eigenvalues. There is no particular order among infinite, generalized eigenvalues.
index is a vector of length 2 . The generalized eigenvalues in elements index[1] through index [2], inclusive, are selected.
leftgeigensystemselecti( $A, B$, index, $X, w, b$ ) mirrors geigensystemselecti(), the difference being that leftgeigensystemi() computes left, generalized eigenvectors.
geigensystemselectf $(A, B, f, X, w, b)$ computes selected generalized eigenvectors of two general, real or complex, square matrices $A$ and $B$ along with their corresponding generalized eigenvalues. Only the generalized eigenvectors corresponding to selected generalized eigenvalues
are computed. Generalized eigenvalues are selected by a user-written function described below. The selected generalized eigenvectors are returned in $X$, and the selected generalized eigenvalues are returned in ( $w, b$ ).
leftgeigensystemselectf $(A, B, f, X, w, b)$ mirrors geigensystemselectf(), the difference being that leftgeigensystemselectf() computes selected left, generalized eigenvectors.
_geigen_la(), _geigensystem_la(), _geigenselectr_la(), _geigenselecti_la(), and _geigenselectf_la() are the interfaces into the LAPACK routines used to implement the above functions; see [M-1] LAPACK. Their direct use is not recommended.

## Syntax

| void | geigensystem $(A, B, X, w, b)$ |
| :--- | ---: |
| void | leftgeigensystem $(A, B, X, w, b)$ |
| void | geigensystemselectr $(A, B$, range $, X, w, b)$ |
| void leftgeigensystemselectr $(A, B$, range $, X, w, b)$ |  |
| void | geigensystemselecti $(A, B$, index, $X, w, b)$ |
| void leftgeigensystemselecti $(A, B$, index, $X, w, b)$ |  |
| void | geigensystemselectf $(A, B, f, X, w, b)$ |
| void leftgeigensystemselectf $(A, B, f, X, w, b)$ |  |

where inputs are
A: numeric matrix
B: numeric matrix
range: real vector (range of generalized eigenvalues to be selected)
index: real vector (indices of generalized eigenvalues to be selected)
$f$ : pointer scalar (points to a function used to select generalized eigenvalues)
and outputs are
$X$ : numeric matrix of generalized eigenvectors
$w$ : numeric vector (numerators of generalized eigenvalues)
$b$ : numeric vector (denominators of generalized eigenvalues)

The following routines are used in implementing the above routines:

```
void _geigensystem_la(numeric matrix H, R,XL,XR,w,b, string scalar side)
void _geigenselectr_la(numeric matrix \(H, R, X L, X R, w, b\), range, string scalar side)
void _geigenselecti_la(numeric matrix \(H, R, X L, X R, w, b\), index, string scalar side)
void _geigenselectf_la(numeric matrix \(H, R, X L, X R, w, b\), pointer scalar \(f\), string scalar side)
```

real scalar _geigen_la(numeric matrix $H, R, X L, X R, w$, select, string scalar side, string scalar howmany)

## Remarks and examples

Remarks are presented under the following headings:

```
Generalized eigenvalues
Generalized eigenvectors
Criterion selection
Range selection
Index selection
```


## Generalized eigenvalues

A scalar, $l$ (usually denoted by lambda), is said to be a generalized eigenvalue of a pair of $n \times n$ square, numeric matrices $(\mathbf{A}, \mathbf{B})$ if there is a nonzero column vector $\mathbf{x}: n \times 1$ (called the generalized eigenvector) such that

$$
\begin{equation*}
\mathbf{A x}=l \mathbf{B x} \tag{1}
\end{equation*}
$$

(1) can also be written as

$$
(\mathbf{A}-l \mathbf{B}) \mathbf{x}=0
$$

A nontrivial solution to this system of $n$ linear homogeneous equations exists if and only if

$$
\begin{equation*}
\operatorname{det}(\mathbf{A}-l \mathbf{B})=0 \tag{2}
\end{equation*}
$$

In practice, the generalized eigenvalue problem for the matrix pair $(\mathbf{A}, \mathbf{B})$ is usually formulated as finding a pair of scalars $(w, b)$ and a nonzero column vector $\mathbf{x}$ such that

$$
w \mathbf{A} \mathbf{x}=b \mathbf{B} \mathbf{x}
$$

The scalar $w / b$ is a finite, generalized eigenvalue if $b$ is not zero. The pair ( $w, b$ ) represents an infinite, generalized eigenvalue if $b$ is zero or numerically close to zero. This situation may arise if $\mathbf{B}$ is singular.

The Mata functions that compute generalized eigenvalues return them in two complex vectors, $\mathbf{w}$ and $\mathbf{b}$, of length $n$. If $\mathrm{b}[i]=0$, the $i$ th generalized eigenvalue is infinite; otherwise, the $i$ th generalized eigenvalue is $\mathrm{w}[i] / \mathrm{b}[i]$.

## Generalized eigenvectors

A column vector, $\mathbf{x}$, is a right, generalized eigenvector or simply a generalized eigenvector of a generalized eigenvalue ( $w, b$ ) for a pair of matrices, $\mathbf{A}$ and $\mathbf{B}$, if

$$
w \mathbf{A} \mathbf{x}=b \mathbf{B} \mathbf{x}
$$

A row vector, $\mathbf{v}$, is a left, generalized eigenvector of a generalized eigenvalue $(w, b)$ for a pair of matrices, $\mathbf{A}$ and $\mathbf{B}$, if

$$
w \mathbf{v A}=b \mathbf{v B}
$$

For instance, let's consider the linear system

$$
\begin{aligned}
& d x / d t=\mathrm{A} 1 \times x+\mathrm{A} 2 \times u \\
& d y / d t=\mathrm{A} 3 \times x+\mathrm{A} 4 \times u
\end{aligned}
$$

where

$$
\begin{aligned}
& : A 1=(-4,-3 \backslash 2,1) \\
& : A 2=(3 \backslash 1) \\
& : A 3=(1,2)
\end{aligned}
$$

and

$$
: A 4=0
$$

The finite solutions of zeros for the transfer function

$$
\begin{equation*}
g(s)=\mathrm{A} 3 \times(s I-\mathrm{A} 1)^{-1} \times \mathrm{A} 2+\mathrm{A} 4 \tag{3}
\end{equation*}
$$

of this linear time-invariant state-space model is given by the finite, generalized eigenvalues of A and $B$ where

$$
: A=(A 1, A 2 \backslash A 3, A 4)
$$

and

$$
: B=(1,0,0 \backslash 0,1,0 \backslash 0,0,0)
$$

We obtain generalized eigenvectors in X and generalized eigenvalues in w and b by using
: geigensystem(A, $B, X=., \mathrm{w}=., \mathrm{b}=$.
: X

| 1 | 2 | 3 |
| ---: | ---: | ---: |
| -1 | 0 | $2.9790 \mathrm{e}-16$ |
| .5 | 0 | $9.9301 \mathrm{e}-17$ |
| .1 | 1 | 1 |

: w

| -1.97989899 | 3.16227766 | 2.23606798 |
| :--- | :--- | :--- |

: b
$\begin{array}{lll}1 & 2 & 3\end{array}$
1

| .7071067812 | 0 | 0 |
| :--- | :--- | :--- |

The only finite, generalized eigenvalue of $A$ and $B$ is

```
: w[1,1]/b[1,1]
    -2.8
```

In this simple example, (3) can be explicitly written out as

$$
g(s)=(5 s+14) /\left(s^{2}+3 s+2\right)
$$

which clearly has the solution of zero at -2.8 .

## Criterion selection

We sometimes want to compute only those generalized eigenvectors whose corresponding generalized eigenvalues satisfy certain criterion. We can use geigensystemselectf () to solve these problems.

We must pass geigensystemselectf() a pointer to a function that implements our conditions. The function must accept two numeric scalar arguments so that it can receive the numerator w and the denominator b of a generalized eigenvalue, and it must return the real value 0 to indicate rejection and a nonzero real value to indicate selection.

In this example, we want to compute only finite, generalized eigenvalues for each of which b is not zero. After deciding that anything smaller than $1 \mathrm{e}-15$ is zero, we define our function to be

```
: real scalar finiteonly(numeric scalar w, numeric scalar b)
> {
> return((abs(b)>=1e-15))
}
```

By using

```
geigensystemselectf(A, B, &finiteonly(), X=., w=., b=.)
```

we get the only finite, generalized eigenvalue of $A$ and $B$ in (w, b) and its corresponding eigenvector in X :
: X
1


## Range selection

We can use geigensystemselectr() to compute only those generalized eigenvectors whose generalized eigenvalues have absolute values that fall in a half-open interval.

For instance,

```
:A = (-132, -88, 84, 104 \ -158.4, -79.2, 76.8, 129.6 \
> 129.6, 81.6, -79.2, -100.8\ \160, 84, -80, -132)
B = (-60, -50, 40, 50 \ -69, -46.4, 38, 58.2 \ 58.8, 46, -37.6, -48\
> 70, 50, -40, -60)
: range = (0.99, 2.1)
```

We obtain generalized eigenvectors in X and generalized eigenvalues in w and b by using

```
: geigensystemselectr(A, B, range, X=., w=., b=.)
X
    1 2
    l|rr|
w
            1 2
    1
b
            1 2
1
```

```
.0141030148 . }170176379
```

```
.0141030148 . }170176379
```

The generalized eigenvalues have absolute values in the half-open interval (0.99, 2.1].

```
abs(w:/b)
            1 2
1
```

```
2 1
```


## Index selection

geigensystemselect() sorts the finite, generalized eigenvalues using their absolute values, in descending order, placing the infinite, generalized eigenvalues after the finite, generalized eigenvalues. There is no particular order among infinite, generalized eigenvalues.

If we want to compute only generalized eigenvalues whose ranks are index [1] through index [2] in the list of generalized eigenvalues obtained by geigensystemselect(), we can use geigensystemseleci().

To compute the first two generalized eigenvalues and generalized eigenvectors in this example, we can specify

```
: index = (1, 2)
: geigensystemselecti(A, B, index, X=., w=., b=.)
```

The results are
: X

| 1 | 2 |
| :--- | :--- |
| 424 | -.059628479 |
| 136 | -.059628479 |
| 2719 | -.02981424 |
| 712 | -.119256959 |

: w

b
$1 \quad 2$
1 .0031622777 .1264911064
: w:/b
12
1


## Conformability

geigensystem $(A, B, X, w, b)$ :
input:

$$
\begin{array}{ll}
A: & \\
B \times n \\
B: & \\
n \times n
\end{array}
$$

output:

$$
\begin{array}{cl}
X: & \\
w: & 1 \times n \\
b: & \\
1 \times n
\end{array}
$$

leftgeigensystem $(A, B, X, w, b)$ :
input:

$$
\begin{array}{ll}
A: & \\
B \times n \\
B: & \\
n \times n
\end{array}
$$

output:

$$
\begin{aligned}
& X: \\
& w: \\
& w: n \\
& b: \\
& 1 \times n
\end{aligned}
$$

geigensystemselectr $(A, B$, range, $X, w, b)$ :
input:
A: $\quad n \times n$
B: $\quad n \times n$

$$
\text { range: } \quad 1 \times 2 \text { or } 2 \times 1
$$

output:

$$
\begin{array}{cl}
X: & n \times m \\
w: & 1 \times m \\
b: & 1 \times m
\end{array}
$$

leftgeigensystemselectr $(A, B$, range, $X, w, b)$ :
input:

$$
\begin{aligned}
A: & n \times n \\
B: & n \times n \\
\text { range: } & 1 \times 2 \text { or } 2 \times 1
\end{aligned}
$$

output:

$$
\begin{array}{cl}
X: & m \times n \\
w: & 1 \times m \\
b: & 1 \times m
\end{array}
$$

geigensystemselecti $(A, B$, index, $X, w, b)$ :
input:

$$
\begin{aligned}
A: & n \times n \\
B: & n \times n \\
\text { index: } & 1 \times 2 \text { or } 2 \times 1
\end{aligned}
$$

output:
$X: \quad n \times m$
$w: \quad 1 \times m$
$b: \quad 1 \times m$
leftgeigensystemselecti ( $A, B$, index, $X, w, b$ ):
input:

$$
\begin{aligned}
A: & n \times n \\
B: & n \times n \\
\text { index: } & 1 \times 2 \text { or } 2 \times 1
\end{aligned}
$$

output:

$$
\begin{array}{cl}
X: & m \times n \\
w: & 1 \times m \\
b: & 1 \times m
\end{array}
$$

geigensystemselectf $(A, B, f, X, w, b)$ :
input:
A: $\quad n \times n$
B: $\quad n \times n$
$f: \quad 1 \times 1$
output:
$X: \quad n \times m$
$w: \quad 1 \times m$
$b: \quad 1 \times m$
leftgeigensystemselectf $(A, B, f, X, w, b)$ :
input:
A: $\quad n \times n$
B: $\quad n \times n$
$f: \quad 1 \times 1$
output:
$X: \quad m \times n$
$w: \quad 1 \times m$
$b: \quad 1 \times m$

## Diagnostics

All functions return missing-value results if $A$ or $B$ has missing values.

## References

Gould, W. W. 2011a. Understanding matrices intuitively, part 1. The Stata Blog: Not Elsewhere Classified. http://blog.stata.com/2011/03/03/understanding-matrices-intuitively-part-1/.

2011b. Understanding matrices intuitively, part 2, eigenvalues and eigenvectors. The Stata Blog: Not Elsewhere Classified. http://blog.stata.com/2011/03/09/understanding-matrices-intuitively-part-2/.

## Also see

[M-1] LAPACK - Linear algebra package (LAPACK) routines
[M-5] ghessenbergd( ) - Generalized Hessenberg decomposition
[M-5] gschurd( ) - Generalized Schur decomposition
[M-4] Matrix - Matrix functions

## Title

[M-5] ghessenbergd() — Generalized Hessenberg decomposition

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

ghessenbergd $(A, B, H, R, U, V)$ computes the generalized Hessenberg decomposition of two general, real or complex, square matrices, $A$ and $B$, returning the upper Hessenberg form matrix in $H$, the upper triangular matrix in $R$, and the orthogonal (unitary) matrices in $U$ and $V$.
_ghessenbergd $(A, B, U, V)$ mirrors ghessenbergd(), the difference being that it returns $H$ in $A$ and $R$ in $B$.
_ghessenbergd_la() is the interface to the LAPACK routines used to implement the above function. Its direct use is not recommended.

## Syntax

void ghessenbergd(numeric matrix $A, B, H, R, U, V)$
void _ghessenbergd(numeric matrix $A, B, \quad U, V$ )

## Remarks and examples

The generalized Hessenberg decomposition of two square, numeric matrices ( $\mathbf{A}$ and $\mathbf{B}$ ) can be written as

$$
\begin{aligned}
& \mathbf{U}^{\prime} \times \mathbf{A} \times \mathbf{V}=\mathbf{H} \\
& \mathbf{U}^{\prime} \times \mathbf{B} \times \mathbf{V}=\mathbf{R}
\end{aligned}
$$

where $\mathbf{H}$ is in upper Hessenberg form, $\mathbf{R}$ is upper triangular, and $\mathbf{U}$ and $\mathbf{V}$ are orthogonal matrices if $\mathbf{A}$ and $\mathbf{B}$ are real or are unitary matrices otherwise.

In the example below, we define A and B , obtain the generalized Hessenberg decomposition, and list H and Q .

```
: A = (6, 2, 8, -1\-3, -4, -6, 4\0, 8, 4, 1\-8, -7, -3, 5)
: B = (8, 0, -8, -1\-6, -2, -6, -1\-7, -6, 2, -6\1, -7, 9, 2)
: ghessenbergd(A, B, H=., R=., U=., V=.)
: H
```

    \(\begin{array}{llll}1 & 2 & 3 & 4\end{array}\)
    | -4.735680169 | 1.363736029 | 5.097381347 | 3.889763589 |
| ---: | ---: | ---: | ---: |
| 9.304479208 | -8.594240253 | -7.993282943 | 4.803411217 |
| 0 | 4.553169015 | 3.236266637 | -2.147709419 |
| 0 | 0 | 6.997043028 | -3.524816722 |

R

| 1 | 2 | 3 | 4 |
| ---: | ---: | ---: | ---: |
| 871 | -1.089095534 | -1.848528639 | -5.398470103 |
| 0 | -5.872766311 | 8.891361089 | 3.86967647 |
| 0 | 0 | 9.056748937 | 1.366322731 |
| 0 | 0 | 0 | 8.357135399 |

## Conformability

ghessenbergd $(A, B, H, R, U, V)$ :
input:

$$
\begin{array}{ll}
A: & \\
B \times n \\
B: & \\
n \times n
\end{array}
$$

output:

$$
\begin{array}{ll}
H: & n \times n \\
R: & \\
n \times n \\
U: & \\
V: & n \times n \\
V: & n \times n
\end{array}
$$

_ghessenbergd $(A, B, U, V)$ :
input:

$$
\begin{array}{ll}
A: & \\
B \times n \\
B: & \\
n \times n
\end{array}
$$

output:
A: $\quad n \times n$
B: $\quad n \times n$
$U: \quad n \times n$
$V: \quad n \times n$

## Diagnostics

_ghessenbergd() aborts with error if $A$ or $B$ is a view.
ghessenbergd() and _ghessenbergd() return missing results if $A$ or $B$ contains missing values.

## Also see

[M-1] LAPACK - Linear algebra package (LAPACK) routines
[M-5] gschurd( ) - Generalized Schur decomposition
[M-4] Matrix - Matrix functions

## Title

[M-5] ghk() — Geweke-Hajivassiliou-Keane (GHK) multivariate normal simulator

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

The ghk*() set of functions provide a Geweke-Hajivassiliou-Keane (GHK) multivariate normal simulator.
$S=$ ghk_init(npts) initializes the simulator with the desired number of simulation points and returns a transmorphic object $S$, which is a handle that should be used in subsequent calls to other ghk*() functions. Calls to ghk_init_method ( $S$, method), ghk_init_start ( $S$, start), ghk_init_pivot ( $S$, pivot), and ghk_init_antithetics ( $S$, anti) prior to calling ghk ( $S$, ...) allow you to modify the simulation algorithm through the object $S$.
$\operatorname{ghk}(S, x, V)$ returns a real scalar containing the simulated value of the multivariate normal (MVN) distribution with variance-covariance $V$ at the point $x$. First, code $S=$ ghk_init (npts) and then use ghk ( $S, \ldots$ ) to obtain the simulated value based on npts simulation points.
ghk ( $S, x, V, d f d x, d f d v$ ) does the same thing but also returns the first-order derivatives of the simulated probability with respect to $x$ in $d f d x$ and the simulated probability derivatives with respect to $\operatorname{vech}(V)$ in $d f d v$. See vech () in [M-5] vec() for details of the half-vectorized operator.

The ghk_query_npts ( $S$ ) function returns the number of simulation points, the same value given in the construction of the transmorphic object $S$.

## Syntax

```
                S = ghk_init(real scalar npts)
(varies) ghk_init_method(S [, string scalar method])
(varies) ghk_init_start(S [, real scalar start])
(varies) ghk_init_pivot(S [, real scalar pivot])
(varies) ghk_init_antithetics(S [, real scalar anti])
real scalar ghk_query_npts(S)
real scalar ghk(S, real vector x,V)
real scalar ghk(S, real vector x,V, real rowvector dfdx, dfdv)
```

where $S$, if declared, should be declared

```
transmorphic S
```

and where method, optionally specified in ghk_init_method(), is
method Description
"halton" Halton sequences
"hammersley" Hammersley's variation of the Halton set
"random" pseudorandom uniforms

## Remarks and examples

Halton and Hammersley point sets are composed of deterministic sequences on $[0,1]$ and, for sets of dimension less than 10 , generally have better coverage than that of the uniform pseudorandom sequences.

Antithetic draws effectively double the number of points and reduce the variability of the simulated probability. For draw $u$, the antithetic draw is $1-u$. To use antithetic draws, call ghk_init_antithetic ( $S, 1$ ) prior to executing ghk().

By default, ghk() will pivot the wider intervals of integration (and associated rows/columns of the covariance matrix) to the interior of the multivariate integration. This improves the accuracy of the quadrature estimate. When ghk() is used in a likelihood evaluator for [R] $\mathbf{m l}$ or [M-5] optimize(), discontinuities may result in the computation of numerical second-order derivatives using finite differencing (for the Newton-Raphson optimization technique) when few simulation points are used, resulting in a non-positive-definite Hessian. To turn off the interval pivoting, call ghk_init_pivot ( $S, 0$ ) prior to executing ghk().

If you are using ghk () in a likelihood evaluator, be sure to use the same sequence with each call to the likelihood evaluator. For a uniform pseudorandom sequence, ghk_init_method("random"), set the seed of the uniform random-variate generator-see rseed() in [M-5] runiform()—to the same value with each call to the likelihood evaluator.

If you are using the Halton or Hammersley point sets, you will want to keep the sequences going with each call to ghk() within one likelihood evaluation. This can be done in one expression executed after each call to ghk(): ghk_init_start ( $S$, ghk_init_start $(S)+$ ghk_query_npts $(S)$ ). With each call to the likelihood evaluator, you will need to reset the starting index to 1 . This last point assumes that the transmorphic object $S$ is not re-created on each call to the likelihood evaluator.

Unlike ghkfast_init ( ) (see [M-5] ghkfast()), the transmorphic object $S$ created by ghk_init () is inexpensive to create, so it is possible to re-create it with each call to your likelihood evaluator instead of storing it as external global and reusing the object with each likelihood evaluation. Alternatively, the initialization function for optimize(), optimize_init_arguments(), can be used.

## Conformability

All initialization functions have $1 \times 1$ inputs and have $1 \times 1$ or void outputs except
ghk_init(npts):
input:

$$
\text { npts: } \quad 1 \times 1
$$

output:
$S: \quad$ transmorphic
ghk_query_npts ( $S$ ):
input:
$S: \quad$ transmorphic
output:

$$
\text { result: } \quad 1 \times 1
$$

$\operatorname{ghk}(S, x, V)$ :
input:
S: $\quad$ transmorphic
$x: \quad 1 \times m$ or $m \times 1$
$V: \quad m \times m$ (symmetric, positive definite)
output:
result: $\quad 1 \times 1$
$\operatorname{ghk}(S, x, V, d f d x, d f d v):$
input:
S: $\quad$ transmorphic
$x: \quad 1 \times m$ or $m \times 1$
$V: \quad m \times m$ (symmetric, positive definite)
output:

$$
\begin{aligned}
\text { result: } & 1 \times 1 \\
d f d x: & 1 \times m \\
d f d v: & 1 \times m(m+1) / 2
\end{aligned}
$$

## Diagnostics

The maximum dimension, $m$, is 20 .
$V$ must be symmetric and positive definite. ghk() will return a missing value when $V$ is not positive definite. When ghk() is used in an ml (or optimize()) likelihood evaluator, return a missing likelihood to ml and let ml take the appropriate action (that is, step halving).

## Also see

[M-5] ghkfast() - GHK multivariate normal simulator using pregenerated points
[M-5] halton( ) - Generate a Halton or Hammersley set
[M-4] Statistical - Statistical functions

## Title

[M-5] ghkfast( ) - GHK multivariate normal simulator using pregenerated points

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

Please see [M-5] ghk(). The routines documented here do the same thing, but ghkfast() can be faster at the expense of using more memory. First, code $S=$ ghkfast_init(...) and then use ghkfast ( $S, \ldots$ ) to obtain the simulated values. There is a time savings because the simulation points are generated once in ghkfast_init(), whereas for ghk() the points are generated on each call to ghk(). Also, ghkfast() can generate simulated probabilities from the generalized Halton sequence; see [M-5] halton().
ghkfast_init ( $n$, npts, dim, method) computes the simulation points to be used by ghkfast (). Inputs $n$, npts, and dim are the number of observations, the number of repetitions for the simulation, and the maximum dimension of the multivariate normal (MVN) distribution, respectively. Input method specifies the type of points to generate and can be one of "halton", "hammersley", "random", or "ghalton".
ghkfast ( $S, X, V$ ) returns an $n \times 1$ real vector containing the simulated values of the MVN distribution with $\operatorname{dim} \times \operatorname{dim}$ variance-covariance matrix $V$ at the points stored in the rows of the $n \times \operatorname{dim}$ matrix $X$.
ghkfast ( $S, X, V, d f d x, d f d v$ ) does the same thing as ghkfast ( $S, X, V$ ) but also returns the first-order derivatives of the simulated probability with respect to the rows of $X$ in $d f d x$ and the simulated probability derivatives with respect to $\operatorname{vech}(V)$ in $d f d v$. See vech () in [M-5] vec() for details of the half-vectorized operator.

The ghk_query_n $(S)$, ghk_query_npts $(S)$, ghk_query_dim $(S)$, and ghk_query_method $(S)$ functions extract the number of observations, number of simulation points, maximum dimension, and method of point-set generation that is specified in the construction of the transmorphic object $S$. Use ghk_query_rseed ( $S$ ) to retrieve the uniform random-variate seed used to generate the "random" or "ghalton" point sets. The ghkfast_query_pointset_i $(S, i)$ function will retrieve the $i$ th point set used to simulate the MVN probability for the $i$ th observation.

The ghkfast_i $(S, X, V, i, \ldots)$ function computes the probability and derivatives for the $i$ th observation, $i=1, \ldots, n$.

## Syntax

$$
S=\text { ghkfast_init(real scalar } n, \text { npts, dim, string scalar method) }
$$

```
(varies) ghkfast_init_pivot(S [, real scalar pivot \(]\) )
(varies) ghkfast_init_antithetics (S [, real scalar anti])
real scalar ghkfast_query_n( \(S\) )
real scalar ghkfast_query_npts ( \(S\) )
real scalar ghkfast_query_dim( \(S\) )
string scalar ghkfast_query_method (S)
string scalar ghkfast_query_rseed ( \(S\) )
real matrix ghkfast_query_pointset_i \((S, i)\)
real colvector ghkfast ( \(S\), real matrix \(X, V\) )
real colvector ghkfast ( \(S\), real matrix \(X, V, d f d x, d f d v\) )
real scalar ghkfast_i \((S\), real matrix \(X, V, i)\)
real scalar ghkfast_i \((S\), real matrix \(X, V, i, d f d x, d f d v)\)
```

where $S$, if it is declared, should be declared

```
transmorphic S
```

and where method specified in ghkfast_init() is

| method | Description |
| :--- | :--- |
| "halton" | Halton sequences |
| "hammersley" | Hammersley’s variation of the Halton set <br> pseudorandom uniforms <br> "random" |
| "ghalton" | generalized Halton sequences |

## Remarks and examples

For problems where repetitive calls to the GHK algorithm are required, ghkfast() might be a preferred alternative to ghk(). Generating the points once at the outset of a program produces a speed increase. For problems with many observations or many simulation points per observation, ghkfast () will be faster than ghk() at the cost of requiring more memory.

If ghkfast() is used within a likelihood evaluator for ml or optimize(), you will need to store the transmorphic object $S$ as an external global and reuse the object with each likelihood evaluation. Alternatively, the initialization function for optimize(), optimize_init_argument(), can be used.

Prior to calling ghkfast(), call ghkfast_init_npivot ( $S, 1$ ) to turn off the integration interval pivoting that takes place in ghkfast (). By default, ghkfast () pivots the wider intervals of integration (and associated rows/columns of the covariance matrix) to the interior of the multivariate integration to improve quadrature accuracy. This option may be useful when ghkfast () is used in a likelihood evaluator for $[R] \mathbf{m l}$ or [M-5] optimize() and few simulation points are used for each observation. Here the pivoting may cause discontinuities when computing numerical second-order derivatives using finite differencing (for the Newton-Raphson technique), resulting in a non-positive-definite Hessian.

Also the sequences "halton", "hammersley", and "random", ghkfast() will use the generalized Halton sequence, "ghalton". Generalized Halton sequences have the same uniform coverage (low discrepancy) as the Halton sequences with the addition of a pseudorandom uniform component. Therefore, "ghalton" sequences are like "random" sequences in that you should set the randomnumber seed before using them if you wish to replicate the same point set; see [M-5] runiform( ).

## Conformability

All initialization functions have $1 \times 1$ inputs and have $1 \times 1$ or void outputs, and all query functions have the transmorphic input and $1 \times 1$ outputs except
ghkfast_init( $n$, npts, dim, method):
input:

| $n:$ | $1 \times 1$ |
| ---: | ---: |
| npts: | $1 \times 1$ |
| dim: | $1 \times 1$ |
| method: | $1 \times 1$ |

output:
result: transmorphic
ghkfast_query_pointset_i $(S, i)$ :
input:

$$
\begin{array}{ll}
S: & \text { transmorphic } \\
i: & 1 \times 1
\end{array}
$$

output:
result: $\quad n p t s \times \operatorname{dim}$
ghkfast ( $S, X, V$ ):
input:
S: transmorphic
X: $\quad n \times \operatorname{dim}$
$V: \quad \operatorname{dim} \times \operatorname{dim}$ (symmetric, positive definite)
output:
result: $\quad n \times 1$
ghkfast ( $S, X, V, d f d x, d f d v)$ :
input:
S: transmorphic
X: $\quad n \times \operatorname{dim}$
$V: \quad \operatorname{dim} \times \operatorname{dim}$ (symmetric, positive definite)
output:

$$
\begin{aligned}
\text { result: } & n \times 1 \\
d f d x: & n \times \operatorname{dim} \\
d f d v: & n \times \operatorname{dim}(\operatorname{dim}+1) / 2
\end{aligned}
$$

ghkfast_i $(S, X, V, i, d f d x, d f d v)$ :
input:

$$
\begin{aligned}
S: & \text { transmorphic } \\
X: & n \times \operatorname{dim} \text { or } 1 \times \operatorname{dim} \\
V: & \operatorname{dim} \times \operatorname{dim}(\text { symmetric, positive definite }) \\
i: & 1 \times 1(1 \leq i \leq n)
\end{aligned}
$$

output:
result: $\quad \mathrm{n} \times 1$
dfdx: $\quad 1 \times \operatorname{dim}$
$d f d v: \quad 1 \times \operatorname{dim}(\operatorname{dim}+1) / 2$

## Diagnostics

ghkfast_init( $n$, npts, dim, method) aborts with error if the dimension, dim, is greater than 20.
ghkfast ( $S, X, V, \ldots$ ) and ghkfast_i $(S, X, V, i, \ldots)$ require that $V$ be symmetric and positive definite. If $V$ is not positive definite, then the returned vector (scalar) is filled with missings.

## Also see

[M-5] ghk( ) - Geweke-Hajivassiliou-Keane (GHK) multivariate normal simulator
[M-5] halton() - Generate a Halton or Hammersley set
[M-4] Statistical - Statistical functions

## Title

[M-5] gschurd( ) - Generalized Schur decomposition

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

$\operatorname{gschurd}(A, B, T, R, U, V, w, b)$ computes the generalized Schur decomposition of two square, numeric matrices, $A$ and $B$, and the generalized eigenvalues. The decomposition is returned in the Schur-form matrix, $T$; the upper-triangular matrix, $R$; and the orthogonal (unitary) matrices, $U$ and $V$. The generalized eigenvalues are returned in the complex vectors $w$ and $b$.
gschurdgroupby $(A, B, f, T, R, U, V, w, b, m)$ computes the generalized Schur decomposition of two square, numeric matrices, $A$ and $B$, and the generalized eigenvalues, and groups the results according to whether a condition on each generalized eigenvalue is satisfied. $f$ is a pointer to the function that implements the condition on each generalized eigenvalue, as discussed below. The number of generalized eigenvalues for which the condition is true is returned in $m$.
_gschurd() mirrors gschurd(), the difference being that it returns $T$ in $A$ and $R$ in $B$.
_gschurdgroupby() mirrors gschurdgroupby(), the difference being that it returns $T$ in $A$ and $R$ in $B$.
_gschurd_la() and _gschurdgroupby_la() are the interfaces into the LAPACK routines used to implement the above functions; see [M-1] LAPACK. Their direct use is not recommended.

## Syntax

```
void gschurd(A,B,T,R,U,V,w,b)
void _gschurd(A,B,U,V,w,b)
void gschurdgroupby (A,B,f,T,R,U,V,w,b,m)
void _gschurdgroupby (A,B,f,U,V,w,b,m)
```


## Remarks and examples

Remarks are presented under the following headings:

## Generalized Schur decomposition

The generalized Schur decomposition of a pair of square, numeric matrices, $\mathbf{A}$ and $\mathbf{B}$, can be written as

$$
\begin{array}{r}
\mathbf{U}^{\prime} \times \mathbf{A} \times \mathbf{V}=\mathbf{T} \\
\mathbf{U}^{\prime} \times \mathbf{B} \times \mathbf{V}=\mathbf{R}
\end{array}
$$

where $\mathbf{T}$ is in Schur form, $\mathbf{R}$ is upper triangular, and $\mathbf{U}$ and $\mathbf{V}$ are orthogonal if $\mathbf{A}$ and $\mathbf{B}$ are real and are unitary if $\mathbf{A}$ or $\mathbf{B}$ is complex. The complex vectors $\mathbf{w}$ and $\mathbf{b}$ contain the generalized eigenvalues.

If $\mathbf{A}$ and $\mathbf{B}$ are real, $\mathbf{T}$ is in real Schur form and $\mathbf{R}$ is a real upper-triangular matrix. If $\mathbf{A}$ or $\mathbf{B}$ is complex, $\mathbf{T}$ is in complex Schur form and $\mathbf{R}$ is a complex upper-triangular matrix.

In the example below, we define A and B , obtain the generalized Schur decomposition, and list T and R.


Generalized eigenvalues can be obtained by typing

|  | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 2.94840508 | .098804579+.442601735i | .098804579-.442601735i | -. 584662287 |

## Grouping the results

gschurdgroupby() reorders the generalized Schur decomposition so that a selected group of generalized eigenvalues appears in the leading block of the pair w and b. It also reorders the generalized Schur form T, R, and orthogonal (unitary) matrices, U and V, correspondingly.

We must pass gschurdgroupby () a pointer to a function that implements our criterion. The function must accept two arguments, a complex scalar and a real scalar, so that it can receive a generalized eigenvalue, and it must return the real value 0 to indicate rejection and a nonzero real value to indicate selection.

In the following example, we use gschurdgroupby () to put the finite, real, generalized eigenvalues first. One of the arguments to schurdgroupby() is a pointer to the function onlyreal() which accepts two arguments, a complex scalar and a real scalar that define a generalized eigenvalue. onlyreal() returns 1 if the generalized eigenvalue is finite and real; it returns zero otherwise.

```
real scalar onlyreal(complex scalar w, real scalar b)
{
if(b==0) return(0)
    if(Im(w/b)==0) return(1)
    return(0)
}
gschurdgroupby(A, B, &onlyreal(), T=., R=., U=., V=., w=., b=., m=.)
```

We obtain

$m$ contains the number of real, generalized eigenvalues

$$
\begin{array}{r}
: m \\
2
\end{array}
$$

## Conformability

$\operatorname{gschurd}(A, B, T, R, U, V, w, b)$ :
input:

$$
\begin{array}{ll}
A: & \\
B \times n \\
B: & \\
n \times n
\end{array}
$$

output:
T: $\quad n \times n$
R: $\quad n \times n$
U: $\quad n \times n$
$V: \quad n \times n$
$w: \quad 1 \times n$
$b: \quad 1 \times n$
$\quad \operatorname{gschurd}(A, B, U, V, w, b):$
input:

$$
\begin{array}{ll}
A: & \\
B \times n \\
B: & \\
n \times n
\end{array}
$$

output:

| $A:$ | $n \times n$ |
| :---: | :--- |
| $B:$ |  |
| $n \times n$ |  |
| $U:$ |  |
| $V:$ | $n \times n$ |
| $W:$ |  |
| $w: n$ |  |
| $b:$ |  |
|  | $1 \times n$ |

gschurdgroupby $(A, B, f, T, R, U, V, w, b, m)$ :
input:

$$
\begin{array}{ll}
A: & \\
B: & \\
f \times n \\
f: & \\
1 \times 1
\end{array}
$$

output:
T: $\quad n \times n$
R: $\quad n \times n$
$U: \quad n \times n$
$V: \quad n \times n$
$w: \quad 1 \times n$
b: $\quad 1 \times n$
$m: \quad 1 \times 1$
_gschurdgroupby $(A, B, f, U, V, w, b, m)$ :
input:
A: $\quad n \times n$
B: $\quad n \times n$
$f: \quad 1 \times 1$
output:
A: $\quad n \times n$
B: $\quad n \times n$
$U: \quad n \times n$
$V: \quad n \times n$
$w: \quad 1 \times n$
b: $\quad 1 \times n$
$m: \quad 1 \times 1$

## Diagnostics

_gschurd() and _gschurdgroupby () abort with error if $A$ or $B$ is a view.
gschurd(), _gschurd(), gschurdgroupby(), and _gschurdgroupby() return missing results if $A$ or $B$ contains missing values.

## Also see

[M-1] LAPACK - Linear algebra package (LAPACK) routines
[M-5] geigensystem () - Generalized eigenvectors and eigenvalues
[M-5] ghessenbergd( ) - Generalized Hessenberg decomposition
[M-4] Matrix - Matrix functions

## Title

[M-5] halton( ) - Generate a Halton or Hammersley set

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | References | Also see |  |

## Description

halton $(n, d)$ returns an $n \times d$ matrix containing a Halton set of length $n$ and dimension $d$. halton ( $n, d$, start) does the same thing, but the first row of the returned matrix contains the sequences starting at index start. The default is start $=1$.
halton( $n, d$, start, hammersley), with hammersley $\neq 0$, returns a Hammersley set of length $n$ and dimension $d$ with the first row of the returned matrix containing the sequences starting at index start. _halton $(x)$ modifies the $n \times d$ matrix $x$ so that it contains a Halton set of dimension $d$ of length $n$. _halton ( $x$, start) does the same thing, but the first row of the returned matrix contains the sequences starting at index start. The default is start $=1$.
_halton( $x$, start, hammersley), with hammersley $\neq 0$, returns a Hammersley set of length $n$ and dimension $d$ with the first row of the returned matrix containing the sequences starting at index start.
ghalton( $n$, base, $u$ ) returns an $n \times 1$ vector containing a (generalized) Halton sequence using base base and starting from scalar $0<u<1$, or a nonnegative index $u=0,1,2, \ldots$

## Syntax

real matrix halton(real scalar $n$, real scalar $d$ )
real matrix halton(real scalar $n$, real scalar d, real scalar start)
real matrix halton(real scalar $n$, real scalar $d$, real scalar start, real scalar hammersley)
void $\quad$ halton (real matrix $x$ )
void halton(real matrix $x$, real scalar start)
void _halton(real matrix $x$, real scalar start, real scalar hammersley)
real colvector ghalton(real scalar $n$, real scalar base, real scalar $u$ )

## Remarks and examples

The Halton sequences are generated from the first $d$ primes and generally have more uniform coverage over the unit cube of dimension $d$ than that of sequences generated from pseudouniform random numbers. However, Halton sequences based on large primes $(d>10)$ can be highly correlated, and their coverage can be worse than that of the pseudorandom uniform sequences.

The Hammersley set contains the sequence $(2 \times i-1) /(2 \times n), i=1, \ldots, n$, in the first dimension and Halton sequences for dimensions $2, \ldots, d$.
_halton() modifies $x$ and can be used when repeated calls are made to generate long sequences in blocks. Here update the start index between calls by using start $=$ start + rows $(x)$.
ghalton() uses the base base, preferably a prime, and generates a Halton sequence using $0<u<$ 1 or a nonnegative index as the starting value. If $u$ is uniform $(0,1)$, the sequence is a randomized Halton sequence. If $u$ is a nonnegative integer, the sequence is the standard Halton sequence starting at index $u+1$.

## Conformability

halton( $n, d$, start, hammersley):
input:

| $n:$ | $1 \times 1$ |  |
| ---: | :--- | :--- |
| $d:$ | $1 \times 1$ |  |
| start: | $1 \times 1$ | (optional) |
| hammersley: | $1 \times 1$ | (optional) |

output:

$$
\text { result: } \quad n \times d
$$

_halton( $x$, start, hammersley):
input:

| $x:$ | $n \times d$ |  |
| ---: | :---: | :--- |
| start: | $1 \times 1$ | (optional) |
| hammersley: | $1 \times 1$ | (optional) |

output:

$$
x: \quad n \times d
$$

ghalton( $n$, base, $u$ ):
input:

| $n:$ |  | $1 \times 1$ |
| ---: | :--- | ---: | :--- |
| base: |  | $1 \times 1$ |
| $u:$ |  | $1 \times 1$ |

output:

$$
\text { result: } \quad n \times 1
$$

## Diagnostics

The maximum dimension, $d$, is 20 . The scalar index start must be a positive integer, and the scalar $u$ must be such that $0<u<1$ or a nonnegative integer. For ghalton(), base must be an integer greater than or equal to 2 .

John Henry Halton (1931-) was born in Brussels, Belgium. He studied mathematics and physics at Cambridge and then at Oxford, where he was a doctoral student of J. M. Hammersley. His career has included positions in government, industry, and academia in both Britain and the United States, latterly as a Professor of Computer Sciences at the University of Wisconsin in Madison and the University of North Carolina in Chapel Hill. Halton's work arises from problems in pure mathematics, theoretical physics, statistics, probability theory, and engineering, and has led to devising and rigorously analyzing algorithms of a combinatorial, probabilistic, and geometric nature. It encompasses tests for statistical relationships between random variables, the hydrodynamic theory of lubrication, probabilistic properties of the Traveling Salesman Problem, and most theoretical aspects of the Monte Carlo method, including the theory and use of both pseudorandom and quasirandom sequences and sets.

John Michael Hammersley (1920-2004) was born in Helensburgh, Dunbartonshire, Scotland. His mathematical studies at Cambridge were interspersed with military service in the Royal Artillery. Hammersley contributed to the use of radar in predicting the flight paths of enemy aircraft and was promoted to the rank of major. After graduation, he moved to Oxford where his research covered several areas of mathematics and statistics. Hammersley is best known for achievements in the study of spatial disorder, especially percolation models, and of stochastic processes generally. He was a pioneer in the use of Monte Carlo methods and was senior author of an important early monograph (Hammersley and Handscomb 1964). Hammersley contributed to reform of mathematical teaching in Britain, emphasizing the value of solving concrete problems. He was elected a Fellow of the Royal Society.

## References

Grimmett, G., and D. Welsh. 2007. John Michael Hammersley, 21 March 1920-2 May 2004. Biographical Memoirs of Fellows of the Royal Society 53: 163-183. https://doi.org/10.1098/rsbm.2007.0001.

Hammersley, J. M., and D. C. Handscomb. 1964. Monte Carlo Methods. London: Methuen.
Kolenikov, S. 2012. Scrambled Halton sequences in Mata. Stata Journal 12: 29-44.

[^8]
## Title

[M-5] hash1() - Jenkins's one-at-a-time hash function

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | References | Also see |  |

## Description

hash1 ( $x$ ) returns Jenkins's one-at-a-time hash calculated over the bytes of $x$; $0 \leq$ hash1 $(x) \leq$ 4,294,967,295.
hash1 $(x, n)$ returns Jenkins's one-at-a-time hash scaled to $1 \leq \operatorname{hash} 1(x, n) \leq n$, assuming $n<$. (missing). hash1 ( $x,$. ) is equivalent to hash1 ( $x$ ).
hash1 ( $x, n$, byteorder) returns hash1 $(x, n)$ performed on the bytes of $x$ ordered as they would be on a HILO computer (byteorder $=1$ ), or as they would be on a LOHI computer (byteorder $=2$ ), or as they are on this computer (byteorder $\geq$.). See [M-5] byteorder() for a definition of byte order.
In all cases, the values returned by hash1() are integers.

## Syntax

$$
\text { real scalar hash1 }(x[\text {, real scalar } n[, \text { real scalar byteorder }]])
$$

where
$x$ : any type except struct and any dimension
$n$ : $\quad 1 \leq n \leq 2,147,483,647$ or .; default is . (missing)
byteorder: 1 (HILO), 2 (LOHI), . (natural byte order); default . (missing)

## Remarks and examples

Calculation is significantly faster using the natural byte order of the computer. Argument byteorder is included for those rare cases when it is important to calculate the same hash value across different computers, which in the case of hash1() is mainly for testing. hash1(), being a one-at-a-time method, is not sufficient for constructing digital signatures. It is sufficient for constructing hash tables; see [M-5] asarray (), in which case, byte order is irrelevant. Also note that because strings occur in the same order on all computers, the value of byteorder is irrelevant when $x$ is a string.

For instance,

```
: hash1("this"), hash1("this",.,1), hash1("this",.,2)
                            1 2 3
1
```



```
: hash1(15), hash1(15,.,1), hash1(15,.,2)
    1 2
1
\(463405819 \quad 3338064604 \quad 463405819\)
```

The computer on which this example was run is evidently byteorder $=2$, meaning LOHI, or leastsignificant byte first.

In a Mata context, it is the two-argument form of hash1 () that is most useful. In that form, the full result is mapped onto $[1, n]$ :

$$
\operatorname{hash} 1(x, n)=\operatorname{floor}((\operatorname{hash} 1(x) / 4294967295) * \mathrm{n})+1
$$

For instance,

```
: hash1("this", 10)
    6
    hash1(15, 10)
    2
```

The result of hash1 $(x, 10)$ could be used directly to index a $10 \times 1$ array.

## Conformability

hash1 ( $x, n$, byteorder):

| $x:$ | $r \times c$ |  |
| ---: | ---: | :--- |
| $n:$ | $1 \times 1$ | (optional) |
| byteorder: | $1 \times 1$ | (optional) |
| result: | $1 \times 1$ |  |

## Diagnostics

None.
Note that hash1 $(x[, \ldots])$ never returns a missing result, even if $x$ is or contains a missing value. In the missing case, the hash value is calculated of the missing value. Also note that $x$ can be a vector or a matrix, in which case the result is calculated over the elements aligned rowwise as if they were a single element. Thus hash1 (("a", "b")) == hash1("ab").

## References

Jenkins, B. 1997. Dr. Dobb's Journal. Algorithm alley: Hash functions. http://www.ddj.com/184410284.
—_. unknown. A hash function for hash table lookup. http://www.burtleburtle.net/bob/hash/doobs.html.

## Also see

[M-5] asarray( ) - Associative arrays
[M-4] Programming - Programming functions

## Title

[M-5] hessenbergd() — Hessenberg decomposition

Description Syntax Remarks and examples Conformability<br>Diagnostics Also see

## Description

hessenbergd $(A, H, Q)$ calculates the Hessenberg decomposition of a square, numeric matrix, $A$, returning the upper Hessenberg form matrix in $H$ and the orthogonal (unitary) matrix in $Q . Q$ is orthogonal if $A$ is real and unitary if $A$ is complex.
_hessenbergd ( $A, Q$ ) does the same as hessenbergd() except that it returns $H$ in $A$.
_hessenbergd_la() is the interface to the LAPACK routines used to implement the above function; see [M-1] LAPACK. Its direct use is not recommended.

## Syntax

```
void hessenbergd(numeric matrix A, H,Q)
void _hessenbergd(numeric matrix A, Q)
```


## Remarks and examples

The Hessenberg decomposition of a matrix, A, can be written as

$$
\mathbf{Q}^{\prime} \times \mathbf{A} \times \mathbf{Q}=\mathbf{H}
$$

where $\mathbf{H}$ is upper Hessenberg; $\mathbf{Q}$ is orthogonal if $\mathbf{A}$ is real or unitary if $\mathbf{A}$ is complex.
A matrix $\mathbf{H}$ is in upper Hessenberg form if all entries below its first subdiagonal are zero. For example, a $5 \times 5$ upper Hessenberg matrix looks like

| 1 | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | 1 | x | x | x | x |
| 2 | x |  |  |  |  |
| 3 | x | x | x | x | x |
| 4 | 0 | x | x | x | x |
| 5 | 0 | 0 | x | x | x |
|  | 0 | 0 | 0 | x | x |
|  |  |  |  |  |  |

For instance,

$$
: \mathrm{A}
$$

| 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- |


| 1 | 3 | 2 | 1 | -2 | -5 |
| :--- | :--- | :--- | :--- | ---: | ---: |
| 2 | 4 | 2 | 1 | 0 | 3 |
| 3 | 4 | 4 | 0 | 1 | -1 |
| 4 | 5 | 6 | 7 | -2 | 4 |
| 5 | 6 | 7 | 1 | 2 | -1 |
|  |  |  |  |  |  |

```
: hessenbergd(A, H=., Q=.)
```

H
$\begin{array}{llll}1 & 2 & 3 & 4\end{array}$

| 1 | 3 | 2.903464745 | -.552977683 | -4.78764119 | -1.530555451 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| 2 | -9.643650761 | 7.806451613 | 2.878001755 | 5.1085876 | 5.580422694 |
| 3 | 0 | -3.454023879 | -6.119229633 | -.2347200215 | 1.467932097 |
| 4 | 0 | 0 | 1.404136249 | -1.715823624 | -.9870601994 |
| 5 | 0 | 0 | 0 | -2.668128952 | -.971398356 |

Q

| 1 | 2 | 3 | 4 | 5 |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
| 1 | 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | -.4147806779 | -.0368006164 | -.4047768558 | -.8140997488 |
| 4 | 0 | -.4147806779 | -.4871239484 | -.5692309155 | .5163752637 |
| 5 | 0 | -.5184758474 | .8096135604 | -.0748449196 | .2647771074 |
|  | 0 | -.6221710168 | -.3253949238 | .7117092805 | -.0221645995 |

Many algorithms use a Hessenberg decomposition in the process of finding another decomposition with more structure.

## Conformability

hessenbergd $(A, H, Q)$ :
input:

$$
A: \quad n \times n
$$

output:

$$
H: \quad n \times n
$$

$Q: \quad n \times n$
_hessenbergd $(A, Q)$ :
input:
A: $\quad n \times n$
output:
A: $\quad n \times n$
Q: $\quad n \times n$

## Diagnostics

_hessenbergd() aborts with error if $A$ is a view.
hessenbergd() and hessenbergd() return missing results if $A$ contains missing values.

Karl Adolf Hessenberg (1904-1959) was born in Frankfurt am Main, Germany. He was an electrical engineer and gained degrees from the Technische Hochschule Darmstadt. His doctoral dissertation, approved in 1942, was on computation of the eigenvalues and eigensolutions of linear systems of equations. In concurrent work, he introduced what are now called Hessenberg matrices. The mathematician Gerhard Hessenberg was a near relative.

## Also see

[M-1] LAPACK - Linear algebra package (LAPACK) routines
[M-5] schurd() — Schur decomposition
[M-4] Matrix - Matrix functions

## Title

[M-5] Hilbert( ) - Hilbert matrices

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | References | Also see |  |

## Description

Hilbert ( $n$ ) returns the $n \times n$ Hilbert matrix, defined as matrix $H$ with elements $H_{i j}=1 /(i+j-1)$.
invHilbert ( $n$ ) returns the inverse of the $n \times n$ Hilbert matrix, defined as the matrix with elements $-1^{i+j}(i+j-1) \times \operatorname{comb}(n+i-1, n-j) \times \operatorname{comb}(n+j-1, n-i) \times \operatorname{comb}(i+j-2, i-1)^{2}$.

## Syntax

real matrix Hilbert (real scalar $n$ )
real matrix invHilbert (real scalar $n$ )

## Remarks and examples

Hilbert ( $n$ ) and invHilbert ( $n$ ) are used in testing Mata. Hilbert matrices are notoriously ill conditioned. The determinants of the first five Hilbert matrices are $1,1 / 12,1 / 2,160,1 / 6,048,000$, and $1 / 266,716,800,000$.

## Conformability

```
Hilbert( \(n\) ), invHilbert ( \(n\) ):
            \(n: \quad 1 \times 1\)
    result: \(\quad n \times n\)
```


## Diagnostics

If $n$ is not an integer, $\operatorname{trunc}(n)$ is used.

David Hilbert (1862-1943) was born near Königsberg, Prussia (now Kaliningrad, Russia), and studied mathematics at the university there. He joined the staff from 1886 to 1895, when he moved to Göttingen, where he stayed despite tempting offers to move. Hilbert was one of the outstanding mathematicians of his time, producing major work in several fields, including invariant theory, algebraic number theory, the foundations of geometry, functional analysis, integral equations, and the calculus of variations. In 1900 he identified 23 key problems in an address to the Second International Congress of Mathematicians in Paris that continues to influence directions in research (Hilbert 1902). Hilbert's most noteworthy contribution is the concept of a Hilbert space. His work on what are now known as Hilbert matrices was published in 1894.

## References

Choi, M.-D. 1983. Tricks or treats with the Hilbert matrix. American Mathematical Monthly 90: 301-312. https://doi.org/10.2307/2975779.

Hilbert, D. 1894. Ein Beitrag zur Theorie des Legendreschen Polynoms. Acta Mathematica 18: 155-159. https://doi.org/10.1007/BF02418278.
_-. 1902. Mathematical problems. Bulletin of the American Mathematical Society 8: 437-479. https://doi.org/10.1090/S0002-9904-1902-00923-3.

Reid, C. 1970. Hilbert. Berlin: Springer.

## Also see

[M-4] Standard - Functions to create standard matrices

## Title

[M-5] I() — Identity matrix

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

$I(n)$ returns the $n \times n$ identity matrix.
I ( $m, n$ ) returns an $m \times n$ matrix with 1 s down its principal diagonal and 0 s elsewhere.

## Syntax

real matrix $\mathrm{I}($ real scalar $n)$
real matrix $I($ real scalar $m$, real scalar $n$ )

## Remarks and examples

I () must be typed in uppercase.

## Conformability

I ( $n$ ):

$$
\begin{aligned}
& n: \\
& \text { result: } \\
& n \times 1 \times n
\end{aligned}
$$

$I(m, n)$ :

| $m:$ | $1 \times 1$ |
| ---: | :--- |
| $n:$ | $1 \times 1$ |
| result: | $m \times n$ |

## Diagnostics

$I(n)$ aborts with error if $n$ is less than 0 or is missing. $n$ is interpreted as trunc $(n)$.
I ( $m, n$ ) aborts with error if $m$ or $n$ are less than 0 or if they are missing. $m$ and $n$ are interpreted as trunc ( $m$ ) and trunc ( $n$ ).

## Also see

[M-4] Standard - Functions to create standard matrices

## Title

[M-5] inbase( ) - Base conversion

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- | Conformability

## Description

inbase (base , $x$ ) returns a string matrix containing the values of $x$ in base base.
inbase (base, $x$, fdigits) does the same; fdigits specifies the maximum number of digits to the right of the base point to appear in the returned result when $x$ has a fractional part. inbase (base, $x$ ) is equivalent to inbase (base, $x, 8$ ).
inbase (base, $x$, fdigits, err) is the same as inbase(base, $x$, fdigits), except that it returns in err the difference between $x$ and the converted result.
$x=$ frombase (base, $s$ ) is the inverse of $s=$ inbase (base, $x$ ). It returns base base number $s$ as a number. We are tempted to say, "as a number in base 10 ", but that is not exactly true. It returns the result as a real, that is, as an IEEE base-2 double-precision float that, when you display it, is displayed in base 10 .

## Syntax

string matrix inbase(real scalar base, real matrix $x[$, real scalar fdigits $[$, err $]]$ ) real matrix frombase (real scalar base, string matrix s)

## Remarks and examples

Remarks are presented under the following headings:

```
Positive integers
Negative integers
Numbers with nonzero fractional parts
Use of the functions
```


## Positive integers

inbase ( 2,1691 ) is 11010011011 ; that is, 1691 base 10 equals 11010011011 base 2 . frombase ( 2 , " 11010011011 ") is 1691.
inbase ( 3,1691 ) is 2022122 ; that is, 1691 base 10 equals 2022122 base 3 . frombase ( 3 , "2022122") is 1691.
inbase ( 16,1691 ) is 69 b ; that is, 1691 base 10 equals 1691 base 16 . frombase ( 16, " 69 b ") is 1691. (The base-16 digits are $0,1,2,3,4,5,6,7,8,9$, a, b, c, d, e, f.)
inbase ( 62,1691 ) is $r h$; that is, 1691 base 10 equals rh base 62 . frombase ( 62, "rh") is 1691. (The base- 62 digits are $0,1,2,3,4,5,6,7,8,9, a, b, \ldots, z, A, B, \ldots, Z$.

There is a one-to-one correspondence between the integers in different bases. The error of the conversion is always zero.

## Negative integers

Negative integers are no different from positive integers. For instance, inbase (2, -1691) is -11010011011 ; that is, -1691 base 10 equals -11010011011 base 2 .frombase (2, " -11010011011 ") is -1691 .

The error of the conversion is always zero.

## Numbers with nonzero fractional parts

inbase ( $2,3.5$ ) is 11.1 ; that is, 3.5 base 10 equals 11.1 base 2 . frombase ( $2, ~ " 11.1 "$ ) is 3.5 . inbase ( $3,3.5$ ) is 10.11111111.
inbase(3, 3.5, 20) is 10.11111111111111111111.
inbase $(3,3.5,30)$ is 10.111111111111111111111111111111.
Therefore, 3.5 base 10 equals $1.1111 \ldots$ in base 3 . There is no exact representation of one-half in base 3. The errors of the above three conversions are $.0000762079,1.433399 \mathrm{e}-10$, and $2.45650 \mathrm{e}-15$. Those are the values that would be returned in err if inbase (3, 3.5, fdigits, err) were coded.
frombase(3, "10.11111111") is 3.499923792 .
frombase(3, "10.11111111111111111111") is 3.4999999998566 .
frombase(3, "10.1111111111111111111111111111111") is 3.49999999999999734.
inbase ( $16,3.5$ ) is 3.8 ; that is, 3.5 base 10 equals 3.8 base 16 . The error is zero. frombase ( 16 , "3.8") is 3.5 .
inbase ( $62,3.5$ ) is $3 . v$; that is, 3.5 base 10 equals 3.v base 62 . frombase( 62, "3.v") is 3.5 . The error is zero.

In inbase (base, $x$, fdigits), fdigits specifies the maximum number of digits to appear to the right of the base point. fdigits is required to be greater than or equal to 1 . inbase ( $16,3.5$, fdigits) will be 3.8 regardless of the value of fdigits because more digits are unnecessary.

The error that is returned in inbase(base, $x$, fdigits, err) can be an understatement. For instance, inbase ( $16, .1,14$, err) is 0.1999999999999 a and returned in err is 0 even though there is no finite-digit representation of 0.1 base 10 in base 16 . That is because the .1 you specified in the call was not actually 0.1 base 10 . The computer that you are using is binary, and it converted the .1 you typed to

$$
0.00011001100110011001100110011001100110011001100110011010 \text { base } 2
$$

before inbase() was ever called. 0.1999999999999 a base 16 is an exact representation of that number.

## Use of the functions

These functions are used mainly for teaching, especially on the sources and avoidance of roundoff error; see Gould (2006).

The functions can have a use in data processing, however, when used with integer arguments. You have a dataset with 10 -digit identification numbers. You wish to record the 10 -digit number, but more densely. You could convert the number to base 62 . The largest 10 -digit ID number possible is 9999999999 , or aUKYOz base 62 . You can record the ID numbers in a six-character string by using inbase(). If you needed the original numbers back, you could use frombase().

In a similar way, Stata internally uses base 36 for naming temporary files, and that was important when filenames were limited to eight characters. Base 36 allows Stata to generate up to $2,821,109,907,455$ filenames before wrapping of filenames occurs.

## Conformability

```
inbase(base, x, fdigits, err):
```

input:

$$
\begin{array}{rll}
\text { base: } & 1 \times 1 & \\
x: & r \times c & \\
\text { fdigits: } & 1 \times 1 & \text { (optional) }
\end{array}
$$

output:

| err: | $r \times c$ | (optional) |
| ---: | :--- | ---: |
| result: | $r \times c$ |  |

frombase (base, $s$ ):

| base: |  | $1 \times 1$ |
| ---: | :--- | ---: | :--- |
| $s:$ |  | $r \times c$ |
| result $:$ |  | $r \times c$ |

## Diagnostics

The digits used by inbase()/frombase() to encode/decode results are

| 00 | 10 a | 20 k | 30 u | 40 E | 50 O | 60 Y |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 11 | 11 b | 21 l | 31 v | 41 F | 51 P | 61 Z |
| 2 2 | 12 c | 22 m | 32 w | 42 G | 52 Q |  |
| 3 3 | 13 d | 23 n | 33 x | 43 H | 53 R |  |
| 44 | 14 e | 24 o | 34 y | 44 I | 54 S |  |
| 55 | 15 f | 25 p | 35 z | 45 J | 55 T |  |
| 66 | 16 g | 26 q | 36 A | 46 K | 56 U |  |
| 77 | 17 h | 27 r | 37 B | 47 L | 57 V |  |
| 88 | 18 i | 28 s | 38 C | 48 M | 58 W |  |
| 99 | 19 j | 29 t | 39 D | 49 N | 59 X |  |

When base $\leq 36$, frombase() treats A, B, C, ..., as if they were $\mathrm{a}, \mathrm{b}, \mathrm{c}, \ldots$.
inbase (base, $x$, fdigits, err) returns . (missing) if base $<2$, base $>62$, base is not an integer, or $x$ is missing. If fdigits is less than 1 or fdigits is missing, results are as if fdigits $=8$ were specified.
frombase (base, $s$ ) returns . (missing) if base $<2$, base $>62$, base is not an integer, or $s$ is missing; if $s$ is not a valid base base number; or if the converted value of $s$ is greater than $8.988 \mathrm{e}+307$ in absolute value.

## Reference

Gould, W. W. 2006. Mata Matters: Precision. Stata Journal 6: 550-560.

## Also see

[M-4] Mathematical - Important mathematical functions

## Title

[M-5] indexnot( ) — Find byte not in list
Description Syntax Conformability Diagnostics Also see

## Description

indexnot ( $s_{1}, s_{2}$ ) returns the position of the first byte of $s_{1}$ not found in $s_{2}$, or it returns 0 if all bytes of $s_{1}$ are found in $s_{2}$. Note that a Unicode character may contain multiple bytes. Use strlen() or ustrlen() to check if $s_{1}$ or $s_{2}$ has more bytes than its number of Unicode characters.

## Syntax

real matrix indexnot (string matrix $s_{1}$, string matrix $s_{2}$ )

## Conformability

```
indexnot ( }\mp@subsup{s}{1}{},\mp@subsup{s}{2}{})\mathrm{ :
    s,
    s}\mp@subsup{s}{2}{}:\quad\mp@subsup{r}{2}{}\times\mp@subsup{c}{2}{},\quad\mp@subsup{s}{1}{}\mathrm{ and }\mp@subsup{s}{2}{}\mathrm{ r-conformable
    result: }\quad\operatorname{max}(\mp@subsup{r}{1}{},\mp@subsup{r}{2}{})\times\operatorname{max}(\mp@subsup{c}{1}{},\mp@subsup{c}{2}{}
```


## Diagnostics

indexnot $\left(s_{1}, s_{2}\right)$ returns 0 if all bytes of $s_{1}$ are found in $s_{2}$.

## Also see

[M-4] String — String manipulation functions

## Title

[M-5] invorder( ) — Permutation vector manipulation

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

invorder ( $p$ ) returns the permutation vector that undoes the permutation performed by $p$.
revorder $(p)$ returns the permutation vector that is the reverse of the permutation performed by $p$.

## Syntax

```
real vector invorder(real vector p)
real vector revorder(real vector p)
```

where $p$ is assumed to be a permutation vector.

## Remarks and examples

See [M-1] Permutation for a description of permutation vectors. To summarize,

1. Permutation vectors $p$ are used to permute the rows or columns of a matrix $X: r \times c$.

If $p$ is intended to permute the rows of $X$, the permuted $X$ is obtained via $Y=X[p$, .].
If $p$ is intended to permute the columns of $X$, the permuted $X$ is obtained via $Y=X[., p]$.
2. If $p$ is intended to permute the rows of $X$, it is called a row-permutation vector. Rowpermutation vectors are $r \times 1$ column vectors.
3. If $p$ is intended to permute the columns of $X$, it is called a column-permutation vector. Column-permutation vectors are $1 \times c$ row vectors.
4. Row-permutation vectors contain a permutation of the integers 1 to $r$.
5. Column-permutation vectors contain a permutation of the integers 1 to $c$.

Let us assume that $p$ is a row-permutation vector, so that

$$
Y=X[p, .]
$$

invorder $(p)$ returns the row-permutation vector that undoes $p$ :

$$
X=Y[\operatorname{invorder}(p), .]
$$

That is, using the matrix notation of [M-1] Permutation,

$$
Y=P X \quad \text { implies } \quad X=P^{-1} Y
$$

If $p$ is the permutation vector corresponding to permutation matrix $P, \operatorname{invorder}(p)$ is the permutation vector corresponding to permutation matrix $P^{-1}$.
revorder $(p)$ returns the permutation vector that reverses the order of $p$. For instance, say that row-permutation vector $p$ permutes the rows of $X$ so that the diagonal elements are in ascending order. Then revorder ( $p$ ) would permute the rows of $X$ so that the diagonal elements would be in descending order.

## Conformability

```
invorder(p), revorder (p):
    p: r}\times1\mathrm{ or }1\times
    result: }r\times1\mathrm{ or }1\times
```


## Diagnostics

invorder ( $p$ ) and revorder ( $p$ ) can abort with error or can produce meaningless results when $p$ is not a permutation vector.

## Also see

[M-1] Permutation - An aside on permutation matrices and vectors
[M-4] Manipulation - Matrix manipulation

## Title

[M-5] invsym( ) - Symmetric real matrix inversion

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

invsym ( $A$ ) returns a generalized inverse of real, symmetric, positive-semidefinite matrix $A$. invsym ( $A$, order) does the same but allows you to specify which columns are to be swept first.
_invsym ( $A$ ) and _invsym ( $A$, order) do the same thing as invsym ( $A$ ) and invsym ( $A$, order ) except that $A$ is replaced with the generalized inverse result rather than the result being returned. _invsym() uses less memory than invsym().
invsym() and _invsym() are the routines Stata uses for calculating inverses of symmetric matrices.
Also see $[\mathrm{M}-5]$ luinv( ), $[\mathrm{M}-5] \operatorname{qrinv}()$, and [M-5] $\operatorname{pinv()~for~general~matrix~inversion.~}$

## Syntax

| real matrix | invsym (real matrix $A)$ |
| :--- | :--- |
| real matrix | invsym $($ real matrix $A$, real vector order $)$ |
| void | -invsym $($ real matrix $A)$ |
| void | _invsym (real matrix $A$, real vector order $)$ |

## Remarks and examples

Remarks are presented under the following headings:

```
Definition of generalized inverse
Specifying the order in which columns are omitted Determining the rank, or counting the number of omitted columns Extracting linear dependencies
```


## Definition of generalized inverse

When the matrix is full rank and positive semidefinite, the matrix $A$ becomes positive definite and the generalized inverse equals the inverse, that is, assuming $A$ is $n \times n$,

$$
\operatorname{invsym}(A) * A=A * \operatorname{invsym}(A)=I(n)
$$

or, at least the above restriction is true up to roundoff error. When $A$ is not full rank, the generalized inverse invsym() satisfies (ignoring roundoff error)

$$
A * \operatorname{invsym}(A) * A=A
$$

$$
\operatorname{invsym}(A) * A * \operatorname{invsym}(A)=\operatorname{invsym}(A)
$$

In the generalized case, there are an infinite number of inverse matrices that can satisfy the above restrictions. The one invsym() chooses is one that sets entire columns (and therefore rows) to 0 , thus treating $A$ as if it were of reduced dimension. Which columns (rows) are selected is determined on the basis of minimizing roundoff error.

In the above we talk as if determining whether a matrix is of full rank is an easy calculation. That is not true. Because of the roundoff error in the manufacturing and recording of $A$ itself, columns that ought to be perfectly collinear will not be and yet you will still want invsym() to behave as if they were. invsym() tolerates a little deviation from collinearity in making the perfectly collinear determination.

## Specifying the order in which columns are omitted

Left to make the decision itself, invsym() will choose which columns to drop (to set to 0) to minimize the overall roundoff error of the generalized inverse calculation. If column 1 and column 3 are collinear, then invsym() will choose to drop column 1 or column 3.

There are occasions, however, when you would like to ensure that a particular column or set of columns are not omitted. Perhaps column 1 corresponds to the intercept of a regression model and you would much rather, if one of columns 1 and 3 has to be omitted, that it be column 3 .

Order allows you to specify the columns of the matrix that you would prefer not be omitted in the generalized inverse calculation. In the above example, to prevent column 1 from being omitted, you could code

```
invsym(A, 1)
```

If you would like to keep columns 1,5 , and 10 from being omitted, you can code

```
invsym(A, (1,5,10))
```

Specifying columns not to be omitted does not guarantee that they will not be omitted because they still might be collinear with each other or they might equal constants. However, if any other column can be omitted to satisfy your desire, it will be.

## Determining the rank, or counting the number of omitted columns

If a column is omitted, 0 will appear on the corresponding diagonal entry. Hence, the rank of the original matrix can be extracted after inversion by invsym():

```
: Ainv = invsym(A)
: rank = rows(Ainv)-diag0cnt(Ainv)
```

See [M-5] diag0cnt().

## Extracting linear dependencies

The linear dependencies can be read from the rows of $A * \operatorname{invsym}(A)$ :

```
: A*invsym(A)
```

| 1 | 2 | 3 |  |
| :--- | ---: | ---: | ---: |
| 1 | 0 | -1 | 1 |
| 2 | 0 | 1 | $-2.22045 \mathrm{e}-16$ |
| 3 | 0 | 0 | 1 |
|  |  |  |  |

The above is interpreted to mean

$$
\begin{aligned}
& x_{1}=-x_{2}+x_{3} \\
& x_{2}=x_{2} \\
& x_{3}=x_{3}
\end{aligned}
$$

ignoring roundoff error.

## Conformability

invsym (A), invsym ( $A$, order):
A: $\quad n \times n$
order: $\quad 1 \times k \quad$ or $\quad k \times 1, k \leq n \quad$ (optional) result: $\quad n \times n$
_invsym ( $A$ ), _invsym ( $A$, order):
A: $\quad n \times n$
order: $\quad 1 \times k \quad$ or $\quad k \times 1, k \leq n \quad$ (optional)
output:
A: $\quad n \times n$

## Diagnostics

$\operatorname{invsym}(A), \operatorname{invsym}(A, \operatorname{order})$, _invsym( $A$ ), and _invsym ( $A$, order) assume that $A$ is symmetric; they do not check. If $A$ is nonsymmetric, they treat it as if it were symmetric and equal to its upper triangle.
invsym() and _invsym() return a result containing missing values if $A$ contains missing values.
_invsym() aborts with error if $A$ is a view. Both functions abort with argument-out-of-range error if order is specified and contains values less than 1, greater than rows ( $A$ ), or the same value more than once.
invsym() and _invsym() return a matrix of zeros if $A$ is not positive semidefinite.

## Also see

[M-5] cholinv( ) - Symmetric, positive-definite matrix inversion
[M-5] diag0cnt( ) - Count zeros on diagonal
[M-5] luinv() - Square matrix inversion
[M-5] $\operatorname{pinv}()$ - Moore-Penrose pseudoinverse
[M-5] qrinv() - Generalized inverse of matrix via QR decomposition
[M-4] Matrix - Matrix functions
[M-4] Solvers - Functions to solve $A X=B$ and to obtain A inverse

## Title

[M-5] invtokens() — Concatenate string rowvector into string scalar

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

invtokens ( $s$ ) returns the elements of $s$, concatenated into a string scalar with the elements separated by spaces. invtokens ( $s$ ) is equivalent to invtokens ( $s$, " ").
invtokens $(s, c)$ returns the elements of $s$, concatenated into a string scalar with the elements separated by $c$.

## Syntax

```
string scalar invtokens(string rowvector s)
string scalar invtokens(string rowvector s, string scalar c)
```


## Remarks and examples

invtokens ( $s$ ) is the inverse of tokens() (see [M-5] tokens()); invtokens() returns the string obtained by concatenating the elements of $s$ into a space-separated list.
invtokens $(s, c)$ places $c$ between the elements of $s$ even when the elements of $s$ are equal to " ". For instance,

```
: s = ("alpha", "", "gamma", "")
invtokens(s, ";")
    alpha;;gamma;
```

To remove separators between empty elements, use select() (see [M-5] select()) to remove the empty elements from $s$ beforehand:

```
: s2 = select(s, strlen(s):>0)
: s2
            1 2
    1
        alpha gamma
invtokens(s2, ";")
    alpha;gamma
```


## Conformability

invtokens $(s, c)$ :

```
            s: }1\times
    c: 1 1 1 (optional)
result: }1\times
```


## Diagnostics

If $s$ is $1 \times 0$, invtokens $(s, c)$ returns " ".

## Also see

[M-5] tokenget() - Advanced parsing
[M-5] tokens() - Obtain tokens from string
[M-5] ustrword() — Obtain Unicode word from Unicode string
[M-4] String - String manipulation functions
[ P$]$ gettoken - Low-level parsing
[P] tokenize - Divide strings into tokens

## Title

[M-5] isascii() — Whether string scalar contains only ASCII codes
Description Syntax Conformability Also see

## Description

isascii $(s)$ returns 1 if string scalar $s$ contains only ASCII codes ( $0-127$ ) and 0 otherwise.

## Syntax

```
real scalar isascii(string scalar s)
```


## Conformability

```
isascii(s):
    s: }\quad1\times
    result: }1\times
```


## Also see

[M-5] ascii( ) - Manipulate ASCII and byte codes
[M-4] String - String manipulation functions

## Title

[M-5] isdiagonal() - Whether matrix is diagonal

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

isdiagonal ( $A$ ) returns 1 if $A$ has only zeros off the principal diagonal and returns 0 otherwise. isdiagonal() may be used with either real or complex matrices.

## Syntax

```
real scalar isdiagonal(numeric matrix A)
```


## Remarks and examples

See [M-5] diag() for making diagonal matrices out of vectors or out of nondiagonal matrices; see [M-5] diagonal() for extracting the diagonal of a matrix into a vector.

## Conformability

isdiagonal(A):
A: $r \times c$
result: $1 \times 1$

## Diagnostics

isdiagonal ( $A$ ) returns 1 if $A$ is void.

## Also see

[M-5] diag() - Create diagonal matrix
[M-5] diagonal() - Extract diagonal into column vector
[M-4] Utility - Matrix utility functions

## Title

[M-5] isfleeting() - Whether argument is temporary

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

isfleeting ( $A$ ) returns 1 if $A$ was constructed for the sole purpose of passing to your function, and returns 0 otherwise. If an argument is fleeting, then you may change its contents and be assured that the caller will not care or even know.

## Syntax

```
real scalar isfleeting(polymorphic matrix A)
```

where $A$ is an argument passed to your function.

## Remarks and examples

Let us assume that you have written function myfunc ( $A$ ) that takes $A$ : $r \times c$ and returns an $r \times c$ matrix. Just to fix ideas, we will pretend that the code for myfunc() reads

```
real matrix myfunc(real matrix A)
{
    real scalar i
    real matrix B
    B=A
    for (i=1; i<=rows(B); i++) B[i,i] = 1
    return(B)
}
```

Function myfunc $(A)$ returns a matrix equal to $A$, but with ones along the diagonal. Now let's imagine myfunc () in use. A snippet of the code might read

```
C = A*myfunc(D)*C
```

Here $D$ is passed to myfunc (), and the argument $D$ is said not to be fleeting. Now consider another code snippet:

$$
\mathrm{D}=\mathrm{A} * \mathrm{my} f \mathrm{unc}_{\mathrm{c}}(\mathrm{D}+\mathrm{E}) * \mathrm{D}
$$

In this code snippet, the argument passed to myfunc() is D+E and that argument is fleeting. It is fleeting because it was constructed for the sole purpose of being passed to myfunc (), and once myfunc () concludes, the matrix containing $\mathrm{D}+\mathrm{E}$ will be discarded.

Arguments that are fleeting can be reused to save memory.
Look carefully at the code for myfunc(). It makes a copy of the matrix that it was passed. It did that to avoid damaging the matrix that it was passed. Making that copy, however, is unnecessary if the argument received was fleeting, because damaging something that would have been discarded anyway does not matter. Had we not made the copy, we would have saved not only computer time but also memory. Function myfunc () could be recoded to read

```
real matrix myfunc(real matrix A)
{
    real scalar i
    real matrix B
    if (isfleeting(A)) {
        for (i=1; i<=rows(A); i++) A[i,i] = 1
        return(A)
    }
    B=A
    for (i=1; i<=rows(B); i++) B[i,i] = 1
    return(B)
}
```

Here we wrote separate code for the fleeting and nonfleeting cases. That is not always necessary. We could use a pointer here to combine the two code blocks:

```
real matrix myfunc(real matrix A)
    {
        real scalar i
        real matrix B
        pointer scalar p
        if (isfleeting(A)) p = &A
        else {
            B=A
            p = &B
            }
        for (i=1; i<=rows(*p); i++) (*p)[i,i] = 1
        return(*p)
}
```

Many official library functions come in two varieties: _foo ( $A, \ldots$ ), which replaces $A$ with the calculated result, and foo ( $A, \ldots$ ), which returns the result leaving $A$ unmodified. Invariably, the code for foo() reads

```
function foo(A, ...)
{
    matrix B
    if (isfleeting(A)) {
        _foo(A, ...)
        return(A)
    }
    _foo(B=A, ...)
    return(B)
}
```

This makes function foo() whoppingly efficient. If foo() is called with a temporary argument-an argument that could be modified without the caller being aware of it-then no extra copy of the matrix is ever made.

## Conformability

```
isfleeting(A):
    A: }\quadr\times
    result: }1\times
```


## Diagnostics

isfleeting $(A)$ returns 1 if $A$ is fleeting and not a view. The value returned is indeterminate if $A$ is not an argument of the function in which it appears, and therefore the value of isfleeting () is also indeterminate when used interactively.

## Also see

[M-4] Programming — Programming functions

## Title

[M-5] isreal() — Storage type of matrix

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

isreal ( $X$ ) returns 1 if $X$ is a real and returns 0 otherwise.
iscomplex $(X)$ returns 1 if $X$ is a complex and returns 0 otherwise.
isstring $(X)$ returns 1 if $X$ is a string and returns 0 otherwise.
ispointer $(X)$ returns 1 if $X$ is a pointer and returns 0 otherwise.

## Syntax

```
real scalar isreal(transmorphic matrix X)
real scalar iscomplex(transmorphic matrix X)
real scalar isstring(transmorphic matrix X)
real scalar ispointer(transmorphic matrix X)
```


## Remarks and examples

These functions base their results on storage type. isreal () is not the way to check whether a number is real, since it might be stored as a complex and yet still be a real number, such as $2+0 \mathrm{i}$. To determine whether $x$ is real, you want to use isrealvalues $(X)$; see [M-5] isrealvalues( ).

## Conformability

```
isreal(X), iscomplex(X), isstring(X), ispointer(X):
    X: r
    result: }1\times
```


## Diagnostics

These functions return 1 or 0 ; they cannot fail.

## Also see

[M-5] eltype( ) - Element type, organizational type, and type name of object
[M-5] isrealvalues() - Whether matrix contains only real values
[M-4] Utility - Matrix utility functions

## Title

[M-5] isrealvalues() - Whether matrix contains only real values

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

isrealvalues $(X)$ returns 1 if $X$ is of storage type real or if $X$ is of storage type complex and all elements are real or missing; 0 is returned otherwise.

## Syntax

real scalar isrealvalues (numeric matrix $X$ )

## Remarks and examples

isrealvalues $(X)$ is logically equivalent to $X==\operatorname{Re}(X)$ but is significantly faster and requires less memory.

## Conformability

```
isrealvalues(X):
            X: r
    result: }1\times
```


## Diagnostics

isrealvalues $(X)$ returns 1 if $X$ is void and complex.

## Also see

[M-5] isreal() - Storage type of matrix
[M-4] Utility - Matrix utility functions

## Title

[M-5] issamefile( ) - Whether two file paths are pointing to the same file
Description Syntax Remarks and examples Conformability Also see

## Description

issamefile (string scalar path1, string scalar path2) returns 1 if pathl and path2 point to the same file (the file must exist in the file system) and 0 otherwise.

## Syntax

real scalar issamefile(string scalar path1, string scalar path2)

## Remarks and examples

issamefile("C:/test/test.do", "./test.do") returns 1 if the current working directory is C:/test/ and file C:/test/test.do exists.
issamefile() returns 0 if path1 and path2 do not point to the same file or if pathl or path2 points to a file that does not exist.

## Conformability

```
issamefile(path1, path2):
    pathl: }\quad1\times
    path2: }1\times
    result: }1\times
```


## Also see

[M-4] IO - I/O functions

## Title

[M-5] issymmetric() — Whether matrix is symmetric (Hermitian)

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Reference | Also see |  |

## Description

issymmetric ( $A$ ) returns 1 if $A==A^{\prime}$ and returns 0 otherwise. (Also see mreldifsym() in [M-5] reldif( ).)
issymmetriconly ( $A$ ) returns 1 if $A==$ transposeonly $(A)$ and returns 0 otherwise.

## Syntax

```
real scalar issymmetric(transmorphic matrix A)
real scalar issymmetriconly(transmorphic matrix A)
```


## Remarks and examples

issymmetric ( $A$ ) and issymmetriconly ( $A$ ) return the same result except when $A$ is complex.
In the complex case, issymmetric ( $A$ ) returns 1 if $A$ is equal to its conjugate transpose, that is, if $A$ is Hermitian, which is the complex analog of symmetric. $A$ is symmetric (Hermitian) if its off-diagonal elements are conjugates of each other and its diagonal elements are real.
issymmetriconly $(A)$, on the other hand, uses the mechanical definition of symmetry: $A$ is symmetriconly [sic] if its off-diagonal elements are equal. issymmetriconly() is uninteresting, mathematically speaking, but can be useful in certain data management programming situations.

## Conformability

```
issymmetric(A), issymmetriconly(A):
    A: }\quad\begin{array}{rl}{r\timesc}\\{\mathrm{ result: }}&{1\times1}
```


## Diagnostics

issymmetric ( $A$ ) returns 0 if $A$ is not square. If $A$ is $0 \times 0$, it is symmetric.
issymmetriconly $(A)$ returns 0 if $A$ is not square. If $A$ is $0 \times 0$, it is symmetriconly.

Charles Hermite (1822-1901) was born in Dieuze in eastern France and early showed an aptitude for mathematics, publishing two papers before entering university. He started studying at the Ecole Polytechnique but left after 1 year because of difficulties from a defect in his right foot. However, Hermite was soon appointed to a post at the same institution and later at the Sorbonne. He made outstanding contributions to number theory, algebra, and especially analysis, and he published the first proof that $e$ is transcendental. Hermite's name carries on in Hermite polynomials, Hermite's differential equation, Hermite's formula of interpolation, and Hermitian matrices.

## Reference

James, I. M. 2002. Remarkable Mathematicians: From Euler to von Neumann. Cambridge: Cambridge University Press.

## Also see

[M-5] makesymmetric () - Make square matrix symmetric (Hermitian)
[M-5] reldif( ) - Relative/absolute difference
[M-4] Utility - Matrix utility functions

## Title

[M-5] isview( ) - Whether matrix is view

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

isview $(X)$ returns 1 if $X$ is a view and 0 otherwise.
See [M-6] Glossary for a definition of a view.

## Syntax

real scalar isview (transmorphic matrix $X$ )

## Remarks and examples

View matrices are created by st_view(); see [M-5] st_view().

## Conformability

## isview $(X)$ :

$$
\begin{aligned}
X: & r \times c \\
\text { result: } & 1 \times 1
\end{aligned}
$$

## Diagnostics

None.

## Also see

[M-5] eltype( ) - Element type, organizational type, and type name of object
[M-4] Utility - Matrix utility functions

## Title

[M-5] J() - Matrix of constants

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

$\mathrm{J}(r, c, v a l)$ returns an $r \times c$ matrix with each element equal to val.
$\mathrm{J}(r, c, m a t)$ returns an $(r * \operatorname{rows}(m a t)) \times(c * \operatorname{cols}(m a t))$ matrix with elements equal to mat.
The first, $\mathrm{J}(r, c, v a l)$, is how J() is commonly used. The first is nothing more than a special case of the second, $\mathrm{J}(r, c$, mat), when mat is $1 \times 1$.

## Syntax

transmorphic matrix J (real scalar $r$, real scalar $c$, scalar val)
transmorphic matrix J (real scalar $r$, real scalar $c$, matrix mat)

## Remarks and examples

Remarks are presented under the following headings:
First syntax: $J(r, c$, val), val a scalar
Second syntax: J(r, c, mat), mat a matrix

## First syntax: J(r, c, val), val a scalar

$J(r, c, v a l)$ creates matrices of constants. For example, $J(2,3,0)$ creates

|  | 1 | 2 | 3 |
| :--- | :--- | :--- | :--- |
|  | 0 0 0 <br> 0 0 0 |  |  |

J () must be typed in uppercase.
$J$ () can create any type of matrix:

| Function | Returns |
| :--- | :--- |
| $J(2,3,4)$ | $2 \times 3$ real matrix, each element $=4$ |
| $J(2,3,4+5 i)$ | $2 \times 3$ complex matrix, each element $=4+5 i$ |
| $J(2,3$, "hi" $)$ | $2 \times 3$ string matrix, each element $=$ "hi" |
| $J(2,3, \& x)$ | $2 \times 3$ pointer matrix, each element $=$ address of $x$ |

Also, J() can create void matrices:

| $\mathrm{J}(0,0,)$. | $0 \times 0$ real |
| :--- | :--- |
| $\mathrm{J}(0,1,)$. | $0 \times 1$ real |
| $\mathrm{J}(1,0,)$. | $1 \times 0$ real |
| $\mathrm{J}(0,0,1 i)$ | $0 \times 0$ complex |
| $\mathrm{J}(0,1,1 i)$ | $0 \times 1$ complex |
| $\mathrm{J}(1,0,1 i)$ | $1 \times 0$ complex |
| $\mathrm{J}(0,0, " ")$ | $0 \times 0$ string |
| $\mathrm{J}(0,1, " ")$ | $0 \times 1$ string |
| $\mathrm{J}(1,0, " ")$ | $1 \times 0$ string |
| $\mathrm{J}(0,0, N U L L)$ | $0 \times 0$ pointer |
| $\mathrm{J}(0,1, N U L L)$ | $0 \times 1$ pointer |
| $\mathrm{J}(1,0$, NULL $)$ | $1 \times 0$ pointer |

When $\mathrm{J}(r, c, v a l)$ is used to create a void matrix, the particular value of the third argument does not matter. Its element type, however, determines the type of matrix produced. Thus, $\mathrm{J}(0,0,$.$) ,$ $J(0,0,1)$, and $J(0,0,1 / 3)$ all create the same result: a $0 \times 0$ real matrix. Similarly, $J(0,0$, ""), $J(0,0$, "name"), and $J(0,0, " ? ")$ all create the same result: a $0 \times 0$ string matrix. See [M-2] void to learn how void matrices are used.

## Second syntax: J(r, c, mat), mat a matrix

$\mathrm{J}(r, c, m a t)$ is a generalization of $\mathrm{J}(r, c, v a l)$. When the third argument is a matrix, that matrix is replicated in the result. For instance, if $X$ is $(1,2 \backslash 3,4)$, then $J(2,3, X)$ creates

| 1 | 2 | 3 | 4 | 5 | 6 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 1 | 1 | 2 | 1 | 2 | 1 |
| 2 |  |  |  |  |  |  |
| 3 | 3 | 4 | 3 | 4 | 3 | 4 |
| 4 | 1 | 2 | 1 | 2 | 1 | 2 |
| 3 | 4 | 3 | 4 | 3 | 4 |  |

$\mathrm{J}(r, c, v a l)$ is a special case of $\mathrm{J}(r, c, m a t)$; it just happens that mat is $1 \times 1$.
The matrix created has $r$ *rows (mat) rows and $c * \operatorname{cols}$ (mat) columns.
Note that $\mathrm{J}(r, c, m a t)$ creates a void matrix if any of $r, c$, rows (mat), or cols (mat) are zero.

## Conformability

$\mathrm{J}(r, c, v a l):$

| $r:$ | $1 \times 1$ |
| ---: | :--- |
| $c:$ | $1 \times 1$ |
| val: | $1 \times 1$ |
| result: | $r \times c$ |

$\mathrm{J}(r, c, m a t):$

| $r:$ |  | 1 | $\times 1$ |
| ---: | :--- | ---: | :--- |
| $c:$ |  | 1 | $\times 1$ |
| mat: |  | $m \times n$ |  |
| result $:$ |  | $r^{*} m \times$ | $\times c^{*} n$ |

## Diagnostics

$\mathrm{J}(r, c$, val $)$ and $\mathrm{J}(r, c$, mat $)$ abort with error if $r<0$ or $c<0$, or if $r \geq$. or $c \geq \ldots$ Arguments $r$ and $c$ are interpreted as trunc ( $r$ ) and trunc (c).

## Also see

[M-5] missingof( ) - Appropriate missing value
[M-4] Standard - Functions to create standard matrices

## Title

[M-5] Kmatrix() - Commutation matrix

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Reference | Also see |  |

## Description

Kmatrix $(m, n)$ returns the $m n \times m n$ commutation matrix K for which $\mathrm{K} * \operatorname{vec}(X)=\operatorname{vec}\left(X^{\prime}\right)$, where $X$ is an $m \times n$ matrix.

## Syntax

real matrix Kmatrix (real scalar m, real scalar $n$ )

## Remarks and examples

Commutation matrices are frequently used in computing derivatives of functions of matrices. Section 9.2 of Lütkepohl (1996) lists many useful properties of commutation matrices.

## Conformability

Kmatrix $(m, n)$ :

| $m:$ | $1 \times 1$ |
| ---: | :--- |
| $n:$ | $1 \times 1$ |
| result: | $m n \times m n$ |

## Diagnostics

Kmatrix $(m, n)$ aborts with error if either $m$ or $n$ is less than 0 or is missing. $m$ and $n$ are interpreted as trunc $(m)$ and trunc $(n)$.

## Reference

Lütkepohl, H. 1996. Handbook of Matrices. New York: Wiley.

## Also see

[M-5] Dmatrix( ) - Duplication matrix
[M-5] Lmatrix( ) - Elimination matrix
[M-5] vec() - Stack matrix columns
[M-4] Standard - Functions to create standard matrices

## Title

[M-5] lapack( ) — Linear algebra package (LAPACK) functions
Description Syntax Remarks and examples Reference Also see

## Description

LA_DGBMV (), LA_DGEBAK (), LA_ZGEBAK (), LA_DGEBAL(), LA_ZGEBAL(), . . are LAPACK functions in original, as-is form. These functions form the basis for many of Mata's linear algebra capabilities. Mata functions such as cholesky(), svd(), and eigensystem() are implemented using these functions; see [M-4] Matrix. Those functions are easier to use. The LA_*() functions provide more capability. The LA-*() functions use only Netlib library routines, not MKL library routines. For details about the two libraries, see [M-1] LAPACK.
_flopin() and _flopout() convert matrices to and from the form required by the LA_*() functions.

## Syntax

void _flopin(numeric matrix A)
void lapack_function (...)
void _flopout (numeric matrix A)
where lapack_function may be
LA_DGBMV ()
LA_DGEBAK () LA_ZGEBAK ()
LA_DGEBAL () LA_ZGEBAL ()
LA_DGEES() LA_ZGEES ()
LA_DGEEV () LA_ZGEEV ()
LA_DGEHRD () LA_ZGEHRD ()
LA_DGGBAK () LA_ZGGBAK ()
LA_DGGBAL() LA_ZGGBAL()
LA_DGGHRD () LA_ZGGHRD ()
LA_DHGEQZ() LA_ZHGEQZ()
LA_DHSEIN() LA_ZHSEIN()
LA_DHSEQR() LA_ZHSEQR()
LA_DLAMCH()
LA_DORGHR()
LA_DSYEVX ()
LA_DTGSEN() LA_ZTGSEN()
LA_DTGEVC() LA_ZTGEVC()
LA_DTREVC() LA_ZTREVC()
LA_DTRSEN() LA_ZTRSEN ()
LA_ZUNGHR()

## Remarks and examples

LAPACK stands for Linear Algebra PACKage and is a freely available set of Fortran 90 routines for solving systems of simultaneous equations, eigenvalue problems, and singular-value problems. The original Fortran routines have six-letter names like DGEHRD, DORGHR, and so on. The Mata functions LA_DGEHRD (), LA_DORGHR (), etc., are a subset of the LAPACK double-precision real and complex routine. All LAPACK double-precision functions will eventually be made available.

Documentation for the LAPACK routines can be found at http://www.netlib.org/lapack/, although we recommend obtaining LAPACK Users' Guide by Anderson et al. (1999).

All functions listed here are implemented with Netlib's LAPACK library. They are not based on the new Intel MKL library, and they are not affected by the settings lapack_mkl and lapack_mkl_cnr; see [M-1] LAPACK.

Remarks are presented under the following headings:

> Mapping calling sequence from Fortran to Mata
> Flopping: Preparing matrices for LAPACK
> Warning on the use of rows( ) and cols() after_flopin()
> Warning: It is your responsibility to check info
> Example

## Mapping calling sequence from Fortran to Mata

LAPACK functions are named with first letter S, D, C, or Z. S means single-precision real, D means double-precision real, C means single-precision complex, and Z means double-precision complex. Mata provides the $\mathrm{D}^{*}$ and $\mathrm{Z}^{*}$ functions. The LAPACK documentation is in terms of $\mathrm{S}^{*}$ and $\mathrm{C}^{*}$. Thus, to find the documentation for LA_DGEHRD, you must look up SGEHRD in the original documentation.

The documentation (Anderson et al. 1999, 227) reads, in part,
SUBROUTINE SGEHRD(N, ILO, IHI, A, LDA, TAU, WORK, LWORK, INFO)

INTEGER IHI, ILO, INFO, LDA, LWORK, N
REAL A(LDA, *), TAU(*), WORK(LWORK)
and the documentation states that SGEHDR reduces a real, general matrix, A, to upper Hessenberg form, $\mathbf{H}$, by an orthogonal similarity transformation: $\mathbf{Q}^{\prime} \times \mathbf{A} \times \mathbf{Q}=\mathbf{H}$.

The corresponding Mata function, LA_DGEHRD (), has the same arguments. In Mata, arguments ihi, ilo, info, lda, lwork, and n are real scalars. Argument A is a real matrix, and arguments tau and work are real vectors.

You can read the rest of the original documentation to find out what is to be placed (or returned) in each argument. It turns out that $\mathbf{A}$ is assumed to be dimensioned LDA $\times$ something and that the routine works on $\mathbf{A}(1,1)$ (using Fortran notation) through $\mathbf{A}(N, N)$. The routine also needs work space, which you are to supply in vector WORK. In the standard LAPACK way, LAPACK offers you a choice: you can preallocate WORK, in which case you have to choose a fairly large dimension for it, or you can do a query to find out how large the dimension needs to be for this particular problem. If you preallocate, the documentation reveals that the WORK must be of size $N$, and you set LWORK equal to $N$. If you wish to query, then you make WORK of size 1 and set LWORK equal to -1 . The LAPACK routine will then return in the first element of WORK the optimal size. Then you call the function again with WORK allocated to be the optimal size and LWORK set to equal the optimal size.

Concerning Mata, the above works. You can follow the LAPACK documentation to the letter. Use $J$ () to allocate matrices or vectors. Alternatively, you can specify all sizes as missing value (.), and Mata will fill in the appropriate value based on the assumption that you are using the entire matrix. Thus, in LA_DGEHRD (), you could specify lda as missing, and the function would run as if you had specified lda equal to cols(A). You could specify $n$ as missing, and the function would run as if you had specified $n$ as rows (A).

Work areas, however, are treated differently. You can follow the standard LAPACK convention outlined above; or you can specify the sizes of work areas (lwork) and specify the work areas themselves (work) as missing values, and Mata will allocate the work areas for you. The allocation will be as you specified.

One feature provided by some LAPACK functions is not supported by the Mata implementation. If a function allows a function pointer, you may not avail yourself of that option.

## Flopping: Preparing matrices for LAPACK

The LAPACK functions provided in Mata are the original LAPACK functions. Mata, which is C based, stores matrices rowwise. LAPACK, which is Fortran based, stores matrices columnwise. Mata and Fortran also disagree on how complex matrices are to be organized.

Functions _flopin() and _flopout() handle these issues. Coding _flopin(A) changes matrix $A$ from the Mata convention to the LAPACK convention. Coding $\quad$ flopout ( $A$ ) changes $A$ from the LAPACK convention to the Mata convention.

The LA_*() functions do not do this for you because LAPACK often takes two or three LAPACK functions run in sequence to achieve the desired result, and it would be a waste of computer time to switch conventions between calls.

## Warning on the use of rows() and cols() after _flopin()

Be careful using the rows () and cols () functions. rows () of a flopped matrix returns the logical number of columns and cols() of a flopped matrix returns the logical number of rows!

The danger of confusion is especially great when using $J()$ to allocate work areas. If a LAPACK function requires a work area of $r \times c$, you code,

$$
\text { _LA_function(..., J }(c, r, .), \ldots)
$$

## Warning: It is your responsibility to check info

The LAPACK functions do not abort with error on failure. They instead store 0 in info (usually the last argument) if successful and store an error code if not successful. The error code is usually negative and indicates the argument that is a problem.

## Example

The following example uses the LAPACK function DGEHRD to obtain the Hessenberg form of matrix A. We will begin with

| 1 | 2 | 3 | 4 |  |
| :--- | ---: | ---: | ---: | ---: |
| 1 | 1 | 2 | 3 | 4 |
| 2 | 4 | 5 | 6 | 7 |
| 3 | 7 | 8 | 9 | 10 |
| 4 | 8 | 9 | 10 | 11 |
|  |  |  |  |  |

The first step is to use _flopin() to put $\mathbf{A}$ in LAPACK order:

```
: _flopin(A)
```

Next we make a work-space query to get the optimal size of the work area.

```
: LA_DGEHRD(., 1, 4, A, ., tau=., work=., lwork=-1, info=0)
: lwork = work[1,1]
: lwork
    128
```

After putting the work-space size in lwork, we can call LA_DGEHRD () again to perform the Hessenberg decomposition:

```
: LA_DGEHRD(., 1, 4, A, ., tau=., work=., lwork, info=0)
```

LAPACK function DGEHRD saves the result in the upper triangle and the first subdiagonal of $\mathbf{A}$. We must use _flopout () to change that back to Mata order, and finally, we extract the result:

```
_flopout(A)
A = A-sublowertriangle(A, 2)
A
```

    \(\begin{array}{llll}1 & 2 & 3 & 4\end{array}\)
    $\begin{array}{|rllr|}1 & -5.370750529 & .0345341258 & .3922322703 \\ -11.35781669 & 25.18604651 & -4.40577178 & -.6561483899 \\ 0 & -1.660145888 & -.1860465116 & .1760901813 \\ 0 & 0 & -8.32667 \mathrm{e}-16 & -5.27356 \mathrm{e}-16\end{array}$

## Reference

Anderson, E., Z. Bai, C. Bischof, S. Blackford, J. Demmel, J. J. Dongarra, J. Du Croz, A. Greenbaum, S. Hammarling, A. McKenney, and D. Sorensen. 1999. LAPACK Users' Guide. 3rd ed. Philadelphia: Society for Industrial and Applied Mathematics.

## Also see

[M-1] LAPACK - Linear algebra package (LAPACK) routines
[M-4] Matrix - Matrix functions
[R] Copyright LAPACK - LAPACK copyright notification

## Title

[M-5] Idl( ) — Bunch-Kaufman decomposition

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

ldl ( $A, L, D, p$ ) returns the Bunch-Kaufman decomposition (with diagonal pivoting) of $A$ in a permuted lower-triangular matrix $L$ and a symmetric block-diagonal matrix $D$ with $1 \times 1$ and $2 \times 2$ diagonal blocks, along with a permutation vector $p$.

With the permutation vector, $L[p,$.$] becomes a lower-triangular matrix with unit diagonal.$
Up to roundoff error, the returned results are such that

$$
A[p, p]=L[p, .] * D * L[p, .]^{\prime}
$$

$\operatorname{ldl}(A, L, D)$ is similar to $\operatorname{ldl}(A, L, D, p)$, but the permutation vector $p$ is omitted from the output.

Up to roundoff error, the returned results are such that

$$
A=L * D * L^{\prime}
$$

## Syntax

```
void ldl(numeric matrix A,L,D)
void ldl(numeric matrix A,L,D, p)
```

where

1. $A$ is symmetric (Hermitian) indefinite.
2. the types of $L, D$, and $p$ are irrelevant; results are returned there.

## Remarks and examples

The Bunch-Kaufman decomposition is a generalization of the Cholesky decomposition.
Bunch-Kaufman decomposition of matrix $A$ can be written as

$$
P A P^{\prime}=P L D L^{\prime} P^{\prime}
$$

where $P$ is a permutation matrix that permutes the rows of A .
$L$ is the permuted lower-triangular matrix. With the permutation matrix $P, P L$ is a lower-triangular matrix with unit diagonal.
$D$ is the symmetric block-diagonal matrix $D$ with $1 \times 1$ and $2 \times 2$ diagonal blocks.
Rather than returning $P$ directly, returned is $p$ corresponding to $P$. Lowercase $p$ is a column vector that contains the subscripts of the rows in the desired order. That is,

$$
P L=L[p, .]
$$

The advantage of this is that $p$ requires less memory than $P$, and the reorganization, should it be desired, can be performed more quickly; see [M-1] Permutation.

## > Example 1: Bunch-Kaufman decomposition

The Bunch-Kaufman decomposition of $A$ can be written as

$$
A=L * D * L^{\prime}
$$

$\operatorname{ldl}(A, L, D)$ will make this calculation:

```
: A
[symmetric]
            1 2 3
    lll
: ldl(A, L = ., D = ., p = .)
: L
```

            \(1 \quad 2\)
                                    23
    \begin{tabular}{|rrr|}
    \hline 1 \& 0 \& 0 <br>
-.1666666667 \& .8333333333 \& 1 <br>
0 \& 1 \& 0 <br>
\hline
\end{tabular}

: D
[symmetric]
$1 \quad 2$
3

|  | 2 |  |  |
| :--- | :--- | :--- | :--- |
| 2 | 2 | 2 |  |
| 3 | 0 | 0 | 1.66666667 |

: p
1
1
$2 \quad 3$
32
: mreldif(A, L * D * L')
$1.11022 \mathrm{e}-16$
: mreldif(A[p, p], L[p, .] * D * L[p, .]')
$1.11022 \mathrm{e}-16$

## Conformability

$\operatorname{ldl}(A, L, D, p)$ :
input:

$$
A: \quad n \times n
$$

output:
L: $\quad n \times n$
D: $\quad n \times n$
p: $\quad n \times 1 \quad$ (optional)

## Diagnostics

$\operatorname{ldl}(A, L, D, p)$ returns missing results if $A$ contains missing values.

## Also see

[M-5] cholesky () - Cholesky square-root decomposition
[M-4] Matrix - Matrix functions

## Title

[M-5] LinearProgram( ) — Linear programming

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | References | Also see |  |

## Description

The LinearProgram() class finds the parameter vector that minimizes or maximizes the linear objective function subject to some restrictions. The restrictions may be linear equality constraints, linear inequality constraints, lower bounds, or upper bounds.

## Syntax

Syntax is presented under the following headings:
Step 1: Initialization
Step 2: Definition of linear programming problem
Step 3: Perform optimization
Step 4: Display or obtain results
Utility function for use in all steps
Definition of $q$
Functions defining the linear programming problem q.setCoefficients( ) and q.getCoefficients( ) q.setMaxOrMin( ) and q.getMaxOrMin()
q.setEquality( ) and q.getEquality( )
q.setInequality( ) and q.getInequality()
q.setBounds( ) and q.getBounds()
q.setMaxiter() and q.getMaxiter()
q.setTol() and q.getTol()
q.setTrace() and q.getTrace()

Performing optimization
q.optimize( )

Functions for obtaining results
q.parameters()
$q$.value()
q.iterations()
q.converged()
q.errorcode( ), q.errortext(), and q.returncode()

Utility function
q.query()

## Step 1: Initialization

$$
q=\text { LinearProgram }()
$$

## Step 2: Definition of linear programming problem

void
void
void
void
void
void
void
void
real rowvector
string scalar
real matrix
real matrix
real matrix
real scalar
real scalar
string scalar
$q$.setCoefficients (real rowvector coef)
$q$.setMaxOrMin(string scalar maxormin)
$q$.setEquality (real matrix ecmat, real colvector rhs)
$q$.setInequality (real matrix iemat, real colvector rhs)
$q$.setBounds (real rowvector lowerbd, real rowvector upperbd)
$q$.setMaxiter (real scalar maxiter)
q.setTol (real scalar tol)
$q$.setTrace (string scalar trace)
$q \cdot$ getCoefficients()
q.getMaxOrMin()
$q \cdot$ getEquality()
$q \cdot$ getInequality()
$q \cdot$ getBounds()
$q$.getMaxiter()
q.getTol()
q.getTrace()

## Step 3: Perform optimization

real scalar
$q$. optimize()

## Step 4: Display or obtain results

real rowvector
real scalar
real scalar
real scalar
real scalar
string scalar
real scalar
q.parameters()
$q$.value()
$q$.iterations()
$q$.converged ()
$q$.errorcode()
q.errortext()
$q . r e t u r n c o d e()$

## Utility function for use in all steps

void
q.query()

## Definition of $\mathbf{q}$

A variable of type LinearProgram is called an instance of the LinearProgram() class. $q$ is an instance of LinearProgram(), a vector of instances, or a matrix of instances. If you are working interactively, you can create an instance of LinearProgram() by typing

$$
q=\text { LinearProgram() }
$$

For a row vector of $n$ LinearProgram() instances, type

$$
\mathrm{q}=\text { LinearProgram }(n)
$$

For an $m \times n$ matrix of LinearProgram() instances, type

$$
\mathrm{q}=\text { LinearProgram }(m, n)
$$

In a function, you would declare one instance of the LinearProgram() class $q$ as a scalar.

```
void myfunc()
{
        class LinearProgram scalar q
        q = LinearProgram()
}
```

Within a function, you can declare $q$ as a row vector of $n$ instances by typing

```
void myfunc()
{
        class LinearProgram rowvector q
        q = LinearProgram(n)
}
```

For an $m \times n$ matrix of instances, type

```
void myfunc()
{
        class LinearProgram matrix q
        q = LinearProgram(m, n)
}
```


## Functions defining the linear programming problem

At a minimum, you need to tell the LinearProgram() class about the coefficients of the linear objective function you wish to optimize. Optionally, you may specify whether to minimize or maximize the objective function, any equality constraints, any inequality constraints, any lower bounds, and any upper bounds. You may also specify the maximum number of iterations allowed, the convergence tolerance, and whether or not to print computational details.

Each pair of functions includes a $q$.set function that specifies a setting and a $q$.get function that returns the current setting.

## q.setCoefficients() and q.getCoefficients()

$q$. setCoefficients (coef) sets the linear objective function coefficients. The coefficients must be set before optimization.
$q \cdot$ getCoefficients() returns the linear objective function coefficients (or an empty vector if not specified).

## q.setMaxOrMin() and q.getMaxOrMin()

q.setMaxOrMin(maxormin) sets whether to perform maximization or minimization. maxormin may be "max" or "min". The default is maximization ("max").
$q$. getMaxOrMin() returns "max" or "min" according to the current setting.

## q.setEquality() and q.getEquality()

The equality constraints for a linear programming problem are in the form of linear system $\mathbf{A}_{\mathrm{EC}} \mathbf{x}=$ $\mathbf{b}_{\mathrm{EC}}$, where $\mathbf{A}_{\mathrm{EC}}$ is the equality-constraints (EC) matrix and $\mathbf{b}_{\mathrm{EC}}$ is the right-hand-side (RHS) vector. $q$.setEquality (ecmat, rhs) sets the EC matrix and the RHS vector.
$q$.getEquality() returns a matrix containing both the EC matrix and the RHS vector. The RHS vector is the last column of the returned matrix. (An empty matrix is returned if equality constraints were not specified.)

## q.setInequality() and q.getInequality()

The inequality constraints for a linear programming problem are in the form of linear system $\mathbf{A}_{\text {IE }} \leq \mathbf{b}_{\text {IE }}$, where $\mathbf{A}_{\text {IE }}$ is the inequality-constraints (IE) matrix and $\mathbf{b}_{\text {IE }}$ is the RHS vector.
$q$.setInequality (iemat, rhs) sets the IE matrix and the RHS vector.
$q$.getInequality() returns a matrix containing both the IE matrix and the RHS vector. The RHS vector is the last column of the returned matrix. (An empty matrix is returned if inequality constraints were not specified.)

## q.setBounds() and q.getBounds()

The parameters may have lower bounds or upper bounds. By default, the lower bound is $-\infty$ and the upper bound is $\infty$.
$q$.setBounds (lowerbd, upperbd) sets the lower and upper bounds. Using a missing value as the lower bound indicates $-\infty$, and using a missing value as the upper bound indicates $\infty$.
$q$.getBounds() returns a two-row matrix containing the lower and upper bounds.

## q.setMaxiter() and q.getMaxiter()

$q$.setMaxiter (maxiter) specifies the maximum number of iterations, which must be an integer greater than 0 . The default value of maxiter is 16000 .
$q \cdot$ getMaxiter() returns the current maximum number of iterations.

## q.setTol() and q.getTol()

$q$.setTol (tol) specifies the convergence-criterion tolerance, which must be greater than 0 . The default value of tol is $1 \mathrm{e}-8$.
$q$. getTol() returns the currently specified tolerance.

## q.setTrace() and q.getTrace()

q.setTrace(trace) sets whether or not to print out computation details. trace may be "on" or "off". The default value is "off".
$q$.getTrace() returns the current trace status.

## Performing optimization

## q.optimize()

$q$.optimize() invokes the optimization process and returns the value of the objective function at the optimum.

## Functions for obtaining results

After performing optimization, the functions below provide results including parameters, the value at the optimum, the number of iterations, whether convergence was achieved, error messages, and return codes.

## q.parameters()

$q \cdot$ parameters() returns the parameter vector that optimizes the objective function; it returns an empty vector prior to performing the optimization.

## q.value()

$q$.value() returns the value of the objective function at the optimum; it returns a missing value prior to performing the optimization.

## q.iterations()

$q$.iterations() returns the number of iterations.
q.converged()
$q$.converged() returns 1 if the optimization converged and 0 if not.

## q.errorcode( ), q.errortext(), and q.returncode()

$q$.errorcode() returns the error code generated during the computation; it returns 0 if no error is found.
q.errortext() returns an error message corresponding to the error code generated during the computation; it returns an empty string if no error is found.
q.returncode() returns the Stata return code corresponding to the error code generated during the computation.

The error codes and the corresponding Stata return codes are as follows:

| Error <br> code | Return <br> code | Error text |
| :---: | :---: | :--- |
| 1 | 430 | problem is infeasible |
| 2 | 430 | problem is unbounded |
| 3 | 430 | maximum number of iterations has been reached |
| 4 | 3499 | dimensions of coefficients, constraints, and bounds do not conform |
| 5 | 111 | dimension of the parameters is 0 |

## Utility function

You can obtain a report of all settings and results currently stored in a class LinearProgram() instance.

## q.query()

$q \cdot$ query () with no return value displays the information stored in the class.

## Remarks and examples

Remarks are presented under the following headings:

## Introduction

Details about the interior-point method
Examples

## Introduction

The LinearProgram() class is a Mata class for linear programming.
LinearProgram() uses Mehrotra's (1992) predictor-corrector primal-dual method to optimize the linear programming problems of the form

$$
\begin{aligned}
\min _{\mathbf{x}} \text { or } \max _{\mathbf{x}} \mathbf{c x}^{\prime} & \\
\text { such that } \mathbf{A}_{\mathrm{EC}} \mathbf{x}^{\prime} & =\mathbf{b}_{\mathrm{EC}} \\
\mathbf{A}_{\mathrm{IE}} \mathbf{x}^{\prime} & \leq \mathbf{b}_{\mathrm{IE}} \\
\text { lowerbd } & \leq \mathbf{x} \leq \text { upperbd }
\end{aligned}
$$

where $\mathbf{c x}^{\prime}$ is the linear objective function, $\mathbf{A}_{\mathrm{EC}} \mathbf{x}^{\prime}=\mathbf{b}_{\mathrm{EC}}$ specifies equality constraints, $\mathbf{A}_{\mathrm{IE}} \mathbf{x}^{\prime} \leq \mathbf{b}_{\mathrm{IE}}$ specifies inequality constraints, lowerbd is the lower bound on $\mathbf{x}$, and upperbd is the upper bound on $\mathbf{x}$.

Mehrotra's (1992) predictor-corrector primal-dual method is much faster than the traditional simplex method for large problems. This method is a version of the interior-point method that is widely used today instead of the older simplex method that was widely used in the past. This speed comes at the cost of some accuracy, but the inaccuracy can be removed in practice by lowering the convergence tolerance. Lowering the convergence tolerance will produce answers that are practically the same as those produced by the simplex method.

## Details about the interior-point method

The simplex method breaks down for large problems, because it repeatedly checks a list of candidate solutions. When the list gets too large, the solution time becomes infeasible.

Instead of repeatedly checking a list of candidate solutions, the interior-point method solves a series of approximations to the original problem. This approximation-based approach makes it feasible to solve large problems that are not feasibly solved by the simplex algorithm. The interior-point method is also much faster than the simplex method on the medium-sized and large-sized problems that are common in statistical applications of linear programming.

Because the simplex algorithm checks a list of candidate solutions, it finds the exact integer solution, when one exists. In contrast, interior-point algorithms find the exact integer solution plus or minus the small positive amount $\epsilon$. Reducing the convergence tolerance reduces $\epsilon$ to a value that is practically 0 . (Technically, it will be on the order of $10^{-16}$.)

See example 1 for an illustration.
For an introduction to class programming in Mata, see [M-2] class.

## Examples

To solve a linear programming problem, you first use LinearProgram() to get an instance of the class. At a minimum, you must also use setCoefficients() to specify the linear objective function coefficients. In the examples below, we demonstrate both basic and more advanced use of the LinearProgram() class.
> Example 1: A first example
Consider the following linear programming problem:

$$
\begin{aligned}
\max _{x_{1}, x_{2}} 5 x_{1}+3 x_{2} & \\
\text { such that }-x_{1}+11 x_{2} & =33 \\
0.5 x_{1}-x_{2} & \leq-3 \\
2 x_{1}+14 x_{2} & \leq 60 \\
2 x_{1}+x_{2} & \leq 14.5 \\
x_{1}-0.4 x_{2} & \leq 5 \\
x_{1} & \geq 0 \\
x_{2} & \geq 0
\end{aligned}
$$

This problem can be written in matrix form,

$$
\begin{aligned}
& \max _{\mathbf{x}} \mathbf{c \mathbf { x } ^ { \prime }} \\
& \text { such that } \mathbf{A}_{\mathrm{EC}} \mathbf{x}^{\prime}=\mathbf{b}_{\mathrm{EC}} \\
& \mathbf{A}_{\mathrm{IE}} \mathbf{x}^{\prime} \leq \mathbf{b}_{\mathrm{IE}} \\
& \mathbf{x} \geq\left[\begin{array}{l}
0 \\
0
\end{array}\right]
\end{aligned}
$$

where $\mathbf{c}=(5,3), \mathbf{A}_{\mathrm{EC}}=(-1,11), \mathbf{b}_{\mathrm{EC}}=33$,

$$
\mathbf{A}_{\mathrm{IE}}=\left[\begin{array}{cc}
0.5 & -1 \\
2 & 14 \\
2 & 1 \\
1 & -0.4
\end{array}\right] \quad \text { and } \quad \mathbf{b}_{\mathrm{IE}}=\left[\begin{array}{c}
-3 \\
60 \\
14.5 \\
5
\end{array}\right]
$$

We first define all the coefficients and constraints in matrix form, respectively:

```
: c = (5, 3)
Aec = (-1, 11)
bec = 33
Aie = (0.5, -1 \ 2, 14 \ 2, 1 \ 1, -0.4)
bie = (-3 \ 60 \ 14.5 \ 5)
lowerbd = (0, 0)
upperbd = (., .)
```

Here we use missing in the upper bound to indicate an infinite upper bound. (A missing value used in the lower bound indicates a minus infinite lower bound.)

We generate q as an instance of the class:

```
: q = LinearProgram()
```

Then we initialize the coefficients and constraints:

```
: q.setCoefficients(c)
: q.setEquality(Aec, bec)
: q.setInequality(Aie, bie)
: q.setBounds(lowerbd, upperbd)
```

Now we can solve the problem:

```
: q.optimize()
    9.000000001
```

We can display the optimal parameters after the solution is found:

```
: q.parameters()

As mentioned above, the exact solution for this problem is \((0,3)\), and the simplex method would find it. Lowering the convergence tolerance produces a solution that is practically \((0,3)\).
```

q.setTol(1e-12)
q.optimize()
9
: q.parameters()
1 2
1
4.94781e-15
3

```
> Example 2: Display information about the linear programming problem
Each instance of the class contains a lot of information about the problem at hand. You can use the member function \(q\).query () to display this information. Several other member functions display specific pieces of information. This example illustrates how to use these functions.

Consider the following linear programming problem:
\[
\begin{aligned}
\min _{x_{1}, x_{2}, x_{3}} x_{1}+x_{2} & \\
\text { such that } x_{1}+x_{2}+x_{3} & =5 \\
x_{1}-x_{2}+2 x_{3} & =8 \\
x_{1} & \geq 1 \\
0 \leq x_{2} & \leq 2 \\
x_{3} & \geq 0
\end{aligned}
\]

We can express the problem in matrix form, as we did with the previous example:
```

: c = (1, 1, 0)
:Aec = (1, 1, 1\ \, -1, 2)
: bec = (5 \ 8)
: lowerbd = (1, 0, 0)
: upperbd = (., 2, .)

```

To calculate the problem, we define the class instance q :
```

: q = LinearProgram()

```

We can show all the default values by using \(q\).query() before we perform any initialization or computation.
```

: q.query()
Settings for LinearProgram()
Version: 1.00
Problem setup:
Perform maximization of the following problem
Objective function size: 1 x 0
Equality constraint matrix size: 0 x 0
Equality constraint right-hand-side size: 0 x 1
Inequality constraint matrix size: 0 x 0
Inequality constraint right-hand-side size: 0 x 1
Lower bound size: 1 x 0
Upper bound size: 1 x 0
Trace: off
Convergence
Maximum iterations: 16000
Tolerance: 1.0000e-08
Current status
Objective function value: .
Converged: no
Note: The function setCoefficients() has not been called.

```

Now we initialize the problem with all the information required. First, we define the coefficients of the objective function:
```

: q.setCoefficients(c)

```

Because this is a minimization problem, we use \(q \cdot \operatorname{setMaxOrMin}()\) to change the setting from the default, "max", to "min".
```

: q.setMaxOrMin("min")

```

We then define the equality constraints and bounds, respectively.
```

: q.setEquality(Aec, bec)
: q.setBounds(lowerbd, upperbd)

```

Our default maximum number of iterations is 16,000 ; Stata will issue a warning message when the maximum number of iterations has been reached. For illustrative purposes, we set the maximum to 2 here and compute the approximation.
```

: q.setMaxiter(2)
: q.optimize()
Warning: Maximum number of iterations has been reached.

```

The solution could not be found using only two iterations. Thus, We see a warning, and a missing value (.) is returned as the value of the objective function.

We switch back to a maximum of 16,000 iterations.
```

: q.setMaxiter(16000)

```

We can set the trace to "on" to see the computation details.
```

: q.setTrace("on")
: q.optimize()
Quadrature trace:
Iteration Current function value Current error estimate
1
2
3
4 -
.3333542428 2.63268e-02
1.333333333
5.202919005 3.44331e+00
.5872208056 6.08052e-01
.3333333396 1.29229e-06
.3333333333 6.46181e-11

```

We turn off the trace by typing
```

```
: q.setTrace("off")
```

```
```

```
: q.setTrace("off")
```

```

Now we can list parameters at the minimum:
```

```
: q.parameters()
```

```
: q.parameters()
1
```

1

```
\begin{tabular}{|lll|}
\hline 1 & .3333333333 & 3.666666667 \\
\hline
\end{tabular}
```

```
                    1 2 3
```

```
                    1 2 3
```

We can also display the value of the objective function at the solution.

```
: q.value()
    1.333333333
```

And, we can display the number of iterations used to find the solution.

```
: q.iterations()
    5
```

No error was found, so the error code and error message are

```
: q.errorcode()
    O
: q.errortext()
```

We can show all the values by using $q$.query () after the computation:

```
: q.query()
Settings for LinearProgram()
Version: 1.00
Problem setup:
    Perform minimization of the following problem
    Objective function size:
    Equality constraint matrix size: 2 x 3
    Equality constraint right-hand-side size: 2 x 1
    Inequality constraint matrix size: 0 x 0
    Inequality constraint right-hand-side size: 0 x 1
    Lower bound size: 1 x 3
    Upper bound size: 1 x 3
Trace:
    off
Convergence
    Maximum iterations: 16000
    Tolerance: 1.0000e-08
Current status
    Objective function value: 1.33333333
    Converged: yes
    Iterations: 5
```

The following three examples use linear programming to solve statistical estimation problems. For some statistical estimation problems, there are many equivalent ways of specifying the corresponding linear programming problems. In our examples, we use a common way of specifying each problem.

## > Example 3: Quantile regression

Linear programming can be used to fit quantile regression models.
We begin with a brief introduction to writing the quantile-regression $(\mathrm{QR})$ estimation problem as a linear programming problem; see [R] qreg, Koenker and Hallock (2001), and Koenker and Bassett (1978) for more details.

Minimizing the sum of squared residuals produces an estimator of the coefficients in a mean regression model. Analogously, minimizing the sum of the check function of the residuals produces an estimator of the coefficients in a QR model. For the QR model, we estimate the coefficients of the $\tau$ th conditional quantile function $\left(\boldsymbol{\beta}_{\tau}\right)$ by solving

$$
\min _{\boldsymbol{\beta}_{\tau}} \sum_{i=1}^{n} c_{\tau}\left(y_{i}-\mathbf{x}_{i} \boldsymbol{\beta}_{\tau}^{\prime}\right)
$$

where $y_{i}$ is the $i$ th observation of the outcome $\mathbf{y}, \mathbf{x}_{i}$ is the $i$ th observation of the vector of covariates $\mathbf{x}, n$ is the number of observations, and $c_{\tau}(\cdot)$ is the check function. The check function $c_{\tau}\left(r_{i}\right)$ of the residual $r_{i}=y_{i}-\mathbf{x}_{i} \boldsymbol{\beta}_{\tau}^{\prime}$ is given by

$$
c_{\tau}\left(r_{i}\right)=\left\{\tau-\mathbb{I}\left(r_{i}<0\right)\right\} r_{i}
$$

where

$$
\mathbb{I}\left(r_{i}<0\right)= \begin{cases}1 & \text { if } r_{i}<0 \\ 0 & \text { otherwise }\end{cases}
$$

This minimization problem for the estimator of the coefficients in the QR model can be written as the following linear programming problem:

$$
\begin{aligned}
\min _{\boldsymbol{\beta}_{\tau}, \mathbf{u}, \mathbf{v}} \tau \mathbf{1}_{n}^{\prime} \mathbf{u} & +(1-\tau) \mathbf{1}_{n}^{\prime} \mathbf{v} \\
\text { such that } \mathbf{y}-\mathbf{X} \boldsymbol{\beta}_{\tau} & =\mathbf{u}-\mathbf{v} \\
\mathbf{u} & \geq \mathbf{0}_{n} \\
\mathbf{v} & \geq \mathbf{0}_{n}
\end{aligned}
$$

where $\mathbf{1}_{n}$ is a vector of $1 \mathrm{~s}, \mathbf{0}_{n}$ is a vector of $0 \mathrm{~s}, \mathbf{X}$ is the matrix of observations of the covariates, $\mathbf{y}$ is the vector of observations of the outcome, and $\mathbf{u}$ and $\mathbf{v}$ are added to the inequality constraint to transform it into an equality (in other words, they are slack variables).

The above problem can be rewritten as

$$
\begin{aligned}
\min _{\boldsymbol{\beta}_{\tau}, \mathbf{u}, \mathbf{v}} \mathbf{c}\left[\begin{array}{c}
\boldsymbol{\beta}_{\tau} \\
\mathbf{u} \\
\mathbf{v}
\end{array}\right] & \\
\text { such that } \mathbf{A}_{\mathrm{EC}}\left[\begin{array}{c}
\boldsymbol{\beta}_{\tau} \\
\mathbf{u} \\
\mathbf{v}
\end{array}\right] & =\mathbf{y} \\
\mathbf{u} & \geq \mathbf{0}_{n} \\
\mathbf{v} & \geq \mathbf{0}_{n}
\end{aligned}
$$

where

$$
\mathbf{c}=\left[\begin{array}{c}
\mathbf{0}_{k} \\
\tau \mathbf{1}_{n} \\
(1-\tau) \mathbf{1}_{n}
\end{array}\right]^{\prime} \quad \text { and } \quad \mathbf{A}_{\mathrm{EC}}=\left[\begin{array}{lll}
\mathbf{X} & \mathbf{I}_{n} & -\mathbf{I}_{n}
\end{array}\right]
$$

where $k$ is the number of covariates in $\mathbf{x}$ and $\mathbf{I}_{n}$ is the identity matrix.
Now let us see an example using linear programming for quantile regression.
We use an extract of the dataset on chief-executive officer (CEO) salaries from Wooldridge (2020). This extract was created and is distributed by the Boston College Economics department (see Wooldridge datasets). In addition to salary (salary), the dataset also contains information on the CEOs' age (age), whether they completed college (college) or graduate school (grad), their years of experience with the company (comten) and as CEOs (ceoten), and the company's current profits as a percentage of sales (profmarg).

```
. use https://www.stata-press.com/data/r18/ceosal2
(CEO salaries)
```

Let's begin by using qreg to perform a quantile regression for the 75 th quantile using salary as the dependent variable, and age, college, grad, comten, ceoten, and profmarg as independent variables.
. qreg salary age college grad comten ceoten profmarg, quant
Iteration 1:
. 75 Quantile regression
Number of obs =
Raw sum of deviations 32811.25 (about 1119)
Min sum of deviations 31396.02 Pseudo R2 $=0.0431$

| salary | Coefficient | Std. err. | t | P>\|t| | [95\% conf. interval] |  |
| ---: | ---: | :--- | ---: | :--- | ---: | ---: |
| age | 5.379105 | 9.441549 | 0.57 | 0.570 | -13.25867 | 24.01688 |
| college | -554.7379 | 419.7605 | -1.32 | 0.188 | -1383.352 | 273.8764 |
| grad | 186.6717 | 140.3615 | 1.33 | 0.185 | -90.40431 | 463.7478 |
| comten | -6.826164 | 6.497454 | -1.05 | 0.295 | -19.65225 | 5.999919 |
| ceoten | 17.84588 | 10.23762 | 1.74 | 0.083 | -2.363353 | 38.05511 |
| profmarg | -.4654744 | 3.791003 | -0.12 | 0.902 | -7.948978 | 7.018029 |
| _cons | 1298.486 | 672.6187 | 1.93 | 0.055 | -29.2744 | 2626.247 |

The output table contains the estimated coefficients that we will now obtain by linear programming. For further interpretation of the output, see $[\mathrm{R}]$ qreg.

We begin by importing the data into Mata and adding a vector of 1 s to covariates for the constant term:

```
mata:
                mata (type end to exit)
X = st_data(., ("age college grad comten ceoten profmarg"))
y = st_data(., ("salary"))
: X = (X, J(rows(X), 1, 1))
```

Now specify that $\tau$ should be 0.75 .

```
: tau = 0.75
```

Then we formulate the problem as a linear programming problem using the formulas at the beginning of this example:

```
: n = rows(X)
k = cols(X)
: c = (J(1, k, 0), tau * J(1, n, 1), (1 - tau) * J(1, n, 1))
: Aec = (X, I(n), -I(n))
: lowerbd = (J(1, k, .), J(1, 2*n, 0))
upperbd = J(1, 2*n + k, .)
```

We then generate an instance of the class and save the required coefficients and the constraints to it:

```
: q = LinearProgram()
q.setCoefficients(c)
q.setEquality(Aec, y)
q.setBounds(lowerbd, upperbd)
q.setMaxOrMin("min")
```

Now we solve it:

```
q.optimize()
    31396.02428
x = q.parameters()
x[1..k]
```

$1 \quad 2$
$3 \quad 4$


These point estimates are the same as those computed by greg.

## > Example 4: Data envelopment analysis

Production theory is a fundamental component of economic analysis. When studying production, we base our analysis on the concept of a production function. The production function describes how inputs in the production process are turned into outputs. Before the work of Debreu (1951), Koopmans (1951), and Farrell (1957), it was assumed that the input-output relationship had no inefficiency. It is now common to study and measure deviations from efficient production (called efficiency analysis).

A way to measure production efficiency is data envelopment analysis DEA; see Cooper, Seiford, and Tone [2007]; Färe [1988]; Grosskopf and Knox Lovell [1994]; and Färe and Primont [1995]). DEA makes no assumptions about the functional form of the production function and is therefore robust to misspecification.

Intuitively, we know the total output produced by each firm, given their inputs. We can then measure the results of each firm relative to the most efficient firms in our sample. The most efficient firms, those that produce the most for a given set of inputs, form a frontier, an envelope. All other firms are underneath and are relatively inefficient. What we obtain when using DEA is a measure of relative efficiency, a number between 0 and 1 , where inefficiency is any number below 1 .

DEA uses the weighted sum of outputs over the weighted sum of inputs to measure efficiency, and it can be written as a linear programming problem. For example, to estimate the efficiency of a firm or, more generally, the efficiency of a unit $k$, we solve

$$
\begin{aligned}
\max _{\mathbf{u}, \mathbf{v}} & \frac{\mathbf{Y}_{k} \mathbf{u}^{\prime}}{\mathbf{X}_{k} \mathbf{v}^{\prime}} \\
\text { such that } & \frac{\mathbf{Y}_{j} \mathbf{u}^{\prime}}{\mathbf{X}_{j} \mathbf{v}^{\prime}}
\end{aligned} \leq 1 \quad j=1, \ldots, n
$$

where $\mathbf{X}_{j}$ and $\mathbf{Y}_{j}$ are the inputs and outputs for unit $j$, respectively. $n$ is the number of units, $m$ is the number of inputs, and $p$ is the number of outputs. This problem can be rewritten as a linear programming problem:

$$
\begin{aligned}
\max _{\mathbf{u}, \mathbf{v}} \mathbf{Y}_{k} \mathbf{u}^{\prime} & \\
\text { such that } \mathbf{X}_{k} \mathbf{v}^{\prime} & =1 \\
\mathbf{Y} \mathbf{u}^{\prime}-\mathbf{X} \mathbf{v}^{\prime} & \leq \mathbf{0}_{n} \\
\mathbf{u} & \geq \mathbf{0}_{p}^{\prime} \\
\mathbf{v} & \geq \mathbf{0}_{m}^{\prime}
\end{aligned}
$$

Using our notation,

$$
\begin{aligned}
\max _{\mathbf{u}, \mathbf{v}} \mathbf{c}\left[\begin{array}{ll}
\mathbf{u} & \mathbf{v}
\end{array}\right]^{\prime} & \\
\text { such that } \mathbf{A}_{\mathrm{EC}}\left[\begin{array}{ll}
\mathbf{u} & \mathbf{v}
\end{array}\right]^{\prime} & =\mathbf{b}_{\mathrm{EC}} \\
\mathbf{A}_{\mathrm{IE}}\left[\begin{array}{ll}
\mathbf{u} & \mathbf{v}
\end{array}\right]^{\prime} & \leq \mathbf{b}_{\mathrm{IE}} \\
\mathbf{u} & \geq \mathbf{0}_{p}^{\prime} \\
\mathbf{v} & \geq \mathbf{0}_{m}^{\prime}
\end{aligned}
$$

We have

$$
\mathbf{c}=\left[\begin{array}{l}
\mathbf{Y}_{k} \\
\mathbf{0}_{m}
\end{array}\right] \quad \mathbf{A}_{\mathrm{EC}}=\left[\begin{array}{ll}
\mathbf{0}_{p}^{\prime} & \mathbf{X}_{k}
\end{array}\right] \quad \mathbf{b}_{\mathrm{EC}}=1 \quad \mathbf{A}_{\mathrm{IE}}=\left[\begin{array}{ll}
\mathbf{Y} & -\mathbf{X}
\end{array}\right] \quad \mathbf{b}_{\mathrm{IE}}=\mathbf{0}_{n}
$$

Here $\mathbf{0}_{m}, \mathbf{0}_{n}$, and $\mathbf{0}_{p}$ are vectors of 0 s .
As an example, we use a sample of 756 fictional firms producing a manufactured good with capital and labor. The firms are hypothesized to use a constant returns-to-scale technology, but the sizes of the firms differ. For more details, see example 2 in $[\mathrm{R}]$ frontier. The inputs for the firms will be capital (lncapital) and labor (lnlabor); the output will be the manufactured good (lnoutput).

```
. use https://www.stata-press.com/data/r18/frontier1
```

We begin by importing the data into Mata:

```
. mata:
: X = st_data(., ("lnlabor", "lncapital"))
: Y = st_data(., "lnoutput")
: n = rows(X)
: m = cols(X)
: p = cols(Y)
```

Now we write the coefficients and constraints as a linear programming problem using the equations above, and we use id $=1$ to indicate the first firm.

```
: id = 1
: c = (Y[id, .], J(1, m, 0))
: Aec = (J(1, p, 0), X[id, .])
: bec = 1
: Aie = (Y, -X)
bie = J(n, 1, 0)
lowerbd = J(1, m + p, 0)
upperbd = J(1, m + p, .)
```

We then generate an instance of the LinearProgram() class and store all the required information.

```
: q = LinearProgram()
: q.setCoefficients(c)
: q.setEquality(Aec, bec)
: q.setInequality(Aie, bie)
q.setBounds(lowerbd, upperbd)
```

Then we can compute the relative efficiency of unit 1 , which we defined above.

```
: q.optimize()
    . }19180371
```

Because the optimal value is less than 1 , we conclude that the first firm is inefficient. If we want to estimate the efficiencies of other firms, we simply change id to the firm of interest. For example, if we change id to 261 , we can get the relative efficiency of firm 261:

```
: q.optimize()
1
```

This shows that firm 261 is efficient.

## > Example 5: Dantzig selector

In a linear model, we model the mean of the outcome $y_{i}$ as the linear combination $\mathbf{x}_{i} \boldsymbol{\beta}^{\prime}$, where $\mathbf{x}_{i}$ are the covariates and $\boldsymbol{\beta}$ are the coefficients. In the standard case, the number of covariates $k$ is small relative to the number of observations $n$. In a high-dimensional regression, $k$ is large relative to $n$, but we must assume that many of the coefficients $\boldsymbol{\beta}$ on $\mathbf{x}_{i}$ are 0 .

The Dantzig selector (Candes and Tao 2007) estimates which of the coefficients are 0 and produces estimates of the coefficients that are not 0 .

In a standard linear model, we estimate $\boldsymbol{\beta}$ by minimizing the sum of the squared residuals. The first-order conditions for this minimization problem are known as the normal equations, and in matrix form, they are

$$
\mathbf{X}^{\prime}(\mathbf{y}-\mathbf{X} \boldsymbol{\beta})=\mathbf{0}
$$

where $\mathbf{X}$ is a matrix containing the observations of the covariates and $\mathbf{y}$ is a vector containing the observations of the outcome.

The Dantzig selector solution to the high-dimensional linear model finds the smallest-in-magnitude coefficients that get close to solving the first-order conditions. In math, the Dantzig selector solves

$$
\begin{gathered}
\min _{\boldsymbol{\beta}}\|\boldsymbol{\beta}\|_{1} \\
\text { such that }\left\|\mathbf{X}^{\prime}(\mathbf{y}-\mathbf{X} \boldsymbol{\beta})\right\|_{\infty} \leq \lambda
\end{gathered}
$$

where $\lambda$ is a constant that parameterizes "close" to solving the first-order conditions. (The least absolute shrinkage and selector operator is another solution to the high-dimensional linear model; see [LASSO] Lasso intro for more details.)

The Dantzig selector problem can be written as the following linear programming problem:

$$
\begin{aligned}
\min _{\boldsymbol{\beta}, \mathbf{u}} \mathbf{1}_{k}^{\prime} \mathbf{u} & \\
\text { such that } \mathbf{X}^{\prime}(\mathbf{y}-\mathbf{X} \boldsymbol{\beta}) & \leq \lambda \mathbf{1}_{k} \\
-\mathbf{X}^{\prime}(\mathbf{y}-\mathbf{X} \boldsymbol{\beta}) & \leq \lambda \mathbf{1}_{k} \\
\boldsymbol{\beta}-\mathbf{u} & \leq \mathbf{0}_{k} \\
-\boldsymbol{\beta}-\mathbf{u} & \leq \mathbf{0}_{k}
\end{aligned}
$$

Here $\mathbf{u}$ is a vector of variables used as the upper bounds of the absolute values of $\boldsymbol{\beta}$.

The above version is closely related to the original motivation for the Dantzig selection, but it is not in an easy-to-implement form. A ready-to-implement form for the above problem is

$$
\min _{\boldsymbol{\beta}, \mathbf{u}} \mathbf{c}\left[\begin{array}{l}
\boldsymbol{\beta} \\
\mathbf{u}
\end{array}\right]
$$

such that $\mathbf{A}_{\text {IE }}\left[\begin{array}{l}\boldsymbol{\beta} \\ \mathbf{u}\end{array}\right] \leq \mathbf{b}_{\mathrm{IE}}$
where

$$
\mathbf{c}=\left[\begin{array}{l}
\mathbf{0}_{k} \\
\mathbf{1}_{k}
\end{array}\right]^{\prime} \quad \mathbf{A}_{\mathrm{IE}}=\left[\begin{array}{cc}
-\mathbf{X}^{\prime} \mathbf{X} & \mathbf{0}_{k \times k} \\
\mathbf{X}^{\prime} \mathbf{X} & \mathbf{0}_{k \times k} \\
\mathbf{I}_{k} & -\mathbf{I}_{k} \\
-\mathbf{I}_{k} & -\mathbf{I}_{k}
\end{array}\right] \quad \mathbf{b}_{\mathrm{IE}}=\left[\begin{array}{c}
-\mathbf{X}^{\prime} \mathbf{y}+\lambda \mathbf{1}_{k} \\
\mathbf{X}^{\prime} \mathbf{y}+\lambda \mathbf{1}_{k} \\
\mathbf{0}_{k} \\
\mathbf{0}_{k}
\end{array}\right]
$$

In this example, we use an extract of the data used in Sunyer et al. (2017) that models how the attention of school children is affected by pollution. Our sample contains 1,096 students, and we model the relationship between the mean-hit reaction time (htime) and other factors, including daily nitrogen dioxide levels (no2), home socioeconomical vulnerability index (sev_home), school socioeconomical vulnerability index (sev_sch), school noise levels (noise_sch), starting age at school (age_start_sch), number of young siblings they live with (youngsibl), and home greenness (ndvi_mn).

We begin by using the dataset and dropping the observations that contain missing values.

```
. use https://www.stata-press.com/data/r18/breathe_lp
. generate byte touse = 1
. markout touse *
. drop if touse == 0
(18 observations deleted)
```

Next we create local macros to hold the names of the outcome variable and the covariates in the model, including some powers and interactions of the original covariates.

```
. local ccontrols "sev_home sev_sch age_start_sch ndvi_mn youngsibl noise_sch"
. local ofinterest "no2"
. local depvar htime
. local indeps 'ofinterest' 'ccontrols' c.('ccontrols')#c.('ccontrols')
```

Then we expand the names of covariates in the model, remove the collinear variables, and save the names back to the same macro.

```
. _rmcoll 'indeps', expand
local indeps 'r(varlist)'
```

Now we import the data into Mata, remove the mean from the outcome variable, and standardize the covariates.

```
. mata:
: X = st_data(., '"'indeps'"', "touse")
: y = st_data(., "'depvar'", "touse")
ybar = mean(y)
y = y :- ybar
```

```
Xbar = mean(X)
X = X :- Xbar
sd_X = sqrt(mean(X: ^2))
X = X :/ sd_X
```

We need a value for $\lambda$ to complete the problem. Prior to writing this example, we used the method of cross-validation to find a good value for $\lambda$; see Hastie, Tibshirani, and Friedman (2009) for an introduction to cross-validation. Cross-validation specifies that we should set $\lambda=4163.558931$.

We are now ready to specify and solve the linear programming problem.

```
: tmpA = quadcross(X, X)
: tmpb = quadcross(X, y)
lambda = 4163.558931
k = cols(X)
c = (J(1, k, 0), J(1, k, 1))
Aec = (-tmpA, J (k,k, 0) \ tmpA, J (k, k, 0) \ I (k), -I (k) \ -I (k), -I (k))
bec = (-tmpb :+ lambda \ tmpb :+ lambda \ J (k, 1, 0) \ J (k, 1, 0))
```

We now define an instance of the LinearProgram() class and put the required information into that instance q.

```
: q = LinearProgram()
: q.setCoefficients(c)
: q.setInequality(Aec, bec)
: q.setMaxOrMin("min")
```

Now solve the problem.

```
q.optimize()
8.21319472
```

We put the estimates of $\boldsymbol{\beta}$ into the Mata vector b by typing

```
: x = q.parameters()
b = x[1..k]
```

Out of the 28 estimated parameters, we only want to see which variables were selected, that is, which variables have estimated coefficients that are greater than 0 in absolute value. To show the variable names that have been selected, we type

```
indep_list = tokens('"'indeps'"')
sel = (abs(b) :>= 1e-6)
: "selected variables are "
selected variables are
select(indep_list, sel)
```

| sev_home | c.sev_home\#c.age_start_sch |
| ---: | ---: |
| 3 | 4 |

c.sev_sch\#c.ndvi_mn c.ndvi_mn\#c.noise_sch

Here we consider a coefficient to be 0 when its absolute value is less than $10^{-6}$. Recall from Details about the interior-point method above and the discussion in example 1 that the interior-point method will not produce solution values that are exactly 0 . Lowering the tolerance will make the coefficients that are practically 0 have values closer to 0 , but it will not change which variables are selected.

The results indicate that the mean-hit reaction time is related to the home socioeconomical vulnerability index, the home socioeconomical vulnerability index times starting age at school, the school socioeconomical vulnerability index times home greenness, and home greenness times school noise levels.

## Conformability

## LinearProgram():

input:

> void
output:

$$
\text { result: } \quad 1 \times 1
$$

LinearProgram ( $n$ ):
input:

$$
n: \quad 1 \times 1
$$

output:

$$
\text { result: } \quad 1 \times n
$$

LinearProgram ( $m, n$ ):
input:

$$
\begin{array}{cl}
m: & \\
n: & \\
n: 1 \times 1 \\
& 1 \times 1
\end{array}
$$

output:

$$
\text { result: } \quad m \times n
$$

setCoefficients(coef):
input:

$$
\text { coef: } \quad 1 \times N
$$

output:
result: void
getCoefficients():
input:

> void
output:
result: $\quad 1 \times N$
setMaxOrMin(maxormin):
input:

$$
\text { object }: \quad 1 \times 1
$$

output:

$$
\text { result: } \quad \text { void }
$$

getMaxOrMin():
input:
void
output:

$$
\text { result: } \quad 1 \times 1
$$

setEquality (ecmat, rhs):
input:

$$
\begin{aligned}
\text { ecmat: } & M 0 \times N \\
\text { rhs: } & M 0 \times 1
\end{aligned}
$$

output:

$$
\text { result: } \quad \text { void }
$$

getEquality():
input:
void
output:
result: $\quad(M 0+1) \times N$
setInequality (iemat, rhs):
input:

$$
\text { iemat: } \quad M 1 \times N
$$

$r h s: \quad M 1 \times 1$
output:
result: void
getInequality():
input:
void
output:
result: $\quad(M 1+1) \times N$
setBounds (lowerbd, upperbd):
input:

$$
\begin{array}{ll}
\text { lowerbd: } & 1 \times N \\
\text { upperbd: } & 1 \times N
\end{array}
$$

output:
result: void
getBounds():
input:
void
output:

$$
\text { result: } \quad 2 \times N
$$

setMaxiter(maxiter):
input:

```
            maxiter: }\quad1\times
```

output:
result: void
getMaxiter():
input:
void
output:
result: $\quad 1 \times 1$
setTol (tol):
input:

$$
\text { tol: } \quad 1 \times 1
$$

output:
result: void
getTol():
input:
void
output:

$$
\text { result: } \quad 1 \times 1
$$

setTrace (trace):
input:

$$
\text { trace: } \quad 1 \times 1
$$

output:
result: void
getTrace():
input:
void
output:

$$
\text { result: } \quad 1 \times 1
$$

optimize():
input:
void
output:

$$
\text { result: } \quad 1 \times 1
$$

parameters():
input:
void
output:

$$
\text { result: } \quad 1 \times N
$$

value():
input:
void
output:

$$
\text { result: } \quad 1 \times 1
$$

iterations():
input:
void
output:

$$
\text { result: } \quad 1 \times 1
$$

converged():
input:
void
output:

$$
\text { result: } \quad 1 \times 1
$$

errorcode():
input:
void
output:

$$
\text { result: } \quad 1 \times 1
$$

errortext():
input:
void
output:

$$
\text { result: } \quad 1 \times 1
$$

returncode():
input:
void
output:

$$
\text { result: } \quad 1 \times 1
$$

query():
input:
void
output:
void

## Diagnostics

LinearProgram(), q.set*(), q.get*(), q.parameters(), q.value(), q.iterations(), $q$.converged(), $q$.errorcode(), q.errortext(), q.returncode(), and $q$.query() functions abort with an error message when used incorrectly.
$q$.optimize() aborts with an error message if it is used incorrectly. If $q$.optimize() runs into numerical difficulties, it returns a missing value and displays a warning message including some details about the problem encountered.

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```
Also see
    [M-2] class - Object-oriented programming (classes)
    [M-5] moptimize() - Model optimization
    [M-5] optimize( ) - Function optimization
    [LASSO] Lasso intro - Introduction to lasso
    [R] frontier - Stochastic frontier models
    [R] qreg - Quantile regression
    [ R\(]\) regress - Linear regression
```


## Title

[M-5] liststruct() — List structure's contents

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

liststruct() lists $x$ 's contents, where $x$ is an instance of structure whatever.

## Syntax

void liststruct (struct whatever matrix $x$ )

## Remarks and examples

liststruct() is often useful in debugging.
The dimension and type of all elements are listed, and the values of scalars are also shown.

## Conformability

## liststruct $(x)$ :

| $x:$ | $r \times c$ |
| ---: | :--- |
| result: | void |

## Diagnostics

None.

## Also see

[M-2] struct - Structures
[M-4] IO - I/O functions

## Title

[M-5] Lmatrix( ) - Elimination matrix

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Reference | Also see |  |

## Description

Lmatrix $(n)$ returns the $n(n+1) / 2 \times n^{2}$ elimination matrix $L$ for which $L * \operatorname{vec}(X)=\operatorname{vech}(X)$, where $X$ is an $n \times n$ symmetric matrix.

## Syntax

real matrix Lmatrix (real scalar $n$ )

## Remarks and examples

Elimination matrices are frequently used in computing derivatives of functions of symmetric matrices.
Section 9.6 of Lütkepohl (1996) lists many useful properties of elimination matrices.

## Conformability

Lmatrix(n):
$\begin{aligned} n: & 1 \times 1 \\ \text { result: } & n(n+1) / 2 \times n^{2}\end{aligned}$

## Diagnostics

Lmatrix ( $n$ ) aborts with error if $n$ is less than 0 or is missing. $n$ is interpreted as trunc $(n)$.

## Reference

Lütkepohl, H. 1996. Handbook of Matrices. New York: Wiley.

## Also see

[M-5] Dmatrix () - Duplication matrix
[M-5] Kmatrix( ) - Commutation matrix
[M-5] vec( ) - Stack matrix columns
[M-4] Standard - Functions to create standard matrices

## Title

[M-5] logit( ) — Log odds and complementary $\log -\log$
Description Syntax Conformability Diagnostics Also see

## Description

$\operatorname{logit}(X)$ returns the $\log$ of the odds ratio of the elements of $X, \ln \{x /(1-x)\}$.
invlogit( $X$ ) returns the inverse of the logit() of the elements of $X, \exp (x) /\{1+\exp (x)\}$. $c \log \log (X)$ returns the complementary $\log -\log$ of the elements of $X, \ln \{-\ln (1-x)\}$. invcloglog $(X)$ returns the elementwise inverse of $c \log \log ()$ of the elements of $X$, $1-\exp \{-\exp (x)\}$.

## Syntax

| real matrix | $\operatorname{logit}($ real matrix $X)$ |
| :--- | :--- |
| real matrix | invlogit(real matrix $X)$ |
| real matrix | $\operatorname{clog} \log ($ real matrix $X)$ |
| real matrix | invcloglog $($ real matrix $X)$ |

## Conformability

All functions return a matrix of the same dimension as input containing element-by-element calculated results.

## Diagnostics

$\operatorname{logit}(X)$ and $c \log \log (X)$ return missing when $x \leq 0$ or $x \geq 1$.

## Also see

[M-4] Statistical - Statistical functions

## Title

[M-5] lowertriangle() — Extract lower or upper triangle

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

lowertriangle() returns the lower triangle of $A$.
uppertriangle() returns the upper triangle of $A$.
_lowertriangle() replaces $A$ with its lower triangle.
_uppertriangle() replaces $A$ with its upper triangle.

## Syntax


where argument $d$ is optional.

## Remarks and examples

Remarks are presented under the following headings:
Optional argument d
Nonsquare matrices

## Optional argument d

Optional argument $d$ specifies the treatment of the diagonal. Specifying $d>=$., or not specifying $d$ at all, means no special treatment; if

$$
A=\left[\begin{array}{lll}
1 & 2 & 3 \\
4 & 5 & 6 \\
7 & 8 & 9
\end{array}\right]
$$

then

$$
\text { lowertriangle }(A, .)=\left[\begin{array}{lll}
1 & 0 & 0 \\
4 & 5 & 0 \\
7 & 8 & 9
\end{array}\right]
$$

If a nonmissing value is specified for $d$, however, that value is substituted for each element of the diagonal, for example,

$$
\text { lowertriangle }(A, 1)=\left[\begin{array}{lll}
1 & 0 & 0 \\
4 & 1 & 0 \\
7 & 8 & 1
\end{array}\right]
$$

## Nonsquare matrices

lowertriangle() and uppertriangle() may be used with nonsquare matrices. If

$$
A=\left[\begin{array}{cccc}
1 & 2 & 3 & 4 \\
5 & 6 & 7 & 8 \\
9 & 10 & 11 & 12
\end{array}\right]
$$

then

$$
\text { lowertriangle }(A)=\left[\begin{array}{ccc}
1 & 0 & 0 \\
5 & 6 & 0 \\
9 & 10 & 11
\end{array}\right]
$$

and

$$
\text { uppertriangle }(A)=\left[\begin{array}{cccc}
1 & 2 & 3 & 4 \\
0 & 6 & 7 & 8 \\
0 & 0 & 11 & 12
\end{array}\right]
$$

_lowertriangle() and _uppertriangle(), however, may not be used with nonsquare matrices.

## Conformability

lowertriangle $(A, d)$ :

$$
\begin{array}{rlrl}
A: & r \times c \\
d: & 1 \times 1 \quad \text { (optional) } \\
\text { result: } & & r \times \min (r, c)
\end{array}
$$

uppertriangle $(A, d)$ :
A: $\quad r \times c$
$d: \quad 1 \times 1 \quad$ (optional)
result: $\quad \min (r, c) \times c$
_lowertriangle $(A, d)$, _uppertriangle $(A, d)$ :
input:

$$
\begin{array}{lll}
A: & n \times n & \\
d: & 1 \times 1 & \text { (optional) }
\end{array}
$$

output:
A: $\quad n \times n$

## Diagnostics

None.

## Also see

[M-4] Manipulation - Matrix manipulation

## Title

[M-5] Iud() - LU decomposition

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

$\operatorname{lud}(A, L, U, p)$ returns the $L U$ decomposition (with partial pivoting) of $A$ in $L$ and $U$ along with a permutation vector $p$. The returned results are such that $A=L[p,] *$.$U up to roundoff error.$
_lud $(L, U, p)$ is similar to lud(), except that it conserves memory. The matrix to be decomposed is passed in $L$, and the same storage location is overwritten with the calculated $L$ matrix.
_lud_la $(A, q)$ is the LAPACK routine that the above functions use to calculate the LU decomposition. See LAPACK routine below.

## Syntax

```
void lud(numeric matrix A,L,U, p)
void _lud(numeric matrix L, U, p)
void _lud_la(numeric matrix A, q)
```

where

1. A may be real or complex and need not be square.
2. The types of $L, U, p$, and $q$ are irrelevant; results are returned there.

## Remarks and examples

Remarks are presented under the following headings:
LU decomposition
LAPACK routine

## LU decomposition

The LU decomposition of matrix $A$ can be written as

$$
P^{\prime} A=L U
$$

where $P^{\prime}$ is a permutation matrix that permutes the rows of $A . L$ is lower triangular and $U$ is upper triangular. The decomposition can also be written as

$$
A=P L U
$$

because, given that $P$ is a permutation matrix, $P^{-1}=P^{\prime}$.

Rather than returning $P$ directly, returned is $p$ corresponding to $P$. Lowercase $p$ is a column vector that contains the subscripts of the rows in the desired order. That is,

$$
P L=L[p, .]
$$

The advantage of this is that $p$ requires less memory than $P$ and the reorganization, should it be desired, can be performed more quickly; see [M-1] Permutation. In any case, the formula defining the LU decomposition can be written as

$$
A=L[p, .] * U
$$

One can also write

$$
B=L U, \text { where } B[p, .]=A
$$

## LAPACK routine

_lud_la $(A, q)$ is the interface to the LAPACK routines that the above functions use to calculate the LU decomposition. You may use it directly if you wish.

Matrix $A$ is decomposed, and the decomposition is placed back in $A . U$ is stored in the upper triangle (including the diagonal) of $A . L$ is stored in the lower triangle of $A$, and it is understood that $L$ is supposed to have ones on its diagonal. $q$ is a column vector recording the row swaps that account for the pivoting. This is the same information as stored in $p$, but in a different format.
$q$ records the row swaps to be made. For instance, $q=(1 \backslash 2 \backslash 2)$ means that (start at the end) the third row is to be swapped with the second row, then the second row is to stay where it is, and finally the first row is to stay where it is. $q$ can be converted into $p$ by the following logic:

```
p = 1::rows(q)
for (i=rows(q); i>=1; i--) {
        hold = p[i]
        p[i] = p[q[i]]
        p[q[i]] = hold
}
```


## Conformability

$\operatorname{lud}(A, L, U, p)$ :
input:
A: $\quad r \times c$
output:
$L: \quad r \times m, \quad m=\min (r, c)$
$U: \quad m \times c$
$p: \quad r \times 1$
_lud $(L, U, p)$ :
input:

$$
L: \quad r \times c
$$

output:
$L: \quad r \times m, \quad m=\min (r, c)$
$U: \quad m \times c$
$p: \quad r \times 1$
_lud_la $(A, q)$ :
input:
A: $\quad r \times c$
output:
A: $\quad r \times c$
$q: \quad r \times 1$

## Diagnostics

$\operatorname{lud}(A, L, U, p)$ returns missing results if $A$ contains missing values; $L$ will have missing values below the diagonal, 1 s on the diagonal, and 0 s above the diagonal; $U$ will have missing values on and above the diagonal and 0 s below. Thus if there are missing values, $U[1,1]$ will contain missing. _lud $(L, U, p)$ sets $L$ and $U$ as described above if the input matrix $L$ contains missing values. _lud_la $(A, q)$ aborts with error if $A$ is a view.

## Also see

[M-5] $\operatorname{det}()$ - Determinant of matrix
[M-5] lusolve( ) - Solve AX=B for X using LU decomposition
[M-4] Matrix - Matrix functions

## Title

[M-5] luinv() — Square matrix inversion

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

luinv ( $A$ ) and luinv ( $A$, tol ) return the inverse of real or complex, square matrix $A$.
_luinv $(A)$ and _luinv $(A$, tol $)$ do the same thing except that, rather than returning the inverse matrix, they overwrite the original matrix $A$ with the inverse.
_luinv_la $(A, b)$ is the interface to the LAPACK routines that do the work. The output $b$ is a real scalar, which is missing if the MKL LAPACK routine was used, is 1 if the Netlib LAPACK routine used a blocked algorithm, and is 0 otherwise. This function uses the MKL LAPACK by default.
_luinv_lapacke ( $A$ ) and _luinv_lapacke ( $A$, tol) are the interfaces to the LAPACK routines that do the work. Both of these function use MKL LAPACK by default.

In all cases, optional argument tol specifies the tolerance for determining singularity; see Remarks and examples below.

## Syntax

| numeric matrix | luinv(numeric matrix A) |
| :---: | :---: |
| numeric matrix | luinv (numeric matrix $A$, real scalar tol) |
| void | _luinv (numeric matrix A) |
| void | _luinv(numeric matrix $A$, real scalar tol) |
| real scalar | _luinv_la(numeric matrix $A, b$ ) |
| real scalar | _luinv_lapacke(numeric matrix $A$ ) |
| real scalar | _luinv_lapacke(numeric matrix A, real scalar tol) |

## Remarks and examples

These routines calculate the inverse of $A$. The inverse matrix $A^{-1}$ of $A$ satisfies the conditions

$$
\begin{aligned}
& A A^{-1}=I \\
& A^{-1} A=I
\end{aligned}
$$

$A$ is required to be square and of full rank. See [M-5] qrinv() and [M-5] pinv() for generalized inverses of nonsquare or rank-deficient matrices. See [M-5] invsym( ) for inversion of real, symmetric matrices.
luinv (A) is logically equivalent to lusolve ( $A$, I (rows (A)) ); see [M-5] lusolve() for details and for use of the optional tol argument.

## Conformability

luinv( $A$, tol):

| $A:$ |  | $n \times n$ |
| ---: | :--- | :--- |
| tol: | $1 \times 1$ |  |
| result: |  | $n \times n$ |

_luinv ( $A$, tol):
input:
A: $\quad n \times n$
tol: $\quad 1 \times 1 \quad$ (optional)
output:
A: $\quad n \times n$
_luinv_la (A, b):
input:
A: $\quad n \times n$
output:
A: $\quad n \times n$
b: $\quad 1 \times 1$
result: $\quad 1 \times 1$
_luinv_lapacke (A, tol):
input:
A: $\quad n \times n$
output:

$$
\begin{array}{rll}
A: & & n \times n \\
\text { tol: } & 1 \times 1 & \text { (optional) } \\
\text { result: } & 1 \times 1 &
\end{array}
$$

## Diagnostics

The inverse returned by these functions is real if $A$ is real and is complex if $A$ is complex. If you use these functions with a singular matrix, returned will be a matrix of missing values. The determination of singularity is made relative to tol. See Tolerance under Remarks and examples in [M-5] lusolve() for details.
luinv ( $A$ ) and _luinv ( $A$ ) return a matrix containing missing if $A$ contains missing values.
-luinv $(A)$ aborts with error if $A$ is a view.
_luinv_la $(A, b)$, _luinv_lapacke $(A)$, and _luinv_lapacke $(A, t o l)$ should not be used directly; use _luinv().

See [M-5] lusolve() and [M-1] Tolerance for information on the optional tol argument.

## Also see

[M-5] cholinv( ) - Symmetric, positive-definite matrix inversion
[M-5] invsym( ) - Symmetric real matrix inversion
[M-5] lud( ) - LU decomposition
[M-5] lusolve( ) - Solve AX=B for X using LU decomposition
[M-5] $\operatorname{pinv}()$ - Moore-Penrose pseudoinverse
[M-5] qrinv() - Generalized inverse of matrix via QR decomposition
[M-4] Matrix - Matrix functions
[M-4] Solvers - Functions to solve $A X=B$ and to obtain A inverse

## Title

[M-5] Iusolve( ) - Solve AX=B for X using LU decomposition

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

lusolve ( $A, B$ ) solves $A X=B$ and returns $X$. lusolve() returns a matrix of missing values if $A$ is singular.
lusolve ( $A, B$, tol) does the same thing but allows you to specify the tolerance for declaring that $A$ is singular; see Tolerance under Remarks and examples below.
_lusolve $(A, B)$ and _lusolve $(A, B$, tol) do the same thing except that, rather than returning the solution $X$, they overwrite $B$ with the solution and, in the process of making the calculation, they destroy the contents of $A$.
_lusolve_la $(A, B)$ and _lusolve_la $(A, B, t o l)$ are the interfaces to the [M-1] LAPACK routines that do the work. They solve $A X=B$ for $X$, returning the solution in $B$ and, in the process, using as workspace (overwriting) $A$. The routines return 1 if $A$ was singular and 0 otherwise. If $A$ was singular, $B$ is overwritten with a matrix of missing values.

## Syntax



## Remarks and examples

The above functions solve $A X=B$ via LU decomposition and are accurate. An alternative is qrsolve() (see [M-5] qrsolve( )), which uses QR decomposition. The difference between the two solutions is not, practically speaking, accuracy. When $A$ is of full rank, both routines return equivalent results, and the LU approach is quicker, using approximately $O\left(2 / 3 n^{3}\right)$ operations rather than $O\left(4 / 3 n^{3}\right)$, where $A$ is $n \times n$.

The difference arises when $A$ is singular. Then the LU-based routines documented here return missing values. The QR-based routines documented in [M-5] qrsolve() return a generalized (least squares) solution.

For more information on $L U$ and QR decomposition, see $[\mathrm{M}-5] \operatorname{lud}()$ and see $[\mathrm{M}-5] \mathbf{q r d}()$.

Remarks are presented under the following headings:

## Derivation

Relationship to inversion
Tolerance

## Derivation

We wish to solve for $X$

$$
\begin{equation*}
A X=B \tag{1}
\end{equation*}
$$

Perform LU decomposition on $A$ so that we have $A=P L U$. Then (1) can be written as

$$
P L U X=B
$$

or, premultiplying by $P^{\prime}$ and remembering that $P^{\prime} P=I$,

$$
\begin{equation*}
L U X=P^{\prime} B \tag{2}
\end{equation*}
$$

Define

$$
\begin{equation*}
Z=U X \tag{3}
\end{equation*}
$$

Then (2) can be rewritten as

$$
\begin{equation*}
L Z=P^{\prime} B \tag{4}
\end{equation*}
$$

It is easy to solve (4) for $Z$ because $L$ is a lower-triangular matrix. Once $Z$ is known, it is easy to solve (3) for $X$ because $U$ is upper triangular.

## Relationship to inversion

Another way to solve

$$
A X=B
$$

is to obtain $A^{-1}$ and then calculate

$$
X=A^{-1} B
$$

It is, however, better to solve $A X=B$ directly because fewer numerical operations are required, and the result is therefore more accurate and obtained in less computer time.

Indeed, rather than thinking about how solving a system of equations can be implemented via inversion, it is more productive to think about how inversion can be implemented via solving a system of equations. Obtaining $A^{-1}$ amounts to solving

$$
A X=I
$$

Thus lusolve() (or any other solve routine) can be used to obtain inverses. The inverse of $A$ can be obtained by coding

```
: Ainv = lusolve(A, I(rows(A)))
```

In fact, we provide luinv() (see [M-5] luinv()) for obtaining inverses via LU decomposition, but luinv() amounts to making the above calculation, although a little memory is saved because the matrix $I$ is never constructed.

Hence, everything said about lusolve() applies equally to luinv().

## Tolerance

The default tolerance used is

$$
e t a=(1 \mathrm{e}-13) * \operatorname{trace}(\operatorname{abs}(U)) / n
$$

where $U$ is the upper-triangular matrix of the LU decomposition of $A: n \times n$. $A$ is declared to be singular if any diagonal element of $U$ is less than or equal to eta.

If you specify tol $>0$, the value you specify is used to multiply eta. You may instead specify tol $\leq$ 0 , and then the negative of the value you specify is used in place of eta; see [M-1] Tolerance.

So why not specify $t o l=0$ ? You do not want to do that because, as matrices become close to being singular, results can become inaccurate. Here is an example:

```
rseed(12345)
A = lowertriangle(runiform(4,4))
A[3,3] = 1e-15
trux = runiform(4,1)
b = A*trux
/* the above created an Ax=b problem, and we have placed the true
    value of x in trux. We now obtain the solution via lusolve()
    and compare trux with the value obtained:
*/
x = lusolve(A, b, 0)
trux, x
\begin{tabular}{l|rr|}
\cline { 2 - 4 } & \begin{tabular}{lll}
.260768733 & .260768733 \\
2 & .0267289389 & .0267289389 \\
3 & .1079423963 & .0989119749 \\
4 & .3666839808 & .3863636364
\end{tabular}\(\quad\)\begin{tabular}{l} 
The discussed numerical \\
instability can cause this \\
output to vary a little
\end{tabular} \\
\begin{tabular}{ll} 
across different computers
\end{tabular}
\end{tabular}
```

We would like to see the second column being nearly equal to the first-the estimated $x$ being nearly equal to the true $x$-but there are substantial differences.

Even though the difference between x and trux is substantial, the difference between them is small in the prediction space:

```
A*trux-b, A*x-b
```

|  | 1 | 2 |
| :--- | ---: | ---: |
| 1 | 0 | 0 |
| 2 | 0 | 0 |
| 3 | 0 | $-2.77556 \mathrm{e}-17$ |
| 4 | 0 | 0 |
|  |  |  |

What made this problem so difficult was the line $A[3,3]=1 e-15$. Remove that and you would find that the maximum absolute difference between x and trux would be $-2.44249 \mathrm{e}-15$.

The degree to which the residuals $\mathrm{A} * \mathrm{x}-\mathrm{b}$ are a reliable measure of the accuracy of $x$ depends on the condition number of the matrix, which can be obtained by [M-5] cond (), which for A, is $4.47684 \mathrm{e}+15$. If the matrix is well conditioned, small residuals imply an accurate solution for $x$. If the matrix is ill conditioned, small residuals are not a reliable indicator of accuracy.

Another way to check the accuracy of $x$ is to set $t o l=0$ and to see how well $x$ could be obtained were $\mathrm{b}=\mathrm{A} * \mathrm{x}$ :

```
: x = lusolve(A, b, 0)
: x2 = lusolve(A, A*x, 0)
```

If $x$ and $x 2$ are virtually the same, then you can safely assume that $x$ is the result of a numerically accurate calculation. You might compare $x$ and $x 2$ with mreldif ( $x 2, x$ ); see [M-5] reldif(). In our example, mreldif $(\mathrm{x} 2, \mathrm{x})$ is .03 , a large difference.

If $A$ is ill conditioned, then small changes in $A$ or $B$ can lead to radical differences in the solution for $X$.

## Conformability

lusolve( $A, B$, tol):
input:

| $A:$ |  | $n \times n$ |  |
| ---: | :--- | :--- | :--- |
| $B:$ | $n \times k$ |  |  |
| tol: $:$ | $1 \times 1$ | (optional) |  |

output:
result: $\quad n \times k$
_lusolve( $A, B$, tol):
input:

| A: |  | $n \times n$ |  |
| ---: | :--- | :--- | :--- |
| $B:$ | $n \times k$ |  |  |
| tol $:$ |  | $1 \times 1$ | (optional) |

output:
A: $\quad 0 \times 0$
B: $\quad n \times k$
_lusolve_la(A, B, tol):
input:

| $A:$ | $n \times n$ |  |
| ---: | :--- | :--- |
| $B:$ | $n \times k$ |  |
| tol: |  | $1 \times 1$ |$\quad$ (optional)

output:

$$
\begin{aligned}
& A: \\
& B: n \times 0 \\
& \text { result: } \\
& 1 \times 1
\end{aligned}
$$

## Diagnostics

lusolve ( $A, B, \ldots$ ), _lusolve $(A, B, \ldots)$, and _lusolve_la( $A, B, \ldots$ ) return a result containing missing if $A$ or $B$ contain missing values. The functions return a result containing all missing values if $A$ is singular.
_lusolve $(A, B, \ldots)$ and _lusolve_la $(A, B, \ldots)$ abort with error if $A$ or $B$ is a view.
_lusolve_la ( $A, B, \ldots$ ) should not be used directly; use _lusolve().

## Also see

[M-5] cholsolve( ) - Solve AX=B for X using Cholesky decomposition
[M-5] lud() - LU decomposition
[M-5] luinv() - Square matrix inversion
[M-5] qrsolve() - Solve AX=B for X using QR decomposition
[M-5] solvelower() - Solve $\mathrm{AX}=\mathrm{B}$ for X , A triangular
[M-5] svsolve( ) - Solve AX=B for X using singular value decomposition
[M-4] Matrix - Matrix functions
[M-4] Solvers - Functions to solve $A X=B$ and to obtain A inverse

## Title

[M-5] makesymmetric( ) - Make square matrix symmetric (Hermitian)

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

makesymmetric ( $A$ ) returns $A$ made into a symmetric (Hermitian) matrix by reflecting elements below the diagonal.
_makesymmetric ( $A$ ) does the same thing but stores the result back in $A$.

## Syntax

```
numeric matrix makesymmetric(numeric matrix A)
void makesymmetric(numeric matrix A)
```


## Remarks and examples

If $A$ is real, elements below the diagonal are copied into their corresponding above-the-diagonal position.

If $A$ is complex, the conjugate of the elements below the diagonal are copied into their corresponding above-the-diagonal positions, and the imaginary part of the diagonal is set to zero.

Whether $A$ is real or complex, roundoff error can make matrix calculations that are supposed to produce symmetric matrices produce matrices that vary a little from symmetry, and makesymmetric() can be used to correct the situation.

## Conformability

```
makesymmetric(A):
    A: 
_makesymmetric(A):
    A: }n\times
```


## Diagnostics

makesymmetric ( $A$ ) and makesymmetric ( $A$ ) abort with error if $A$ is not square. Also, makesymmetric() aborts with error if $A$ is a view.

## Also see

[M-5] issymmetric() - Whether matrix is symmetric (Hermitian)
[M-4] Manipulation - Matrix manipulation

## Title

[M-5] matexpsym( ) - Exponentiation and logarithms of symmetric matrices

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

matexpsym $(A)$ returns the matrix exponential of the symmetric (Hermitian) matrix $A$.
matlogsym $(A)$ returns the matrix natural logarithm of the symmetric (Hermitian) matrix $A$.
_matexpsym ( $A$ ) and _matlogsym ( $A$ ) do the same thing as matexpsym() and matlogsym(), but instead of returning the result, they store the result in $A$.

## Syntax

| numeric matrix | matexpsym (numeric matrix $A$ ) |
| :--- | :--- |
| numeric matrix | matlogsym(numeric matrix $A$ ) |
| void | matexpsym (numeric matrix $A$ ) |
| void | matlogsym (numeric matrix $A)$ |

## Remarks and examples

Do not confuse matexpsym (A) with $\exp (A)$, nor matlogsym $(A)$ with $\log (A)$.
matexpsym ( $2 *$ matlogsym $(A)$ ) produces the same result as $A * A . \exp ()$ and $\log ()$ return elementwise results.

Exponentiated results and logarithms are obtained by extracting the eigenvectors and eigenvalues of $A$, performing the operation on the eigenvalues, and then rebuilding the matrix. That is, first $X$ and $L$ are found such that

$$
\begin{equation*}
A X=X \times \operatorname{diag}(L) \tag{1}
\end{equation*}
$$

For symmetric (Hermitian) matrix $A, X$ is orthogonal, meaning $X^{\prime} X=X X^{\prime}=I$. Thus

$$
\begin{equation*}
A=X \times \operatorname{diag}(L) \times X^{\prime} \tag{2}
\end{equation*}
$$

matexpsym $(A)$ is then defined

$$
\begin{equation*}
A=X \times \operatorname{diag}(\exp (L)) \times X^{\prime} \tag{3}
\end{equation*}
$$

and matlogsym ( $A$ ) is defined

$$
\begin{equation*}
A=X \times \operatorname{diag}(\log (L)) \times X^{\prime} \tag{4}
\end{equation*}
$$

(1) is obtained via symeigensystem(); see [M-5] eigensystem().

## Conformability

matexpsym $(A)$, matlogsym $(A)$ :

| A: |  |
| ---: | :--- |
| result: |  |
| $n \times n$ |  |
|  | $n \times n$ |

_matexpsym (A), matlogsym (A):
input:
A: $\quad n \times n$
output:
A: $\quad n \times n$

## Diagnostics

matexpsym $(A)$, matlogsym ( $A$ ), _matexpsym ( $A$ ), and _matlogsym ( $A$ ) return missing results if $A$ contains missing values.

Also:

1. These functions do not check that $A$ is symmetric or Hermitian. If $A$ is a real matrix, only the lower triangle, including the diagonal, is used. If $A$ is a complex matrix, only the lower triangle and the real parts of the diagonal elements are used.
2. These functions return a matrix of the same storage type as $A$.

For symatlog $(A)$, this means that if $A$ is real and the result cannot be expressed as a real, a matrix of missing values is returned. If you want the generalized solution, code matlogsym $(C(A))$. This is the same rule as with scalars: $\log (-1)$ evaluates to missing, but $\log (C(-1))$ is $3.14159265 i$.
3. These functions are guaranteed to return a matrix that is numerically symmetric, Hermitian, or symmetriconly if theory states that the matrix should be symmetric, Hermitian, or symmetriconly. See [M-5] matpowersym() for a discussion of this issue.

For the functions here, real function $\exp (x)$ is defined for all real values of $x$ (ignoring overflow), and thus the matrix returned by matexpsym() will be symmetric (Hermitian).

The same is not true for matlogsym(). $\log (x)$ is not defined for $x=0$, so if any of the eigenvalues of $A$ are 0 or very small, a matrix of missing values will result. Also, $\log (x)$ is complex for $x<0$, and thus if any of the eigenvalues are negative, the resulting matrix will be (1) missing if $A$ is real stored as real, (2) symmetriconly if $A$ contains reals stored as complex, and (3) general if $A$ is complex.

## Also see

[M-5] eigensystem() - Eigenvectors and eigenvalues
[M-5] matpowersym() - Powers of a symmetric matrix
[M-4] Matrix - Matrix functions

## Title

[M-5] matpowersym( ) — Powers of a symmetric matrix

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

matpowersym $(A, p)$ returns $A^{p}$ for symmetric matrix or Hermitian matrix $A$. The matrix returned is real if $A$ is real and complex is $A$ is complex.
matpowersym $(A, p)$ does the same thing, but instead of returning the result, it stores the result in $A$.

## Syntax

```
numeric matrix matpowersym(numeric matrix A, real scalar p)
void _matpowersym(numeric matrix A, real scalar p)
```


## Remarks and examples

Do not confuse matpowersym ( $A, p$ ) and $A:^{\wedge} p$. If $p==2$, the first returns $A * A$ and the second returns $A$ with each element squared.

Powers can be positive, negative, integer, or noninteger. Thus matpowersym ( $A, .5$ ) is a way to find the square-root matrix $R$ such that $R * R==A$, and matpowersym $(A,-1)$ is a way to find the inverse. For inversion, you could obtain the result more quickly using other routines.

Powers are obtained by extracting the eigenvectors and eigenvalues of $A$, raising the eigenvalues to the specified power, and then rebuilding the matrix. That is, first $X$ and $L$ are found such that

$$
\begin{equation*}
A X=X \times \operatorname{diag}(L) \tag{1}
\end{equation*}
$$

For symmetric (Hermitian) matrix $A, X$ is orthogonal, meaning $X^{\prime} X=X X^{\prime}=I$. Thus

$$
\begin{equation*}
A=X \times \operatorname{diag}(L) X^{\prime} \tag{2}
\end{equation*}
$$

$A^{p}$ is then defined

$$
\begin{equation*}
A=X \times \operatorname{diag}\left(L:^{\wedge} p\right) \times X^{\prime} \tag{3}
\end{equation*}
$$

(1) is obtained via symeigensystem(); see [M-5] eigensystem().

## Conformability

matpowersym $(A, p)$ :

| $A:$ |  | $n \times n$ |
| ---: | :--- | ---: | :--- |
| $p:$ |  | $1 \times 1$ |
| result: |  | $n \times n$ |
| owersym $(A$, | $p):$ |  |

input:

$$
\begin{array}{ll}
A: & n \times n \\
p: & 1 \times 1
\end{array}
$$

output:

$$
A: \quad n \times n
$$

## Diagnostics

matpowersym $(A, p)$ and _matpowersym $(A, p)$ return missing results if $A$ contains missing values. Also:

1. These functions do not check that $A$ is symmetric or Hermitian. If $A$ is a real matrix, only the lower triangle, including the diagonal, is used. If $A$ is a complex matrix, only the lower triangle and the real parts of the diagonal elements are used.
2. These functions return a matrix of the same storage type as $A$. That means that if $A$ is real and $A^{p}$ cannot be expressed as a real, a matrix of missing values is returned. If you want the generalized solution, code matpowersym $(C(A), p)$. This is the same rule as with scalars: $(-1)^{\wedge} .5$ is missing, but $C(-1)^{\wedge} .5$ is 1 i .
3. These functions are guaranteed to return a matrix that is numerically symmetric, Hermitian, or symmetriconly if theory states that the matrix should be symmetric, Hermitian, or symmetriconly.

Concerning theory, the returned result is not necessarily symmetric (Hermitian). The eigenvalues $L$ of a symmetric (Hermitian) matrix are real. If $L:^{\wedge} p$ are real, then the returned matrix will be symmetric (Hermitian), but otherwise, it will not. Think of a negative eigenvalue and $p=.5$ : this results in a complex eigenvalue for $A^{p}$. Then if the original matrix was real (the eigenvectors were real), the resulting matrix will be symmetriconly. If the original matrix was complex (the eigenvectors were complex), the resulting matrix will have no special structure.

## Also see

[M-5] eigensystem() - Eigenvectors and eigenvalues
[M-5] matexpsym( ) - Exponentiation and logarithms of symmetric matrices
[M-4] Matrix - Matrix functions

## Title

[M-5] mean( ) - Means, variances, and correlations

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

mean ( $X, w$ ) returns the weighted-or-unweighted column means of data matrix $X$. mean() uses quad precision in forming sums and so is very accurate.
variance $(X, w)$ returns the weighted-or-unweighted variance matrix of $X$. In the calculation, means are removed and those means are calculated in quad precision, but quad precision is not otherwise used.
quadvariance $(X, w)$ returns the weighted-or-unweighted variance matrix of $X$. Calculation is highly accurate; quad precision is used in forming all sums.
meanvariance $(X, w)$ returns mean $(X, w) \backslash \operatorname{variance}(X, w)$.
quadmeanvariance $(X, w)$ returns mean $(X, w)$ \quadvariance $(X, w)$.
correlation ( $X, w$ ) returns the weighted-or-unweighted correlation matrix of $X$. correlation() obtains the variance matrix from variance().
quadcorrelation $(X, w)$ returns the weighted-or-unweighted correlation matrix of $X$. quadcorrelation() obtains the variance matrix from quadvariance().

In all cases, $w$ specifies the weight. Omit $w$, or specify $w$ as 1 to obtain unweighted means.
In all cases, rows of $X$ or $w$ that contain missing values are omitted from the calculation, which amounts to casewise deletion.

## Syntax

| real rowvector | mean $(X[, w])$ |
| :--- | :--- |
| real matrix | variance $(X[, w])$ |
| real matrix | quadvariance $(X[, w])$ |
| real matrix | meanvariance $(X[, w])$ |
| real matrix | quadmeanvariance $(X[, w])$ |
| real matrix | correlation $(X[, w])$ |
| real matrix | quadcorrelation $(X[, w])$ |

where
$X: \quad$ real matrix $X$ (rows are observations, columns are variables)
$w$ : real colvector $w$ and is optional

## Remarks and examples

1. There is no quadmean() function because mean(), in fact, is quadmean(). The fact that mean() defaults to the quad-precision calculation reflects our judgment that the extra computational cost in computing means in quad precision is typically justified.
2. The fact that variance() and correlation() do not default to using quad precision for their calculations reflects our judgment that the extra computational cost is typically not justified. The emphasis on this last sentence is on the word typically.

It is easier to justify means in part because the extra computational cost is less: there are only $k$ means but $k(k+1) / 2$ variances and covariances.
3. If you need just the mean or just the variance matrix, call mean() or variance() (or quadvariance()). If you need both, there is a CPU-time savings to be had by calling meanvariance() instead of the two functions separately (or quadmeanvariance() instead of calling mean() and quadvariance()).

The savings is not great-one mean() calculation is saved-but the greater rows $(X)$, the greater the savings.

Upon getting back the combined result, it can be efficiently broken into its components via

```
: var = meanvariance(X)
: means = var[1,.]
: var = var[|2,1 .,.|]
```


## Conformability

```
mean \((X, w)\) :
    \(X: \quad n \times k\)
    \(w: \quad n \times 1 \quad\) or \(\quad 1 \times 1 \quad\) (optional, \(w=1\) assumed)
    result: \(\quad 1 \times k\)
variance \((X, w)\), quadvariance \((X, w)\), correlation \((X, w)\), quadcorrelation \((X, w)\) :
            \(X: \quad n \times k\)
            \(w: \quad n \times 1 \quad\) or \(\quad 1 \times 1 \quad\) (optional, \(w=1\) assumed)
    result: \(\quad k \times k\)
meanvariance \((X, w)\), quadmeanvariance \((X, w)\) :
            \(X: \quad n \times k\)
            \(w: \quad n \times 1 \quad\) or \(\quad 1 \times 1 \quad\) (optional, \(w=1\) assumed)
    result: \(\quad(k+1) \times k\)
```


## Diagnostics

All functions omit from the calculation rows that contain missing values unless all rows contain missing values. Then the returned result contains all missing values.

## Also see

[M-4] Statistical - Statistical functions

## Title

[M-5] mindouble() - Minimum and maximum nonmissing value

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Reference | Also see |  |

## Description

mindouble() returns the largest negative, nonmissing value.
maxdouble() returns the largest positive, nonmissing value.
smallestdouble() returns the smallest full-precision value of $e, e>0$. The largest full-precision value of $e, e<0$ is - smallestdouble().

## Syntax

```
real scalar mindouble()
real scalar maxdouble()
real scalar smallestdouble()
```


## Remarks and examples

All nonmissing values $x$ fulfill mindouble() $\leq x \leq$ maxdouble().
All missing values $m$ fulfill $m>$ maxdouble()
Missing values also fulfill $m \geq$.
On all computers on which Stata and Mata are currently implemented, which are computers following IEEE standards:

| Function | Exact | Approximate <br> decimal value |
| :--- | :---: | :---: |
| mindouble() | $-1 . f f f f f f f f f f f f f \mathrm{X}+3 \mathrm{ff}$ | $-1.7977 \mathrm{e}+308$ |
| smallestdouble() | $+1.0000000000000 \mathrm{X}-3 \mathrm{fe}$ | $2.2251 \mathrm{e}-308$ |
| epsilon(1) | $+1.0000000000000 \mathrm{X}-034$ | $2.2205 \mathrm{e}-016$ |
| maxdouble() | $+1 . f f f f f f f f f f f f f \mathrm{X}+3 \mathrm{fe}$ | $8.9885 \mathrm{e}+307$ |

The smallest missing value $(.<. \mathrm{a}<\cdots<. \mathrm{z})$ is $+1.0000000000000 \mathrm{X}+3 \mathrm{ff}$.
Do not confuse smallestdouble() with the more interesting value epsilon(1). smallestdouble() is the smallest full-precision value of $e, e>0$. epsilon(1) is the smallest value of $e, \mathrm{e}+1$ $>1$; see [M-5] epsilon().

## Conformability

```
mindouble(), maxdouble(), smallestdouble():
    result: }1\times
```


## Diagnostics

None.

## Reference

Linhart, J. M. 2008. Mata Matters: Overflow, underflow and the IEEE floating-point format. Stata Journal 8: 255-268.

## Also see

[M-5] epsilon( ) - Unit roundoff error (machine precision)
[M-4] Utility - Matrix utility functions

## Title

[M-5] minindex() — Indices of minimums and maximums

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

minindex $(v, k, i, w)$ returns in $i$ and $w$ the indices of the $k$ minimums of $v$.
maxindex $(v, k, i, w)$ does the same, except that it returns the indices of the $k$ maximums. minindex() may be called with $k<0$; it is then equivalent to maxindex(). maxindex() may be called with $k<0$; it is then equivalent to minindex().

## Syntax

void minindex (real vector $v$, real scalar $k, i, w$ )
void maxindex (real vector $v$, real scalar $k, i, w$ )

Results are returned in $i$ and $w$.
$i$ will be a real colvector .
$w$ will be a $K \times 2$ real matrix, $K \leq|k|$.

## Remarks and examples

Remarks are presented under the following headings:
Use of functions when $v$ has all unique values Use of functions when $v$ has repeated (tied) values Summary

Remarks are cast in terms of minindex() but apply equally to maxindex().

## Use of functions when $\mathbf{v}$ has all unique values

Consider $v=(3,1,5,7,6)$.

1. minindex (v, $1, i, w)$ returns $i=2$, which means that $v[2]$ is the minimum value in $v$.
2. minindex ( $\mathrm{v}, 2$, $\mathrm{i}, \mathrm{w}$ ) returns $\mathrm{i}=(2,1)^{\prime}$, which means that $\mathrm{v}[2]$ is the minimum value of v and that $\mathrm{v}[1]$ is the second minimum.
3. minindex (v, 5, i, w) returns $i=(2,1,3,5,4)^{\prime}$, which means that the ordered values in $v$ are $v[2], v[1], v[3], v[5]$, and $v[4]$.
4. minindex (v, 6, i, w), minindex (v, 7, i, w), and so on, return the same as (5), because there are only five minimums in a five-element vector.

When v has unique values, the values returned in w are irrelevant.

- In (1), w will be $(1,1)$.
- In (2), w will be $(1,1 \backslash 2,1)$.
- ...
- In (5), w will be $(1,1 \backslash 2,1 \backslash 3,1 \backslash 4,1 \backslash 5,1)$.

The second column of $w$ records the number of tied values. Since the values in $v$ are unique, the second column of $w$ will be ones. If you have a problem where you are uncertain whether the values in $v$ are unique, code

```
if (!allof(w[,2], 1)) {
    /* uniqueness assumption false */
}
```


## Use of functions when $\mathbf{v}$ has repeated (tied) values

Consider $\mathrm{v}=(3,2,3,2,3,3)$.

1. minindex (v, 1 , $i, w)$ returns $i=(2,4)^{\prime}$, which means that there is one minimum value and that it is repeated in two elements of v , namely, v [2] and v [4].

Here, w will be $(1,2)$, but you can ignore that. There are two values in i corresponding to the same minimum.

When $k==1$, rows (i) equals the number of observations in $v$ corresponding to the minimum, as does $\mathrm{w}[1,2]$.
2. minindex (v, 2, i, w) returns $i=(2,4,1,3,5,6)^{\prime}$ and $\mathrm{w}=(1,2 \backslash 3,4)$.

Begin with w . The first row of w is $(1,2)$, which states that the indices of the first minimums of $v$ start at $i[1]$ and consist of two elements. Thus the indices of the first minimums are $i[1]$ and $i[2]$ (the minimums are $v[i[1]]$ and $v[i[2]]$, which of course are equal).

The second row of w is $(3,4)$, which states that the indices of the second minimums of v start at $i[3]$ and consist of four elements: $i[3]$, $i[4], i[5]$, and $i[6]$ (which are 1, 3, 5 , and 6).

In summary, rows (w) records the number of minimums returned. w [m,1] records where in $i$ the mth minimum begins (it begins at $i[w[m, 1]]$ ). $w[m, 2]$ records the total number of tied values. Thus one could step across the minimums and the tied values by coding

```
minindex(v, k, i, w)
for (m=1; m<=rows(w); m++) {
    for (j=w[m,1]; j<w[m,1]+w[m,2]; j++) {
        /* i [j] is the index in v of an mth minimum */
    }
}
```

3. minindex (v, 3, i, w), minindex (v, 4, i, w), and so on, return the same as (2) because, with $\mathrm{v}=(3,2,3,2,3,3)$, there are only two minimums.

## Summary

Consider minindex $(v, k, i, w)$. Returned will be

$$
\left.\begin{array}{l}
w=\left[\begin{array}{cc}
i_{1} & n_{1} \\
i_{2} & n_{2} \\
\cdot & \cdot \\
\cdot & \cdot
\end{array}\right] \quad w: K \times 2, \quad K \leq|k| \\
i=\left[\begin{array}{c}
j_{1} \\
j_{2} \\
j_{3} \\
j_{4} \\
\cdot \\
\cdot \\
\cdot
\end{array}\right] \leftarrow i\left[i_{1}\right] \text { is start of first minimums } \\
\left.i: 1 \times m, \quad m=n_{1}+i_{2}\right] \text { is start of second minimums } \\
\text { etc. } \\
i: \cdots
\end{array}\right\} \text { has } n_{1} \text { values } n_{2} \text { values }
$$

$j_{1}, j_{2}, \ldots$, are indices into $v$.

## Conformability

minindex $(v, k, i, w)$, maxindex $(v, k, i, w)$ :
input:

$$
\begin{array}{ll}
v: & n \times 1 \text { or } 1 \times n \\
k: & 1 \times 1
\end{array}
$$

output:

$$
i: \quad L \times 1, \quad L \geq K
$$

$$
w: \quad K \times 2, \quad K \leq|k|
$$

## Diagnostics

$\operatorname{minindex}(v, k, i, w)$ and maxindex $(v, k, i, w)$ abort with error if $i$ or $w$ is a view.
In minindex $(v, k, i, w)$ and maxindex $(v, k, i, w)$, missing values in $v$ are ignored in obtaining minimums and maximums.

In the examples above, we have shown input vector $v$ as a row vector. It can also be a column vector; it makes no difference.

In minindex $(v, k, i, w)$, input argument $k$ specifies the number of minimums to be obtained. $k$ may be zero. If $k$ is negative, $-k$ maximums are obtained.

Similarly, in maxindex $(v, k, i, w)$, input argument $k$ specifies the number of maximums to be obtained. $k$ may be zero. If $k$ is negative, $-k$ minimums are obtained.
minindex () and maxindex () are designed for use when $k$ is small relative to length ( $v$ ) ; otherwise, see order () in [M-5] sort( ).

## Also see

[M-5] minmax () - Minimums and maximums
[M-4] Utility - Matrix utility functions

## Title

[M-5] minmax( ) - Minimums and maximums

## Description Syntax Conformability Diagnostics Also see

## Description

These functions return the indicated minimums and maximums of $X$.
$\operatorname{rowmin}(X)$ returns the minimum of each row of $X, \operatorname{colmin}(X)$ returns the minimum of each column, and $\min (X)$ returns the overall minimum. Elements of $X$ that contain missing values are ignored.
rowmax $(X)$ returns the maximum of each row of $X, \operatorname{colmax}(X)$ returns the maximum of each column, and $\max (X)$ returns the overall maximum. Elements of $X$ that contain missing values are ignored.
rowminmax $(X)$ returns the minimum and maximum of each row of $X$ in an $r \times 2$ matrix; colminmax $(X)$ returns the minimum and maximum of each column in a $2 \times c$ matrix; and $\operatorname{minmax}(X)$ returns the overall minimum and maximum. Elements of $X$ that contain missing values are ignored.

The two-argument versions of rowminmax(), colminmax (), and minmax () allow you to specify how missing values are to be treated. Specifying a second argument with value 0 is the same as using the single-argument versions of the functions. In the two-argument versions, if the second argument is not zero, missing values are treated like all other values in determining the minimums and maximums: nonmissing $<.<. \mathrm{a}<. \mathrm{b}<\cdots<. \mathrm{z}$.
rowmaxabs $(A)$ and colmaxabs $(A)$ return the same result as rowmax (abs $(A)$ ) and col$\max (\operatorname{abs}(A))$. The advantage is that matrix $\operatorname{abs}(A)$ is never formed or stored, and so these functions use less memory.

## Syntax

```
real colvector rowmin(real matrix X)
real rowvector colmin(real matrix X)
real scalar min(real matrix X)
real colvector rowmax(real matrix X)
real rowvector colmax(real matrix X)
real scalar max(real matrix X)
real matrix rowminmax (real matrix X)
real matrix colminmax(real matrix X)
real rowvector minmax(real matrix X)
real matrix rowminmax(real matrix X, real scalar usemiss)
real matrix colminmax(real matrix X, real scalar usemiss)
real rowvector minmax(real matrix X, real scalar usemiss)
real colvector rowmaxabs(numeric matrix A)
real rowvector colmaxabs(numeric matrix A)
```


## Conformability

```
rowmin}(X), rowmax (X)
    X: r
    result: }\quadr\times
colmin(X), colmax(X):
    X: }\quadr\times
    result: }1\times
min}(X),\operatorname{max}(X)
    X: }\quadr\times
    result: }\quad1\times
rowminmax( }X\mathrm{ , usemiss):
            X: r
    usemiss: }\quad1\times
        result: r}<<
colminmax( }X\mathrm{ , usemiss)
            X: r}\times
            usemiss: }\quad1\times
            result: }2\times
minmax(X, usemiss)
            X: r
            usemiss: }\quad1\times
            result: }1\times
```

rowmaxabs (A):

$$
A: \quad r \times c
$$

            result: \(\quad r \times 1\)
    colmaxabs (A):
$\begin{aligned} \text { A: } & \\ \text { result: } & \\ & 1 \times c\end{aligned}$

## Diagnostics

row*() functions return missing values for the corresponding minimum or maximum when the entire row contains missing.
$\mathrm{col}^{*}()$ functions return missing values for the corresponding minimum or maximum when the entire column contains missing. $\min ()$ and $\max ()$ return missing values when the entire matrix contains missing values.

## Also see

[M-5] minindex () - Indices of minimums and maximums
[M-4] Mathematical - Important mathematical functions
[M-4] Utility - Matrix utility functions

## Title

[M-5] missing() — Count missing and nonmissing values

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

These functions return the indicated count of missing or nonmissing values.
colmissing $(X)$ returns the count of missing values of each column of $X$, rowmissing $(X)$ returns the count of missing values of each row, and missing $(X)$ returns the overall count.
colnonmissing $(X)$ returns the count of nonmissing values of each column of $X$, rownonmissing $(X)$ returns the count of nonmissing values of each row, and nonmissing $(X)$ returns the overall count.
hasmissing $(X)$ returns 1 if $X$ has a missing value or 0 if $X$ does not have a missing value.

## Syntax

| real rowvector | colmissing(numeric matrix $X$ ) |
| :--- | :--- |
| real colvector | rowmissing(numeric matrix $X$ ) |
| real scalar | missing(numeric matrix $X$ ) |
| real rowvector | colnonmissing(numeric matrix $X$ ) |
| real colvector | rownonmissing(numeric matrix $X$ ) |
| real scalar | nonmissing(numeric matrix $X$ ) |
| real scalar | hasmissing(numeric matrix $X$ ) |

## Remarks and examples

```
colnonmissing(X) = rows(X) :- colmissing(X)
rownonmissing}(X)=\operatorname{cols}(X) :- rowmissing(X
    nonmissing(X) = rows (X)*\operatorname{cols}(X) - missing(X)
```


## Conformability

colmissing $(X)$, colnonmissing $(X)$ :
$X: \quad r \times c$
result: $\quad 1 \times c$
rowmissing $(X)$, rownonmissing $(X)$ :
$X: \quad r \times c$
result: $\quad r \times 1$
missing $(X)$, nonmissing $(X)$, hasmissing $(X)$ :
$X: \quad r \times c$
result: $\quad 1 \times 1$

## Diagnostics

None.

## Also see

[M-5] editmissing() — Edit matrix for missing values
[M-4] Utility - Matrix utility functions

## Title

[M-5] missingof( ) - Appropriate missing value

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

missingof $(A)$ returns a missing of the same element type as $A$ :

- if $A$ is real, a real missing is returned;
- if $A$ is complex, a complex missing is returned;
- if $A$ is pointer, NULL is returned;
- if $A$ is string, " " is returned.


## Syntax

transmorphic scalar missingof (transmorphic matrix A)

## Remarks and examples

missingof () is useful when creating empty matrices of the same type as another matrix; for example,

```
newmat = J(rows(x), cols(x), missingof(x))
```


## Conformability

missingof ( $A$ )

| $A:$ |  | $r \times c$ |
| ---: | :--- | ---: |
| result: |  | $1 \times 1$ |

## Diagnostics

None.

## Also see

[M-4] Utility - Matrix utility functions

## Title

[M-5] $\bmod ()$ - Modulus
Description Syntax Conformability Diagnostics
Also see

## Description

$\bmod (x, y)$ returns the elementwise modulus of $x$ with respect to $y \cdot \bmod ()$ is defined

$$
\bmod (x, y)=x-y * \operatorname{trunc}(x / y)
$$

## Syntax

real matrix $\bmod ($ real matrix $x$, real matrix $y)$

## Conformability

$\bmod (x, y):$

```
            x: }\quad\mp@subsup{r}{1}{}\times\mp@subsup{c}{1}{
            y: }\quad\mp@subsup{r}{2}{}\times\mp@subsup{c}{2}{},\quadx\mathrm{ and }y\mathrm{ r-conformable
result: }\quad\operatorname{max}(\mp@subsup{r}{1}{},\mp@subsup{r}{2}{})\times\operatorname{max}(\mp@subsup{c}{1}{},\mp@subsup{c}{2}{})\quad\mathrm{ (elementwise calculation)
```


## Diagnostics

$\bmod (x, y)$ returns missing when either argument is missing or when $y=0$.

## Also see

[M-4] Scalar - Scalar mathematical functions

## Title

[M-5] moptimize( ) - Model optimization

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | References | Also see |  |

## Description

The moptimize() functions find coefficients $\left(\mathbf{b}_{1}, \mathbf{b}_{2}, \ldots, \mathbf{b}_{m}\right)$ that maximize or minimize $f\left(\mathbf{p}_{1}, \mathbf{p}_{2}, \ldots, \mathbf{p}_{m}\right)$, where $\mathbf{p}_{i}=\mathbf{X}_{i} \times \mathbf{b}_{i}^{\prime}$, a linear combination of $\mathbf{b}_{i}$ and the data. The user of moptimize() writes a Mata function or Stata program to evaluate $f\left(\mathbf{p}_{1}, \mathbf{p}_{2}, \ldots, \mathbf{p}_{m}\right)$. The data can be in Mata matrices or in the Stata dataset currently residing in memory.
moptimize() is especially useful for obtaining solutions for maximum likelihood models, minimum $\chi^{2}$ models, minimum squared-residual models, and the like.

## Syntax

If you are reading this entry for the first time, skip down to Description and to Remarks and examples, and more especially, to Mathematical statement of the moptimize() problem under Remarks and examples.

Syntax is presented under the following headings:
Step 1: Initialization
Step 2: Definition of maximization or minimization problem
Step 3: Perform optimization or perform a single function evaluation
Step 4: Post, display, or obtain results
Utility functions for use in all steps
Definition of $M$
Setting the sample
Specifying dependent variables
Specifying independent variables
Specifying constraints
Specifying weights or survey data
Specifying clusters and panels
Specifying optimization technique
Specifying initial values
Performing one evaluation of the objective function
Performing optimization of the objective function
Tracing optimization
Specifying convergence criteria
Accessing results
Stata evaluators
Advanced functions
Syntax of evaluators
Syntax of type lf evaluators
Syntax of type d evaluators
Syntax of type $1 f^{*}$ evaluators
Syntax of type gf evaluators
Syntax of type q evaluators
Passing extra information to evaluators
Utility functions

## Step 1: Initialization

$$
M=\text { moptimize_init() }
$$

## Step 2: Definition of maximization or minimization problem

In each of the functions, the last argument is optional. If specified, the function sets the value and returns void. If not specified, no change is made, and instead what is currently set is returned.

```
(varies) moptimize_init_which(M, { "max"| "min" })
(varies) moptimize_init_evaluator(M, &functionname())
(varies) moptimize_init_evaluator(M, "programname")
(varies) moptimize_init_evaluatortype(M, evaluatortype)
(varies) moptimize_init_negH(M, { "off"|"on" })
```

(varies) moptimize_init_touse(M, "tousevarname")
(varies) moptimize_init_ndepvars $(M, D)$
(varies) moptimize_init_depvar $(M, j, y)$
(varies) moptimize_init_eq_n( $M, m$ )
(varies) moptimize_init_eq_indepvars( $M, i, X$ )
(varies) moptimize_init_eq_cons(M,,$\{$ "on"|"off" $\}$ )
(varies) moptimize_init_eq_offset ( $M, i, o$ )
(varies) moptimize_init_eq_exposure( $M, i, t$ )
(varies) moptimize_init_eq_name( $M, i$, name)
(varies) moptimize_init_eq_colnames( $M, i$, names)
(varies) moptimize_init_eq_freeparm(M,, \{ "on"|"off" \})
(varies) moptimize_init_eq_coefs $(M, i, b 0)$
(varies) moptimize_init_constraints(M, Cc)
(varies) moptimize_init_search ( $M$, \{ "on"|"off" $\}$ )
(varies) moptimize_init_search_random(M, \{"off"|"on" \})
(varies) moptimize_init_search_repeat ( $M, n r$ )
(varies) moptimize_init_search_bounds ( $M, i$, minmax)
(varies) moptimize_init_search_rescale( $M,\{$ "on"|"off" $\}$ )
(varies) moptimize_init_weight( $M, w$ )
(varies) moptimize_init_weighttype(M, weighttype)
(varies) moptimize_init_cluster ( $M, c$ )
(varies) moptimize_init_svy (M, \{"off"|"on"\})
(varies) moptimize_init_by(M, by)
(varies) moptimize_init_nuserinfo(M, n_user)
(varies) moptimize_init_userinfo( $M, l, Z$ )
(varies) moptimize_init_technique( $M$, technique)
(varies) moptimize_init_vcetype(M, vcetype)
(varies) moptimize_init_nmsimplexdeltas ( $M$, delta)
(varies) moptimize_init_gnweightmatrix (M, W)
(varies) moptimize_init_singularHmethod(M, singularHmethod)
(varies) moptimize_init_conv_maxiter(M, maxiter)
(varies) moptimize_init_conv_warning(M, \{"on"|"off"\})
(varies) moptimize_init_conv_ptol(M, ptol)
(varies) moptimize_init_conv_vtol(M, vtol)
(varies) moptimize_init_conv_nrtol(M, nrtol)
(varies) moptimize_init_conv_ignorenrtol(M, \{"off"|"on" $\}$ )
(varies) moptimize_init_iterid(M, id)
(varies) moptimize_init_valueid(M, id)
(varies) moptimize_init_tracelevel(M, tracelevel)
(varies) moptimize_init_trace_ado( $M,\{" o f f " \mid " o n "\})$
(varies) moptimize_init_trace_dots(M, \{"off"|"on"\})
(varies) moptimize_init_trace_value (M, \{"on"|"off"\})
(varies) moptimize_init_trace_tol(M, \{"off"|"on"\})
(varies) moptimize_init_trace_step(M, \{"off"|"on"\})
(varies) moptimize_init_trace_coefdiffs(M, \{"off"|"on"\})
(varies) moptimize_init_trace_coefs (M, \{"off"|"on" $\}$ )
(varies) moptimize_init_trace_gradient(M, \{"off"|"on"\})
(varies) moptimize_init_trace_Hessian(M, \{"off"|"on"\})

```
(varies) moptimize_init_evaluations(M, {"off"|"on" })
(varies) moptimize_init_verbose(M, {"on"|"off"})
(varies) moptimize_init_deriv_usemin(M, {"off"|"on" })
(varies) moptimize_init_deriv_min(M [, real rowvector min])
```


## Step 3: Perform optimization or perform a single function evaluation

```
void moptimize(M)
real scalar_moptimize(M)
void moptimize_evaluate(M)
real scalar_moptimize_evaluate(M)
```


## Step 4: Post, display, or obtain results

```
void
                                    moptimize_result_post(M [, vcetype])
void
                                    moptimize_result_display([M[,vcetype]])
real scalar
    moptimize_result_value(M)
real scalar moptimize_result_value0(M)
real rowvector moptimize_result_eq_coefs(M [, i])
real rowvector moptimize_result_coefs(M)
string matrix moptimize_result_colstripe(M [, i])
real matrix moptimize_result_scores(M)
real rowvector moptimize_result_gradient(M [, i])
real matrix moptimize_result_Hessian(M [, i])
real matrix moptimize_result_V(M [, i])
string scalar moptimize_result_Vtype(M)
real matrix moptimize_result_V_oim(M [, i])
real matrix moptimize_result_V_opg(M [, i])
real matrix moptimize_result_V_robust(M [, i])
```

```
real scalar moptimize_result_iterations(M)
real scalar moptimize_result_converged(M)
real colvector moptimize_result_iterationlog( }M\mathrm{ )
real rowvector moptimize_result_evaluations(M)
real scalar moptimize_result_errorcode(M)
string scalar moptimize_result_errortext(M)
real scalar moptimize_result_returncode(M)
void moptimize_ado_cleanup (M)
```


## Utility functions for use in all steps

```
void
    moptimize_query(M)
real matrix
    moptimize_util_eq_indices(M,i[,i2])
(varies) moptimize_util_depvar(M,j)
    returns y set by moptimize_init_depvar(M,j, y), which is usually a real
    colvector
real colvector moptimize_util_xb(M, b, i)
real scalar moptimize_util_sum(M, real colvector v)
real rowvector moptimize_util_vecsum(M, i, real colvector s, real scalar value)
real matrix moptimize_util_matsum(M, i, i2, real colvector }s\mathrm{ ,
    real scalar value)
real matrix moptimize_util_matbysum(M, i, real colvector a, real colvector b,
    real scalar value)
real matrix
    moptimize_util_matbysum(M, i, i2, real colvector a,
    real colvector b, real colvector c, real scalar value)
```

pointer scalar moptimize_util_by(M)

## Definition of M

$M$, if it is declared, should be declared transmorphic. $M$ is obtained from moptimize_init() and then passed as an argument to the other moptimize() functions.
moptimize_init() returns $M$, called an moptimize() problem handle. The function takes no arguments. $M$ holds the information about the optimization problem.

## Setting the sample

Various moptimize_init_*() functions set values for dependent variables, independent variables, etc. When you set those values, you do that either by specifying Stata variable names or by specifying Mata matrices containing the data themselves. Function moptimize_init_touse() specifies the sample to be used when you specify Stata variable names.
moptimize_init_touse ( $M$, "tousevarname") specifies the name of the variable in the Stata dataset that marks the observations to be included. Observations for which the Stata variable is nonzero are included. The default is " ", meaning all observations are to be used.

You need to specify tousevarname only if you specify Stata variable names in the other moptimize_init_*() functions, and even then it is not required. Setting tousevar when you specify the data themselves via Mata matrices, whether views or not, has no effect.

## Specifying dependent variables

$D$ and $j$ index dependent variables:

$$
\text { index } \quad \text { Description }
$$

$D \quad$ number of dependent variables, $D \geq 0$
$D$ and $j$ are real scalars.
You set the dependent variables one at a time. In a particular optimization problem, you may have no dependent variables or have more dependent variables than equations.
moptimize_init_depvar $(M, j, y)$ sets the $j$ th dependent variable to be $y . y$ may be a string scalar containing a Stata variable name that in turn contains the values of the $j$ th dependent variable, or $y$ may be a real colvector directly containing the values.
moptimize_init_ndepvars $(M, D)$ sets the total number of dependent variables. You can set $D$ before defining dependent variables, and that speeds execution slightly, but it is not necessary because $D$ is automatically set to the maximum $j$.

## Specifying independent variables

Independent variables are defined within parameters or, equivalently, equations. The words parameter and equation mean the same thing. $m, i$, and $i 2$ index parameters:

| index | Description |
| :--- | :--- |
| $m$ | number of parameters (equations), $m \geq 1$ |
| $i$ | equation index, $1 \leq i \leq m$ |
| $i 2$ | equation index, $1 \leq i 2 \leq m$ |

$m, i$, and $i 2$ are real scalars.
The function to be optimized is $f\left(p_{1}, p_{2}, \ldots, p_{m}\right)$. The $i$ th parameter (equation) is defined as

$$
p i=X i \times b i^{\prime}+o i+\ln (t i):+b 0 i
$$

where

| $p i: N i \times 1$ | (ith parameter) |
| :---: | :--- |
| $X i: N i \times k i$ | (Ni observations on $k i$ independent variables) |
| $b i: \quad 1 \times k i$ | (coefficients to be fit) |
| $o i: N i \times 1$ | (exposure/offset in offset form, optional) |
| $t i: N i \times 1$ | (exposure/offset in exposure form, optional) |
| $b 0 i: \quad 1 \times 1$ | (constant or intercept, optional) |

Any of the terms may be omitted. The most common forms for a parameter are $p i=X i \times b i^{\prime}+b 0 i$ (standard model), $p i=X i \times b i^{\prime}$ (no-constant model), and $p i=b 0 i$ (constant-only model).

In addition, define $b: 1 \times K$ as the entire coefficient vector, which is to say,

$$
b=(b 1,[b 01,] \quad b 2,[b 02,] \quad \ldots)
$$

That is, because $b i$ is $1 \times k i$ for $i=1,2, \ldots, m$, then $b$ is $1 \times K$, where $K=\sum_{i} k i+c i$, where $c i$ is 1 if equation $i$ contains an intercept and is 0 otherwise. Note that $b i$ does not contain the constant or intercept, if there is one, but $b$ contains all the coefficients, including the intercepts. $b$ is called the full set of coefficients.

Parameters are defined one at a time by using the following functions:
moptimize_init_eq_n $(M, m)$ sets the number of parameters. Use of this function is optional; $m$ will be automatically determined from the other moptimize_init_eq_*() functions you issue.
moptimize_init_eq_indepvars ( $M, i, X$ ) sets $X$ to be the data (independent variables) for the $i$ th parameter. $X$ may be a $1 \times k i$ string rowvector containing Stata variable names, or $X$ may be a string scalar containing the same names in space-separated format, or $X$ may be an $N i \times k i$ real matrix containing the data for the independent variables. Specify $X$ as " " to omit term $X i \times b i^{\prime}$, for instance, as when fitting a constant-only model. The default is "".
moptimize_init_eq_cons $(M, i,\{$ "on" | "off" $\}$ ) specifies whether the equation for the $i$ th parameter includes $b 0 i$, a constant or intercept. Specify "on" to include $b 0 i$, "off" to exclude it. The default is "on".
moptimize_init_eq_offset $(M, i, o)$ specifies $o i$ in the equation for the $i$ th parameter. $o$ may be a string scalar containing a Stata variable name, or $o$ may be an $N i \times 1$ real colvector containing the offsets. The default is " ", meaning term oi is omitted. Parameters may not have both $o i$ and $\ln (t i)$ terms.
moptimize_init_eq_exposure ( $M, i, t$ ) specifies $t i$ in term $\ln (t i)$ of the equation for the $i$ th parameter. $t$ may be a string scalar containing a Stata variable name, or $t$ may be an $N i \times 1$ real colvector containing the exposure values. The default is " ", meaning term $\ln (t i)$ is omitted.
moptimize_init_eq_name ( $M, i$, name) specifies a string scalar, name, to be used in the output to label the $i$ th parameter. The default is to use an automatically generated name.
moptimize_init_eq_colnames ( $M, i$, names) specifies a $1 \times k i$ string rowvector, names, to be used in the output to label the coefficients for the $i$ th parameter. The default is to use automatically generated names.
moptimize_init_eq_freeparm ( $M, i$, $\{$ "on" $\mid$ "off" $\}$ ) specifies whether the equation for the $i$ th parameter is to be treated as a free parameter. This setting is ignored if there are independent variables or an offset attached to the $i$ th parameter. Free parameters have a shortcut notation that distinguishes them from constant linear equations. The free parameter notation for an equation labeled name is /name. The corresponding notation for a constant linear equation is name:_cons.

## Specifying constraints

Linear constraints may be placed on the coefficients, $b$, which may be either within equation or between equations.
moptimize_init_constraints ( $M, C c$ ) specifies an $R \times K+1$ real matrix, $C c$, that places $R$ linear restrictions on the $1 \times K$ full set of coefficients, $b$. Think of $C c$ as being ( $C, c$ ), $C: R \times K$ and $c: R \times 1$. Optimization will be performed subject to the constraint $C b^{\prime}=c$. The default is no constraints.

## Specifying weights or survey data

You may specify weights, and once you do, everything is automatic, assuming you implement your evaluator by using the provided utility functions.
moptimize_init_weight $(M, w)$ specifies the weighting variable or data. $w$ may be a string scalar containing a Stata variable name, or $w$ may be a real colvector directly containing the weight values. The default is "", meaning no weights.
moptimize_init_weighttype ( $M$, weighttype) specifies how $w$ is to be treated. weighttype may be "fweight", "aweight", "pweight", or "iweight". You may set $w$ first and then weighttype, or the reverse. If you set $w$ without setting weighttype, then "fweight" is assumed. If you set weighttype without setting $w$, then weighttype is ignored. The default weighttype is "fweight".

Alternatively, you may inherit the full set of survey settings from Stata by using moptimize_init_svy(). If you do this, do not use moptimize_init_weight(), moptimize_init_weighttype(), or moptimize_init_cluster(). When you use the survey settings, everything is nearly automatic, assuming you use the provided utility functions to implement your evaluator. The proviso is that your evaluator must be of evaluatortype lf, lf*, gf, or $q$.
moptimize_init_svy (M, \{ "off"|"on" \}) specifies whether Stata's survey settings should be used. The default is "off". Using the survey settings changes the default vcetype to "svy", which is equivalent to "robust".

## Specifying clusters and panels

Clustering refers to possible nonindependence of the observations within groups called clusters. A cluster variable takes on the same value within a cluster and different values across clusters. After setting the cluster variable, there is nothing special you have to do, but be aware that clustering is allowed only if you use a type lf, lf*, gf, or q evaluator, moptimize_init_cluster() allows you to set a cluster variable.

Panels refer to likelihood functions or other objective functions that can only be calculated at the panel level, for which there is no observation-by-observation decomposition. Unlike clusters, these panel likelihood functions are difficult to calculate and require the use of type d or gf evaluator. A panel variable takes on the same value within a panel and different values across panels. moptimize_init_by() allows you to set a panel variable.

You may set both a cluster variable and a panel variable, but be careful because, for most likelihood functions, panels are mathematically required to be nested within cluster.
moptimize_init_cluster $(M, c)$ specifies a cluster variable. $c$ may be a string scalar containing a Stata variable name, or $c$ may be a real colvector directly containing the cluster values. The default is "", meaning no clustering. If clustering is specified, the default vcetype becomes "robust".
moptimize_init_by ( $M$, by) specifies a panel variable and specifies that only panel-level calculations are meaningful. by may be a string scalar containing a Stata variable name, or by may be a real colvector directly containing the panel ID values. The default is " ", meaning no panels. If panels are specified, the default vcetype remains unchanged, but if the opg variance estimator is used, the opg calculation is modified so that it is clustered at the panel level.

## Specifying optimization technique

Technique refers to the numerical methods used to solve the optimization problem. The default is Newton-Raphson maximization.
moptimize_init_which $(M,\{$ "max" $\mid$ "min" $\})$ sets whether the maximum or minimum of the objective function is to be found. The default is "max".
moptimize_init_technique ( $M$, technique) specifies the technique to be used to find the coefficient vector $b$ that maximizes or minimizes the objective function. Allowed values are

| technique | Description |
| :--- | :--- |
| "nr" | modified Newton-Raphson |
| "dfp" | Davidon-Fletcher-Powell |
| "bfgs" | Broyden-Fletcher-Goldfarb-Shanno |
| "bhhh" | Berndt-Hall-Hall-Hausman |
| "nm" | Nelder-Mead |
| "gn" | Gauss-Newton (quadratic optimization) |

The default is " nr ".

You can switch between "nr", "dfp", "bfgs", and "bhhh" by specifying two or more of them in a space-separated list. By default, moptimize() will use an algorithm for five iterations before switching to the next algorithm. To specify a different number of iterations, include the number after the technique. For example, specifying moptimize_init_technique ( $M$, "bhhh 10 nr 1000 ") requests that moptimize() perform 10 iterations using the Berndt-Hall-Hall-Hausman algorithm, followed by 1,000 iterations using the modified Newton-Raphson algorithm, and then switch back to Berndt-Hall-Hall-Hausman for 10 iterations, and so on. The process continues until convergence or until maxiter is exceeded.
moptimize_init_singularHmethod ( $M$, singularHmethod) specifies the action to be taken during optimization if the Hessian is found to be singular and the technique requires the Hessian be of full rank. Allowed values are

## singularHmethod Description

"m-marquardt" modified Marquardt algorithm
"hybrid" mixture of steepest descent and Newton

## The default is "m-marquardt".

"hybrid" is equivalent to ml's difficult option; see [R] ml.
moptimize_init_nmsimplexdeltas ( $M$, delta) is for use with Nelder-Mead, also known as technique nm . This function sets the values of delta to be used, along with the initial parameters, to build the simplex required by Nelder-Mead. Use of this function is required only in the Nelder-Mead case. The values in delta must be at least 10 times larger than ptol. The initial simplex will be $\left\{p, p+\left(d_{1}, 0, \ldots, 0\right), p+\left(0, d_{2}, 0, \ldots, 0\right), \ldots, p+\left(0,0, \ldots, 0, d_{K}\right)\right\}$.

## Specifying initial values

Initial values are values you optionally specify that via a search procedure result in starting values that are then used for the first iteration of the optimization technique. That is,

## (optimization

initial values $\xrightarrow{\text { (searching) }}$ starting values $\xrightarrow{\text { technique) }}$ final results
Initial values are specified parameter by parameter.
moptimize_init_eq_coefs $(M, i, b 0)$ sets the initial values of the coefficients for the $i$ th parameter to be $b 0: 1 \times(k i+c i)$. The default is $(0,0, \ldots, 0)$.

The following functions control whether searching is used to improve on the initial values to produce better starting values. In addition to searching a predetermined set of hardcoded starting values, there are two other methods that can be used to improve on the initial values: random and rescaling. By default, random is off and rescaling is on. You can use one, the other, or both.
moptimize_init_search ( $M$, \{"on"|"off"\}) determines whether any attempts are to be made to improve on the initial values via a search technique. The default is "on". If you specify "off", the initial values become the starting values.
moptimize_init_search_random( $M$, \{"off"|"on" $\}$ ) determines whether the random method of improving initial values is to be attempted. The default is "off". Use of the random method is recommended when the initial values are or might be infeasible. Infeasible means that the function cannot be evaluated, which mechanically corresponds to the user-written
evaluator returning a missing value. The random method is seldom able to improve on feasible initial values. It works well when the initial values are or might be infeasible.
moptimize_init_search_repeat ( $M, n r$ ) controls how many times random values are tried if the random method is turned on. The default is 10 .
moptimize_init_search_bounds ( $M, i$, minmax) specifies the bounds for the random search. minmax is a $1 \times 2$ real rowvector containing the minimum and maximum values for the $i$ th parameter (equation). The default is (., .), meaning no lower and no upper bounds.
moptimize_init_search_rescale( $M,\{$ "on" | "off" $\}$ ) determines whether rescaling is attempted. The default is "on". Rescaling is a deterministic (not random) method. It also usually improves initial values, and usually reduces the number of subsequent iterations required by the optimization technique.

## Performing one evaluation of the objective function

moptimize_evaluate ( $M$ ) and _moptimize_evaluate $(M)$ perform one evaluation of the function evaluated at the initial values. Results can be accessed by using moptimize_result_*(), including first- and second-derivative-based results.
moptimize_evaluate() and _moptimize_evaluate() do the same thing, differing only in that moptimize_evaluate() aborts with a nonzero return code if things go badly, whereas _moptimize_evaluate() returns the real scalar error code. An infeasible initial value is an error.

The evaluation is performed at the initial values, not the starting values, and this is true even if search is turned on. If you want to perform an evaluation at the starting values, then perform optimization with maxiter set to 0 .

## Performing optimization of the objective function

moptimize ( $M$ ) and moptimize ( $M$ ) perform optimization. Both routines do the same thing; they differ only in their behavior when things go badly moptimize() returns nothing and aborts with error. _moptimize() returns a real scalar error code. moptimize() is best for interactive use and often adequate for use in programs that do not want to consider the possibility that optimization might fail.

The optimization process is as follows:

1. The initial values are used to create starting values. The value of the function at the starting values is calculated. If that results in a missing value, the starting values are declared infeasible. moptimize() aborts with return code 430; moptimize() returns a nonzero error code, which maps to 430 via moptimize_result_returncode(). This step is called iteration 0 .
2. The starting values are passed to the technique to produce better values. Usually this involves the technique calculating first and second derivatives, numerically or analytically, and then stepping multiple times in the appropriate direction, but techniques can vary on this. In general, the technique performs what it calls one iteration, the result of which is to produce better values. Those new values then become the starting values and the process repeats.

An iteration is said to fail if the new coefficient vector is infeasible (results in a missing value). Then attempts are made to recover and, if those attempts are successful, optimization
continues. If they fail, moptimize() aborts with error and _moptimize() returns a nonzero error code.

Other problems may arise, such as singular Hessians or the inability to find better values. Various fix-ups are made and optimization continues. These are not failures.

This step is called iterations 1,2 , and so on.
3. Step 2 continues either until the process converges or until the maximum number of iterations (maxiter) is exceeded. Stopping due to maxiter is not considered an error. Upon completion, programmers should check moptimize_result_converged().

If optimization succeeds, which is to say, if moptimize() does not abort or moptimize() returns 0 , you can use the moptimize_result_*() functions to access results.

## Tracing optimization

moptimize() and _moptimize() will produce output like

```
Iteration 0: f(p) = ........
Iteration 1: f(p) = ........
```

You can change the $f(p)$ to be "Log likelihood" or whatever else you want. You can also change "Iteration".
moptimize_init_iterid ( $M$, id) sets the string to be used to label the iterations in the iteration log. id is a string scalar. The default is "Iteration".
moptimize_init_valueid ( $M$, id) sets the string to be used to label the objective function value in the iteration log. id is a string scalar. The default is " $f(p)$ ".

Additional results can be displayed during optimization, which can be useful when you are debugging your evaluator. This is called tracing the execution.
moptimize_init_tracelevel( $M$, tracelevel) specifies the output to be displayed during the optimization process. Allowed values are
tracelevel To be displayed each iteration

| "none" | nothing |
| :--- | :--- |
| "value" | function value |
| "tolerance" | previous + convergence values |
| "step" | previous + stepping information |
| "coefdiffs" | previous + parameter relative differences |
| "paramdiffs" | same as "coefdiffs" |
| "coefs" | previous + parameter values |
| "params" | same as "coefs" |
| "gradient" | previous + gradient vector |
| "hessian" | previous + Hessian matrix |

Setting tracelevel is a shortcut. The other trace functions allow you to turn on and off individual features. In what follows, the documented defaults are the defaults when tracelevel is "value".
moptimize_init_trace_ado (M, \{ "off"|"on" \}) traces the execution of evaluators written as ado-files. This topic is not discussed in this manual entry. The default is "off".
moptimize_init_trace_dots ( $M$, \{ "off"|"on" \}) displays a dot each time your evaluator is called. The default is "off".
moptimize_init_trace_value ( $M$, $\{$ "on" | "off" $\}$ ) displays the function value at the start of each iteration. The default is "on".
moptimize_init_trace_tol ( $M$, \{ "off"|"on" \}) displays the value of the calculated result that is compared with the effective convergence criterion at the end of each iteration. The default is "off".
moptimize_init_trace_step ( $M$, \{ "off"|"on" \}) displays the steps within iteration. Listed are the value of objective function along with the word forward or backward. The default is "off".
moptimize_init_trace_coefdiffs ( $M$, \{ "off"|"on" \}) displays the coefficient relative differences from the previous iteration that are greater than the coefficient tolerance ptol. The default is "off".
moptimize_init_trace_coefs ( $M,\{$ "off"|"on" $\}$ ) displays the coefficients at the start of each iteration. The default is "off".
moptimize_init_trace_gradient ( $M$, \{"off"|"on" $\}$ ) displays the gradient vector at the start of each iteration. The default is "off".
moptimize_init_trace_Hessian( $M$, \{ "off"|"on" $\}$ ) displays the Hessian matrix at the start of each iteration. The default is "off".

## Specifying convergence criteria

Convergence is based on several rules controlled by four parameters: maxiter, ptol, vtol, and nrtol. The first rule is not a convergence rule, but a stopping rule, and it is controlled by maxiter.
moptimize_init_conv_maxiter ( $M$, maxiter) specifies the maximum number of iterations. If this number is exceeded, optimization stops and results are posted where they are accessible by using the moptimize_result_*() functions, just as if convergence had been achieved. moptimize_result_converged(), however, is set to 0 rather than 1 . The default maxiter is Stata's c(maxiter), which is 300 by default.
moptimize_init_conv_warning ( $M$, \{ "on"|"off" $\}$ ) specifies whether the warning message "convergence not achieved" is to be displayed when this stopping rule is invoked. The default is "on".

Usually, convergence occurs before the stopping rule comes into effect. The convergence criterion is a function of three real scalar values: ptol, vtol, and nrtol. Let

$$
\begin{aligned}
b & =\text { full set of coefficients } \\
b \_ \text {prior } & =\text { value of } b \text { from prior iteration } \\
v & =\text { value of objective function } \\
\text { v_prior } & =\text { value of } v \text { from prior iteration } \\
g & =\text { gradient vector from this iteration } \\
H & =\text { Hessian matrix from this iteration }
\end{aligned}
$$

Define, for maximization,
C_ptol: mreldif $\left(b, b \_p r i o r\right) \leq p t o l$
C_vtol: $\quad$ reldif $(v, v$ prior $) \leq v t o l$
C_nrtol: $\quad g \times \operatorname{invsym}(-H) \times g^{\prime}<n r t o l$
C_concave: $-H$ is positive semidefinite
For minimization, think in terms of maximization of $-f(p)$. Convergence is declared when

$$
\text { (C_ptol } \mid \text { C_vtol) \& C_nrtol \& C_concave }
$$

The above applies in cases of derivative-based optimization, which currently is all techniques except " nm " (Nelder-Mead). In the Nelder-Mead case, the criterion is

$$
\begin{array}{rr}
\text { C_ptol: } & \text { mreldif }(\text { vertices of } R) \leq \text { ptol } \\
\text { C_vtol: } & \text { reldif }(R) \leq \text { vtol }
\end{array}
$$

where $R$ is the minimum and maximum values on the simplex. Convergence is declared when C_ptol $\mid$ C_vtol.

The values of ptol, vtol, and nrtol are set by the following functions:
moptimize_init_conv_ptol( $M$, ptol) sets ptol. The default is $1 \mathrm{e}-6$.
moptimize_init_conv_vtol( $M$, vtol) sets vtol. The default is $1 \mathrm{e}-7$.
moptimize_init_conv_nrtol( $M$, nrtol) sets nrtol. The default is $1 \mathrm{e}-5$.
moptimize_init_conv_ignorenrtol ( $M$, \{ "off"|"on" $\}$ ) sets whether $C \_n r t o l$ should always be treated as true, which in effect removes the nrtol criterion from the convergence rule. The default is "off".

## Accessing results

Once you have successfully performed optimization, or you have successfully performed a single function evaluation, you may display results, post results to Stata, or access individual results.

To display results, use moptimize_result_display().
moptimize_result_display $(M)$ displays estimation results. Standard errors are shown using the default vcetype.
moptimize_result_display ( $M$, vcetype) displays estimation results. Standard errors are shown using the specified vcetype.

Also there is a third syntax for use after results have been posted to Stata, which we will discuss below.
moptimize_result_display() without arguments (not even $M$ ) displays the estimation results currently posted in Stata.
vcetype specifies how the variance-covariance matrix of the estimators (VCE) is to be calculated. Allowed values are

| vcetype | Description |
| :--- | :--- |
| "" | use default for technique |
| "oim" | observed information matrix |
| "opg" | outer product of gradients |
| "robust" | Huber/White/sandwich estimator |
| "svy" | survey estimator; equivalent to robust |

The default vcetype is oim except for technique bhhh, where it is opg.
If survey, pweights, or clusters are used, the default becomes robust or svy.

As an aside, if you set moptimize_init_vcetype() during initialization, that changes the default. moptimize_init_vcetype ( $M$, vcetype), vcetype being a string scalar, resets the default vcetype.

To post results to Stata, use moptimize_result_post ().
moptimize_result_post ( $M$ ) posts estimation results to Stata where they can be displayed with Mata function moptimize_result_post () (without arguments) or with Stata command ereturn display (see [P] ereturn). The posted VCE will be of the default vcetype.
moptimize_result_post ( $M$, vcetype) does the same thing, except the VCE will be of the specified vcetype.

The remaining moptimize_result_*() functions simply return the requested result. It does not matter whether results have been posted or previously displayed.
moptimize_result_value $(M)$ returns the real scalar value of the objective function.
moptimize_result_value0 ( $M$ ) returns the real scalar value of the objective function at the starting values.
moptimize_result_eq_coefs $(M[, i])$ returns the $1 \times(k i+c i)$ coefficient rowvector for the $i$ th equation. If $i \geq$. or argument $i$ is omitted, the $1 \times K$ full set of coefficients is returned.
moptimize_result_coefs $(M)$ returns the $1 \times K$ full set of coefficients.
moptimize_result_colstripe $(M[, i])$ returns a $(k i+c i) \times 2$ string matrix containing, for the $i$ th equation, the equation names in the first column and the coefficient names in the second. If $i \geq$. or argument $i$ is omitted, the result is $K \times 2$.
moptimize_result_scores $(M)$ returns an $N \times m$ (evaluator types lf and lf*), or an $N \times K$ (evaluator type gf), or an $L \times K$ (evaluator type q) real matrix containing the observation-byobservation scores. For all other evaluator types, $J(0,0,$.$) is returned. For evaluator types$ lf and lf*, scores are defined as the derivative of the objective function with respect to the parameters. For evaluator type gf, scores are defined as the derivative of the objective function with respect to the coefficients. For evaluator type q, scores are defined as the derivatives of the $L$ independent elements with respect to the coefficients.
moptimize_result_gradient $(M[, i])$ returns the $1 \times(k i+c i)$ gradient rowvector for the $i$ th equation. If $i \geq$. or argument $i$ is omitted, the $1 \times K$ gradient corresponding to the full set
of coefficients is returned. Gradient is defined as the derivative of the objective function with respect to the coefficients.
moptimize_result_Hessian $(M[, i])$ returns the $(k i+c i) \times(k i+c i)$ Hessian matrix for the $i$ th equation. If $i \geq$. or argument $i$ is omitted, the $K \times K$ Hessian corresponding to the full set of coefficients is returned. The Hessian is defined as the second derivative of the objective function with respect to the coefficients.
moptimize_result_V $(M[, i])$ returns the appropriate $(k i+c i) \times(k i+c i)$ submatrix of the full variance matrix calculated according to the default vcetype. If $i \geq$. or argument $i$ is omitted, the full $K \times K$ variance matrix corresponding to the full set of coefficients is returned.
moptimize_result_Vtype $(M)$ returns a string scalar containing the default vcetype.
moptimize_result_V_oim $(M[, i])$ returns the appropriate $(k i+c i) \times(k i+c i)$ submatrix of the full variance matrix calculated as the inverse of the negative Hessian matrix (the observed information matrix). If $i \geq$. or argument $i$ is omitted, the full $K \times K$ variance matrix corresponding to the full set of coefficients is returned.
moptimize_result_V_opg $(M[, i])$ returns the appropriate $(k i+c i) \times(k i+c i)$ submatrix of the full variance matrix calculated as the inverse of the outer product of the gradients. If $i \geq$. or argument $i$ is omitted, the full $K \times K$ variance matrix corresponding to the full full set of coefficients is returned. If moptimize_result_V_opg() is used with evaluator types other than $l f, l f *$, $g f$, or $q$, an appropriately dimensioned matrix of zeros is returned.
moptimize_result_V_robust $(M[, i])$ returns the appropriate $(k i+c i) \times(k i+c i)$ submatrix of the full variance matrix calculated via the sandwich estimator. If $i \geq$. or argument $i$ is omitted, the full $K \times K$ variance matrix corresponding to the full set of coefficients is returned. If moptimize_result_V_robust() is used with evaluator types other than lf, lf*, gf, or q , an appropriately dimensioned matrix of zeros is returned.
moptimize_result_iterations $(M)$ returns a real scalar containing the number of iterations performed.
moptimize_result_converged $(M)$ returns a real scalar containing 1 if convergence was achieved and 0 otherwise.
moptimize_result_iterationlog( $M$ ) returns a real colvector containing the values of the objective function at the end of each iteration. Up to the last 20 iterations are returned, one to a row.
moptimize_result_errorcode $(M)$ returns the real scalar containing the error code from the most recently run optimization or function evaluation. The error code is 0 if there are no errors. This function is useful only after _moptimize() or moptimize_evaluate() because the nonunderscore versions aborted with error if there were problems.
moptimize_result_errortext $(M)$ returns a string scalar containing the error text corresponding to moptimize_result_errorcode().
moptimize_result_returncode $(M)$ returns a real scalar containing the Stata return code corresponding to moptimize_result_errorcode().

The following error codes and their corresponding Stata return codes are for moptimize() only. To see other error codes and their corresponding Stata return codes, see [M-5] optimize().

| Error <br> code | Return <br> code | Error text |
| :---: | :---: | :--- |
| 400 | 1400 | could not find feasible values |
| 401 | 491 | Stata program evaluator returned an error <br> views are required when the evaluator is a |
| 402 | 198 | Stata program |
| 403 | 198 | Stata program evaluators require a touse variable |

## Stata evaluators

The following function is useful only when your evaluator is a Stata program instead of a Mata function.
moptimize_ado_cleanup ( $M$ ) removes all the global macros with the ML_ prefix. A temporary weight variable is also dropped if weights were specified.

## Advanced functions

These functions are not really advanced, they are just seldomly used.
moptimize_init_verbose ( $M$, $\{$ "on"|"off" $\}$ ) specifies whether error messages are to be displayed. The default is "on".
moptimize_init_deriv_usemin( $M$, \{"off"|"on" $\}$ ) specifies whether to use the minimum values of step sizes for computing numerical derivatives. The default is "off", meaning minimum values will not be used. If you specify "on", minimum values will be used; when minimum values are not specified by moptimize_init_deriv_min(), default minimum values are $1 \mathrm{e}-6$ and $1 \mathrm{e}-4$ for first- and second-order derivatives, respectively.
moptimize_init_deriv_min $(M, \min )$ specifies the minimum values of step sizes used for computing numerical derivatives. $\min$ is a $1 \times 2$ real row vector; the first column specifies the minimum for first-order derivatives, and the second column specifies the minimum for second-order derivatives. If there is a missing value in the row vector, the default values of $1 \mathrm{e}-6$ for first-order derivatives and $1 \mathrm{e}-4$ for second-order derivatives are used. If this function is used, moptimize_init_deriv_usemin() is automatically set to "on".
moptimize_init_deriv_min $(M)$ returns a $1 \times 2$ real row vector containing the user-specified min values.
moptimize_init_evaluations ( $M,\{$ "off" | "on" $\}$ ) specifies whether the system is to count the number of times the evaluator is called. The default is "off".
moptimize_result_evaluations $(M)$ returns a $1 \times 3$ real rowvector containing the number of times the evaluator was called, assuming moptimize_init_evaluations() was set on. Contents are the number of times called for the purposes of 1) calculating the objective function, 2) calculating the objective function and its first derivative, and 3) calculating the objective function and its first and second derivatives. If moptimize_init_evaluations() was set off, returned is $(0,0,0)$.

## Syntax of evaluators

An evaluator is a program you write that calculates the value of the function being optimized and optionally calculates the function's first and second derivatives. The evaluator you write is called by the moptimize() functions.

There are five styles in which the evaluator can be written, known as types lf, d, lf*, gf, and $q$. evaluatortype, optionally specified in moptimize_init_evaluatortype(), specifies the style in which the evaluator is written. Allowed values are

| evaluatortype | Description |
| :--- | :--- |
| "lf" | function() returns $N \times 1$ colvector value |
| "d0" | function() returns scalar value |
| "d1" | same as "d0" and returns gradient rowvector |
| "d2" | same as "d1" and returns Hessian matrix |
| "d1debug" | same as "d1" but checks gradient |
| "d2debug" | same as "d2" but checks gradient and Hessian |
| "lf0" | function() returns $N \times 1$ colvector value |
| "lf1" | same as "lf0" and return equation-level score matrix |
| "lf2" | same as "lf1" and returns Hessian matrix |
| "lf1debug" | same as "lf1" but checks gradient |
| "lf2debug" | same as "lf2" but checks gradient and Hessian |
| "gf0" | function() returns $N \times 1$ colvector value |
| "gf1" | same as "gf0" and returns score matrix |
| "gf2" | same as "gf1" and returns Hessian matrix |
| "gf1debug" | same as "gf1" but checks gradient |
| "gf2debug" | same as "gf2" but checks gradient and Hessian |
| "q0" | function() returns colvector value |
| "q1" | same as "q0" and returns score matrix |
| "q1debug" | same as "q1" but checks gradient |

The default is "lf" if not set.
"q" evaluators are used with technique "gn".
Returned gradients are $1 \times K$ rowvectors.
Returned Hessians are $K \times K$ matrices.
Examples of each of the evaluator types are outlined below.
You must tell moptimize() the identity and type of your evaluator, which you do by using the moptimize_init_evaluator() and moptimize_init_evaluatortype() functions.
moptimize_init_evaluator ( $M$, \&functionname()) sets the identity of the evaluator function that you write in Mata.
moptimize_init_evaluator( $M$, "programname") sets the identity of the evaluator program that you write in Stata.
moptimize_init_evaluatortype( $M$, evaluatortype) informs moptimize() of the style of evaluator you have written. evaluatortype is a string scalar from the table above. The default is "lf".
moptimize_init_negH ( $M,\{$ "off" | "on" $\}$ ) sets whether the evaluator you have written returns $H$ or $-H$, the Hessian or the negative of the Hessian, if it returns a Hessian at all. This is for backward compatibility with prior versions of Stata's ml command (see [R] ml). Modern evaluators return $H$. The default is "off".

## Syntax of type If evaluators

lfeval $(M, b, f v)$ :
inputs:
M: problem definition
b: coefficient vector
outputs:
fv: $\quad N \times 1, N=$ \# of observations
Notes:

1. The objective function is $f()=\operatorname{colsum}(f v)$.
2. In the case where $f()$ is a log-likelihood function, the values of the log likelihood must be summable over the observations.
3. For use with any technique except gn.
4. May be used with robust, clustering, and survey.
5. Returns $f v$ containing missing $(f v=$.$) if evaluation is not possible.$

## Syntax of type d evaluators

```
deval(M, todo, b, fv, g, H):
```

inputs:
M: problem definition
todo: real scalar containing 0,1 , or 2
$b$ : coefficient vector
outputs:
fv: real scalar
g: $\quad 1 \times K$, gradients, $K=\#$ of coefficients
$H: \quad K \times K$, Hessian
Notes:

1. The objective function is $f()=f v$.
2. For use with any log-likelihood function, or any function.
3. For use with any technique except gn and bhhh.
4. Cannot be used with robust, clustering, or survey.
5. deval() must always fill in $f v$, and fill in $g$ if todo $\geq 1$, and fill in $H$ if todo $=2$. For type d0, todo will always be 0 . For type d 1 and d1debug, todo will be 0 or 1 . For type d 2 and d2debug, todo will be 0,1 , or 2 .
6. Returns $f v=$. if evaluation is not possible.

## Syntax of type If ${ }^{\star}$ evaluators

lfeval ( $M$, todo, $b, f v, S, H$ ):
inputs:
M: problem definition
todo: real scalar containing 0,1 , or 2
$b$ : coefficient vector
outputs:

$$
\begin{aligned}
f v: & N \times 1, N=\# \text { of observations } \\
S: & N \times m, \text { scores, } m=\text { \# of equations (parameters) } \\
H: & K \times K, \text { Hessian, } K=\text { \# of coefficients }
\end{aligned}
$$

Notes:

1. The objective function is $f()=\operatorname{colsum}(f v)$.
2. Type lf* is a variation of type lf that allows the user to supply analytic derivatives. Although lf* could be used with an arbitrary function, it is intended for use when $f()$ is a log-likelihood function and the log-likelihood values are summable over the observations.
3. For use with any technique except gn.
4. May be used with robust, clustering, and survey.
5. Always returns $f v$, returns $S$ if todo $\geq 1$, and returns $H$ if todo $=2$. For type lf0, todo will always be 0 . For type lf1 and lf1debug, todo will be 0 or 1 . For type lf2 and lf2debug, todo will be 0,1 , or 2 .
6. Returns $f v$ containing missing $(f v=$.$) if evaluation is not possible.$

## Syntax of type gf evaluators

$\operatorname{gfeval}(M$, todo $, b, f v, S, H)$ :
inputs:
M: problem definition
todo: real scalar containing 0,1 , or 2
$b$ : coefficient vector
outputs:
fv: $\quad L \times 1$, values, $L=$ \# of independent elements
$S: \quad L \times K$, scores, $K=$ \# of coefficients
$H: \quad K \times K$, Hessian

Notes:

1. The objective function is $f()=\operatorname{colsum}(f v)$.
2. Type gf is a variation on type lf* that relaxes the requirement that the log-likelihood function be summable over the observations. gf is especially useful for fitting panel-data models with technique bhhh. Then $L$ is the number of panels.
3. For use with any gf is especially useful for fitting panel-data models with except gn.
4. May be used with robust, clustering, and survey.
5. Always returns $f v$, returns $S$ if todo $\geq 1$, and returns $H$ if todo $=2$. For type gf0, todo will always be 0 . For type gf1 and gf1debug, todo will be 0 or 1 . For type gf2 and gf 2 debug , todo will be 0,1 , or 2 .
6. Returns $f v=$. if evaluation is not possible.

## Syntax of type q evaluators

$$
\text { qeval }(M, \text { todo }, b, r, S)
$$

inputs:
M: problem definition
todo: real scalar containing 0 or 1
$b$ : coefficient vector
outputs:
$r: \quad L \times 1$ of independent elements
$S: \quad L \times K$, scores, $K=$ \# of coefficients
Notes:

1. Type q is for quadratic optimization. The objective function is $f()=r^{\prime} W r$, where $r$ is returned by qeval() and $W$ has been previously set by using moptimize_init_gnweightmatrix(), described below.
2. For use only with techniques gn and nm .
3. Always returns $r$ and returns $S$ if todo $=1$. For type q0, todo will always be 0 . For type q1 and q1debug, todo will be 0 or 1 . There is no type q2.
4. Returns $r$ containing missing, or $r=$. if evaluation is not possible.

Use moptimize_init_gnweightmatrix() during initialization to set matrix $W$.
moptimize_init_gnweightmatrix $(M, W)$ sets real matrix $W: L \times L$, which is used only by type q evaluators. The objective function is $r^{\prime} W r$. If $W$ is not set and if observation weights $w$ are set by using moptimize_init_weight (), then $W=\operatorname{diag}(w)$. If $w$ is not set, then $W$ is the identity matrix.
moptimize() does not produce a robust VCE when you set $W$ with
moptimize_init_gnweight().

## Passing extra information to evaluators

In addition to the arguments the evaluator receives, you may arrange that extra information be sent to the evaluator. Specify the extra information to be sent by using moptimize_init_userinfo().
moptimize_init_userinfo $(M, l, Z)$ specifies that the $l$ th piece of extra information is $Z$. $l$ is a real scalar. The first piece of extra information should be 1 ; the second piece, 2 ; and so on. $Z$ can be anything. No copy of $Z$ is made.
moptimize_init_nuserinfo( $M, n \_$user $)$specifies the total number of extra pieces of information to be sent. Setting n_user is optional; it will be automatically determined from the moptimize_init_userinfo() calls you issue.

Inside your evaluator, you access the information by using moptimize_util_userinfo(). moptimize_util_userinfo( $M, l$ ) returns the $Z$ set by moptimize_init_userinfo().

## Utility functions

There are various utility functions that are helpful in writing evaluators and in processing results returned by the moptimize_result_*() functions.

The first set of utility functions are useful in writing evaluators, and the first set return results that all evaluators need.
moptimize_util_depvar $(M, j)$ returns an $N j \times 1$ colvector containing the values of the $j$ th dependent variable, the values set by moptimize_init_depvar $(M, j, \ldots)$.
moptimize_util_xb $(M, b, i)$ returns the $N i \times 1$ colvector containing the value of the $i$ th parameter, which is usually $X i \times b i^{\prime}:+b 0 i$, but might be as complicated as $X i \times b i^{\prime}+o i+$ $\ln (t i):+b 0 i$.

Once the inputs of an evaluator have been processed, the following functions assist in making the calculations required of evaluators.
moptimize_util_sum $(M, v)$ returns the "sum" of colvector $v$. This function is for use in evaluators that require you to return an overall objective function value rather than observation-by-observation results. Usually, moptimize_util_sum () returns sum ( $v$ ), but in cases where you have specified a weight by using moptimize_init_weight() or there is an implied weight due to use of moptimize_init_svy (), the appropriately weighted sum is returned. Use moptimize_util_sum() to sum log-likelihood values.
moptimize_util_vecsum ( $M, i, s$, value) is like moptimize_util_sum(), but for use with gradients. The gradient is defined as the vector of partial derivatives of $f()$ with respect to the coefficients bi. Some evaluator types require that your evaluator be able to return this vector. Nonetheless, it is usually easier to write your evaluator in terms of parameters rather than coefficients, and this function handles the mapping of parameter gradients to the required coefficient gradients.

Input $s$ is an $N i \times 1$ colvector containing $d f / d p i$ for each observation. $d f / d p i$ is the partial derivative of the objective function, but with respect to the $i$ th parameter rather than the $i$ th set of coefficients. moptimize_util_vecsum() takes $s$ and returns the $1 \times(k i+c i)$ summed gradient. Also weights, if any, are factored into the calculation.

If you have more than one equation, you will need to call moptimize_util_vecsum() $m$ times, once for each equation, and then concatenate the individual results into one vector.
value plays no role in moptimize_util_vecsum()'s calculations. value, however, should be specified as the result obtained from moptimize_util_sum(). If that is inconvenient, make value any nonmissing value. If the calculation from parameter space to vector space cannot be performed, or if your original parameter space derivatives have any missing values, value will be changed to missing. Remember, when a calculation cannot be made, the evaluator is to return a missing value for the objective function. Thus storing the value of the objective function in value ensures that your evaluator will return missing if it is supposed to.
moptimize_util_matsum ( $M, i, i 2, s$, value) is similar to moptimize_util_vecsum(), but for Hessians (matrix of second derivatives).

Input $s$ is an $N i \times 1$ colvector containing $d^{2} f / d p i d p i 2$ for each observation. moptimize_util_matsum() returns the $(k i+c i) \times(k i 2+c i 2)$ summed Hessian. Also weights, if any, are factored into the calculation.

If you have $m>1$ equations, you will need to call moptimize_util_matsum () $m \times(m+1) / 2$ times and then join the results into one symmetric matrix.
value plays no role in the calculation and works the same way it does in moptimize_util_vecsum().
moptimize_util_matbysum() is an added helper for making moptimize_util_matsum() calculations in cases where you have panel data and the log-likelihood function's values exists only at the panel level. moptimize_util_matbysum ( $M, i, a, b, v a l u e$ ) is for making diagonal calculations and moptimize_util_matbysum ( $M, i, i 2, a, b, c, v a l u e$ ) is for making off-diagonal calculations.

This is an advanced topic; see Pitblado, Poi, and Gould (2024, 156-158) for a full description of it. In applying the chain rule to translate results from parameter space to coefficient space, moptimize_util_matsum() can be used to make some of the calculations, and moptimize_util_matbysum() can be used to make the rest. value plays no role and works just as it did in the other helper functions. moptimize_util_matbysum() is for use sometimes when by has been set, which is done via moptimize_init_by ( $M$, by). moptimize_util_matbysum() is never required unless by has been set.

The formula implemented in moptimize_util_matbysum ( $M, i, a, b$, value) is

$$
\sum_{j=1}^{N}\left(\sum_{t=1}^{T_{j}} a_{j t}\right)\left(\sum_{t=1}^{T_{j}} b_{j t} \mathbf{x}_{1 j t}^{\prime}\right)\left(\sum_{t=1}^{T_{j}} b_{j t} \mathbf{x}_{1 j t}\right)
$$

The formula implemented in moptimize_util_matbysum ( $M, i, i 2, a, b, c, v a l u e)$ is

$$
\sum_{j=1}^{N}\left(\sum_{t=1}^{T_{j}} a_{j t}\right)\left(\sum_{t=1}^{T_{j}} b_{j t} \mathbf{x}_{1 j t}^{\prime}\right)\left(\sum_{t=1}^{T_{j}} c_{j t} \mathbf{x}_{2 j t}\right)
$$

moptimize_util_by() returns a pointer to the vector of group identifiers that were set using moptimize_init_by(). This vector can be used with panelsetup() to perform panel level calculations.

The other utility functions are useful inside or outside of evaluators. One of the more useful is moptimize_util_eq_indices(), which allows two or three arguments.
moptimize_util_eq_indices $(M, i)$ returns a $1 \times 2$ vector that can be used with range subscripts to extract the portion relevant for the $i$ th equation from any $1 \times K$ vector, that is, from any vector conformable with the full coefficient vector.
moptimize_util_eq_indices $(M, i, i 2)$ returns a $2 \times 2$ matrix that can be used with range subscripts to exact the portion relevant for the $i$ th and $i 2$ th equations from any $K \times K$ matrix, that is, from any matrix with rows and columns conformable with the full variance matrix.

For instance, let b be the $1 \times K$ full coefficient vector, perhaps obtained by being passed into an evaluator, or perhaps obtained from $\mathrm{b}=$ moptimize_result_coefs $(M)$. Then b [|moptimize_util_eq_indices $(M, i) \mid]$ is the $1 \times(k i+c i)$ vector of coefficients for the $i$ th equation.

Let V be the $K \times K$ full variance matrix obtained by $\mathrm{V}=$ moptimize_result_V $(M)$. Then $\mathrm{V}[\mid$ moptimize_util_eq_indices $(M, i, i) \mid]$ is the $(k i+c i) \times(k i+c i)$ variance matrix for the $i$ th equation. V[|moptimize_util_eq_indices $(M, i, j) \mid]$ is the $(k i+c i) \times(k j+c j)$ covariance matrix between the $i$ th and $j$ th equations.

Finally, there is one more utility function that may help when you become confused: moptimize_query().
moptimize_query $(M)$ displays in readable form everything you have set via the moptimize_init_*() functions, along with the status of the system.

## Remarks and examples

Remarks are presented under the following headings:

```
Relationship of moptimize() to Stata's ml and to Mata's optimize()
Mathematical statement of the moptimize() problem
Filling in moptimize() from the mathematical statement
The type lf evaluator
The type d, lf*, gf, and q evaluators
Example using type d
Example using type lf*
```


## Relationship of moptimize() to Stata's $\mathbf{m l}$ and to Mata's optimize()

moptimize() is Mata's and Stata's premier optimization routine. This is the routine used by most of the official optimization-based estimators implemented in Stata.

That said, Stata's ml command—see [R] ml—provides most of the capabilities of Mata's moptimize (), and ml is easier to use. In fact, ml uses moptimize() to perform the optimization, and ml amounts to little more than a shell providing a friendlier interface. If you have a maximum likelihood model you wish to fit, we recommend you use ml instead of moptimize(). Use moptimize() when you need or want to work in the Mata environment, or when you wish to implement a specialized system for fitting a class of models.

Also make note of Mata's optimize() function; see [M-5] optimize(). moptimize() finds coefficients $\left(\mathbf{b}_{1}, \mathbf{b}_{2}, \ldots, \mathbf{b}_{m}\right)$ that maximize or minimize $f\left(\mathbf{p}_{1}, \mathbf{p}_{2}, \ldots, \mathbf{p}_{m}\right)$, where $\mathbf{p}_{i}=\mathbf{X}_{i} \times \mathbf{b}_{i}$. optimize () handles a simplified form of the problem, namely, finding constant ( $p_{1}, p_{2}, \ldots, p_{m}$ ) that maximizes or minimizes $f()$. moptimize() is the appropriate routine for fitting a Weibull model, but
if all you need to estimate are the fixed parameters of the Weibull distribution for some population, moptimize() is overkill and optimize() will prove easier to use.

These three routines are all related. Stata's ml uses moptimize() to do the numerical work. moptimize(), in turn, uses optimize() to perform certain calculations, including the search for parameters. There is nothing inferior about optimize() except that it cannot efficiently deal with models in which parameters are given by linear combinations of coefficients and data.

## Mathematical statement of the moptimize() problem

We mathematically describe the problem moptimize() solves not merely to fix notation and ease communication, but also because there is a one-to-one correspondence between the mathematical notation and the moptimize*() functions. Simply writing your problem in the following notation makes obvious the moptimize*() functions you need and what their arguments should be.

In what follows, we are going to simplify the mathematics a little. For instance, we are about to claim $\mathbf{p}_{i}=\mathbf{X}_{i} \times \mathbf{b}_{i}:+c_{i}$, when in the syntax section, you will see that $\mathbf{p}_{i}=\mathbf{X}_{i} \times \mathbf{b}_{i}+\mathbf{o}_{i}+\ln \left(\mathbf{t}_{i}\right):+c_{i}$. Here we omit $\mathbf{o}_{i}$ and $\ln \left(\mathbf{t}_{i}\right)$ because they are seldom used. We will omit some other details, too. The statement of the problem under Syntax, above, is the full and accurate statement. We will also use typefaces a little differently. In the syntax section, we use italics following programming convention. In what follows, we will use boldface for matrices and vectors, and italics for scalars so that you can follow the math more easily. So in this section, we will write $\mathbf{b}_{i}$, whereas under syntax we would write $b i$; regardless of typeface, they mean the same thing.

Function moptimize() finds coefficients

$$
\mathbf{b}=\left(\left(\mathbf{b}_{1}, c_{1}\right),\left(\mathbf{b}_{2}, c_{2}\right), \ldots,\left(\mathbf{b}_{m}, c_{m}\right)\right)
$$

where

$$
\begin{array}{llll}
\mathbf{b}_{1}: 1 \times k_{1}, & \mathbf{b}_{2}: 1 \times k_{2}, & \ldots, & \mathbf{b}_{m}: 1 \times k_{m} \\
c_{1}: 1 \times 1, & c_{2}: 1 \times 1, & \ldots, & c_{m}: 1 \times 1
\end{array}
$$

that maximize or minimize function

$$
f\left(\mathbf{p}_{1}, \mathbf{p}_{2}, \ldots, \mathbf{p}_{m} ; \mathbf{y}_{1}, \mathbf{y}_{2}, \ldots, \mathbf{y}_{D}\right)
$$

where

$$
\begin{array}{ll}
\mathbf{p}_{1}=\mathbf{X}_{1} \times \mathbf{b}_{1}^{\prime}:+c_{1}, & \mathbf{X}_{1}: N_{1} \times k_{1} \\
\mathbf{p}_{2}=\mathbf{X}_{2} \times \mathbf{b}_{2}^{\prime}:+c_{2}, & \mathbf{X}_{2}: N_{2} \times k_{2} \\
& \\
\mathbf{p}_{m}=\mathbf{X}_{m} \times \mathbf{b}_{m}^{\prime}:+c_{m}, & \mathbf{X}_{m}: N_{m} \times k_{m}
\end{array}
$$

and where $\mathbf{y}_{1}, \mathbf{y}_{2}, \ldots, \mathbf{y}_{D}$ are of arbitrary dimension.
Usually, $N_{1}=N_{2}=\cdots=N_{m}$, and the model is said to be fit on data of $N$ observations. Similarly, column vectors $\mathbf{y}_{1}, \mathbf{y}_{2}, \ldots, \mathbf{y}_{D}$ are usually called dependent variables, and each is also of $N$ observations.

As an example, let's write the maximum likelihood estimator for linear regression in the above notation. We begin by stating the problem in the usual way, but in Mata-ish notation:
Given data $\mathbf{y}: N \times 1$ and $\mathbf{X}: N \times k$, obtain $\left((\mathbf{b}, c), s^{2}\right)$ to fit

$$
\mathbf{y}=\mathbf{X} \times \mathbf{b}^{\prime}:+c+\mathbf{u}
$$

where the elements of $\mathbf{u}$ are distributed $N\left(0, s^{2}\right)$. The log-likelihood function is

$$
\ln L=\sum_{j} \ln \left(\text { normalden }\left(\mathbf{y}_{j}-\left(\mathbf{X}_{j} \times \mathbf{b}^{\prime}:+c\right), 0, \operatorname{sqrt}\left(s^{2}\right)\right)\right)
$$

where normalden $(x$, mean, $s d$ ) returns the density at $x$ of the Gaussian normal with the specified mean and standard deviation; see [M-5] normal().

The above is a two-parameter or, equivalently, two-equation model in moptimize() jargon. There may be many coefficients, but the likelihood function can be written in terms of two parameters, namely $\mathbf{p}_{1}=\mathbf{X} \times \mathbf{b}^{\prime}:+c$ and $\mathbf{p}_{2}=s^{2}$. Here is the problem stated in the moptimize() notation:

Find coefficients

$$
\mathbf{b}=\left(\left(\mathbf{b}_{1}, c_{1}\right),\left(c_{2}\right)\right)
$$

where

$$
\begin{aligned}
& \mathbf{b}_{1}: 1 \times k \\
& c_{1}: 1 \times 1, \quad c_{2}: 1 \times 1
\end{aligned}
$$

that maximize

$$
f\left(\mathbf{p}_{1}, \mathbf{p}_{2} ; \mathbf{y}\right)=\sum \ln \left(\operatorname{normalden}\left(\mathbf{y}-\mathbf{p}_{1}, 0, \operatorname{sqrt}\left(\mathbf{p}_{2}\right)\right)\right.
$$

where

$$
\begin{array}{ll}
\mathbf{p}_{1}=\mathbf{X} \times \mathbf{b}_{1}^{\prime}:+c_{1}, \quad \mathbf{X}: N \times k \\
\mathbf{p}_{2}=c_{2} &
\end{array}
$$

and where $y$ is $N \times 1$.

Notice that, in this notation, the regression coefficients ( $\mathbf{b}_{1}, c_{1}$ ) play a secondary role, namely, to determine $\mathbf{p}_{1}$. That is, the function, $f()$, to be optimized-a log-likelihood function here-is written in terms of $\mathbf{p}_{1}$ and $\mathbf{p}_{2}$. The program you will write to evaluate $f()$ will be written in terms of $\mathbf{p}_{1}$ and $\mathbf{p}_{2}$, thus abstracting from the particular regression model being fit. Whether the regression is mpg on weight or log income on age, education, and experience, your program to calculate $f()$ will remain unchanged. All that will change are the definitions of $\mathbf{y}$ and $\mathbf{X}$, which you will communicate to moptimize() separately.

There is another advantage to this arrangement. We can trivially generalize linear regression without writing new code. Note that the variance $s^{2}$ is given by $\mathbf{p}_{2}$, and currently, we have $\mathbf{p}_{2}=c_{2}$, that is, a constant. moptimize () allows parameters to be constant, but it equally allows them to be given by a linear combination. Thus rather than defining $\mathbf{p}_{2}=c_{2}$, we could define $\mathbf{p}_{2}=\mathbf{X}_{2} \times \mathbf{b}_{2}^{\prime}:+c_{2}$. If we did that, we would have a second linear equation that allowed the variance to vary observation by observation. As far as moptimize() is concerned, that problem is the same as the original problem.

## Filling in moptimize() from the mathematical statement

The mathematical statement of our sample problem is the following:
Find coefficients

$$
\begin{aligned}
\mathbf{b} & =\left(\left(\mathbf{b}_{1}, c_{1}\right),\left(c_{2}\right)\right) \\
\mathbf{b}_{1} & : 1 \times k \\
c_{1} & : 1 \times 1,
\end{aligned}
$$

that maximize

$$
f\left(\mathbf{p}_{1}, \mathbf{p}_{2} ; \mathbf{y}\right)=\sum \ln \left(\operatorname{normalden}\left(\mathbf{y}-\mathbf{p}_{1}, 0, \operatorname{sqrt}\left(\mathbf{p}_{2}\right)\right)\right.
$$

where

$$
\begin{aligned}
& \mathbf{p}_{1}=\mathbf{X} \times \mathbf{b}_{1}^{\prime}:+c_{1}, \quad \mathbf{X}: N \times k \\
& \mathbf{p}_{2}=c_{2}
\end{aligned}
$$

and where $y$ is $N \times 1$.

The corresponding code to perform the optimization is

```
. sysuse auto
(1978 automobile data)
mata:
mata (type ond to exit)
    function linregeval(transmorphic M, real rowvector b, real colvector lnf)
{
real colvector p1, p2
> real colvector y1
> real colvector
> p1 = moptimize_util_xb(M, b, 1)
> p2 = moptimize_util_xb(M, b, 2)
> y1 = moptimize_util_depvar(M, 1)
> lnf = ln(normalden(y1:-p1, 0, sqrt(p2)))
}
M = moptimize_init()
moptimize_init_evaluator(M, &linregeval())
: moptimize_init_depvar(M, 1, "mpg")
moptimize_init_eq_indepvars(M, 1, "weight foreign")
: moptimize_init_eq_indepvars(M, 2, "")
: moptimize(M)
Initial: f(p) = -<inf> (could not be evaluated)
Feasible: f(p) = -12949.708
Rescale: f(p) = -243.04355
Rescale eq: f(p) = -236.58999
Iteration 0: f(p) = -236.58999 (not concave)
Iteration 1: f(p) = -227.46735
Iteration 2: f(p) = -205.73129 (backed up)
Iteration 3: f(p) = -195.72216
Iteration 4: f(p) = -194.20847
Iteration 5: f(p) = -194.18313
Iteration 6: f(p) = -194.18306
Iteration 7: f(p) = -194.18306
: moptimize_result_display(M)
```

| mpg | Coefficient | Std. err. | z | $P>\|z\|$ | [95\% con | interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| eq1 |  |  |  |  |  |  |
| weight | -. 0065879 | . 0006241 | -10.56 | 0.000 | -. 007811 | -. 0053647 |
| foreign | -1.650029 | 1.053958 | -1.57 | 0.117 | -3.715749 | . 4156903 |
| _cons | 41.6797 | 2.121197 | 19.65 | 0.000 | 37.52223 | 45.83717 |
| eq2 |  |  |  |  |  |  |
| _cons | 11.13746 | 1.830987 | 6.08 | 0.000 | 7.54879 | 14.72613 |

## The type If evaluator

Let's now interpret the code we wrote, which was

```
: function linregeval(transmorphic M, real rowvector b,
    real colvector lnf)
{
        real colvector p1, p2
        real colvector y1
        p1 = moptimize_util_xb(M, b, 1)
        p2 = moptimize_util_xb(M, b, 2)
        y1 = moptimize_util_depvar(M, 1)
        lnf = ln(normalden(y1:-p1, 0, sqrt(p2)))
}
: M = moptimize_init()
: moptimize_init_evaluator(M, &linregeval())
: moptimize_init_depvar(M, 1, "mpg")
: moptimize_init_eq_indepvars(M, 1, "weight foreign")
: moptimize_init_eq_indepvars(M, 2, "")
: moptimize(M)
: moptimize_result_display(M)
```

We first defined the function to evaluate our likelihood function-we named the function linregeval(). The name was of our choosing. After that, we began an optimization problem by typing $M=$ moptimize_init(), described the problem with moptimize_init_*() functions, performed the optimization by typing moptimize(), and displayed results by using moptimize_result_display().

Function linregeval () is an example of a type lf evaluator. There are several different evaluator types, including d0, d1, d2, through q1. Of all of them, type lf is the easiest to use and is the one moptimize() uses unless we tell it differently. What makes lf easy is that we need only calculate the likelihood function; we are not required to calculate its derivatives. A description of lf appears under the heading Syntax of type if evaluators under Syntax above.

In the syntax diagrams, you will see that type lf evaluators receive three arguments, $M, b$, and $f v$, although in linregeval (), we decided to call them M , b , and $\operatorname{lnf}$. The first two arguments are inputs, and your evaluator is expected to fill in the third argument with observation-by-observation values of the log-likelihood function.

The input arguments are M and $\mathrm{b} . \mathrm{M}$ is the problem handle, which we have not explained yet. Basically, all evaluators receive $M$ as the first argument and are expected to pass $M$ to any moptimize*() subroutines that they call. $M$ in fact contains all the details of the optimization problem. The second argument, b, is the entire coefficient vector, which in the linregeval() case will be all the coefficients of our regression, the constant (intercept), and the variance of the residual. Those details are unimportant. Instead, your evaluator will pass $M$ and $b$ to moptimize() utility programs that will give you back what you need.

Using one of those utilities is the first action our linregeval() evaluator performs:
p1 = moptimize_util_xb(M, b, 1)

That returns observation-by-observation values of the first parameter, namely, $X \times b_{1}:+c_{1}$. moptimize_util_xb(x, b, 1) returns the first parameter because the last argument specified is 1 . We obtained the second parameter similarly:

$$
\text { p2 = moptimize_util_xb }(M, b, 2)
$$

To evaluate the likelihood function, we also need the dependent variable. Another moptimize*() utility returns that to us:

$$
\text { y1 = moptimize_util_depvar }(\mathrm{M}, 1)
$$

Having p1, p2, and y1, we are ready to fill in the log-likelihood values:

$$
\operatorname{lnf}=\ln (\operatorname{normalden}(y 1:-p 1,0, \operatorname{sqrt}(p 2)))
$$

For a type lf evaluator, you are to return observation-by-observation values of the log-likelihood function; moptimize() itself will sum them to obtain the overall log likelihood. That is exactly what the line $\operatorname{lnf}=\ln ($ normalden $(\mathrm{y} 1:-\mathrm{p} 1,0, \operatorname{sqrt}(\mathrm{p} 2)))$ did. Note that y 1 is $N \times 1, \mathrm{p} 1$ is $N \times 1$, and p2 is $N \times 1$, so the $\operatorname{lnf}$ result we calculate is also $N \times 1$. Some of the other evaluator types are expected to return a scalar equal to the overall value of the function.

With the evaluator defined, we can estimate a linear regression by typing

```
: M = moptimize_init()
: moptimize_init_evaluator(M, &linregeval())
: moptimize_init_depvar(M, 1, "mpg")
: moptimize_init_eq_indepvars(M, 1, "weight foreign")
: moptimize_init_eq_indepvars(M, 2, "")
: moptimize(M)
: moptimize_result_display(M)
```

All estimation problems begin with
M = moptimize_init()

The returned value $M$ is called a problem handle, and from that point on, you will pass $M$ to every other moptimize() function you call. M contains the details of your problem. If you were to list $M$, you would see something like

```
: M
    0x15369a
```

$0 \times 15369$ a is in fact the address where all those details are stored. Exactly how M works does not matter, but it is important that you understand what M is. M is your problem. In a more complicated problem, you might need to perform nested optimizations. You might have one optimization problem, and right in the middle of it, even right in the middle of evaluating its log-likelihood function, you might need to set up and solve another optimization problem. You can do that. The first problem you would set up as M1 = moptimize_init(). The second problem you would set up as M2 = moptimize_init(). moptimize() would not confuse the two problems because it would know to which problem you were referring by whether you used M1 or M2 as the argument of the moptimize() functions. As another example, you might have one optimization problem, $M=$ moptimize_init(), and halfway through it, decide you want to try something wild. You could code $M 2=M$, thus making a copy of the problem, use the moptimize*() functions with M2, and all the while your original problem would remain undisturbed.

Having obtained a problem handle, that is, having coded $M=$ moptimize_init(), you now need to fill in the details of your problem. You do that with the moptimize_init_*() functions. The order in which you do this does not matter. We first informed moptimize() of the identity of the evaluator function:

```
: moptimize_init_evaluator(M, &linregeval())
```

We must also inform moptimize() as to the type of evaluator function linregeval() is, which we could do by coding

```
: moptimize_init_evaluatortype(M, "lf")
```

We did not bother, however, because type lf is the default.
After that, we need to inform moptimize() as to the identity of the dependent variables:

```
: moptimize_init_depvar(M, 1, "mpg")
```

Dependent variables play no special role in moptimize(); they are merely something that are remembered so that they can be passed to the evaluator function that we write. One problem might have no dependent variables and another might have lots of them. moptimize_init_depvar ( $M$, $i, y$ )'s second argument specifies which dependent variable is being set. There is no requirement that the number of dependent variables match the number of equations. In the linear regression case, we have one dependent variable and two equations.

Next we set the independent variables, or equivalently, the mapping of coefficients, into parameters. When we code

```
: moptimize_init_eq_indepvars(M, 1, "weight foreign")
```

we are stating that there is a parameter, $\mathbf{p}_{1}=\mathbf{X}_{1} \times \mathbf{b}_{1}:+c_{1}$, and that $\mathbf{X}_{1}=$ (weight, foreign). Thus $\mathbf{b}_{1}$ contains two coefficients, that is, $\mathbf{p}_{1}=$ (weight, foreign) $\times\left(b_{11}, b_{12}\right)^{\prime}:+c_{1}$. Actually, we have not yet specified whether there is a constant, $c_{1}$, on the end, but if we do not specify otherwise, the constant will be included. If we want to suppress the constant, after coding moptimize_init_eq_indepvars(M, 1, "weight foreign"), we would code moptimize_init_eq_cons(M, 1, "off"). The 1 says first equation, and the "off" says to turn the constant off.

As an aside, we coded moptimize_init_eq_indepvars(M, 1, "weight foreign") and so specified that the independent variables were the Stata variables weight and foreign, but the independent variables do not have to be in Stata. If we had a $74 \times 2$ matrix named data in Mata that we wanted to use, we would have coded moptimize_init_eq_indepvars ( $\mathrm{M}, 1$, data).

To define the second parameter, we code

```
: moptimize_init_eq_indepvars(M, 2, "")
```

Thus we are stating that there is a parameter, $\mathbf{p}_{2}=\mathbf{X}_{2} \times \mathbf{b}_{2}:+c_{2}$, and that $\mathbf{X}_{2}$ does not exist, leaving $\mathbf{p}_{2}=c_{2}$, meaning that the second parameter is a constant.

Our problem defined, we code

```
: moptimize(M)
```

to obtain the solution, and we code

```
: moptimize_result_display(M)
```

to see the results. There are many moptimize_result_*() functions for use after the solution is obtained.

## The type d, If*, gf, and q evaluators

Above we wrote our evaluator function in the style of type lf. moptimize() provides four other evaluator types-called types $\mathrm{d}, \mathrm{lf} *$, gf , and q-and each have their uses.

Using type lf above, we were required to calculate the observation-by-observation log likelihoods and that was all. Using another type of evaluator, say, type d, we are required to calculate the overall log likelihood, and optionally, its first derivatives, and optionally, its second derivatives. The corresponding evaluator types are called d0, d1, and d2. Type $d$ is better than type lf because if we do calculate the derivatives, then moptimize() can execute more quickly and it can produce a slightly more accurate result (more accurate because numerical derivatives are not involved). These speed and accuracy gains justify type d1 and d2, but what about type d0? For many optimization problems, type d 0 is redundant and amounts to nothing more than a slight variation on type lf. In these cases, type d0's justification is that if we want to write a type d1 or type d2 evaluator, then it is usually easiest to start by writing a type d0 evaluator. Make that work, and then add to the code to convert our type d0 evaluator into a type d1 evaluator; make that work, and then, if we are going all the way to type d 2 , add the code to convert our type d 1 evaluator into a type d 2 evaluator.

For other optimization problems, however, there is a substantive reason for type d0's existence. Type lf requires observation-by-observation values of the log-likelihood function, and for some likelihood functions, those simply do not exist. Think of a panel-data model. There may be observations within each of the panels, but there is no corresponding log-likelihood value for each of them. The loglikelihood function is defined only across the entire panel. Type lf cannot handle problems like that. Type d0 can.

That makes type d 0 seem to be a strict improvement on type lf. Type d 0 can handle any problem that type lf can handle, and it can handle other problems to boot. Where both can handle the problem, the only extra work to use type d0 is that we must sum the individual values we produce, and that is not difficult. Type lf, however, has other advantages. If you write a type lf evaluator, then without writing another line of code, you can obtain the robust estimates of variance, adjust for clustering, account for survey design effects, and more. moptimize() can do all that because it has the results of an observation-by-observation calculation. moptimize() can break into the assembly of those observation-by-observation results and modify how that is done. moptimize() cannot do that for d0.

So there are advantages both ways.
Another provided evaluator type is type $1 \mathrm{f} *$. Type $l f *$ is a variation on type lf . It also comes in the subflavors $\operatorname{lf} 0, \operatorname{lf} 1$, and $\operatorname{lf} 2$. Type $l f *$ allows you to make observation-level derivative calculations, which means that results can be obtained more quickly and more accurately. Type lf* is designed to always work where lf is appropriate, which means panel-data estimators are excluded. In return, it provides all the ancillary features provided by type lf, meaning that robust standard errors, clustering, and survey-data adjustments are available. You write the evaluator function in a slightly different style when you use type lf* rather than type d.

Type gf is a variation on type lf* that relaxes the requirement that the log-likelihood function be summable over the observations. Thus type gf can work with panel-data models and resurrect the features of robust standard errors, clustering, and survey-data adjustments. Type gf evaluators, however, are more difficult to write than type $1 f *$ evaluators.

Type $q$ is for the special case of quadratic optimization. You either need it, and then only type $q$ will do, or you do not.

## Example using type d

Let's return to our linear regression maximum-likelihood estimator. To remind you, this is a twoparameter model, and the log-likelihood function is

$$
f\left(\mathbf{p}_{1}, \mathbf{p}_{2} ; \mathbf{y}\right)=\sum \ln \left(\text { normalden }\left(\mathbf{y}-\mathbf{p}_{1}, 0, \operatorname{sqrt}\left(\mathbf{p}_{2}\right)\right)\right.
$$

This time, however, we are going to parameterize the variance parameter $\mathbf{p}_{2}$ as the $\log$ of the standard deviation, so we will write

$$
f\left(\mathbf{p}_{1}, \mathbf{p}_{2} ; \mathbf{y}\right)=\sum \ln \left(\text { normalden }\left(\mathbf{y}-\mathbf{p}_{1}, 0, \exp \left(\mathbf{p}_{2}\right)\right)\right.
$$

It does not make any conceptual difference which parameterization we use, but the log parameterization converges a little more quickly, and the derivatives are easier, too. We are going to implement a type d2 evaluator for this function. To save you from pulling out pencil and paper, let's tell you the derivatives:

$$
\begin{aligned}
d f / d \mathbf{p}_{1} & =\mathbf{z}: / \mathbf{s} \\
d f / d \mathbf{p}_{2} & =\mathbf{z}: \wedge 2:-1 \\
& \\
d^{2} f / d \mathbf{p}_{1}^{2} & =-1: / \mathbf{s}: \wedge 2 \\
d^{2} f / d \mathbf{p} 2^{2} & =-2 \times \mathbf{z}: \wedge 2 \\
d^{2} f / d \mathbf{p} d \mathbf{p}_{2} & =-2 \times \mathbf{z}: / \mathbf{s}
\end{aligned}
$$

where

$$
\begin{aligned}
& \mathbf{z}=\left(\mathbf{y}:-\mathbf{p}_{1}\right): / \mathbf{s} \\
& \mathbf{s}=\exp \left(\mathbf{p}_{2}\right)
\end{aligned}
$$

The d2 evaluator function for this problem is

```
function linregevald2(transmorphic M, real scalar todo,
    real rowvector b, fv, g, H)
{
        y1 = moptimize_calc_depvar(M, 1)
        p1 = moptimize_calc_xb(M, b, 1)
        p2 = moptimize_calc_xb(M, b, 2)
        s = exp(p2)
        z = (y1:-p1):/s
        fv = moptimize_util_sum(M, ln(normalden(y1:-p1, 0, s)))
        if (todo>=1) {
            s1 = z:/s
            s2 = z:^2 :- 1
            g1 = moptimize_util_vecsum(M, 1, s1, fv)
            g2 = moptimize_util_vecsum(M, 2, s2, fv)
            g = (g1, g2)
```

```
    if (todo==2) {
    h11 = -1:/s:^2
    h22 = -2*z:^2
    h12 = -2*z:/s
    H11 = moptimize_util_matsum(M, 1,1, h11, fv)
    H22 = moptimize_util_matsum(M, 2,2, h22, fv)
    H12 = moptimize_util_matsum(M, 1,2, h12, fv)
    H = (H11, H12 \ H12', H22)
}
}
}
```

The code to fit a model of mpg on weight and foreign reads

```
: M = moptimize_init()
: moptimize_init_evaluator(M, &linregevald2())
: moptimize_init_evaluatortype(M, "d2")
: moptimize_init_depvar(M, 1, "mpg")
: moptimize_init_eq_indepvars(M, 1, "weight foreign")
: moptimize_init_eq_indepvars(M, 2, "")
: moptimize(M)
: moptimize_result_display(M)
```

By the way, function linregevald2() will work not only with type d2, but also with types d1 and d0. Our function has the code to calculate first and second derivatives, but if we use type d1, todo will never be 2 and the second derivative part of our code will be ignored. If we use type d0, todo will never be 1 or 2 and so the first and second derivative parts of our code will be ignored. You could delete the unnecessary blocks if you wanted.

It is also worth trying the above code with types d1debug and d2debug. Type d1debug is like d 1 ; the second derivative code will not be used. Also type d1debug will almost ignore the first derivative code. Our program will be asked to make the calculation, but moptimize() will not use the results except to report a comparison of the derivatives we calculate with numerically calculated derivatives. That way, we can check that our program is right. Once we have done that, we move to type d2debug, which will check our second-derivative calculation.

## Example using type If*

The lf 2 evaluator function for the linear-regression problem is almost identical to the type d 2 evaluator. It differs in that rather than return the summed $\log$ likelihood, we return the observation-level log likelihoods. And rather than return the gradient vector, we return the equation-level scores that when used with the chain-rule can be summed to produce the gradient. The conversion from d2 to lf2 was possible because of the observation-by-observation nature of the linear-regression problem; if the evaluator was not going to be implemented as lf, it always should have been implemented as lf1 or lf 2 instead of d 1 or d 2 . In the d 2 evaluator above, we went to extra work-summing the scores-the result of which was to eliminate moptimize() features such as being able to automatically adjust for clusters and survey data. In a more appropriate type d problem-a problem for which a type lf* evaluator could not have been implemented-those scores never would have been available in the first place.

The 1 f 2 evaluator is

```
function linregevallf2(transmorphic M, real scalar todo,
real rowvector b, fv, S, H)
{
    y1 = moptimize_calc_depvar(M, 1)
    p1 = moptimize_calc_xb(M, b, 1)
    p2 = moptimize_calc_xb(M, b, 2)
    s = exp(p2)
    z = (y1:-p1):/s
    fv = ln(normalden(y1:-p1, 0, s))
    if (todo>=1) {
        s1 = z:/s
        s2 = z:^2 :- 1
        S = (s1, s2)
        if (todo==2) {
            h11 = -1:/s:^2
            h22 = -2*z:^2
            h12 = -2*z:/s
            mis = 0
            H11 = moptimize_util_matsum(M, 1,1, h11, mis)
            H22 = moptimize_util_matsum(M, 2,2, h22, mis)
            H12 = moptimize_util_matsum(M, 1,2, h12, mis)
            H = (H11, H12 \ H12', H22)
            }
    }
}
```

The code to fit a model of mpg on weight and foreign reads nearly identically to the code we used in the type d 2 case. We must specify the name of our type lf 2 evaluator and specify that it is type lf 2 :

```
: M = moptimize_init()
: moptimize_init_evaluator(M, &linregevallf2())
: moptimize_init_evaluatortype(M, "lf2")
: moptimize_init_depvar(M, 1, "mpg")
: moptimize_init_eq_indepvars(M, 1, "weight foreign")
: moptimize_init_eq_indepvars(M, 2, "")
: moptimize(M)
: moptimize_result_display(M)
```


## Conformability

See Syntax above.

## Diagnostics

All functions abort with error when used incorrectly. moptimize() aborts with error if it runs into numerical difficulties. _moptimize() does not; it instead returns a nonzero error code.

The moptimize_result*() functions abort with error if they run into numerical difficulties when called after moptimize() or moptimize_evaluate(). They do not abort when run after _moptimize() or moptimize_evaluate(). They instead return a properly dimensioned missing result and set moptimize_result_errorcode() and moptimize_result_errortext().

Ludwig Otto Hesse (1811-1874) was born in Königsberg, Prussia, which is now Kaliningrad, Russia. He obtained a PhD from the University of Königsberg in 1840. His many contributions are reflected in the mathematical terms named after him, including the Hessian matrix, Hessian group, and Hesse configuration. His doctoral work was supervised by Carl Gustav Jacob Jacobi, for whom the Jacobian matrix and Jacobi transform are named. Hesse worked heavily on algebraic functions and invariants and published a lot of his work in Crelles Journal. One of his notable works was the principle of transfer, which was later generalized by Wilhelm Meyer and Élie Cartan.

Hesse supervised the doctoral work of several well-known mathematicians, including Jacob Lüroth and Max Noether. He was a member of three science academies and was granted the Steiner prize by the Berlin Academy of Sciences. The Bavarian Academy of Sciences, who granted Hesse membership in 1869, published his collected works in 1897.

Donald Wesley Marquardt (1929-1997) was born in New York and obtained degrees in physics, mathematics, and statistics from Columbia and the University of Delaware. For 39 years, he worked at DuPont as a statistician and founding manager of the company's Quality Management and Technology Center. In retirement, Marquardt set up his own consultancy and remained an international leader in establishing standards for quality management and quality assurance. His work on nonlinear estimation is highly cited. Marquardt also made major contributions to ridge and generalized inverse regression, mixture designs, and analysis of unequally spaced time series.

## References

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[^9]
## Title

[M-5] more() - Create -more- condition

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

more() displays -more- and waits for a key to be pressed. That is, more() does that if more is on, and it does nothing otherwise. more can be turned on and off by Stata's set more command (see [R] more) or by the functions below.
setmore() returns whether more is on or off, encoded as 1 and 0 .
setmore (onoff) sets more on if onoff $\neq 0$ and sets more off otherwise.
setmoreonexit (onoff) sets more on or off when the current execution ends. It has no effect on the current setting. The specified setting will take effect when control is passed back to the Mata prompt or to the calling ado-file or to Stata itself, and it will take effect regardless of whether execution ended because of a return, exit(), error, or abort. Only the first call to setmoreonexit() has that effect. Later calls have no effect whatsoever.

## Syntax

```
void more()
real scalar setmore()
void setmore(real scalar onoff)
void setmoreonexit(real scalar onoff)
```


## Remarks and examples

setmoreonexit () is used to ensure that the more setting is restored if a program wants to temporarily reset it:

```
setmoreonexit(setmore())
setmore(0)
```

Only the first invocation of setmoreonexit() has any effect. This way, a subroutine that is used in various contexts might also contain

```
setmoreonexit(setmore())
setmore(0)
```

and that will not cause the wrong more setting to be restored if an earlier routine had already done that and yet still cause the right setting to be restored if the subroutine is the first to issue setmoreonexit().

## Conformability

more() takes no arguments and returns void.
setmore():
result: $\quad 1 \times 1$
setmore(onoff), setmoreonexit (onoff):
onoff: $\quad 1 \times 1$
result: void

## Diagnostics

None.

## Also see

[P] more - Pause until key is pressed
[M-4] IO - I/O functions

## Title

[M-5] mvnormal() - Compute multivariate normal distributions and derivatives

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Conformability | References | Also see |

## Description

mvnormal ( $U, R$ ) returns the cumulative multivariate normal distributions with lower limits $-\infty$, upper limits $U$, and vectorized correlation matrices $R$ (only the lower halves are recorded).
mvnormal ( $L, U, R$ ) returns the multivariate normal distributions with lower limits $L$, upper limits $U$, and vectorized correlation matrices $R$ (only the lower halves are recorded).
mvnormalcv ( $L, U, M, V$ ) returns the multivariate normal distributions with lower limits $L$, upper limits $U$, means $M$, and vectorized covariance matrices $V$ (only the lower halves are recorded).
mvnormalderiv ( $U, R, d U, d R$ ) computes the derivatives $d U$ and $d R$ of cumulative multivariate normal distributions mvnormal $(U, R)$ with upper limits $U$ (lower limits $-\infty$ ) and vectorized correlation matrices $R$ (only the lower halves are recorded), respectively, with no return values.
mvnormalderiv ( $L, U, R, d L, d U, d R$ ) computes the derivatives $d L, d U$, and $d R$ of multivariate normal distributions mvnormal $(L, U, R)$ with lower limits $L$, upper limits $U$, and vectorized correlation matrices $R$ (only the lower halves are recorded), respectively, with no return values.
mvnormalcvderiv( $L, U, M, V, d L, d U, d M, d V$ ) computes the derivatives $d L, d U, d M$, and $d V$ of multivariate normal distributions mvnormalcv $(L, U, M, V)$ with lower limits $L$, upper limits $U$, means $M$, and vectorized covariance matrices $V$ (only the lower halves are recorded), respectively, with no return values.
mvnormal(), mvnormalcv(), mvnormalderiv(), and mvnormalcvderiv() use 128 quadrature points by default for dimensions greater than 3 and use 10 quadrature points by default for dimensions less than or equal to 3 .
mvnormalqp( $U, R, q$ ), mvnormalqp( $L, U, R, q$ ), mvnormalcvqp( $L, U, M, V, q$ ), mvnormalderivqp $(U, R, d U, d R, q)$, mvnormalderivqp $(L, U, R, d L, d U, d R, q)$ and mvnormalcvderivqp $(L, U, M, V, d L, d U, d M, d V, q)$ do the same things except that, rather than using the default number of quadrature points in the calculations, they allow you to specify the number of quadrature points in $q . q$ must be between 3 and 5,000 . If $q$ is not an integer, the integer part of $q$ is taken as the number of quadrature points.

Note that mvnormal(), mvnormalcv(), mvnormalqp(), and mvnormalcvqp() use normal() when the dimension equals 1 and binormal() when the dimension equals 2 , in which cases the number of quadrature points is irrelevant. The computation of the cumulative multivariate normal distributions and their derivatives can be slow when the dimension is large.

The matrices $L, U, M, R$, and $V$ parameterize the integrals that are approximated and returned in a column vector. See Conformability for the general rules of conformability and see Remarks and examples for examples.

## Syntax

| real colvector | mvnormal $(U, R)$ |
| :--- | :--- |
| real colvector | mvnormal $(L, U, R)$ |
| real colvector | mvnormalcv $(L, U, M, V)$ |
| real colvector | mvnormalqp $(U, R, q)$ |
| real colvector | mvnormalqp $(L, U, R, q)$ |
| real colvector | mvnormalcvqp $L, U, M, V, q)$ |
| void | mvnormalderiv( $U, R, d U, d R)$ |
| void | mvnormalderiv $(L, U, R, d L, d U, d R)$ |
| void | mvnormalcvderiv $(L, U, M, V, d L, d U, d M, d V)$ |
| void | mvnormalderivqp $(U, R, d U, d R, q)$ |
| void | mvnormalderivqp $(L, U, R, d L, d U, d R, q)$ |
| void | mvnormalcvderivqp $(L, U, M, V, d L, d U, d M, d V, q)$ |

where
L: real matrix $L$
$U: \quad$ real matrix $U$
M: real matrix $M$
$R$ : $\quad$ real matrix $R$
$V$ : real matrix $V$
$q$ : real scalar $q$
The types of $d L, d U, d M, d R$, and $d V$ are irrelevant; results are returned there as real matrices.

## Remarks and examples

The functions documented here approximate the multivariate normal distributions and their derivatives.
Remarks are presented under the following headings:

```
Multivariate normal distribution functions mvnormal( \(U, R\) ) and mvnormalqp \((U, R, q)\) mvnormal( \(L, U, R\) ) and mvnormalqp \((L, U, R, q)\) mvnormalcv(L, U, M, V) and mvnormalcvqp(L, U, M, V, q)
Derivatives of multivariate normal distribution functions mvnormalderiv( \(U, R, d U, d R\) ) and mvnormalderivqp \((U, R, d U, d R, q)\) mvnormalderiv \((L, U, R, d L, d U, d R)\) and mvnormalderivqp( \(L, U, R, d L, d U, d R, q)\) mvnormalcvderiv( \(L, U, M, V, d L, d U, d M, d V)\) and mvnormalcvderivqp(L, \(U, M, V, d L, d U, d M, d V, q)\)
```


## Multivariate normal distribution functions

mvnormal(U, R) and mvnormalqp(U, R, q)

real colvector mvnormal (real matrix $U$, real matrix $R$ )
real colvector mvnormalqp(real matrix $U$, real matrix $R$, real scalar $q$ )
mvnormal ( $U, R$ ) computes the cumulative multivariate normal distributions with lower limits $-\infty$, upper limits $U$, and vectorized correlation matrices $R$. mvnormalqp $(U, R, q$ ) allows you to specify the number of quadrature points in $q$ between 3 and 5,000. By default, $q=128$ for $d>3$ and $q=10$ for $d \leq 3$ ( $d$ is the dimension of the integral between 1 and 50).

The distribution function of a multivariate normal random vector $\mathbf{x}$ with mean 0 and correlation matrix $\mathbf{R}$ with upper limits $\mathbf{u}$ is

$$
\Phi_{d}(\mathbf{u}, \mathbf{R})=\frac{1}{\sqrt{(2 \pi)^{d}|\mathbf{R}|}} \int_{-\infty}^{u_{1}} \cdots \int_{-\infty}^{u_{d}} \exp \left(-\frac{1}{2} \mathbf{x}^{T} \mathbf{R}^{-1} \mathbf{x}\right) \quad d x_{1}, \ldots, d x_{d}
$$

where $\mathbf{u}=\left\{u_{1}, u_{2}, \ldots, u_{d}\right\}$ and $\mathbf{x}=\left\{x_{1}, x_{2}, \ldots, x_{d}\right\}$.
Each row of $U$ specifies an upper limit, $U[j,]=.\mathbf{u}$, where $j=1,2, \ldots, \operatorname{rows}(U)$. Each row of $R$ specifies the correlation matrix of the distribution. When there is a single correlation matrix $W$, $R=\operatorname{vech}(\mathrm{W})$ '. (We only need the lower half of $W$, because $W$ must be symmetric.) When there are multiple correlation matrices $W_{1}, \ldots, W_{J}$, the $j$ th row of $R$ contains the transpose of the vech() of the $j$ th correlation matrix $W_{j}$; in other words, $R[j,]=.\operatorname{vech}\left(W_{j}\right)^{\prime}$, where $j=1,2, \ldots, J$.
> Example 1: One limit with one correlation matrix
We want to calculate the cumulative multivariate normal distribution with upper limits defined in the vector $\mathbf{u}$,

```
: U = (3, 0, 2, -1)
: U
    1
```


and the correlation matrix $\mathbf{W}$,


We begin by storing the transpose of the vectorized $W$ in $R$,

```
: R = vech(W)'
: R
1 \begin{tabular}{cccccccccc|}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
\cline { 2 - 8 } & 1 & .5 & .5 & .5 & 1 & .5 & .5 & 1 & .5 \\
\hline
\end{tabular}
```

Then, we supply $U$ and $R$ to mvnormal().

```
: mvnormal(U, R)
    . }127346563
```

The result is the cumulative multivariate normal distribution of standard normal variates with lower limit $-\infty$, upper limit $U$, and a correlation matrix $W$.

## > Example 2: One limit with multiple correlation matrices

We now calculate the cumulative multivariate normal distributions for the same upper limits in $U$ for multiple correlation matrices. We defined $U$ and $W$ in example 1 . We copy $W$ to $W 1$ and define $W 2$ here.

| $: W 1=W$ |  |
| :--- | :--- |
| $:$ | $W 2=(1,0.1,0.2,0.25 \backslash$ |
| $>$ | $0.1,1,0.3,0.35 \backslash$ |
| $>$ | $0.2,0.3,1,0.15 \backslash$ |
| $>$ | $0.25,0.35,0.15,1)$ |

We now store the transposes of the vectorized $W 1$ and $W 2$ in $R$.

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | . 5 | . 5 | . 5 | 1 | . 5 | . 5 | 1 | . 5 | 1 |
| 2 | 1 | . 1 | . 2 | . 25 | 1 | . 3 | . 35 | 1 | . 15 | 1 |

Now, we calculate distributions from $U$ and $R$,

```
: mvnormal(U, R)
```

                        1
    | 1 | .1273465633 |
| :--- | :--- |
| 2 | .1122053302 |

Using the same $U$ and computing the cumulative multivariate normal distributions of each row of $R$ individually produces the same results.

```
R1 = R[1, .]
R2 = R[2, .]
: (mvnormal(U, R1) \ mvnormal(U, R2))
                                    1
1
```

```
. }127346563
```

. }127346563
. }112205330

```
    . }112205330
```


## > Example 3: Multiple limits with multiple correlation matrices

We can calculate cumulative multivariate normal distributions when both the upper limits and the correlation matrices vary. Each vector of upper limits is a row in the matrix $U 2$. Each vectorized correlation matrix is a row in the matrix $R$.

We reuse $R$ from the previous example, and we copy $U$ from that example into the new $U 2$ and add a row.

```
: U2 = (U \ -0.5, 2, 3, 10)
: U2
\begin{tabular}{l|rrrr|}
\multicolumn{1}{c}{} & 1 & 2 & 3 & 4 \\
\cline { 2 - 5 } 1 & 3 & 0 & 2 & -1 \\
2 & -.5 & 2 & 3 & 10 \\
\hline
\end{tabular}
```

We calculate the distributions implied by $U 2$ and $R$.

```
: mvnormal(U2, R)
    1
    |.1273465633
```

We would get the same answers if we calculated the cumulative multivariate normal distributions with each row of $U$ and $R$ individually.
mvnormal() and mvnormalcv() uses the quadrature method derived in Miwa, Hayter, and Kuriki (2003) to approximate the cumulative distribution function numerically. Increasing the number of quadrature points increases the accuracy of the approximation. There are two points to keep in mind. First, increasing the number of quadrature points can dramatically increase the computation time. Second, the additional accuracy obtained declines as you increase the number of quadrature points. These points together imply that, after a while, adding quadrature points does not add meaningful accuracy though it still adds to computation time.
mvnormalqp ( $U, R, q$ ) allows you to specify the number of quadrature points $q$. (When $q$ is not an integer, the integer part is taken as the number of quadrature points.) The default number of 128 quadrature points works well in most cases.
$>$ Example 4: Specifying the number of quadrature points
We reuse $U$ and $R$ from example 1 and use 256 as the number of quadrature points.

```
: mvnormalqp(U, R, 256)
    1
        . }127346563
        . }112205419
```

The results are essentially the same as those in example 1 ; the difference is around $10^{-7}$.

## mvnormal(L, U, R) and mvnormalqp(L, U, R, q)

real colvector mvnormal (real matrix L, real matrix $U$, real matrix $R$ )
real colvector mvnormalqp (real matrix $L$, real matrix $U$, real matrix $R$, real scalar $q$ )
mvnormal ( $L, U, R$ ) computes the multivariate normal distributions with lower limits $L$, upper limits $U$, and vectorized correlation matrices $R$. mvnormalqp $(L, U, R, q)$ allows you to specify the number of quadrature points in $q$.

The probability that a multivariate normal random vector $\mathbf{x}$ with mean 0 and correlation matrix $\mathbf{R}$ falls in the region defined by lower limits $\ell$ and upper limits $\mathbf{u}$ is

$$
\Phi_{d}(\ell, \mathbf{u}, \mathbf{R})=\frac{1}{\sqrt{(2 \pi)^{d}|\mathbf{R}|}} \int_{\ell_{1}}^{u_{1}} \cdots \int_{\ell_{d}}^{u_{d}} \exp \left(-\frac{1}{2} \mathbf{x}^{T} \mathbf{R}^{-1} \mathbf{x}\right) \quad d x_{1}, \ldots, d x_{d}
$$

where $\boldsymbol{\ell}=\left\{\ell_{1}, \ell_{2}, \ldots, \ell_{d}\right\}, \mathbf{u}=\left\{u_{1}, u_{2}, \ldots, u_{d}\right\}$, and $\mathbf{x}=\left\{x_{1}, x_{2}, \ldots, x_{d}\right\}$.
Each row of $L$ and $U$ is a pair of limits: $L[j,]=.\ell$, where $j=1,2, \ldots, \operatorname{rows}(L)$, and $U[j,]=.\mathbf{u}$, where $j=1,2, \ldots, \operatorname{rows}(U)$. Each row of $R$ is the transpose of a vectorized correlation matrix (only use the lower half); that is, $R[j,]=.\operatorname{vech}(\mathrm{W})$ ', where $j=1,2, \ldots$, rows $(R)$.

The usage of mvnormal $(L, U, R)$ is similar to that of mvnormal $(U, R)$.
> Example 5: Multiple limits with multiple correlation matrices
To calculate multiple cumulative multivariate normal distributions for matrices $L$ and $U$ with multiple rows and correlation matrices $W 1$ and $W 2$, we reuse $U 2, W 1, W 2$, and $V$ from example 3 and define $L$.


Then, we supply $L, U 2$, and $R$ to mvnormal().

```
: mvnormal(L, U2, R)
                            1
. }185254885
    .0352619752
```

The usage of mvnormalqp $(L, U, R, q)$ is similar to that of mvnormal $(L, U, R)$, except that the former function allows you to specify a scalar value as the number of quadrature points instead of using the defaults.

## mvnormalcv(L, U, M, V) and mvnormalcvqp(L, U, M, V, q)

real colvector mvnormalcv(real matrix $L$, real matrix $U$, real matrix $M$, real matrix $V$ ) real colvector mvnormalcvqp (real matrix $L$, real matrix $U$, real matrix $M$, real matrix $V$, real scalar $q$ )
mvnormalcv $(L, U, M, V)$ computes multivariate normal probabilities with lower limits $L$ and upper limits $U$. The means of the distribution are in $M$, and the vectorized covariance matrices are in $V$.

Each row of $L$ and $U$ is a pair of limits; each row of $M$ is the corresponding mean. Note that here each row of $V$ is the transpose of a vectorized covariance matrix instead of a correlation matrix. mvnormalcvqp $(L, U, M, V, q)$ allows you to specify the number of quadrature points in $q$.

The probability that a multivariate normal random vector x with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$ falls in the region defined by lower limits $\ell$ and upper limits $\mathbf{u}$ is

$$
\begin{aligned}
\Phi_{d}(\boldsymbol{\ell}, \mathbf{u}, \boldsymbol{\mu}, \boldsymbol{\Sigma}) & = \\
& \frac{1}{\sqrt{(2 \pi)^{d}}|\boldsymbol{\Sigma}|} \int_{\ell_{1}}^{u_{1}} \cdots \int_{\ell_{d}}^{u_{d}} \exp \left\{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^{T} \boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\right\} \quad d x_{1}, \ldots, d x_{d}
\end{aligned}
$$

where $\ell=\left\{\ell_{1}, \ell_{2}, \ldots, \ell_{d}\right\}, \mathbf{u}=\left\{u_{1}, u_{2}, \ldots, u_{d}\right\}, \boldsymbol{\mu}=\left\{\mu_{1}, \mu_{2}, \ldots, \mu_{d}\right\}$, and $\mathbf{x}=\left\{x_{1}, x_{2}, \ldots, x_{d}\right\}$. Each row of $L$ and $U$ is a pair of limits: $L[j,]=.\ell$, where $j=1,2, \ldots, \operatorname{rows}(L)$, and $U[j,]=.\mathbf{u}$, where $j=1,2, \ldots, \operatorname{rows}(U)$. Each row of $M$ is the corresponding mean with those limits: $M[j,]=.\boldsymbol{\mu}$, where $j=1,2, \ldots$, rows $(M)$. Each row of $V$ is the transpose of a vectorized covariance matrix (only use the lower half); that is, $V[j,]=.\operatorname{vech}(\Sigma)$ ', where $j=1,2, \ldots$, rows $(V)$.

The usage of mvnormalcv $(L, U, M, V)$ is similar to that of mvnormal ( $U, R$ ) and mvnormal ( $L$, $U, R)$.
> Example 6: Multiple limits with multiple covariance matrices
To calculate multiple cumulative multivariate normal distributions for matrices $L, U$, and $M$ with multiple rows and covariance matrices $Z 1$ and $Z 2$, we reuse $L$ and $U 2$ from example 5 and define $M$, $Z 1$, and $Z 2$.

```
:M = (0, 0, 0, 0 \ 0.2, 3, 2.5, -1)
: M
            1 2 
1
\begin{tabular}{rrrr|}
0 & 0 & 0 & 0 \\
.2 & 3 & 2.5 & -1 \\
\hline
\end{tabular}
```

```
Z1 = (1, 0.5, 0.5, 0.5 \
```

Z1 = (1, 0.5, 0.5, 0.5 \
0.5, 1, 0.5, 0.5 \
0.5, 1, 0.5, 0.5 \
0.5,0.5,1,0.5\
0.5,0.5,1,0.5\
0.5, 0.5, 0.5, 1)
0.5, 0.5, 0.5, 1)
Z2 = (3, 0.1, 0.2, 0.25 \
Z2 = (3, 0.1, 0.2, 0.25 \
0.1, 2, 0.3, 0.35 \
0.1, 2, 0.3, 0.35 \
0.2, 0.3, 1.2, 0.15\
0.2, 0.3, 1.2, 0.15\
0.25,0.35, 0.15, 1.2)

```
    0.25,0.35, 0.15, 1.2)
```

Now, we take the transposes of the vectorized $Z 1$ and $Z 2$ and store them in $V$.


Then, we supply $L, U, M$, and $V$ to mvnormal().
: mvnormalcv(L, U2, M, V)
1

| 1 | .1852548854 |
| :--- | :--- |
| 2 | .0403852144 |
|  |  |

The usage of mvnormalcvqp $(L, U, M, V, q)$ is similar to that of mvnormalcv $(L, U, M, V)$, except that the former function allows you to specify a scalar value for the number of quadrature points instead of using the defaults.

## Derivatives of multivariate normal distribution functions

mvnormalderiv(U, R, dU, dR) and mvnormalderivqp(U, R, dU, dR, q) void mvnormalderiv(real matrix $U$, real matrix $R$, real matrix $d U$, real matrix $d R$ ) void mvnormalderivqp(real matrix $U$, real matrix $R$, real matrix $d U$, real matrix $d R$, real scalar $q$ )

mvnormalderiv( $U, R, d U, d R$ ) with no return values computes the derivatives $d U$ and $d R$ of cumulative multivariate normal distributions mvnormal $(U, R)$ with upper limits $U$ (lower limits $-\infty)$ and vectorized correlation matrices $R$.
$d U$ contains the $r \times d$ matrix of derivatives mvnormal $(U, R)$ with respect to $U$, where $r$ is the number of rows in the vector returned by mvnormal $(U, R)$ and $d$ is the dimension of the integral.
$d R$ contains the $r \times d(d+1) / 2$ matrix of derivatives mvnormal $(U, R)$ with respect to $R$.
Each row of $U$ is a limit, and each row of $R$ is the transpose of a vectorized correlation matrix where we only use the lower halves.

Each row of $d U$ contains the derivative of mvnormal ( $U, R$ ) with respect to each row of the limits $U$, whereas each row of $d R$ is the derivative of $\operatorname{mvnormal}(U, R)$ with respect to each row of $R$. mvnormalderivqp $(U, R, d U, d R, q$ ) allows you to specify the number of quadrature points in $q$.

Note that this function has no return values. In addition, because the corresponding diagonal elements in the vectorized correlation matrices $R$ are constant (equal to 1 ), the derivatives of those elements are 0 .

## > Example 7: One limit with one correlation matrix

To calculate derivatives of a single cumulative multivariate normal distribution for matrix $U$ with one row and correlation matrix $W$, we first define $U$ and $W$. We then take the transpose of the vectorized $W$ and store it in $R$ as we did in previous examples.

```
: U = (0.5, 1, 0, -0.5)
: W = (1, 0.1, 0.1, 0.1 \
    0.1, 1, 0.1, 0.1\
    0.1, 0.1, 1, 0.1 \
    0.1, 0.1, 0.1, 1)
R = vech(W)'
```

The computed derivatives are stored in $d U$ and $d R$, and the values at which they are initialized do not matter. Following standard practice, we initialize $d U$ and $d R$ to missing values. Then, we supply $U, R, d U$, and $d R$ to mvnormalderiv().

```
: mvnormalderiv(U, R, dU=., dR=.)
: dU
```

| 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- |


dR

|  | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | . 0113351496 | . 0351339293 | . 0520695051 | 0 |
|  | 6 | 7 | 8 | 9 | 10 |
| 1 | . 0185285518 | . 0273357123 | 0 | . 0863381056 | 0 |

The 0 s correspond to the 1 s in the correlation matrix.

## > Example 8: Multiple limits with one correlation matrix

To calculate derivatives of multiple cumulative multivariate normal distributions for matrix $U$ with multiple rows and correlation matrix $W$, we reuse $W$ and $R$ from example 7 and define $U$.

```
: U = (0.5, 1, 0, -0.5 \ -1, 1.5, 0.3, 1)
: U
\begin{tabular}{lrrrr}
\multicolumn{1}{c}{} & 1 & 2 & 3 & 4 \\
\cline { 2 - 5 } 1 & .5 & 1 & 0 & -.5 \\
2 & -1 & 1.5 & .3 & 1
\end{tabular}
```

We use missing values as inputs of $d U$ and $d R$ as in example 7 . Then, we may supply $U, R, d U$, and $d R$ to mvnormalderiv().

```
mvnormalderiv(U, R, dU=., dR=.)
dU
```

| 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- |


|  | 1 | .0465441005 | .0239977303 | .0782867096 |
| :--- | :--- | :--- | :--- | :--- |
|  | .1328088257 | .0084957344 | .0473086864 | .0195949727 |
|  |  |  |  |  |


| dR | 1 |  |  | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 2 | 3 |  |  |
| 1 | 0 | . 0113351496 | . 0351339293 | . 0520695051 | 0 |
| 2 | 0 | . 0135452556 | . 0726269847 | . 0306888519 | 0 |
|  | 6 | 7 | 8 | 9 | 10 |
| 1 | . 0185285518 | . 0273357123 | 0 | . 0863381056 | 0 |
| 2 | . 0051677173 | . 0022765085 | 0 | . 0115490127 | 0 |

The results are the same as if we had computed the derivatives of $R$ and each row of $U$ individually.

## Example 9: Multiple limits with multiple correlation matrices

To calculate derivatives of multiple cumulative multivariate normal distributions for matrix $U$ with multiple rows and correlation matrices $W 1$ and $W 2$, we reuse $U$ from example 8, copy $W$ from example 7 to $W 1$, and define $W 2$. We then take the transposes of the vectorized $W 1$ and $W 2$ and store them in $R$ as we did in previous examples.

```
: W1 = W
: W2 = (1, 0.3, 0.5, 0.1 \
    0.3, 1, 0.2, 0.15 \
> 0.5, 0.2, 1, 0.2 \
0.1,0.15,0.2, 1)
R = (vech(W1)' \ vech(W2)')
```

We use missing values as inputs of $d U$ and $d R$. Then, we may supply $U, R, d U$, and $d R$ to mvnormalderiv().


The results are the same as if we had computed the derivatives of each row of $U$ and $R$ individually.

The usage of mvnormalderivqp ( $U, R, d U, d R, q$ ) is similar to that of mvnormalderiv ( $U$, $R, d U, d R)$, except that the former function allows you to specify a scalar value for the number of quadrature points instead of using the defaults.

mvnormalderiv(L, $\mathrm{U}, \mathrm{R}, \mathrm{dL}, \mathrm{dU}, \mathrm{dR}$ ) and mvnormalderivqp(L, $\mathrm{U}, \mathrm{R}, \mathrm{dL}, \mathrm{dU}, \mathrm{dR}, \mathbf{q})$

void mvnormalderiv(real matrix $L$, real matrix $U$, real matrix $R$, real matrix $d L$, real matrix $d U$, real matrix $d R$ )
void mvnormalderiv(real matrix $L$, real matrix $U$, real matrix $R$, real matrix $d L$, real matrix $d U$, real matrix $d R$, real scalar $q$ )
mvnormalderiv ( $L, U, R, d L, d U, d R$ ) with no return values computes the derivatives $d L, d U$, and $d R$ of cumulative multivariate normal distributions mvnormal ( $L, U, R$ ) with lower limits $L$, upper limits $U$, and correlation matrices $R$. Each row of $L$ and $U$ is a pair of limits, and each row of $R$ is the transpose of a vectorized positive-definite correlation matrix (only use the lower half). Each row of $d L$ and $d U$ are the derivatives of mvnormal $(L, U, R)$ with respect to each row of the limits $L$ and $U$, respectively, whereas each row of $d R$ is the derivative of mvnormal $(L, U, R)$ with respect to each row of $R$. mvnormalderivqp $(L, U, R, d L, d U, d R, q$ ) allows you to specify the number of quadrature points in $q$.

The derivatives in this function are similar to the ones in mvnormalderiv $(U, R, d U, d R)$. Note that this function has no return values. In addition, because the corresponding diagonal elements in the vectorized correlation matrix $R$ are constant, the derivatives of those elements are 0 .

## > Example 10: Multiple limits with multiple correlation matrices

To calculate derivatives of multiple cumulative multivariate normal distributions for matrices $L$ and $U$ with multiple rows and correlation matrix $W 1$ and $W 2$, we reuse $U, W 1, W 2$, and $R$ from example 9 and define $L$.

```
: L = (0, -1, -2, -2 \ -2, 1, 0, 0.2)
L
\begin{tabular}{l|rrrr} 
& 1 & 2 & 3 & 4 \\
\cline { 2 - 5 } \\
2 & \begin{tabular}{rrrr}
0 & -1 & -2 & -2 \\
-2 & 1 & 0 & .2 \\
\cline { 2 - 5 }
\end{tabular} &
\end{tabular}
```

We use missing values as inputs of $d L, d U$, and $d R$. Then, we may supply $L, U, R, d L, d U$, and $d R$ to mvnormalderiv().
: mvnormalderiv(L, $\mathrm{U}, \mathrm{R}, \mathrm{dL}=., \mathrm{dU}=., \mathrm{dR}=$.
: dL

|  | 1 | 2 | 3 | 4 |
| :--- | ---: | ---: | ---: | ---: |
| 1 | -.0396970253 | -.0073300085 | -.0021848589 | -.0034955646 |
| 2 | -.0000351832 | -.0005292068 | -.0006843267 | -.0002516705 |
|  |  |  |  |  |

$\begin{array}{llll}1 & 2 & 3 & 4\end{array}$

|  | .0323093485 | .0056353816 | .0146886254 | .0223104617 |
| :--- | :--- | :--- | :--- | :--- |
| 2 | .0004527324 | .0002251999 | .0005241578 | .0001821562 |

dR

|  | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | . 0018005222 | -. 0038626517 | -. 0062264941 |
| 2 | 0 | -. 0006487396 | -. 000259649 | -. 0001645526 |
|  | 5 | 6 | 7 | 8 |
| 1 | 0 | -. 0003285816 | -. 0007853533 | 0 |
| 2 | 0 | . 0002784176 | . 0001375648 | 0 |
|  | 9 | 10 |  |  |
| 1 | . 0138456858 | 0 |  |  |
| 2 | . 0000954914 | 0 |  |  |

The usage of mvnormalderivqp ( $L, U, R, d L, d U, d R, q$ ) is similar to that of mvnormalderiv ( $L$, $U, R, d L, d U, d R)$, except that the former function allows you to specify a scalar value for the number of quadrature points instead of using the defaults.

## mvnormalcvderiv(L, U, M, V, dL, dU, dM, dV) and

mvnormalcvderivqp(L, $\mathbf{U}, \mathrm{M}, \mathrm{V}, \mathrm{dL}, \mathrm{dU}, \mathrm{dM}, \mathrm{dV}, \mathrm{q})$
void mvnormalcvderiv(real matrix $L$, real matrix $U$, real matrix $M$, real matrix $V$, real matrix $d L$, real matrix $d U$, real matrix $d M$, real matrix $d V$ )
void mvnormalcvderivqp (real matrix $L$, real matrix $U$, real matrix $M$, real matrix $V$, real matrix $d L$, real matrix $d U$, real matrix $d M$, real matrix $d V$, real scalar $q$ )
mvnormalcvderiv ( $L, U, M, V, d L, d U, d M, d V$ ) with no return values computes the derivatives $d L, d U, d M$, and $d V$ of cumulative multivariate normal distributions mvnormalcv $(L, U, M, V)$ with lower limits $L$, upper limits $U$, means $M$, and covariance matrices $V$. Each row of $L$ and $U$ is a pair of limits, each row of $M$ is the corresponding mean, and each row of $V$ is the transpose of a vectorized covariance matrix (only use the lower half). Each row of $d L$ and $d U$ is the derivatives of mvnormal ( $L, U, M, V$ ) with respect to each row of the limits $L$ and $U$, respectively; each row of $d M$ is the derivative of mvnormalcv $(L, U, M, V)$ with respect to each row of $M$; each row of $d V$ is the derivative of mvnormalcv $(L, U, M, V)$ with respect to each row of $V$. mvnormalcvderivqp ( $L$, $U, M, V, d L, d U, d M, d V, q)$ allows you to specify the number of quadrature points in $q$.

The calculation of the derivatives of mvnormalcv $(L, U, M, V)$ with respect to means is done by first standardizing the function using the mean and covariance matrix so that each individual variate becomes standard normal and then applying the chain rule. In this function, because the corresponding diagonal elements in the vectorized covariance matrices $V$ are not constant, the derivatives of those elements are not 0 . Here is a short example.

## > Example 11: Multiple limits with multiple covariance matrices

To calculate derivatives of multiple cumulative multivariate normal distributions for matrices $L$ and $U, M$ with multiple rows, and covariance matrices $Z 1$ and $Z 2$, we reuse $L$ and $U$ from example 10 and define $M, Z 1, Z 2$, and $V$.

```
:M = (0, 0, 0, 0 \ -1.5, 1, 0.5, 0.8)
: M
\begin{tabular}{l|rrrr|}
\multicolumn{1}{c}{} & 1 & 2 & 3 & 4 \\
\cline { 2 - 5 } 1 \\
2 & 0 & 0 & 0 & 0 \\
-1.5 & 1 & .5 & .8 \\
\cline { 2 - 5 } &
\end{tabular}
: Z1 = (1, 0.5, 0.5, 0.5\
> 0.5,1,0.5,0.5\
> 0.5, 0.5,1,0.5\
> 0.5, 0.5, 0.5, 1)
: Z2 = (0.5, 0.1, 0.2, 0.25 \
> 0.1, 1, 0.3,0.35\
> 0.2,0.3,1.2,0.15\
> 0.25, 0.35, 0.15, 0.9)
V = (vech(Z1)' \ vech(Z2)')
V
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|}
\hline & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
\hline 1 & 1 & . 5 & . 5 & . 5 & 1 & . 5 & . 5 & 1 & . 5 & 1 \\
\hline 2 & . 5 & . 1 & . 2 & . 25 & 1 & . 3 & . 35 & 1.2 & . 15 & . 9 \\
\hline
\end{tabular}
```

We use missing values as inputs of $d L, d U, d M$, and $d V$. Then, we may supply $L, U, M, V, d L, d U$, $d M$, and $d V$ to mvnormalderiv().
: mvnormalcvderiv(L, $\mathrm{U}, \mathrm{M}, \mathrm{V}, \mathrm{dL}=., \mathrm{dU}=., \mathrm{dM}=., \mathrm{dV}=$.
: dL
$\begin{array}{llll}1 & 2 & 3 & 4\end{array}$
$-.0493454772-.0096189442-.0013933579-.0020835731$
$-.0034517505-.0081925241-.0116407426-.0036636013$
: dU

|  | 1 | 2 | 3 | 4 |
| ---: | ---: | ---: | ---: | ---: |
| 1 | .0280753135 | .0030781644 | .0158206252 | .0279108292 |
| 2 | .0026620096 | .0064431981 | .0131152322 | .0050610202 |
|  |  |  |  |  |

: dM
$\begin{array}{llll}1 & 2 & 3 & 4\end{array}$

| 1 | .0212701637 | .0065407798 | -.0144272673 | -.0258272561 |
| :--- | ---: | ---: | ---: | ---: |
| .0007897409 | .0017493259 | -.0014744896 | -.001397419 |  |

: dV


The usage of mvnormalcvderivqp $(L, U, M, V, d L, d U, d M, d V, q)$ is similar to that of mvnormalcvderivqp $(L, U, M, V, d L, d U, d M, d V)$, except that the former function allows you to specify a scalar value for the number of quadrature points instead of using the defaults.

## Conformability

Let $d$ be the dimension between 1 and 50 . Let $t=(d+1) \times d / 2$, and let $q$ be the number of quadrature points between 3 and 5,000 . By default, $q=128$ for $d>3$ and $q=10$ for $d \leq 3$. Let $r=\max$ (rows $(L)$, rows $(U)$, rows $(M)$, rows $(R)$, rows $(V)$ ).
mvnormal $(U, R)$ :
input:

$$
\begin{array}{llll}
U: & 1 \times d & \text { or } & r \times d, r>1 \\
R: & 1 \times t & \text { or } & r \times t, r>1
\end{array}
$$

output:

$$
\text { result: } 1 \times 1 \quad \text { or } \quad r \times 1, r>1
$$

mvnormal $(L, U, R)$ :
input:

$$
\begin{array}{llll}
L: & 1 \times d & \text { or } & r \times d, r>1 \\
U: & 1 \times d & \text { or } & r \times d, r>1 \\
R: & 1 \times t & \text { or } & r \times t, r>1
\end{array}
$$

output:

$$
\text { result: } 1 \times 1 \quad \text { or } \quad r \times 1, r>1
$$

mvnormalcv $(L, U, M, V)$ :
input:

$$
\begin{array}{clll}
L: & 1 \times d & \text { or } & r \times d, r>1 \\
U: & 1 \times d & \text { or } & r \times d, r>1 \\
M: & 1 \times d & \text { or } & r \times d, r>1 \\
V: & 1 \times t & \text { or } & r \times t, r>1
\end{array}
$$

output:

$$
\text { result: } 1 \times 1 \quad \text { or } \quad r \times 1, r>1
$$

mvnormalqp $(U, R, q)$ :
input:

$$
\begin{array}{clll}
U: & 1 \times d & \text { or } & r \times d, r>1 \\
R: & 1 \times t & \text { or } & r \times t, r>1 \\
q: & 1 \times 1 & &
\end{array}
$$

output:

$$
\text { result: } \quad 1 \times 1 \quad \text { or } \quad r \times 1, r>1
$$

mvnormalqp $(L, U, R, q)$ :
input:

$$
\begin{array}{clll}
L: & 1 \times d & \text { or } & r \times d, r>1 \\
U: & 1 \times d & \text { or } & r \times d, r>1 \\
R: & 1 \times t & \text { or } & r \times t, r>1 \\
q: & 1 \times 1 & &
\end{array}
$$

output:

$$
\text { result: } 1 \times 1 \quad \text { or } \quad r \times 1, r>1
$$

mvnormalcvqp $(L, U, M, V, q)$ :
input:

$$
\begin{array}{clll}
L: & 1 \times d & \text { or } & r \times d, r>1 \\
U: & 1 \times d & \text { or } & r \times d, r>1 \\
M: & 1 \times d & \text { or } & r \times d, r>1 \\
V: & 1 \times t & \text { or } & r \times t, r>1 \\
q: & 1 \times 1 & &
\end{array}
$$

output:

$$
\text { result: } 1 \times 1 \quad \text { or } \quad r \times 1, r>1
$$

mvnormalderiv( $U, R, d U, d R$ ):
input:

$$
\begin{array}{llll}
U: & 1 \times d & \text { or } & r \times d, r>1 \\
R: & 1 \times t & \text { or } & r \times t, r>1
\end{array}
$$

output:

$$
d U: \quad 1 \times d \quad \text { or } \quad r \times d, r>1
$$

$$
d R: \quad 1 \times t \quad \text { or } \quad r \times t, r>1
$$

result: void
mvnormalderiv( $L, U, R, d L, d U, d R$ ):
input:

| $L:$ | $1 \times d$ | or | $r \times d, r>1$ |
| :---: | :--- | :--- | :--- |
| $U:$ | $1 \times d$ | or | $r \times d, r>1$ |
| $R:$ | $1 \times t$ | or | $r \times t, r>1$ |

output:
$d L: \quad 1 \times d \quad$ or $\quad r \times d, r>1$
$d U: \quad 1 \times d \quad$ or $\quad r \times d, r>1$
$d R: \quad 1 \times t \quad$ or $\quad r \times t, r>1$
result: void
mvnormalcvderiv( $L, U, M, V, d L, d U, d M, d V)$ :
input:

$$
\begin{array}{clll}
L: & 1 \times d & \text { or } & r \times d, r>1 \\
U: & 1 \times d & \text { or } & r \times d, r>1 \\
M: & 1 \times d & \text { or } & r \times d, r>1 \\
V: & 1 \times t & \text { or } & r \times t, r>1
\end{array}
$$

output:
$d L: \quad 1 \times d \quad$ or $\quad r \times d, r>1$
$d U: \quad 1 \times d \quad$ or $\quad r \times d, r>1$
$d M: 1 \times d \quad$ or $\quad r \times d, r>1$
$d V: \quad 1 \times t \quad$ or $\quad r \times t, r>1$
result: void
mvnormalderivqp $(U, R, d U, d R, q)$ :
input:

$$
\begin{array}{clll}
U: & 1 \times d & \text { or } & r \times d, r>1 \\
R: & 1 \times t & \text { or } & r \times t, r>1 \\
q: & 1 \times 1 & &
\end{array}
$$

output:

$$
d U: \quad 1 \times d \quad \text { or } \quad r \times d, r>1
$$

$$
d R: \quad 1 \times t \quad \text { or } \quad r \times t, r>1
$$

result: void
mvnormalderivqp $(L, U, R, d L, d U, d R, q)$ :
input:

$$
\begin{array}{clll}
L: & 1 \times d & \text { or } & r \times d, r>1 \\
U: & 1 \times d & \text { or } & r \times d, r>1 \\
R: & 1 \times t & \text { or } & r \times t, r>1 \\
q: & 1 \times 1 & &
\end{array}
$$

output:

$$
\begin{array}{rlll}
d L: & 1 \times d & \text { or } & r \times d, r>1 \\
d U: & 1 \times d & \text { or } & r \times d, r>1 \\
d R: & 1 \times t & \text { or } & r \times t, r>1 \\
\text { result: } & \text { void } & &
\end{array}
$$

mvnormalcvderivqp $(L, U, M, V, d L, d U, d M, d V, q)$ :
input:

$$
\begin{array}{clll}
L: & 1 \times d & \text { or } & r \times d, r>1 \\
U: & 1 \times d & \text { or } & r \times d, r>1 \\
M: & 1 \times d & \text { or } & r \times d, r>1 \\
V: & 1 \times t & \text { or } & r \times t, r>1 \\
q: & 1 \times 1 & &
\end{array}
$$

output:

$$
\begin{array}{rlll}
d L: & 1 \times d & \text { or } & r \times d, r>1 \\
d U: & 1 \times d & \text { or } & r \times d, r>1 \\
d M: & 1 \times d & \text { or } & r \times d, r>1 \\
d V: & 1 \times t & \text { or } & r \times t, r>1 \\
\text { result: } & \text { void } & &
\end{array}
$$

## References

Drezner, Z. 1994. Computation of the trivariate normal integral. Mathematics of Computation 62: 289-294. http://doi.org/10.2307/2153409.

Miwa, T., A. J. Hayter, and S. Kuriki. 2003. The evaluation of general non-centred orthant probabilities. Journal of the Royal Statistical Society, Series B 65: 223-234. https://doi.org/10.1111/1467-9868.00382.

Miwa, T., A. J. Hayter, and W. Liu. 2000. Calculations of level probabilities for normal random variables with unequal variances with applications to Bartholomew's test in unbalanced one-way models. Computational Statistics and Data Analysis 34: 17-32. https://doi.org/10.1016/S0167-9473(99)00075-4.

Plackett, R. L. 1954. A reduction formula for normal multivariate integrals. Biometrika 41: 351-360. https://doi.org/10.2307/2332716.

## Also see

[M-5] normal() - Cumulatives, reverse cumulatives, and densities
[M-5] ghk() - Geweke-Hajivassiliou-Keane (GHK) multivariate normal simulator
[M-4] Statistical - Statistical functions

## Title

[M-5] _negate() — Negate real matrix

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

_negate $(X)$ speedily replaces $X=-X$.

## Syntax

void _negate (real matrix $X$ )

## Remarks and examples

Coding _negate ( X ) executes more quickly than coding $\mathrm{X}=-\mathrm{X}$.
However, coding

$$
\begin{aligned}
& B=A \\
& \text { _negate (B) }
\end{aligned}
$$

does not execute more quickly than coding

$$
B=-A
$$

## Conformability

## _negate $(X)$ :

$$
\begin{aligned}
X: & r \times c \\
\text { result: } & \text { void }
\end{aligned}
$$

## Diagnostics

None. $X$ may be a view.

## Also see

[M-4] Utility - Matrix utility functions

## Title

[M-5] norm( ) — Matrix and vector norms

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

norm ( $A$ ) returns norm ( $A, 2$ ).
norm $(A, p)$ returns the value of the norm of $A$ for the specified $p$. The possible values and the meaning of $p$ depend on whether $A$ is a vector or a matrix.

When $A$ is a vector, norm $(A, p)$ returns

```
sum(abs(A): ^p)^ (1/p) if 1\leqp<.
max(abs(A)) if p\geq.
```

When $A$ is a matrix, returned is

```
p norm(A, p)
0 sqrt(trace(\operatorname{conj (A)'A))}
1 max(colsum(abs(A)))
2 max(svdsv(A))
. max(rowsum(abs(A)))
```


## Syntax

```
real scalar norm(numeric matrix A)
real scalar norm(numeric matrix A, real scalar p)
```


## Remarks and examples

norm $(A)$ and norm $(A, p)$ calculate vector norms and matrix norms. $A$ may be real or complex and need not be square when it is a matrix.

The formulas presented above are not the actual ones used in calculation. In the vector-norm case when $1 \leq p<$. , the formula is applied to $A: / \max (\operatorname{abs}(A))$ and the result then multiplied by $\max (\operatorname{abs}(A))$. This prevents numerical overflow. A similar technique is used in calculating the matrix norm for $p=0$, and that technique also avoids storage of $\operatorname{conj}(A)^{\prime} A$.

## Conformability

norm ( $A$ ):

| $A:$ | $r \times c$ |
| ---: | :--- |
| result: | $1 \times 1$ |

$\operatorname{norm}(A, p)$ :

| $A:$ | $r \times c$ |
| ---: | :--- |
| $p:$ | $1 \times 1$ |
| result: | $1 \times 1$ |

## Diagnostics

The norm() is defined to return 0 if $A$ is void and missing if any element of $A$ is missing.
norm $(A, p)$ aborts with error if $p$ is out of range. When $A$ is a vector, $p$ must be greater than or equal to 1 . When $A$ is a matrix, $p$ must be $0,1,2$, or . (missing).
$\operatorname{norm}(A)$ and norm $(A, p)$ return missing if the 2-norm is requested and the singular value decomposition does not converge, an event not expected to occur; see [M-5] svd().

## Also see

[M-4] Matrix - Matrix functions

## Title

[M-5] normal( ) - Cumulatives, reverse cumulatives, and densities

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

The below functions return density values, cumulatives, reverse cumulatives, inverse cumulatives, and in one case, derivatives of the indicated probability density function. These functions mirror the Stata functions of the same name and in fact are the Stata functions.

See [FN] Statistical functions for details. In the syntax diagram above, some arguments have been renamed in hope of aiding understanding, but the function arguments match one to one with the underlying Stata functions.

## Syntax

Gaussian normal

$$
\begin{aligned}
d & =\operatorname{normalden}(z) \\
d & =\operatorname{normalden}(x, s d) \\
d & =\operatorname{normalden}(x, \text { mean }, s d) \\
p & =\operatorname{normal}(z) \\
z & =\operatorname{invnormal}(p) \\
\ln (d) & =\ln n o r m a l d e n \\
\ln (d) & =\ln n o r m a l d e n \\
\ln (d) & =\ln \text { normalden }(x, \text { mean }, s d) \\
\ln (p) & =\ln \operatorname{normal}(z)
\end{aligned}
$$

Binormal

$$
p=\operatorname{binormal}\left(z_{1}, z_{2}, \text { rho }\right)
$$

Multivariate normal

$$
\ln (d)=\operatorname{lnmvnormalden}(M, V, X)
$$

Beta

$$
\begin{aligned}
d & =\operatorname{betaden}(a, b, x) \\
p & =\operatorname{ibeta}(a, b, x) \\
q & =\operatorname{ibetatail}(a, b, x) \\
x & =\operatorname{invibeta}(a, b, p) \\
x & =\operatorname{invibetatail}(a, b, q)
\end{aligned}
$$

Binomial

$$
\begin{aligned}
p k & =\operatorname{binomialp}(n, k, p i) \\
p & =\operatorname{binomial}(n, k, p i) \\
q & =\operatorname{binomialtail}(n, k, p i) \\
p i & =\operatorname{invbinomial}(n, k, p) \\
p i & =\operatorname{invbinomialtail}(n, k, q)
\end{aligned}
$$

Cauchy

$$
\begin{aligned}
d & =\operatorname{cauchyden}(a, b, x) \\
p & =\operatorname{cauchy}(a, b, x) \\
q & =\operatorname{cauchytail}(a, b, x) \\
x & =\operatorname{invcauchy}(a, b, p) \\
x & =\operatorname{invcauchytail}(a, b, q) \\
\ln (d) & =\operatorname{lncauchyden}(a, b, x)
\end{aligned}
$$

$\chi^{2}$

$$
\begin{aligned}
d & =\operatorname{chi} 2 \operatorname{den}(d f, x) \\
p & =\operatorname{chi2}(d f, x) \\
q & =\operatorname{chi} 2 \operatorname{tail}(d f, x) \\
x & =\operatorname{invchi2}(d f, p) \\
x & =\operatorname{invchi2tail}(d f, q)
\end{aligned}
$$

Dunnett's multiple range

$$
\begin{aligned}
& p=\operatorname{dunnettprob}(k, d f, x) \\
& x=\operatorname{invdunnettprob}(k, d f, p)
\end{aligned}
$$

Exponential

$$
\begin{aligned}
d & =\operatorname{exponentialden}(b, x) \\
p & =\operatorname{exponential}(b, x) \\
q & =\operatorname{exponentialtail}(b, x) \\
x & =\operatorname{invexponential}(b, p) \\
x & =\operatorname{invexponentialtail}(b, q)
\end{aligned}
$$

F

$$
\begin{aligned}
d & =\operatorname{Fden}\left(d f_{1}, d f_{2}, f\right) \\
p & =\operatorname{F}\left(d f_{1}, d f_{2}, f\right) \\
q & =\operatorname{Ftail}\left(d f_{1}, d f_{2}, f\right) \\
f & =\operatorname{invF}\left(d f_{1}, d f_{2}, p\right) \\
f & =\operatorname{invFtail}\left(d f_{1}, d f_{2}, q\right)
\end{aligned}
$$

Gamma and inverse gamma

$$
\begin{aligned}
d & =\operatorname{gammaden}(a, b, g, x) \\
p & =\operatorname{gammap}(a, x) \\
q & =\operatorname{gammaptail}(a, x) \\
x & =\operatorname{invgammap}(a, p) \\
x & =\operatorname{invgammaptail}(a, q) \\
d g / d a & =\operatorname{dgammapda}(a, x) \\
d g / d x & =\operatorname{dgammapdx}(a, x) \\
d 2 g / d a 2 & =\operatorname{dgammapdada}(a, x) \\
d 2 g / d a d x & =\operatorname{dgammapdadx}(a, x) \\
d 2 g / d x 2 & =\operatorname{dgammapdxdx}(a, x) \\
\ln (d) & =\operatorname{lnigammaden}(a, b, x)
\end{aligned}
$$

Hypergeometric

$$
\begin{aligned}
p k & =\text { hypergeometricp }(N, K, n, k) \\
p & =\text { hypergeometric }(N, K, n, k)
\end{aligned}
$$

Inverse Gaussian

$$
\begin{aligned}
d & =\operatorname{igaussianden}(m, a, x) \\
p & =\operatorname{igaussian}(m, a, x) \\
q & =\operatorname{igaussiantail}(m, a, x) \\
x & =\operatorname{invigaussian}(m, a, p) \\
x & =\operatorname{invigaussiantail}(m, a, q) \\
\ln (d) & =\operatorname{lnigaussianden}(m, a, x)
\end{aligned}
$$

Laplace

$$
\begin{aligned}
d & =\operatorname{laplaceden}(m, b, x) \\
p & =\operatorname{laplace}(m, b, x) \\
q & =\operatorname{laplacetail}(m, b, x) \\
x & =\operatorname{invlaplace}(m, b, p) \\
x & =\operatorname{invlaplacetail}(m, b, q) \\
\ln (d) & =\ln \operatorname{laplaceden}(m, b, x)
\end{aligned}
$$

Logistic

$$
\begin{aligned}
d & =\operatorname{logisticden}(x) \\
d & =\operatorname{logisticden}(s, x) \\
d & =\operatorname{logisticden}(m, s, x) \\
p & =\operatorname{logistic}(x) \\
p & =\operatorname{logistic}(s, x) \\
p & =\operatorname{logistic}(m, s, x) \\
q & =\operatorname{logistictail}(x) \\
q & =\operatorname{logistictail}(s, x) \\
q & =\operatorname{logistictail}(m, s, x) \\
x & =\operatorname{invlogistic}(p) \\
x & =\operatorname{invlogistic}(s, p) \\
x & =\operatorname{invlogistic}(m, s, p) \\
x & =\operatorname{invlogistictail}(q) \\
x & =\operatorname{invlogistictail}(s, q) \\
x & =\operatorname{invlogistictail}(m, s, q)
\end{aligned}
$$

Negative binomial

$$
\begin{aligned}
p k & =\operatorname{nbinomialp}(n, k, p i) \\
p & =\operatorname{nbinomial}(n, k, p i) \\
q & =\operatorname{nbinomialtail}(n, k, p i) \\
p i & =\operatorname{invnbinomial}(n, k, p) \\
p i & =\operatorname{invnbinomialtail}(n, k, q)
\end{aligned}
$$

Noncentral beta

$$
\begin{aligned}
& d=\operatorname{nbetaden}(a, b, n p, x) \\
& p=\operatorname{nibeta}(a, b, n p, x) \\
& x=\operatorname{invnibeta}(a, b, n p, p)
\end{aligned}
$$

Noncentral $\chi^{2}$

$$
\begin{aligned}
d & =\operatorname{nchi2den}(d f, n p, x) \\
p & =\operatorname{nchi2}(d f, n p, x) \\
q & =\operatorname{nchi2tail}(d f, n p, x) \\
x & =\operatorname{invnchi2}(d f, n p, p) \\
x & =\operatorname{invnchi2tail}(d f, n p, q) \\
n p & =\operatorname{npnchi2}(d f, x, p)
\end{aligned}
$$

Noncentral F

$$
\begin{aligned}
d & =\mathrm{nFden}\left(d f_{1}, d f_{2}, n p, f\right) \\
p & =\operatorname{nF}\left(d f_{1}, d f_{2}, n p, f\right) \\
q & =\operatorname{nFtail}\left(d f_{1}, d f_{2}, n p, f\right) \\
f & =\operatorname{invnF}\left(d f_{1}, d f_{2}, n p, p\right) \\
f & =\operatorname{invnFtail}\left(d f_{1}, d f_{2}, n p, q\right) \\
n p & =\operatorname{npnF}\left(d f_{1}, d f_{2}, f, p\right)
\end{aligned}
$$

Noncentral Student's $t$

$$
\begin{aligned}
d & =\operatorname{ntden}(d f, n p, t) \\
p & =\operatorname{nt}(d f, n p, t) \\
q & =\operatorname{nttail}(d f, n p, t) \\
t & =\operatorname{invnt}(d f, n p, p) \\
t & =\operatorname{invnttail}(d f, n p, q) \\
n p & =\operatorname{npnt}(d f, t, p)
\end{aligned}
$$

Poisson

$$
\begin{aligned}
p k & =\text { poissonp }(\text { mean }, k) \\
p & =\text { poisson }(\text { mean }, k) \\
q & =\text { poissontail }(\text { mean }, k) \\
m & =\operatorname{invpoisson~}(k, p) \\
m & =\operatorname{invpoissontail}(k, q)
\end{aligned}
$$

Student's $t$

$$
\begin{aligned}
d & =\operatorname{tden}(d f, t) \\
p & =\operatorname{t}(d f, t) \\
q & =\operatorname{ttail}(d f, t) \\
t & =\operatorname{invt}(d f, p) \\
t & =\operatorname{invttail}(d f, q)
\end{aligned}
$$

Tukey's Studentized range

$$
\begin{aligned}
p & =\operatorname{tukeyprob}(k, d f, x) \\
x & =\operatorname{invtukeyprob}(k, d f, p)
\end{aligned}
$$

Weibull

$$
\begin{aligned}
d & =\text { weibullden }(a, b, x) \\
d & =\text { weibullden }(a, b, g, x) \\
p & =\text { weibull }(a, b, x) \\
p & =\text { weibull }(a, b, g, x) \\
q & =\text { weibulltail }(a, b, x) \\
q & =\text { weibulltail }(a, b, g, x) \\
x & =\operatorname{invweibull}(a, b, p) \\
x & =\operatorname{invweibull}(a, b, g, p) \\
x & =\operatorname{invweibulltail}(a, b, q) \\
x & =\operatorname{invweibulltail}(a, b, g, q)
\end{aligned}
$$

Weibull (proportional hazards)

$$
\left.\left.\begin{array}{l}
d=\text { weibullphden }(a, b, x) \\
d=\text { weibullphden }(a, b, g, x) \\
p=\text { weibullph }(a, b, x) \\
p=\text { weibullph }(a, b, g, x) \\
q=\text { weibullphtail }(a, b, x) \\
q=\text { weibullphtail }(a, b, g, x) \\
x=\operatorname{invweibullph}(a, b, p) \\
x=\operatorname{invweibullph}(a, b, g, p) \\
x=\operatorname{invweibullphtail}(a, b, q) \\
x
\end{array}\right)=\operatorname{invweibullphtail}(a, b, g, q)\right)
$$

Wishart and inverse Wishart

$$
\begin{aligned}
& \ln (d)=\operatorname{lnwishartden}(d f, V, X) \\
& \ln (d)=\operatorname{lniwishartden}(d f, V, X)
\end{aligned}
$$

where

1. All functions return real and all arguments are real or real matrices.
2. The left-hand-side notation is used to assist in interpreting the meaning of the returned value:

$$
\begin{array}{ll}
d & =\text { density value } \\
p k & =\text { probability of discrete outcome } K=\operatorname{Pr}(K=k) \\
p & =\text { left cumulative } \\
& =\operatorname{Pr}(- \text { infinity }<\text { statistic } \leq x) \text { (continuous) } \\
& =\operatorname{Pr}(0 \leq K \leq k) \text { (discrete) } \\
q & =\text { right cumulative } \\
& =1-p \text { (continuous) } \\
& =\operatorname{Pr}(K \geq k)=1-p+p k \text { (discrete) } \\
n p & =\text { noncentrality parameter } \\
\ln (p) & =\log \text { cumulative } \\
\ln (d) & =\log \text { density }
\end{array}
$$

3. Hypergeometric distribution:
$N=$ number of objects in the population
$K=$ number of objects in the population with the characteristic of interest, $K<N$
$n=$ sample size, $n<N$
$k=$ number of objects in the sample with the characteristic of interest, $\max (0, n-N+K) \leq k \leq \min (K, n)$
4. Negative binomial distribution: $n>0$ and may be nonintegral.
5. Multivariate normal, Wishart, and inverse-Wishart distributions:
$M=$ mean vector
$V=$ covariance or scale matrix
$X=$ random vector for multivariate normal or random matrix for Wishart and inverse Wishart

## Remarks and examples

Remarks are presented under the following headings:
$R$-conformability
A note concerning invbinomial( ) and invbinomialtail( )
A note concerning ibeta()
A note concerning gammap()

## R-conformability

The above functions are usually used with scalar arguments and then return a scalar result:

```
: x = chi2(10, 12)
: x
    .7149434997
```

The arguments may, however, be vectors or matrices. For instance,

```
: x = chi2((10,11,12), 12)
: x
```

    1 2 3
    \(1 \quad .7149434997 \quad .6363567795 \quad .5543203586\)
    $: \mathrm{x}=\operatorname{chi2}(10,(12,12.5,13))$
: x
$\begin{array}{lll}1 & 2 & 3\end{array}$

    1 \begin{tabular}{|l|l|}
    \hline .7149434997 \& .7470146767 <br>
\hline
\end{tabular}

$: x=\operatorname{chi} 2((10,11,12),(12,12.5,13))$
: x
$\begin{array}{lll}1 & 2 & 3\end{array}$
1
.7149434997 .6727441644 .6309593164

In the last example, the numbers correspond to $\operatorname{chi} 2(10,12)$, $\operatorname{chi} 2(11,12.5)$, and $\operatorname{chi2}(12,13)$.
Arguments are required to be r-conformable (see [M-6] Glossary), and thus,

```
: x = chi2((10\11\12), (12,12.5,13))
: x
```

|  | 1 |  | 2 |
| :--- | ---: | ---: | ---: |
| 1 | .7149434997 | .7470146767 | .7763281832 |
| 2 | .6363567795 | .6727441644 | .7066745906 |
| 3 | .5543203586 | .593595966 | .6309593164 |
|  |  |  |  |

which corresponds to
$\begin{array}{lll}1 & 2 & 3\end{array}$

```
chi2(10,12) chi2(10,12.5) chi2(10,13)
chi2(11,12) chi2(11,12.5) chi2(11,13)
chi2(12,12) chi2(12,12.5) chi2(12,13)
```


## A note concerning invbinomial( ) and invbinomialtail( )

invbinomial ( $n, k, p$ ) invbinomialtail ( $n, k, q$ ) are useful for calculating confidence intervals for $p i$, the probability of a success. invbinomial() returns the probability pi such that the probability of observing $k$ or fewer successes in $n$ trials is $p$. invbinomialtail() returns the probability pi such that the probability of observing $k$ or more successes in $n$ trials is $q$.

## A note concerning ibeta()

ibeta $(a, b, x)$ is known as the cumulative beta distribution, and it is known as the incomplete beta function $I_{x}(a, b)$.

## A note concerning gammap()

$\operatorname{gammap}(a, x)$ is known as the cumulative gamma distribution, and it is known as the incomplete gamma function $P(a, x)$.

## Conformability

All functions require that arguments be r-conformable; see $R$-conformability above. Returned is matrix of max (argument rows) rows and max (argument columns) columns containing element-by-element calculated results.

## Diagnostics

All functions return missing when arguments are out of range.

## Also see

[M-5] mvnormal( ) - Compute multivariate normal distributions and derivatives
[M-4] Statistical - Statistical functions

## Title

[M-5] optimize() - Function optimization

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | References | Also see |  |

## Description

These functions find parameter vector or scalar $p$ such that function $f(p)$ is a maximum or a minimum. optimize_init() begins the definition of a problem and returns $S$, a problem-description handle set to contain default values.

The optimize_init_* $(S, \ldots)$ functions then allow you to modify those defaults. You use these functions to describe your particular problem: to set whether you wish maximization or minimization, to set the identity of function $f()$, to set initial values, and the like.
optimize ( $S$ ) then performs the optimization. optimize() returns real rowvector $p$ containing the values of the parameters that produce a maximum or minimum.

The optimize_result_*( $S$ ) functions can then be used to access other values associated with the solution.

Usually you would stop there. In other cases, you could restart optimization by using the resulting parameter vector as new initial values, change the optimization technique, and restart the optimization:

```
optimize_init_params(S, optimize_result_params(S))
optimize_init_technique(S, "dfp")
optimize(S)
```

Aside: The optimize_init_* $(S, \ldots)$ functions have two modes of operation. Each has an optional argument that you specify to set the value and that you omit to query the value. For instance, the full syntax of optimize_init_params() is
void optimize_init_params ( $S$, real rowvector initialvalues)
real rowvector optimize_init_params $(S)$
The first syntax sets the initial values and returns nothing. The second syntax returns the previously set (or default, if not set) initial values.

All the optimize_init_* $(S, \ldots)$ functions work the same way.

## Syntax

$$
S=\text { optimize_init() }
$$

(varies) optimize_init_which (S [, \{"max"|"min" \}])
(varies) optimize_init_evaluator (S [, \&function()])
(varies) optimize_init_evaluatortype ( $S$ [, evaluatortype $]$ )
(varies) optimize_init_negH (S, \{ "off"|"on" \})
(varies) optimize_init_params (S [, real rowvector initialvalues])
(varies) optimize_init_nmsimplexdeltas (S [, real rowvector delta])
(varies) optimize_init_argument ( $S$, real scalar $k[, X]$ )
(varies) optimize_init_narguments (S [, real scalar K])
(varies) optimize_init_cluster ( $S, c$ )
(varies) optimize_init_colstripe(S [, stripe])
(varies) optimize_init_technique (S [, technique $]$ )
(varies) optimize_init_singularHmethod (S [, singularHmethod $]$ )
(varies) optimize_init_conv_maxiter (S [, real scalar maxiter $]$ )
(varies) optimize_init_conv_warning(S, \{"on"|"off"\})
(varies) optimize_init_conv_ptol(S [, real scalar ptol])
(varies) optimize_init_conv_vtol (S [, real scalar vtol])
(varies) optimize_init_conv_nrtol(S [, real scalar nrtol])
(varies) optimize_init_conv_ignorenrtol(S, \{"off"|"on"\})
(varies) optimize_init_iterid(S [, string scalar id])
(varies) optimize_init_valueid(S [, string scalar id $]$ )
(varies) optimize_init_tracelevel ( $S$ [, tracelevel $]$ )
(varies) optimize_init_trace_dots ( $S,\{$ "off"|"on" $\}$ )
(varies) optimize_init_trace_value(S, \{ "on"|"off" \})
(varies) optimize_init_trace_tol (S, \{"off"|"on" \})
(varies) optimize_init_trace_step(S, \{"off"|"on"\})
(varies) optimize_init_trace_paramdiffs(S, \{"off"|"on"\})
(varies) optimize_init_trace_params (S, \{"off"|"on"\})
(varies) optimize_init_trace_gradient (S, \{"off"|"on"\})
(varies) optimize_init_trace_Hessian(S, \{"off"|"on" \})

| (varies) | optimize_init_evaluations (S, \{ "off"\|"on" \}) |
| :---: | :---: |
| (varies) | optimize_init_constraints ( $S$ [, real matrix $C c]$ ) |
| (varies) | optimize_init_verbose ( $S$ [, real scalar verbose]) |
| (varies) | optimize_init_deriv_usemin(S [, \{ "off"\|"on" \}]) |
| (varies) | optimize_init_deriv_min $(S$ [, real rowvector min $]$ ) |
| real rowvector | optimize ( $S$ ) |
| real scalar | _optimize (S) |
| void | optimize_evaluate (S) |
| real scalar | _optimize_evaluate( $S$ ) |
| real rowvector | optimize_result_params ( $S$ ) |
| real scalar | optimize_result_value ( $S$ ) |
| real scalar | optimize_result_value0 (S) |
| real rowvector | optimize_result_gradient ( $S$ ) |
| real matrix | optimize_result_scores ( $S$ ) |
| real matrix | optimize_result_Hessian(S) |
| real matrix | optimize_result_V(S) |
| string scalar | optimize_result_Vtype ( $S$ ) |
| real matrix | optimize_result_V_oim( $S$ ) |
| real matrix | optimize_result_V_opg (S) |
| real matrix | optimize_result_V_robust ( $S$ ) |
| real scalar | optimize_result_iterations( $S$ ) |
| real scalar | optimize_result_converged ( $S$ ) |
| real colvector | optimize_result_iterationlog( $S$ ) |
| real rowvector | optimize_result_evaluations( $S$ ) |
| real scalar | optimize_result_errorcode ( $S$ ) |
| string scalar | optimize_result_errortext ( $S$ ) |
| real scalar | optimize_result_returncode( $S$ ) |
| void | optimize_query ( $S$ ) |

where $S$, if it is declared, should be declared
transmorphic $S$
and where evaluatortype optionally specified in optimize_init_evaluatortype() is
evaluatortype Description

| "d0" | function() returns scalar value |
| :--- | :--- |
| "d1" | same as "d0" and returns gradient rowvector |
| "d2" | same as "d1" and returns Hessian matrix |
| "d1debug" | same as "d1" but checks gradient |
| "d2debug" | same as "d2" but checks gradient and Hessian |
| "gf0" | function() returns colvector value |
| "gf1" | same as "gf0" and returns score matrix |
| "gf2" | same as "gf1" and returns Hessian matrix |
| "gf1debug" | same as "gf1" but checks gradient |
| "gf2debug" | same as "gf2" but checks gradient and Hessian |

The default is "d0" if not set.
and where technique optionally specified in optimize_init_technique() is

| technique | Description |
| :--- | :--- |
| "nr" | modified Newton-Raphson |
| "dfp" | Davidon-Fletcher-Powell |
| "bfgs" | Broyden-Fletcher-Goldfarb-Shanno |
| "bhhh" | Berndt-Hall-Hall-Hausman |
| "nm" | Nelder-Mead |

The default is " nr ".
and where singularHmethod optionally specified in optimize_init_singularHmethod() is singularHmethod Description
"m-marquardt" modified Marquardt algorithm
"hybrid" mixture of steepest descent and Newton
The default is "m-marquardt" if not set;
"hybrid" is equivalent to ml's difficult option; see [R] ml.
and where tracelevel optionally specified in optimize_init_tracelevel() is
tracelevel

```
"none" nothing
"value" function value
"tolerance" previous + convergence values
"step" previous + stepping information
"paramdiffs" previous + parameter relative differences
"params" previous + parameter values
"gradient" previous + gradient vector
"hessian" previous + Hessian matrix
```

The default is "value" when set iterlog is on and "none" when set iterlog is off; see [R] set iter.

## Remarks and examples

Remarks are presented under the following headings:

```
First example
Notation
Type d evaluators
Example of d0, d1, and d2
d1debug and d2debug
Type gf evaluators
Example of gf0,gf1, and gf2
Functions
    optimize_init()
    optimize_init_which()
    optimize_init_evaluator() and optimize_init_evaluatortype()
    optimize_init_negH()
    optimize_init_params()
    optimize_init_nmsimplexdeltas()
    optimize_init_argument() and optimize_init_narguments()
    optimize_init_cluster()
    optimize_init_colstripe()
    optimize_init_technique()
    optimize_init_singularHmethod()
    optimize_init_conv_maxiter()
    optimize_init_conv_warning()
    optimize_init_conv_ptol(), . . _vtol(), . . _nrtol()
    optimize_init_conv_ignorenrtol()
    optimize_init_iterid()
    optimize_init_valueid()
    optimize_init_tracelevel()
    optimize_init_trace_dots( ), . . _value( ), . . __tol( ), . . . _step( ), . . _paramdiffs( ),
        ..._params(), ... _gradient(), ... _Hessian()
    optimize_init_evaluations()
    optimize_init_constraints()
    optimize_init_verbose()
    optimize_init_deriv_usemin( ) and optimize_init_deriv_min()
    optimize()
    _optimize()
    optimize_evaluate()
    _optimize_evaluate()
    optimize_result_params()
```

```
optimize_result_value() and optimize_result_value0( )
optimize_result_gradient()
optimize_result_scores()
optimize_result_Hessian()
optimize_result_V() and optimize_result_Vtype()
optimize_result_V_oim( ), . . _ _opg( ), . . . _robust()
optimize_result_iterations()
optimize_result_converged()
optimize_result_iterationlog()
optimize_result_evaluations()
optimize_result_errorcode( ), . . _errortext(), and . . ._returncode()
optimize_query()
```


## First example

The optimization functions may be used interactively.
Below we use the functions to find the value of $x$ that maximizes $y=\exp \left(-x^{2}+x-3\right)$ :

```
void myeval(todo, x, y, g, H)
{
> y = exp(-x^2 + x - 3)
> }
note: argument todo unused
note: argument g unused
note: argument H unused
: S = optimize_init()
: optimize_init_evaluator(S, &myeval())
: optimize_init_params(S, 0)
: x = optimize(S)
Iteration 0: f(p) = . 04978707
Iteration 1: f(p) = . 04978708
Iteration 2: f(p) = . 06381186
Iteration 3: f(p) = . 06392786
Iteration 4: f(p) = . 06392786
: x
. }
```


## Notation

We wrote the above in the way that mathematicians think, that is, optimizing $y=f(x)$. Statisticians, on the other hand, think of optimizing $s=f(b)$. To avoid favoritism, we will write $v=f(p)$ and write the general problem with the following notation:

Maximize or minimize $v=f(p)$,
$v$ : a scalar
$p: 1 \times n p$
subject to the constraint $C p^{\prime}=c$,
$C: n c \times n p \quad(n c=0$ if no constraints)
$c: n c \times 1$
where $g$, the gradient vector, is $g=f^{\prime}(p)=d f / d p$,

$$
g: \quad 1 \times n p
$$

and $H$, the Hessian matrix, is $H=f^{\prime \prime}(p)=d^{2} f / d p d p^{\prime}$

$$
H: n p \times n p
$$

## Type d evaluators

You must write an evaluator function to calculate $f()$ before you can use the optimization functions. The example we showed above was of what is called a type d evaluator. Let's stay with that.

The evaluator function we wrote was

```
void myeval(todo, x, y, g, H)
{
        y = exp(-x^2 + x - 3)
}
```

All type d evaluators open the same way,

```
void evaluator(todo, x, y,g,H)
```

although what you name the arguments is up to you. We named the arguments the way that mathematicians think, although we could just as well have named them the way that statisticians think:

$$
\text { void evaluator (todo, } b, s, g, H)
$$

To avoid favoritism, we will write them as
void evaluator (todo, $p, v, g, H$ )
that is, we will think in terms of optimizing $v=f(p)$.
Here is the full definition of a type d evaluator:
void evaluator (real scalar todo, real rowvector $p, v, g, H$ )
where $v, g$, and $H$ are values to be returned:
$v$ : real scalar
$g$ : real rowvector
$H$ : real matrix
evaluator () is to fill in $v$ given the values in $p$ and optionally to fill in $g$ and $H$, depending on the value of todo:
todo Required action by evaluator ()
$0 \quad$ calculate $v=f(p)$ and store in $v$
1 calculate $v=f(p)$ and $g=f^{\prime}(p)$ and store in $v$ and $g$
2 calculate $v=f(p), g=f^{\prime}(p)$, and $H=f^{\prime \prime}(p)$ and store in $v, g$, and $H$
evaluator () may return $v=$. if $f()$ cannot be evaluated at $p$. Then $g$ and $H$ need not be filled in even if requested.

An evaluator does not have to be able to do all of this. In the first example, myeval () could handle only todo $=0$. There are three types of type d evaluators:

| d type | Capabilities expected of evaluator () |
| :--- | :--- |
| d 0 | can calculate $v=f(p)$ |
| d 1 | can calculate $v=f(p)$ and $g=f^{\prime}(p)$ |
| d 2 | can calculate $v=f(p)$ and $g=f^{\prime}(p)$ and $H=f^{\prime \prime}(p)$ |

myeval() is a type d0 evaluator. Type d0 evaluators are never asked to calculate $g$ or $H$. Type d0 is the default type but, if we were worried that it was not, we could set the evaluator type before invoking optimize() by coding

```
optimize_init_evaluatortype(S, "d0")
```

Here are the code outlines of the three types of evaluators:

```
void dO_evaluator(todo, p, v, g, H)
{
    v = ...
}
void dl_evaluator(todo, p, v, g, H)
{
    v = ...
    if (todo>=1) {
        g = ...
    }
}
```

void d2_evaluator(todo, $\mathrm{p}, \mathrm{v}, \mathrm{g}, \mathrm{H}$ )
\{
v = ...
if (todo>=1) \{
g = ...
if (todo==2) \{
H = ...
\}
\}
\}

Here is the code outline where there are three additional user arguments to the evaluator:

```
void d0_user3_eval(todo, p, u1, u2, u3, v, g, H)
{
    v = ...
}
```


## Example of d0, d1, and d2

We wish to find the $p_{1}$ and $p_{2}$ corresponding to the maximum of

$$
v=\exp \left(-p_{1}^{2}-p_{2}^{2}-p_{1} p_{2}+p_{1}-p_{2}-3\right)
$$

A d0 solution to the problem would be

```
: void evalO(todo, p, v, g, H)
> {
> v = exp(-p[1]^2 - p[2]^2 - p[1]*p[2] + p[1] - p[2] - 3)
> }
note: argument todo unused
note: argument g unused
note: argument h unused
: S = optimize_init()
: optimize_init_evaluator(S, &evalO())
: optimize_init_params(S, (0,0))
: p = optimize(S)
Iteration 0: f(p) = . 04978707 (not concave)
Iteration 1: f(p) = . }1251302
Iteration 2: f(p) = . }1349588
Iteration 3: f(p) = . }1353352
Iteration 4: f(p) = . }1353352
p
\begin{tabular}{|cc|}
\hline 1 & 2 \\
\hline 1 & -1 \\
\hline
\end{tabular}
```

A d1 solution to the problem would be

```
: void eval1(todo, p, v, g, H)
{
> v = exp(-p[1]^2 - p[2]^2 - p[1]*p[2] + p[1] - p[2] - 3)
> if (todo==1) {
> g[1] = (-2*p[1] - p[2] + 1)*v
> g[2] = (-2*p[2] - p[1] - 1)*v
> }
> }
note: argument H unused
: S = optimize_init()
: optimize_init_evaluator(S, &eval1())
: optimize_init_evaluatortype(S, "d1")}\leftarrow < important
: optimize_init_params(S, (0,0))
```

```
: p = optimize(S)
Iteration 0: f(p) = . 04978707 (not concave)
Iteration 1: f(p) = . }1251302
Iteration 2: f(p) = . }1349688
Iteration 3: f(p) = . }1353352
Iteration 4: f(p) = . }1353352
: p
    1 2
1
\begin{tabular}{ll}
1 & -1 \\
\hline
\end{tabular}
```

The d 1 solution is better than the d 0 solution because it runs faster and usually is more accurate. Type d 1 evaluators require more code, however, and deriving analytic derivatives is not always possible.

A d2 solution to the problem would be

```
: void eval2(todo, p, v, g, H)
{
        v = exp(-p[1]^2 - p[2]^2 - p[1]*p[2] + p[1] - p[2] - 3)
        if (todo>=1) {
            g[1] = (-2*p[1] - p[2] + 1)*v
            g[2] = (-2*p[2] - p[1] - 1)*v
            if (todo==2) {
                H[1,1] = -2*v + (-2*p[1]-p[2]+1)*g[1]
                H[2,1] = -1*v + (-2*p[2]-p[1]-1)*g[1]
                H[2,2] = -2*v + (-2*p[2]-p[1]-1)*g[2]
                        _makesymmetric(H)
            }
    }
}
S = optimize_init()
optimize_init_evaluator(S, &eval2())
optimize_init_evaluatortype(S, "d2") }\leftarrow importan
optimize_init_params(S, (0,0))
p = optimize(S)
Iteration 0: f(p) = . 04978707 (not concave)
Iteration 1: f(p) = . }1251302
Iteration 2: f(p) = . }1349688
Iteration 3: f(p) = . }1353352
Iteration 4: f(p) = . }1353352
p
\begin{tabular}{|cc|}
\hline 1 & 2 \\
\hline 1 & -1 \\
\hline
\end{tabular}
```

A d2 solution is best because it runs fastest and usually is the most accurate. Type d2 evaluators require the most code, and deriving analytic derivatives is not always possible.

In the d 2 evaluator eval2(), note our use of makesymmetric(). Type d 2 evaluators are required to return $H$ as a symmetric matrix; filling in just the lower or upper triangle is not sufficient. The easiest way to do that is to fill in the lower triangle and then use makesymmetric() to reflect the lower off-diagonal elements; see [M-5] makesymmetric().

## d1debug and d2debug

In addition to evaluator types "d0", "d1", and "d2" that are specified in
optimize_init_evaluatortype( $S$, evaluatortype), there are two more: "d1debug" and "d2debug". They assist in coding d1 and d2 evaluators.

In Example of d0, d1, and d2 above, we admit that we did not correctly code the functions eval1() and eval2() at the outset, before you saw them. In both cases, that was because we had taken the derivatives incorrectly. The problem was not with our code but with our math. d1debug and d2debug helped us find the problem.
d1debug is an alternative to d1. When you code
optimize_init_evaluatortype ( $S$, "d1debug"), the derivatives you calculate are not taken seriously. Instead, optimize() calculates its own numerical derivatives and uses those. Each time optimize() does that, however, it compares your derivatives to the ones it calculated and gives you a report on how they differ. If you have coded correctly, they should not differ by much.
d2debug does the same thing, but for d2 evaluators. When you code
optimize_init_evaluatortype ( $S$, "d2debug"), optimize() uses numerical derivatives but, each time, optimize() gives you a report on how much your results for the gradient and for the Hessian differ from the numerical calculations.

For each comparison, optimize() reports just one number: the mreldif() (see [M-5] reldif()) between your results and the numerical ones. When you have done things right, gradient vectors will differ by approximately $1 \mathrm{e}-12$ or less and Hessians will differ by $1 \mathrm{e}-7$ or less.

When differences are large, you will want to see not only the summary comparison but also the full vectors and matrices so that you can compare your results element by element with those calculated numerically. Sometimes the error is in one element and not the others. To do this, set the trace level with optimize_init_tracelevel ( $S$, tracelevel) before issuing optimize(). Code optimize_init_tracelevel ( $S$, "gradient") to get a full report on the gradient comparison, or set optimize_init_tracelevel ( $S$, "hessian") to get a full report on the gradient comparison and the Hessian comparison.

## Type gf evaluators

In some statistical applications, you will find $g f 0, g f 1$, and $g f 2$ more convenient to code than d 0 , d1, and d2. The $g f$ stands for general form.

In statistical applications, one tends to think of a dataset of values arranged in matrix $X$, the rows of which are observations. A function $h(p, X[i,]$.$) can be calculated for each row separately, and it is$ the sum of those resulting values that forms the function $f(p)$ that is to be maximized or minimized.

The $g f 0, g f 1$, and $g f 2$ methods are for such cases.
In a type do evaluator, you return scalar $v=f(p)$.
In a type gf 0 evaluator, you return a column vector $v$ such that $\operatorname{col} \operatorname{sum}(v)=f(p)$.
In a type d1 evaluator, you return $v=f(p)$ and you return a row vector $g=f^{\prime}(p)$.
In a type gf 1 evaluator, you return $v$ such that $\operatorname{colsum}(v)=f(p)$ and you return matrix $g$ such that $\operatorname{colsum}(g)=f^{\prime}(p)$.

In a type d2 evaluator, you return $v=f(p), g=f^{\prime}(p)$, and you return $H=f^{\prime \prime}(p)$.

In a type gf2 evaluator, you return $v$ such that colsum $(v)=f(p), g$ such that colsum $(g)=f^{\prime}(p)$, and you return $H=f^{\prime \prime}(p)$. This is the same $H$ returned for d2.

The code outline for type gf evaluators is the same as those for d evaluators. For instance, the outline for a gf2 evaluator is

```
void gf2_evaluator(todo, p, v, g, H)
{
    v = ...
    if (todo>=1) {
        g = ...
        if (todo==2) {
            H = ...
        }
    }
}
```

The above is the same as the outline for d 2 evaluators. All that differs is that $v$ and $g$, which were real scalar and real rowvector in the d 2 case, are now real colvector and real matrix in the gf 2 case. The same applies to gf1 and gf0.

The type gf evaluators arise in statistical applications and, in such applications, there are data; that is, just knowing $p$ is not sufficient to calculate $v, g$, and $H$. Actually, that same problem can arise when coding type d evaluators as well.

You can pass extra arguments to evaluators, whether they be $\mathrm{d} 0, \mathrm{~d} 1$, or d 2 or $\mathrm{gf} 0, \mathrm{gf} 1$, or gf 2 . The first line of all evaluators, regardless of style, is
void evaluator (todo, $p, v, g, H$ )
If you code
optimize_init_argument ( $S, 1, X$ )
the first line becomes
void evaluator (todo, $p, X, v, g, H$ )
If you code
optimize_init_argument ( $S, 1, X$ )
optimize_init_argument $(S, 2, Y)$
the first line becomes
void evaluator (todo, $p, X, Y, v, g, H)$
and so on, up to nine extra arguments. That is, you can specify extra arguments to be passed to your function. These extra arguments should be placed right after the parameter vector.

## Example of gf0, gf1, and gf2

You have the following data:

| x | 1 |
| :---: | :---: |
|  |  |
| 1 | . 35 |
| 2 | . 29 |
| 3 | . 3 |
| 4 | . 3 |
| 5 | . 65 |
| 6 | . 56 |
| 7 | . 37 |
| 8 | . 16 |
| 9 | . 26 |
| 10 | . 19 |

You believe that the data are the result of a beta distribution process with fixed parameters alpha and beta and you wish to obtain the maximum likelihood estimates of alpha and beta ( $a$ and $b$ in what follows). The formula for the density of the beta distribution is

$$
\operatorname{density}(x)=\frac{\Gamma(a+b)}{\Gamma(a) \Gamma(b)} x^{a-1}(1-x)^{b-1}
$$

The gf0 solution to this problem is

```
: void lnbetadenO(todo, p, x, lnf, S, H)
> {
> a = p[1]
> b = p[2]
> lnf = lngamma(a+b) :- lngamma(a) :- lngamma(b) :+
> (a-1)*log(x) :+ (b-1)*log(1:-x)
> }
note: argument todo unused
note: argument S unused
note: argument H unused
: S = optimize_init()
: optimize_init_evaluator(S, &lnbetaden0())
: optimize_init_evaluatortype(S, "gf0")
: optimize_init_params(S, (1,1))
: optimize_init_argument(S, 1, x)}\leftarrow important
: p = optimize(S)
Iteration 0: f(p) = 0
Iteration 1: f(p) = 5.7294728
Iteration 2: f(p) = 5.7646641
Iteration 3: f(p) = 5.7647122
Iteration 4: f(p) = 5.7647122
    p
        1 2
    1
    3.714209592 7.014926315
```

Note the following:

1. Rather than calling the returned value v , we called it lnf . You can name the arguments as you please.
2. We arranged for an extra argument to be passed by coding optimize_init_argument (S, 1 , x ). The extra argument is the vector x , which we listed previously for you. In our function, we received the argument as $x$, but we could have used a different name, just as we used lnf rather than v .
3. We set the evaluator type to "gf0".

This being a statistical problem, we should be interested not only in the estimates p but also in their variance. We can get this from the inverse of the negative Hessian, which is the observed information matrix:

```
: optimize_result_V_oim(S)
[symmetric]
```

1
2

```
2.556301184
4.498194785 9.716647065
```

The gf 1 solution to this problem is

```
: void lnbetaden1(todo, p, x, lnf, S, H)
> {
> a = p[1]
> b = p[2]
> lnf = lngamma(a+b) :- lngamma(a) :- lngamma(b) :+
    (a-1)*log(x) :+ (b-1)*log(1:-x)
    if (todo >= 1) {
                                    S = J(rows(x), 2, .)
                                    S[.,1] = log(x) :+ digamma(a+b) :- digamma(a)
                            S[.,2] = log(1:-x) :+ digamma(a+b) :- digamma(b)
    }
}
note: argument H unused
: S = optimize_init()
: optimize_init_evaluator(S, &lnbetaden1())
: optimize_init_evaluatortype(S, "gf1")
: optimize_init_params(S, (1,1))
: optimize_init_argument(S, 1, x)
: p = optimize(S)
Iteration 0: f(p) = 0
Iteration 1: f(p) = 5.7297061
Iteration 2: f(p) = 5.7641349
Iteration 3: f(p) = 5.7647121
Iteration 4: f(p) = 5.7647122
p
    1 2
1
3.714209343 7.014925751
```

```
: optimize_result_V_oim(S)
[symmetric]
```

$1 \quad 2$

```
2.556299425
    4.49819212 9.716643068
```

Note the following:

1. We called the next-to-last argument of lnbetaden1() S rather than g in accordance with standard statistical jargon. What is being returned is in fact the observation-level scores, which sum to the gradient vector.
2. We called the next-to-last argument $S$ even though that name conflicted with $S$ outside the program, where $S$ is the problem handle. Perhaps we should have renamed the outside $S$, but there is no confusion on Mata's part.
3. In our program, we allocated $S$ for ourselves: $\mathrm{S}=\mathrm{J}$ (rows (x), 2, .). It is worth comparing this with the example of d1 in Example of d0, d1, and d2, where we did not need to allocate g. In d1, optimize() preallocates $g$ for us. In gf1, optimize() cannot do this because it has no idea how many "observations" we have.

The gf 2 solution to this problem is

```
void lnbetaden2(todo, p, x, lnf, S, H)
{
a = p[1]
    b}=p[2
    lnf = lngamma(a+b) :- lngamma(a) :- lngamma(b) :+
                        (a-1)*\operatorname{log}(x) :+ (b-1)*\operatorname{log}(1:-x)
        if (todo >= 1) {
            S = J(rows (x), 2, .)
            S[.,1] = log(x) :+ digamma(a+b) :- digamma(a)
            S[.,2] = log(1:-x) :+ digamma(a+b) :- digamma(b)
            if (todo==2) {
                    n = rows(x)
                    H[1,1] = n*(trigamma(a+b) - trigamma(a))
                    H[2,1] = n*(trigamma(a+b))
                    H[2,2] = n*(trigamma(a+b) - trigamma(b))
                            _makesymmetric(H)
            }
    }
}
S = optimize_init()
optimize_init_evaluator(S, &lnbetaden2())
: optimize_init_evaluatortype(S, "gf2")
: optimize_init_params(S, (1,1))
: optimize_init_argument(S, 1, x)
p = optimize(S)
Iteration 0: f(p) = 0
Iteration 1: f(p) = 5.7297061
Iteration 2: f(p) = 5.7641349
Iteration 3: f(p) = 5.7647121
Iteration 4: f(p) = 5.7647122
```

: p
$\qquad$

1

```
3.714209343
7.014925751
```

: optimize_result_V_oim(S)
[symmetric]
$1 \quad 2$
2.556299574
$4.498192412 \quad 9.716643651$

## Functions

## optimize_init()

transmorphic optimize_init()
optimize_init () is used to begin an optimization problem. Store the returned result in a variable name of your choosing; we have used $S$ in this documentation. You pass $S$ as the first argument to the other optimize*() functions.
optimize_init() sets all optimize_init_*() values to their defaults. You may use the query form of the optimize_init_*() to determine an individual default, or you can use optimize_query () to see them all.

The query form of optimize_init_*() can be used before or after optimization performed by optimize().

## optimize_init_which()

void optimize_init_which(S, \{"max"|"min"\})
string scalar optimize_init_which (S)
optimize_init_which ( $S$, which) specifies whether optimize() is to perform maximization or minimization. The default is maximization if you do not invoke this function.
optimize_init_which $(S)$ returns "max" or "min" according to which is currently set.

## optimize_init_evaluator() and optimize_init_evaluatortype()

void optimize_init_evaluator ( $S$, pointer(real function) scalar fptr)
void optimize_init_evaluatortype ( $S$, evaluatortype)
pointer(real function) scalar optimize_init_evaluator $(S)$
string scalar optimize_init_evaluatortype ( $S$ )
optimize_init_evaluator $(S, f p t r)$ specifies the function to be called to evaluate $f(p)$. Use of this function is required. If your function is named myf cn() , you code optimize_init_evaluator ( $S$, \&myfcn()).
optimize_init_evaluatortype ( $S$, evaluatortype) specifies the capabilities of the function that has been set using optimize_init_evaluator(). Alternatives for evaluatortype are "d0", "d1", "d2", "d1debug", "d2debug", "gf0", "gf1", "gf2", "gf1debug", and "gf2debug". The default is "dO" if you do not invoke this function.
optimize_init_evaluator $(S)$ returns a pointer to the function that has been set.
optimize_init_evaluatortype ( $S$ ) returns the evaluatortype currently set.

## optimize_init_negH()

optimize_init_negH ( $S$, \{ "off"|"on" \}) sets whether the evaluator you have written returns $H$ or $-H$, the Hessian or the negative of the Hessian, if it returns a Hessian at all. This is for backward compatibility with prior versions of Stata's ml command (see [R] ml). Modern evaluators return H . The default is "off".

## optimize_init_params()

```
    void optimize_init_params(S, real rowvector initialvalues)
    real rowvector optimize_init_params(S)
```

optimize_init_params ( $S$, initialvalues) sets the values of $p$ to be used at the start of the first iteration. Use of this function is required.
optimize_init_params ( $S$ ) returns the initial values that will be (or were) used.

## optimize_init_nmsimplexdeltas()

```
void optimize_init_nmsimplexdeltas(S, real rowvector delta)
real rowvector optimize_init_nmsimplexdeltas(S)
```

optimize_init_nmsimplexdeltas ( $S$, delta) sets the values of delta to be used, along with the initial parameters, to build the simplex required by technique "nm" (Nelder-Mead). Use of this function is required only in the Nelder-Mead case. The values in delta must be at least 10 times larger than ptol, which is set by optimize_init_conv_ptol(). The initial simplex will be $\left\{p, p+\left(d_{1}, 0\right), \ldots, 0, p+\left(0, d_{2}, 0, \ldots, 0\right), \ldots, p+\left(0,0, \ldots, 0, d_{k}\right)\right\}$.
optimize_init_nmsimplexdeltas $(S)$ returns the deltas that will be (or were) used.

## optimize_init_argument( ) and optimize_init_narguments()

| void | optimize_init_argument $(S$, real scalar $k, X)$ |
| :--- | :--- |
| void | optimize_init_narguments $(S$, real scalar $K)$ |
| pointer scalar | optimize_init_argument $(S$, real scalar $k)$ |
| real scalar | optimize_init_narguments $(S)$ |

optimize_init_argument $(S, k, X)$ sets the $k$ th extra argument of the evaluator function to be $X$, where $k$ can only $1,2,3, \ldots, 9 . X$ can be anything, including a view matrix or even a pointer to a function. No copy of $X$ is made; it is a pointer to $X$ that is stored, so any changes you make to $X$ between setting it and $X$ being used will be reflected in what is passed to the evaluator function.
optimize_init_narguments $(S, K)$ sets the number of extra arguments to be passed to the evaluator function. This function is useless and included only for completeness. The number of extra arguments is automatically set as you use optimize_init_argument().
optimize_init_argument ( $S$ ) returns a pointer to the object that was previously set.
optimize_init_narguments $(S)$ returns the number of extra arguments that are passed to the evaluator function.

## optimize_init_cluster()

optimize_init_cluster $(S, c)$ specifies a cluster variable. $c$ may be a string scalar containing a Stata variable name, or $c$ may be real colvector directly containing the cluster values. The default is "", meaning no clustering. If clustering is specified, the default vcetype becomes "robust".

## optimize_init_colstripe()

optimize_init_colstripe ( $S$ [, stripe]) sets the string matrix to be associated with the parameter vector. See matrix colnames in $[\mathrm{P}]$ matrix rownames.

## optimize_init_technique()

```
void optimize_init_technique(S, string scalar technique)
string scalar optimize_init_technique(S)
```

optimize_init_technique ( $S$, technique) sets the optimization technique to be used. Current choices are

| technique | Description |
| :--- | :--- |
| "nr" | modified Newton-Raphson |
| "dfp" | Davidon-Fletcher-Powell |
| "bfgs" | Broyden-Fletcher-Goldfarb-Shanno |
| "bhhh" | Berndt-Hall-Hall-Hausman |
| "nm" | Nelder-Mead |

The default is "nr".
optimize_init_technique ( $S$ ) returns the technique currently set.
Aside: All techniques require optimize_init_params() be set. Technique "nm" also requires that optimize_init_nmsimplexdeltas() be set. Parameters (and delta) can be set before or after the technique is set.

You can switch between "nr", "dfp", "bfgs", and "bhhh" by specifying two or more of them in a space-separated list. By default, optimize() will use an algorithm for five iterations before switching to the next algorithm. To specify a different number of iterations, include the number after the technique. For example, specifying optimize_init_technique (M, "bhhh 10 nr 1000 ") requests that optimize() perform 10 iterations using the Berndt-Hall-Hall-Hausman algorithm, followed by 1,000 iterations using the modified Newton-Raphson algorithm, and then switch back to Berndt-Hall-Hall-Hausman for 10 iterations, and so on. The process continues until convergence or until maxiter is exceeded.

## optimize_init_singularHmethod()

void optimize_init_singularHmethod ( $S$, string scalar singularHmethod)
string scalar optimize_init_singularHmethod ( $S$ )
optimize_init_singularHmethod ( $S$, singularHmethod) specifies what the optimizer should do when, at an iteration step, it finds that $H$ is singular. Current choices are

## singularHmethod Description

"m-marquardt" modified Marquardt algorithm
"hybrid" mixture of steepest descent and Newton
The default is "m-marquardt" if not set;
"hybrid" is equivalent to ml's difficult option; see $[R]$ ml.
optimize_init_technique $(S)$ returns the singularHmethod currently set.

## optimize_init_conv_maxiter()

void optimize_init_conv_maxiter ( $S$, real scalar maxiter)
real scalar optimize_init_conv_maxiter $(S)$
optimize_init_conv_maxiter ( $S$, maxiter) sets the maximum number of iterations to be performed before optimization() is stopped; results are posted to optimize_result_*() just as if convergence were achieved, but optimize_result_converged() is set to 0 . The default maxiter is Stata's c (maxiter), which is 300 by default.
optimize_init_conv_maxiter $(S)$ returns the maxiter currently set.

## optimize_init_conv_warning()

optimize_init_conv_warning ( $S$, \{ "on"|"off" $\}$ ) specifies whether the warning message "convergence not achieved" is to be displayed when this stopping rule is invoked. The default is "on".

```
optimize_init_conv_ptol(), . . _vtol(), . . _nrtol()
    void optimize_init_conv_ptol(S, real scalar ptol)
    void optimize_init_conv_vtol(S, real scalar vtol)
    void optimize_init_conv_nrtol(S, real scalar nrtol)
    real scalar optimize_init_conv_ptol(S)
    real scalar optimize_init_conv_vtol(S)
    real scalar optimize_init_conv_nrtol(S)
```

The two-argument form of these functions set the tolerances that control optimize()'s convergence criterion. optimize() performs iterations until the convergence criterion is met or until the number of iterations exceeds optimize_init_conv_maxiter(). When the convergence criterion is met, optimize_result_converged() is set to 1 . The default values of ptol, vtol, and nrtol are $1 \mathrm{e}-6$, $1 \mathrm{e}-7$, and $1 \mathrm{e}-5$, respectively.

The single-argument form of these functions return the current values of ptol, vtol, and nrtol.
Optimization criterion: In all cases except optimize_init_technique $(S)==" n m$ ", that is, in all cases except Nelder-Mead, that is, in all cases of derivative-based maximization, the optimization criterion is defined as follows:

Define
C_ptol: mreldif ( $p$, p_prior $)<$ ptol
C_vtol: $\quad$ reldif $\left(v, v \_\right.$prior $)<v t o l$
C_nrtol: $\quad g * \operatorname{invsym}(-H) * g^{\prime}<n r t o l$
C_concave: $-H$ is positive semidefinite
The above definitions apply for maximization. For minimization, think of it as maximization of $-f(p)$. optimize() declares convergence when

$$
\text { (C_ptol } \mid \text { C_vtol) \& C_concave \& C_nrtol }
$$

For optimize_init_technique ( $S$ )=="nm" (Nelder-Mead), the criterion is defined as follows:
Let $R$ be the minimum and maximum values on the simplex and define

$$
\begin{aligned}
& \text { C_ptol: mreldif }(\text { vertices of } R)<\text { ptol } \\
& \text { C_vtol: reldif }(R)<\text { vtol }
\end{aligned}
$$

optimize() declares successful convergence when

$$
\text { C_ptol } \mid \text { C_vtol }
$$

## optimize_init_conv_ignorenrtol()

optimize_init_conv_ignorenrtol (S, \{ "off"|"on" \}) sets whether C_nrtol should simply be treated as true in all cases, which in effects removes the nrtol criterion from the convergence rule. The default is "off".

## optimize_init_iterid()

void optimize_init_iterid ( $S$, string scalar id)
string scalar optimize_init_iterid(S)
By default, optimize() shows an iteration log, a line of which looks like

$$
\text { Iteration 1: } f(p)=5.7641349
$$

See optimize_init_tracelevel() below.
optimize_init_iterid ( $S, i d$ ) sets the string used to label the iteration in the iteration log. The default is "Iteration".
optimize_init_iterid $(S)$ returns the $i d$ currently in use.

## optimize_init_valueid()

void optimize_init_valueid ( $S$, string scalar id)
string scalar optimize_init_valueid(S)
By default, optimize() shows an iteration log, a line of which looks like

$$
\text { Iteration 1: } f(p)=5.7641349
$$

See optimize_init_tracelevel() below.
optimize_init_valueid ( $S, i d$ ) sets the string used to identify the value. The default is " $f(\mathrm{p})$ ". optimize_init_valueid $(S)$ returns the $i d$ currently in use.

## optimize_init_tracelevel()

void optimize_init_tracelevel (S, string scalar tracelevel)
string scalar optimize_init_tracelevel ( $S$ )
optimize_init_tracelevel ( $S$, tracelevel) sets what is displayed in the iteration log. Allowed values of tracelevel are
tracelevel To be displayed each iteration

| "none" | nothing (suppress the log) |
| :--- | :--- |
| "value" | function value |
| "tolerance" | previous + convergence values |
| "step" | previous + stepping information |
| "paramdiffs" | previous + parameter relative differences <br> previous + parameter values |
| "params" | previous + gradient vector |
| "hessian" | previous + Hessian matrix |

The default is "value" when set iterlog is on and "none" when set iterlog is off; see [R] set iter.
optimize_init_tracelevel $(S)$ returns the value of tracelevel currently set.
optimize_init_trace_dots(), ..._value( ), ..._tol(), ..._step(), ..._paramdiffs(), ..._params(), ..._gradient(), ..._Hessian()
optimize_init_trace_dots $(S,\{$ "off"|"on" $\}$ ) displays a dot each time your evaluator is called. The default is "off".
optimize_init_trace_value ( $S$, \{ "on" | "off" $\}$ ) displays the function value at the start of each iteration. The default is "on".
optimize_init_trace_tol ( $S$, \{ "off"|"on" \}) displays the value of the calculated result that is compared to the effective convergence criterion at the end of each iteration. The default is "off".
optimize_init_trace_step ( $S$, \{ "off"|"on" $\}$ ) displays the steps within iteration. Listed are the value of objective function along with the word forward or backward. The default is "off".
optimize_init_trace_paramdiffs (S, \{"off"|"on" \}) displays the parameter relative differences from the previous iteration that are greater than the parameter tolerance ptol. The default is "off".
optimize_init_trace_params ( $S,\{$ "off" | "on" $\}$ ) displays the parameters at the start of each iteration. The default is "off".
optimize_init_trace_gradient (S, \{"off"|"on" \}) displays the gradient vector at the start of each iteration. The default is "off".
optimize_init_trace_Hessian(S, \{"off"|"on" \}) displays the Hessian matrix at the start of each iteration. The default is "off".

## optimize_init_evaluations()

optimize_init_evaluations ( $S$, $\{$ "off" $\mid$ "on" $\}$ ) specifies whether the system is to count the number of times the evaluator is called. The default is "off".

## optimize_init_constraints()

```
void optimize_init_constraints(S, real matrix Cc)
real matrix optimize_init_constraints(S)
```

$n c$ linear constraints may be imposed on the $n p$ parameters in $p$ according to $C p^{\prime}=c, C: n c \times n p$ and $c: n c \times 1$. For instance, if there are four parameters and you wish to impose the single constraint $p_{1}=p_{2}$, then $C=(1,-1,0,0)$ and $c=(0)$. If you wish to add the constraint $p_{4}=2$, then $C=(1,-1,0,0 \backslash 0,0,0,1)$ and $c=(0 \backslash 2)$.
optimize_init_constraints ( $S, C c$ ) allows you to impose such constraints where $C c=(C, c)$. Use of this function is optional. If no constraints have been set, then $C c$ is $0 \times(n p+1)$.
optimize_init_constraints $(S)$ returns the current $C c$ matrix.

## optimize_init_verbose()

void optimize_init_verbose ( $S$, real scalar verbose)
real scalar optimize_init_verbose ( $S$ )
optimize_init_verbose ( $S$, verbose) sets whether error messages that arise during the execution of optimize() or _optimize() are to be displayed. verbose $=1$ means that they are; 0 means that they are not. The default is 1 . Setting verbose to 0 is of interest only to users of _optimize(). If you wish to suppress all output, code

```
optimize_init_verbose(S, 0)
optimize_init_tracelevel(S, "none")
```

optimize_init_verbose $(S)$ returns the current value of verbose.
optimize_init_deriv_usemin() and optimize_init_deriv_min()

```
void optimize_init_deriv_usemin(S, \{"off"|"on" \})
void optimize_init_deriv_min(S, real rowvector min)
string scalar optimize_init_deriv_usemin( \(S\) )
real rowvector optimize_init_deriv_min(S)
```

optimize_init_deriv_usemin( $S$, \{ "off"|"on" \}) specifies whether to use the minimum values of step sizes for computing numerical derivatives. The default is "off", meaning minimum values will not be used. If you specify "on", minimum values will be used; when minimum values are not specified by optimize_init_deriv_min(), default minimum values are $1 \mathrm{e}-6$ and $1 \mathrm{e}-4$ for first- and second-order derivatives, respectively.
optimize_init_deriv_min $(S, \min )$ sets the minimum values of step sizes used for computing numerical derivatives. $\min$ is a $1 \times 2$ real row vector; the first column specifies the minimum for first-order derivatives, and the second column specifies the minimum for second-order derivatives. If there is a missing value in the row vector, the default values of $1 \mathrm{e}-6$ for first-order derivatives and $1 \mathrm{e}-4$ for second-order derivatives are used. If this function is used, optimize_init_deriv_usemin() is automatically set to "on".
optimize_init_deriv_usemin( $S$ ) returns the current value "off" or "on", indicating whether to use minimum values of the step size.
optimize_init_deriv_min(S) returns the current value min.

## optimize()

real rowvector optimize ( $S$ )
optimize ( $S$ ) invokes the optimization process and returns the resulting parameter vector. If something goes wrong, optimize() aborts with error.

Before you can invoke optimize(), you must have defined your evaluator function evaluator() and you must have set initial values:

```
S = optimize_init()
optimize_init_evaluator(S, &evaluator())
optimize_init_params(S, (...))
```

The above assumes that your evaluator function is d0. Often you will also have coded

```
optimize_init_evaluatortype(S, "..."))
```

and you may have coded other optimize_init_*() functions as well.
Once optimize() completes, you may use the optimize_result_*() functions. You may also continue to use the optimize_init_*() functions to access initial settings, and you may use them to change settings and restart optimization (that is, invoke optimize () again) if you wish. If you do that, you will usually want to use the resulting parameter values from the first round of optimization as initial values for the second. If so, do not forget to code

```
optimize_init_params(S, optimize_result_params(S))
```


## _optimize()

real scalar _optimize ( $S$ )
_optimize $(S)$ performs the same actions as optimize $(S)$ except that, rather than returning the resulting parameter vector, _optimize() returns a real scalar and, rather than aborting if numerical issues arise, _optimize() returns a nonzero value. _optimize() returns 0 if all went well. The returned value is called an error code.
optimize() returns the resulting parameter vector $p$. It can work that way because optimization must have gone well. Had it not, optimize() would have aborted execution.
_optimize() returns an error code. If it is 0 , optimization went well and you can obtain the parameter vector by using optimize_result_params(). If optimization did not go well, you can use the error code to diagnose what went wrong and take the appropriate action.

Thus _optimize $(S)$ is an alternative to optimize $(S)$. Both functions do the same thing. The difference is what happens when there are numerical difficulties.
optimize () and _optimize () work around most numerical difficulties. For instance, the evaluator function you write is allowed to return $v$ equal to missing if it cannot calculate the $f()$ at the current values of $p$. If that happens during optimization, optimize() and _optimize() will back up to the last value that worked and choose a different direction. optimize(), however, cannot tolerate that happening with the initial values of the parameters because optimize() has no value to back up to. optimize() issues an error message and aborts, meaning that execution is stopped. There can be advantages in that. The calling program need not include complicated code for such instances, figuring that stopping is good enough because a human will know to address the problem.
_optimize(), however, does not stop execution. Rather than aborting, _optimize() returns a nonzero value to the caller, identifying what went wrong.

Programmers implementing advanced systems will want to use _optimize() instead of optimize(). Everybody else should use optimize().

Programmers using _optimize() will also be interested in the functions

```
optimize_init_verbose()
optimize_result_errorcode()
optimize_result_errortext()
optimize_result_returncode()
```

If you perform optimization by using _optimize(), the behavior of all optimize_result_*() functions is altered. The usual behavior is that, if calculation is required and numerical problems arise, the functions abort with error. After _optimize(), however, a properly dimensioned missing result is returned and optimize_result_errorcode() and optimize_result_errortext() are set appropriately.

The error codes returned by _optimize() are listed under the heading optimize_result_errorcode() below.

## optimize_evaluate()

void optimize_evaluate ( $S$ )
optimize_evaluate $(S)$ evaluates $f()$ at optimize_init_params() and posts results to optimize_result_*() just as if optimization had been performed, meaning that all optimize_result_*() functions are available for use. optimize_result_converged() is set to 1.

The setup for running this function is the same as for running optimize():

```
S = optimize_init()
optimize_init_evaluator(S, &evaluator())
optimize_init_params(S, (...))
```

Usually, you will have also coded

```
optimize_init_evaluatortype(S,...))
```

The other optimize_init_*() settings do not matter.

## _optimize_evaluate()

```
real scalar _optimize_evaluate(S)
```

The relationship between _optimize_evaluate() and optimize_evaluate() is the same as that between _optimize() and optimize(); see _optimize().
_optimize_evaluate() returns an error code.

## optimize_result_params()

real rowvector optimize_result_params ( $S$ )
optimize_result_params $(S)$ returns the resulting parameter values. These are the same values that were returned by optimize() itself. There is no computational cost to accessing the results, so rather than coding

$$
p=\operatorname{optimize}(S)
$$

if you find it more convenient to code

```
(void) optimize(S)
p = optimize_result_params(S)
```

then do so.

## optimize_result_value() and optimize_result_value0()

real scalar optimize_result_value ( $S$ )
real scalar optimize_result_value0 (S)
optimize_result_value $(S)$ returns the value of $f()$ evaluated at $p$ equal to
optimize_result_params().
optimize_result_value $0(S)$ returns the value of $f()$ evaluated at $p$ equal to optimize_init_params().

These functions may be called regardless of the evaluator or technique used.

## optimize_result_gradient()

real rowvector optimize_result_gradient ( $S$ )
optimize_result_gradient $(S)$ returns the value of the gradient vector evaluated at $p$ equal to optimize_result_params (). This function may be called regardless of the evaluator or technique used.

## optimize_result_scores()

```
real matrix optimize_result_scores(S)
```

optimize_result_scores $(S)$ returns the value of the scores evaluated at $p$ equal to
optimize_result_params (). This function may be called only if a type gf evaluator is used, but regardless of the technique used.

## optimize_result_Hessian()

real matrix optimize_result_Hessian(S)
optimize_result_Hessian $(S)$ returns the value of the Hessian matrix evaluated at $p$ equal to optimize_result_params (). This function may be called regardless of the evaluator or technique used.

## optimize_result_V() and optimize_result_Vtype()

real matrix optimize_result_V (S)
string scalar optimize_result_Vtype( $S$ )
optimize_result_V $(S)$ returns optimize_result_V_oim $(S)$
or optimize_result_V_opg $(S)$, depending on which is the natural conjugate for the optimization technique used. If there is no natural conjugate, optimize_result_V_oim( $S$ ) is returned.

```
optimize_result_Vtype(S) returns "oim" or "opg".
optimize_result_V_oim( ), .. _opg(), . . _robust( )
    real matrix optimize_result_V_oim(S)
    real matrix optimize_result_V_opg(S)
    real matrix optimize_result_V_robust(S)
```

These functions return the variance matrix of $p$ evaluated at $p$ equal to optimize_result_params (). These functions are relevant only for maximization of log-likelihood functions but may be called in any context, including minimization.
optimize_result_V_oim $(S)$ returns invsym $(-H)$, which is the variance matrix obtained from the observed information matrix. For minimization, returned is invsym $(H)$.
optimize_result_V_opg $(S)$ returns invsym $\left(S^{\prime} S\right)$, where $S$ is the $N \times n p$ matrix of scores. This is known as the variance matrix obtained from the outer product of the gradients. optimize_result_V_opg() is available only when the evaluator function is type gf, but regardless of the technique used.
optimize_result_V_robust $(S)$ returns $\operatorname{invsym}(-H) *\left(S^{\prime} S\right) * \operatorname{invsym}(-H)$, which is the robust estimate of variance, also known as the sandwich estimator of variance.
optimize_result_V_robust() is available only when the evaluator function is type gf, but regardless of the technique used.

## optimize_result_iterations()

real scalar optimize_result_iterations ( $S$ )
optimize_result_iterations $(S)$ returns the number of iterations used in obtaining results.

## optimize_result_converged()

real scalar optimize_result_converged ( $S$ )
optimize_result_converged $(S)$ returns 1 if results converged and 0 otherwise.
See optimize_init_conv_ptol() for the definition of convergence.

## optimize_result_iterationlog()

real colvector optimize_result_iterationlog(S)
optimize_result_iterationlog $(S)$ returns a column vector of the values of $f()$ at the start of the final 20 iterations, or, if there were fewer, however many iterations there were. Returned vector is min(optimize_result_iterations(), 20) $\times 1$.

## optimize_result_evaluations()

optimize_result_evaluations $(S)$ returns a $1 \times 3$ real rowvector containing the number of times the evaluator was called, assuming optimize_init_evaluations() was set on. Contents are the number of times called for the purposes of 1) calculating the objective function, 2) calculating the objective function and its first derivative, and 3) calculating the objective function and its first and second derivatives. If optimize_init_evaluations() was set to "off", returned is ( $0,0,0$ ).
optimize_result_errorcode( ), . . . _errortext( ), and . . . _returncode( )
real scalar optimize_result_errorcode $(S)$
string scalar optimize_result_errortext $(S)$
real scalar optimize_result_returncode $(S)$
These functions are for use after _optimize().
optimize_result_errorcode ( $S$ ) returns the error code of _optimize(),
_optimize_evaluate(), or the last optimize_result_*() run after either of the first two functions. The value will be zero if there were no errors. The error codes are listed directly below.
optimize_result_errortext ( $S$ ) returns a string containing the error message corresponding to the error code. If the error code is zero, the string will be " ".
optimize_result_returncode $(S)$ returns the Stata return code corresponding to the error code. The mapping is listed directly below.

In advanced code, these functions might be used as

```
(void) _optimize(S)
if (ec = optimize_result_returncode(S)) {
    errprintf("{p}\n")
    errprintf("%s\n", optimize_result_errortext(S))
    errprintf("{p_end}\n")
    exit(optimize_result_returncode(S))
    /*NOTREACHED*/
}
```

The error codes and their corresponding Stata return codes are

| Error <br> code | Return code | Error text |
| :---: | :---: | :---: |
| 1 | 1400 | initial values not feasible |
| 2 | 412 | redundant or inconsistent constraints |
| 3 | 430 | missing values returned by evaluator |
| 4 | 430 | Hessian is not positive semidefinite or Hessian is not negative semidefinite |
| 5 | 430 | could not calculate numerical derivatives-discontinuous region with missing values encountered |
| 6 | 430 | could not calculate numerical derivatives-flat or discontinuous region encountered |
| 7 | 430 | could not calculate improvement-discontinuous region encountered |
| 8 | 430 | could not calculate improvement-flat region encountered |
| 9 | 430 | Hessian could not be updated-Hessian is unstable |
| 10 | 111 | technique unknown |
| 11 | 111 | incompatible combination of techniques |
| 12 | 111 | singular H method unknown |
| 13 | 198 | matrix stripe invalid for parameter vector |
| 14 | 198 | negative convergence tolerance values are not allowed |
| 15 | 503 | invalid starting values |
| 16 | 111 | optimize() subroutine not found |
| 17 | 111 | simplex delta required |
| 18 | 3499 | simplex delta not conformable with parameter vector |
| 19 | 198 | simplex delta value too small (must be greater than $10 \times$ ptol in absolute value) |
| 20 | 198 | evaluator type requires the nr technique |
| 21 | 198 | evaluator type not allowed with specified technique |
| 22 | 111 | optimize() subroutine not found |
| 23 | 198 | evaluator type not allowed with bhhh technique |
| 24 | 111 | evaluator functions required |
| 25 | 198 | starting values for parameters required |
| 26 | 198 | missing parameter values not allowed |
| 27 | 198 | invalid evaluator type |

Notes: 1. Error 1 can occur only when evaluating $f()$ at initial parameters.
2. Error 2 can occur only if constraints are specified.
3. Error 3 can occur only if the technique is "nm".
4. Error 9 can occur only if technique is "bfgs" or "dfp".

## optimize_query()

void optimize_query ( $S$ )
optimize_query ( $S$ ) displays a report on all optimize_init_*() and optimize_result*() values. optimize_query() may be used before or after optimize() and is useful when using optimize() interactively or when debugging a program that calls optimize() or _optimize().

## Conformability

All functions have $1 \times 1$ inputs and have $1 \times 1$ or void outputs except the following:

```
optimize_init_params(S, initialvalues):
```

$S$ : transmorphic
initialvalues: $\quad 1 \times n p$
result: void
optimize_init_params(S):
S: transmorphic
result: $\quad 1 \times n p$
optimize_init_argument ( $S, k, X$ ):
$S$ : transmorphic
$k: \quad 1 \times 1$
$X: \quad$ anything
result: void
optimize_init_nmsimplexdeltas ( $S$, delta):
$S$ : transmorphic
delta: $1 \times n p$
result: void
optimize_init_nmsimplexdeltas $(S)$ :
$S$ : transmorphic
result: $\quad 1 \times n p$
optimize_init_constraints(S, Cc):
$S$ : transmorphic
Cc: $\quad n c \times(n p+1)$
result: void
optimize_init_constraints $(S)$ :
$S$ : transmorphic
result: $\quad n c \times(n p+1)$
optimize_init_deriv_min(S, min):
$S$ : transmorphic
min: $1 \times 2$
result: void
optimize_init_deriv_min( $S$ ):
$S$ : transmorphic
result: $1 \times 2$

```
optimize(S):
```

| S: | transmorphic |
| ---: | :--- |
| result: | $1 \times n p$ |

optimize_result_params (S):
S: transmorphic result: $\quad 1 \times n p$
optimize_result_gradient $(S)$, optimize_result_evaluations $(S)$ :
S: transmorphic
result: $\quad 1 \times n p$
optimize_result_scores ( $S$ ):
S: transmorphic
result: $\quad N \times n p$
optimize_result_Hessian( $S$ ):
$S: \quad$ transmorphic
result: $\quad n p \times n p$
optimize_result_V $(S)$, optimize_result_V_oim $(S)$, optimize_result_V_opg $(S)$, optimize_result_V_robust ( $S$ ):

S: transmorphic
result: $\quad n p \times n p$
optimize_result_iterationlog( $S$ ):
S: transmorphic
result: $\quad L \times 1, L \leq 20$
For optimize_init_cluster $(S, c)$ and optimize_init_colstripe $(S)$, see Syntax above.

## Diagnostics

All functions abort with error when used incorrectly.
optimize() aborts with error if it runs into numerical difficulties. _optimize() does not; it instead returns a nonzero error code.
optimize_evaluate() aborts with error if it runs into numerical difficulties.
_optimize_evaluate() does not; it instead returns a nonzero error code.
The optimize_result_*() functions abort with error if they run into numerical difficulties when called after optimize() or optimize_evaluate(). They do not abort when run after _optimize() or _optimize_evaluate(). They instead return a properly dimensioned missing result and set optimize_result_errorcode() and optimize_result_errortext().

The formula $x_{i+1}=x_{i}-f\left(x_{i}\right) / f^{\prime}\left(x_{i}\right)$ and its generalizations for solving $f(x)=0$ (and its generalizations) are known variously as Newton's method or the Newton-Raphson method. The real history is more complicated than these names imply and has roots in the earlier work of Arabic algebraists and François Viète.

Newton's first formulation dating from about 1669 refers only to solution of polynomial equations and does not use calculus. In his Philosophiae Naturalis Principia Mathematica, first published in 1687, the method is used, but not obviously, to solve a nonpolynomial equation. Raphson's work, first published in 1690, also concerns polynomial equations, and proceeds algebraically without using calculus, but lays more stress on iterative calculation and so is closer to present ideas. It was not until 1740 that Thomas Simpson published a more general version explicitly formulated in calculus terms that was applied to both polynomial and nonpolynomial equations and to both single equations and systems of equations. Simpson's work was in turn overlooked in influential later accounts by Lagrange and Fourier, but his contribution also deserves recognition.

Isaac Newton (1643-1727) was an English mathematician, astronomer, physicist, natural philosopher, alchemist, theologian, biblical scholar, historian, politician and civil servant. He was born in Lincolnshire and later studied there and at the University of Cambridge, where he was a fellow of Trinity College and elected Lucasian Professor in 1669. Newton demonstrated the generalized binomial theorem, did major work on power series, and deserves credit with Gottfried Leibniz for the development of calculus. They entered a longstanding priority dispute in 1711, which lasted until Leibniz died in 1716.

Newton described universal gravitation and the laws of motion central to classical mechanics and showed that the motions of objects on Earth and beyond are subject to the same laws. Newton invented the reflecting telescope and developed a theory of color that was based on the fact that a prism splits white light into a visible spectrum. He also studied cooling and the speed of sound and proposed a theory of the origin of stars. Much of his later life was spent in London, including brief spells as member of Parliament and longer periods as master of the Mint and president of the Royal Society. He was knighted in 1705. Although undoubtedly one of the greatest mathematical and scientific geniuses of all time, Newton was also outstandingly contradictory, secretive, and quarrelsome.

Joseph Raphson (1648-1715) was an English or possibly Irish mathematician. No exact dates are known for his birth or death years. He appears to have been largely self-taught and was awarded a degree by the University of Cambridge after the publication of his most notable work, Analysis Aequationum Universalis (1690), and his election as a fellow of the Royal Society.

Thomas Simpson (1710-1761) was born in Market Bosworth, Leicestershire, England. Although he lacked formal education, he managed to teach himself mathematics. Simpson moved to London and worked as a teacher in London coffee houses (as did De Moivre) and then at the Royal Military Academy at Woolwich. He published texts on calculus, astronomy, and probability. His legacy includes work on interpolation and numerical methods of integration; namely Simpson's Rule, which Simpson learned from Newton. Simpson was also a fellow of the Royal Society.

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## Also see

[M-5] moptimize( ) - Model optimization
[M-5] Quadrature() - Numerical integration
[M-4] Mathematical - Important mathematical functions
[M-4] Statistical - Statistical functions
[R] set iter - Control iteration settings

## Title

[M-5] panelsetup() — Panel-data processing

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

These functions assist with the processing of panel data. The idea is to make it easy and fast to write loops like

```
for (i=1; i<=number_of_panels; i++) {
    X = matrix corresponding to panel i
    ... (calculations using X) ...
}
```

Using these functions, this loop could become

```
st_view(Vid, ., "idvar", "touse")
st_view(V, ., ("x1", "x2"), "touse")
info = panelsetup(Vid, 1)
for (i=1; i<=rows(info); i++) {
    X = panelsubmatrix(V, i, info)
    ... (calculations using X) ...
    ...
}
```

panelsetup ( $V$, idcol, ...) sets up panel processing. It returns a matrix (info) that is passed to other panel-processing functions.
panelstats (info) returns a row vector containing the number of panels, number of observations, minimum number of observations per panel, and maximum number of observations per panel. panelsubmatrix ( $V, i$, info) returns a matrix containing the contents of $V$ for panel $i$.
panelsubview ( $S V, V, i$, info) does nearly the same thing. Rather than returning a matrix, however, it places the matrix in $S V$. If $V$ is a view, then the matrix placed in $S V$ will be a view.

## Syntax

```
info = panelsetup ( V,idcol)
info = panelsetup (V,idcol, minobs)
info = panelsetup (V,idcol, minobs, maxobs)
```

real rowvector panelstats (info)
real matrix panelsubmatrix ( $V, i$, info $)$
void
panelsubview ( $S V, V, i$, info)
where
$V: \quad$ real or string matrix, possibly a view
idcol: real scalar
minobs: real scalar
maxobs: real scalar
info: real matrix
$i$ : real scalar
SV: matrix to be created, possibly as view

## Remarks and examples

Remarks are presented under the following headings:

```
Definition of panel data
Definition of problem
Preparation
Use of panelsetup()
Using panelstats()
Using panelsubmatrix()
Using panelsubview()
```


## Definition of panel data

Panel data include multiple observations on subjects, countries, etc.:

| Subject ID | Time ID | x 1 | x 2 |
| :--- | :---: | :---: | :---: |
| 1 | 1 | 4.2 | 3.7 |
| 1 | 2 | 3.2 | 3.7 |
| 1 | 3 | 9.2 | 4.2 |
| 2 | 1 | 1.7 | 4.0 |
| 2 | 2 | 1.9 | 5.0 |
| 3 | 1 | 9.5 | 1.3 |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |

In the above dataset, there are three observations for subject 1 , two for subject 2 , etc. We labeled the identifier within subject to be time, but that is only suggestive, and in any case, the secondary identifier will play no role in what follows.

If we speak about the first panel, we are discussing the first 3 observations of this dataset. If we speak about the second, that corresponds to observations 4 and 5.

It is common to refer to panel numbers with the letter $i$. It is common to refer to the number of observations in the $i$ th panel as $T_{i}$ even when the data within panel have nothing to do with repeated observations over time.

## Definition of problem

We want to calculate some statistic on panel data. The calculation amounts to

$$
\sum_{i=1}^{K} f\left(X_{i}\right)
$$

where the sum is performed across panels, and $X_{i}$ is the data matrix for panel $i$. For instance, given the example in the previous section

$$
X_{1}=\left[\begin{array}{ll}
4.2 & 3.7 \\
3.2 & 3.7 \\
9.2 & 4.2
\end{array}\right]
$$

and $X_{2}$ is a similarly constructed $2 \times 2$ matrix.
Depending on the nature of the calculation, there will be problems for which

1. we want to use all the panels,
2. we want to use only panels for which there are two or more observations, and
3. we want to use the same number of observations in all the panels (balanced panels).

In addition to simple problems of the sort,

$$
\sum_{i=1}^{K} f\left(X_{i}\right)
$$

you may also need to deal with problems of the form,

$$
\sum_{i=1}^{K} f\left(X_{i}, Y_{i}, \ldots\right)
$$

That is, you may need to deal with problems where there are multiple matrices per subject.
We use the sum operator purely for illustration, although it is the most common. Your problem might be

$$
F\left(X_{1}, Y_{1}, \ldots, X_{2}, Y_{2}, \ldots\right)
$$

## Preparation

Before using the functions documented here, create a matrix or matrices containing the data. For illustration, it will be sufficient to create $V$ containing all the data in our example problem:

$$
V=\left[\begin{array}{cccc}
1 & 1 & 4.2 & 3.7 \\
1 & 2 & 3.2 & 3.7 \\
1 & 3 & 9.2 & 4.2 \\
2 & 1 & 1.7 & 4.0 \\
2 & 2 & 1.9 & 5.0 \\
3 & 1 & 9.5 & 1.3 \\
\vdots & \vdots & \vdots & \vdots
\end{array}\right]
$$

But you will probably find it more convenient (and we recommend) if you create at least two matrices, one containing the subject identifier and the other containing the $x$ variables (and omit the within-subject "time" identifier altogether):

$$
V 1=\left[\begin{array}{c}
1 \\
1 \\
1 \\
2 \\
2 \\
3 \\
\vdots
\end{array}\right] \quad V 2=\left[\begin{array}{cc}
4.2 & 3.7 \\
3.2 & 3.7 \\
9.2 & 4.2 \\
1.7 & 4.0 \\
1.9 & 5.0 \\
9.5 & 1.3 \\
\vdots & \vdots
\end{array}\right]
$$

In the above, matrix $V 1$ contains the subject identifier, and matrix $V 2$ contains the data for all the $X_{i}$ matrices in

$$
\sum_{i=1}^{K} f\left(X_{i}\right)
$$

If your calculation is

$$
\sum_{i=1}^{K} f\left(X_{i}, Y_{i}, \ldots\right)
$$

create additional $V$ matrices, $V 3$ corresponding to $Y_{i}$, and so on.
To create these matrices, use [M-5] st_view()

```
st_view(V1, ., "idvar", "touse")
st_view(V2, ., ("x1", "x2"), "touse")
```

although you could use [M-5] st_data() if you preferred. Using st_view() will save memory. You can also construct $V 1, V 2, \ldots$, however you wish; they are just matrices. Be sure that the matrices align, for example, that row 4 of one matrix corresponds to row 4 of another. We did that above by assuming a touse variable had been included (or constructed) in the dataset.

## Use of panelsetup()

panelsetup ( $V$, idcol, ...) sets up panel processing, returning a $K \times 2$ matrix that contains a row for each panel. The row records the first and last observation numbers (row numbers in $V$ ) that correspond to the panel.

For instance, with our example, panelsetup() will return

$$
\left[\begin{array}{cc}
1 & 3 \\
4 & 5 \\
6 & 9 \\
\vdots & \vdots
\end{array}\right]
$$

The first panel is recorded in observations 1 to 3 ; it contains $3-1+1=3$ observations. The second panel is recorded in observations 4 to 5 and it contains $5-4+1=2$ observations, and so on. We recorded the third panel as being observations 6 to 9 , although we did not show you enough of the original data for you to know that 9 was the last observation with ID 3.
panelsetup() has many more capabilities in constructing this result, but it is important to appreciate that returning this observation-number matrix is all that panelsetup() does. This matrix is all that other panel functions need to know. They work with the information produced by panelsetup(), but they will equally well work with any two-column matrix that contains observation numbers. Correspondingly, panelsetup() engages in no secret behavior that ties up memory, puts you in a mode, or anything else. panelsetup() merely produces this matrix.

The number of rows of the matrix panelsetup() returns equals $K$, the number of panels.
The syntax of panelsetup() is

$$
\text { info }=\text { panelsetup }(V, \text { idcol }, \text { minobs }, \text { maxobs })
$$

The last two arguments are optional.
The required argument $V$ specifies a matrix containing at least the panel identification numbers and required argument idcol specifies the column of $V$ that contains that ID. Here we will use the matrix $V 1$, which contains only the identification number:

```
info = panelsetup(Vl, 1)
```

The two optional arguments are minobs and maxobs. minobs specifies the minimum number of observations within panel that we are willing to tolerate; if a panel has fewer observations, we want to omit it entirely. For instance, were we to specify

$$
\text { info }=\text { panelsetup }(V 1,1,3)
$$

then the matrix panelsetup() would contain fewer rows. In our example, the returned info matrix would contain

$$
\left[\begin{array}{cc}
1 & 3 \\
6 & 9 \\
\vdots & \vdots
\end{array}\right]
$$

Observations 4 and 5 are now omitted because they correspond to a two-observation panel, and we said only panels with three or more observations should be included.

We chose three as a demonstration. In fact, it is most common to code

$$
\text { info }=\text { panelsetup }(V 1,1,2)
$$

because that eliminates the singletons (panels with one observation).
The final optional argument is maxobs. For example,

```
info = panelsetup(Vl, 1, 2, 5)
```

means to include only up to five observations per panel. Any observations beyond five are to be trimmed. If we code

```
info = panelsetup(V1, 1, 3, 3)
```

then all the panels contained in info would have three observations. If a panel had fewer than three observations, it would be omitted entirely. If a panel had more than three observations, only the first three would be included.

Panel datasets with the same number of observations per panel are said to be balanced. panelsetup() also provides panel-balancing capabilities. If you specify maxobs as 0 , then

1. panelsetup() first calculates the $\min \left(T_{i}\right)$ among the panels with minobs observations or more. Call that number $m$.
2. panelsetup () then returns panelsetup ( $V 1, i d c o l, m, m$ ), thus creating balanced panels of size $m$ and producing a dataset that has the maximum number of within-panel observations given it has the maximum number of panels.

If we coded

```
info = panelsetup(V1, 1, 2, 0)
```

then panelsetup() would create the maximum number of panels with the maximum number of within-panel observations subject to the constraint of no singletons and the panels being balanced.

## Using panelstats()

panelstats (info) can be used on any two-column matrix that contains observation numbers. panelstats() returns a row vector containing

```
panelstats()[1] = number of panels (same as rows(info))
panelstats() [2] = number of observations
panelstats()[3] = min( (Ti)
panelstats()[4] = max( (Ti)
```


## Using panelsubmatrix()

Having created an info matrix using panelsetup (), you can obtain the matrix corresponding to the $i$ th panel using

```
X = panelsubmatrix( }V,i,info
```

It is not necessary that panelsubmatrix() be used with the same matrix that was used to produce info. We created matrix $V 1$ containing the ID numbers, and we created matrix $V 2$ containing the $x$ variables

```
st_view(V1, ., "idvar", "touse")
st_view(V2, ., ("x1", "x2"), "touse")
```

and we create info using V1:

```
info = panelsetup(V1, 1)
```

We can now create the corresponding $X$ matrix by coding

$$
X=\text { panelsubmatrix }(V 2, i, \text { info })
$$

and, had we created a $V 3$ matrix corresponding to $Y_{i}$, we could also code

$$
Y=\text { panelsubmatrix }(V 3, i, \text { info })
$$

and so on.

## Using panelsubview()

panelsubview() works much like panelsubmatrix(). The difference is that rather than coding

```
X = panelsubmatrix(V, i, info)
```

you code

```
panelsubview(X, V, i, info)
```

The matrix to be defined becomes the first argument of panelsubview (). That is because panelsubview () is designed especially to work with views. panelsubmatrix() will work with views, but panelsubview() does something special. Rather than returning an ordinary matrix (an array, in the jargon), if $V$ is a view, panelsubview () returns a view in its first argument. Views save memory.

Views can save much memory, so it would seem that you would always want to use panelsubview() in place of panelsubmatrix(). What is not always appreciated, however, is that it takes Mata longer to access the data recorded in views, and so there is a tradeoff.

If the panels are likely to be large, you want to use panelsubview(). Conserving memory trumps all other considerations.

In fact, the panels that occur in most datasets are not that large, even when the dataset itself is. If you are going to make many calculations on $X$, you may wish to use panelsubmatrix().

Both panelsubmatrix() and panelsubview() work with view and nonview matrices. panelsubview () produces a regular matrix when the base matrix $V$ is not a view, just as does panelsubmatrix(). The difference is that panelsubview() will produce a view when $V$ is a view, whereas panelsubmatrix() always produces a nonview matrix.

## Conformability

panelsetup ( $V$, idcol, minobs, maxobs):
$V: \quad r \times c$
idcol: $\quad 1 \times 1$
minobs: $1 \times 1 \quad$ (optional)
maxobs: $\quad 1 \times 1 \quad$ (optional)
result: $\quad K \times 2, \quad K=$ number of panels
panelstats (info):
info: $\quad K \times 2$
result: $1 \times 4$
panelsubmatrix ( $V, i$, info):

$$
\begin{aligned}
V: & r \times c \\
i: & 1 \times 1, \quad 1 \leq i \leq \operatorname{rows}(\text { info }) \\
\text { info: } & K \times 2 \\
\text { result: } & t \times c, \quad t=\text { number of obs. in panel }
\end{aligned}
$$

panelsubview ( $S V, V, i$, info):
input:

$$
\begin{aligned}
S V: & \text { irrelevant } \\
V: & r \times c \\
i: & 1 \times 1, \quad 1 \leq i \leq \text { rows (info) } \\
\text { info: } & K \times 2 \\
\text { result: } & t \times c, \quad t=\text { number of obs. in panel }
\end{aligned}
$$

output:
$S V: \quad t \times c, \quad t=$ number of obs. in panel

## Diagnostics

panelsubmatrix ( $V, i$, info) and panelsubview ( $S V, V, i$, info) abort with error if $i<1$ or $i>$ rows(info).
panelsetup() can return a $0 \times 2$ result.

## Also see

[M-4] Utility - Matrix utility functions

## Title

[M-5] panelsum( ) - Panel sums

```
Description Syntax Conformability Diagnostics Also see
```


## Description

panelsum() computes within-panel sums of the columns of $X$ according to the panel information in info.

## Syntax

$$
\text { real matrix }=\text { panelsum }(X,[\text { weights },] \text { info })
$$

where
X: real matrix, possibly a view
weights: real colvector
info: real matrix, from panelsetup()

## Conformability

```
panelsum( \(X\), weights, info):
            \(X: \quad r \times c\)
    weights: \(\quad r \times 1 \quad\) (optional)
        info: \(\quad K \times 2, \quad K=\) number of panels
    result: \(\quad K \times c\)
```


## Diagnostics

panelsum ( $X$, weights, info) aborts with error if $X$ or optional weights is not compatible with the panel information object info, such as deficient rows.

## Also see

[M-4] Utility - Matrix utility functions

## Title

[M-5] pathjoin() — File path manipulation

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

pathjoin(path1, path2) forms, logically speaking, pathl/path2, but does so in the appropriate style. For instance, pathl might be a URL and path2 a Windows dirname $\backslash$ filename, and the two paths will, even so, be joined correctly. All issues of whether pathl ends with a directory separator, path2 begins with one, etc., are handled automatically.
pathsplit (path, pathl, path2) performs the inverse operation, removing the last element of the path (which is typically a filename) and storing it in path2 and storing the rest in path1.
pathbasename (path) returns the last element of path.
pathsuffix (path) returns the file suffix, with leading dot, if there is one, and returns " " otherwise. For instance, pathsuffix("this ${ }^{\text {that.ado") returns ".ado". }}$
pathrmsuffix (path) returns path with the suffix removed, if there was one. For instance, pathrmsuffix("this ${ }^{\text {that.ado") returns "this } \backslash \text { that". }}$
pathisurl (path) returns 1 if path is a URL and 0 otherwise.
pathisabs (path) returns 1 if path is absolute and 0 if relative. $c: \backslash$ this is an absolute path. this $\backslash$ that is a relative path. URLS are considered to be absolute.
pathasciisuffix(path) and pathstatasuffix (path) are more for StataCorp use than anything else. pathasciisuffix () returns 1 if the file is known to be text, based on its file suffix. StataCorp uses this function in Stata's net command to decide whether end-of-line characters, which differ across operating systems, should be modified during downloads. pathstatasuffix() is the function used by Stata's net and update commands to decide whether a file belongs in the official directories. pathstatasuffix("example.ado") is true, but pathstatasuffix("example.do") is false because do-files do not go in system directories.
pathlist (dirlist) returns a row vector, each element of which contains an element of a semicolonseparated path list dirlist. For instance, pathlist (" $a ; b ; c$ ") returns (" $a$ ", " $b ", ~ " c$ ").
pathlist() without arguments returns pathlist(c("adopath")), the broken-out elements of the official Stata ado-path.
pathsubsysdir (pathlist) returns pathlist with any elements that are Stata system directories' shorthands, such as PLUS, PERSONAL, substituted with the actual directory names. For instance, the right way to obtain the official directories over which Stata searches for files is pathsubsysdir (pathlist()).
pathsearchlist ( $f n$ ) returns a row vector. The elements are full paths/filenames specifying all the locations, in order, where Stata would look for $f n$ along the official Stata ado-path.
pathresolve(basepath, path) returns a resolved path of path against basepath. For example, pathresolve("c:/test", "../test1") returns "c:/test1".
pathgetparent (path) returns the parent path of path. For example, pathgetparent("c:/test/test.do") returns "c:/test". Neither $\mathrm{c}: / \mathrm{test}$ nor $\mathrm{c}: /$ test/test. do need exist.

## Syntax

| string scalar void | pathjoin(string scalar path1, string scalar path2) pathsplit(string scalar path, path1, path2) |
| :---: | :---: |
| string scalar | pathbasename (string scalar path) |
| string scalar | pathsuffix (string scalar path) |
| string scalar | pathrmsuffix(string scalar path) |
| real scalar | pathisurl (string scalar path) |
| real scalar | pathisabs (string scalar path) |
| real scalar | pathasciisuffix(string scalar path) |
| real scalar | pathstatasuffix(string scalar path) |
| string rowvector | pathlist(string scalar dirlist) |
| string rowvector | pathlist() |
| string rowvector | pathsubsysdir (string rowvector pathlist) |
| string rowvector | pathsearchlist (string scalar fn) |
| string scalar | pathresolve(string scalar basepath, path) |
| string scalar | pathgetparent (string scalar path) |

## Remarks and examples

Using these functions, you are more likely to produce code that works correctly regardless of operating system.
pathremove() returns an error code if you do not have the necessary permission to remove the directory or its contents.
pathresolve(basepath, path) simply returns path if it is an absolute path.
pathresolve (basepath, path)'s behavior is undefined if basepath has no root element, for example, . ./.

## Conformability

```
pathjoin(pathl, path2):
path1: }1\times
path2: }1\times
result: }1\times
```

pathsplit(path, pathl, path2): input:
path: $\quad 1 \times 1$
output:
pathl: $\quad 1 \times 1$
path2: $1 \times 1$
pathbasename(path), pathsuffix(path), pathrmsuffix(path):
path: $\quad 1 \times 1$
result: $\quad 1 \times 1$
pathisurl(path), pathisabs(path), pathasciisuffix(path), pathstatasuffix(path):
path: $\quad 1 \times 1$
result: $\quad 1 \times 1$
pathlist(dirlist):
$\begin{array}{rll}\text { dirlist: } & 1 \times 1 & \text { (optional) } \\ \text { result: } & 1 \times k & \end{array}$
pathsubsysdir(pathlist):
pathlist: $\quad 1 \times k$
result: $\quad 1 \times k$
pathsearchlist(fn):

| fn: |  |
| ---: | :--- |
| result: |  |
|  | $1 \times 1 \times k$ |

pathresolve(basepath, path):

| basepath: |  |
| ---: | :--- |
| path: | $1 \times 1$ |
| result: |  |
| $1 \times 1$ |  |

pathgetparent (path):

| path: |  |
| ---: | :--- |
| result: |  |
| $1 \times 1 \times 1$ |  |

## Diagnostics

All routines abort with error if the path is too long for the operating system.

## Also see

[M-4] IO - I/O functions

## Title

[M-5] $\operatorname{Pdf}^{\star}()$ — Create a PDF file

Description Syntax Remarks and examples Also see

## Description

The Pdf*() classes are used to programmatically create a PDF file. The PdfDocument class creates the overall file and is required. The other classes are PdfParagraph, which adds a paragraph to a PdfDocument; PdfText, which adds customized text; and PdfTable, which adds a table.

## Syntax

The syntax diagrams describe a set of Mata classes for creating a PDF file. For help with class programming in Mata, see [M-2] class.

Syntax is presented under the following headings:

```
PdfDocument
PdfParagraph
PdfText
PdfTable
```


## PdfDocument

The PdfDocument class is the foundation for creating a PDF file. Using the PdfDocument class is mandatory, but the remaining classes are not.

```
PdfDocument()
void save(string scalar filename)
void close()
void setPageSize(string scalar sz)
void setLandscape(real scalar landscape)
void setMargins(real scalar left, real scalar top,real scalar right, real scalar bottom)
void setHAlignment(string scalar a)
void setLineSpace(real scalar sz)
void setBgColor(real scalar r, real scalar g, real scalar b)
void setColor(real scalar r, real scalar g, real scalar b)
void setFont(string scalar fontname [, string scalar style])
void setFontSize(real scalar sz)
void addImage(string scalar filename [, real scalar cx, real scalar cy])
```

```
void addParagraph(class PdfParagraph scalar p)
void addTable(class PdfTable scalar t)
void addNewPage()
void addLineBreak()
```


## PdfParagraph

The PdfParagraph class can be used to add a paragraph to a PdfDocument or PdfTable.

```
PdfParagraph()
void addString(string scalar s)
void addText(class PdfText scalar t)
void addLineBreak()
void clearContent()
void setFirstIndent(real scalar sz)
void setLeftIndent(real scalar sz)
void setRightIndent(real scalar sz)
void setTopSpacing(real scalar sz)
void setBottomSpacing(real scalar sz)
void setBgColor(real scalar r, real scalar g, real scalar b)
void setColor(real scalar r, real scalar g, real scalar b)
void setFont(string scalar fontname [, string scalar style])
void setFontSize(real scalar sz)
void setUnderline()
void setStrikethru()
void setHAlignment(string scalar a)
void setVAlignment(string scalar a)
void setLineSpace(real scalar sz)
```


## PdfText

The PdfText class can be used to add customized text to a PdfParagraph.

```
PdfText()
void setBgColor(real scalar r, real scalar g, real scalar b)
void setColor(real scalar r, real scalar g, real scalar b)
void setFont(string scalar fontname [, string scalar style])
void setFontSize(real scalar sz)
void setUnderline()
void setStrikethru()
void setSuperscript()
void setSubscript()
void addString(string scalar s)
void clearContent()
```


## PdfTable

The PdfTable class can be used to add a table to a PdfDocument.

```
PdfTable()
void init(real scalar rows, real scalar cols)
void setTotalWidth(real scalar sz)
void setColumnWidths(real rowvector pct_v)
void setWidthPercent(real scalar pct)
void setIndentation(real scalar sz)
void setHAlignment(string scalar a)
void setBorderWidth(real scalar sz [, string scalar bordername])
void setBorderColor(real scalar r, real scalar g, real scalar b
                                    [, string scalar bordername])
void setTopSpacing(real scalar sz)
void setBottomSpacing(real scalar sz)
void setCellContentString(real scalar i, real scalar j, string scalar s)
void setCellContentParagraph(real scalar i, real scalar j, class
                                    PdfParagraph scalar p)
void setCellContentImage(real scalar i, real scalar j, string scalar filename)
```

setCellContentTable(real scalar $i$, real scalar $j$, class PdfTable scalar tbl)
void setCellHAlignment (real scalar $i$, real scalar $j$, string scalar a)
void setCellVAlignment (real scalar $i$, real scalar $j$, string scalar a)
void setCellBgColor (real scalar $i$, real scalar $j$, real scalar $r$, real scalar $g$, real scalar b)
void setCellBorderWidth (real scalar $i$, real scalar $j$, real scalar $s z$ [, string scalar bordername])
void setCellBorderColor(real scalar $i$, real scalar $j$, real scalar $r$, real scalar $g$, real scalar $b$ [, string scalar bordername])
void setCellMargin(real scalar $i$, real scalar $j$, real scalar sz [, string scalar marginname $]$ )
void setCellFont(real scalar $i$, real scalar $j$, string scalar fontname [, string scalar style])
void setCellFontSize(real scalar $i$, real scalar $j$, real scalar size)
void setCellColor (real scalar $i$, real scalar $j$, real scalar $r$, real scalar $g$, real scalar $b$ )
void setCellSpan(real scalar $i$, real scalar $j$, real scalar rowcount, real scalar colcount)
void setCellRowSpan(real scalar $i$, real scalar $j$, real scalar count)
void setCellColSpan(real scalar $i$, real scalar $j$, real scalar count)
void setRowSplit(real scalar $i$, real scalar split)
void addRow (real scalar $i$ )
void delRow (real scalar $i$ )
void addColumn (real scalar $j$ )
void delColumn (real scalar $j$ )
void fillStataMatrix (string scalar name, real scalar colnames, real scalar rownames $[$, string scalar fimt $]$ )
void fillMataMatrix (real matrix name [, real scalar $i$, real scalar $j$, string scalar fmt])
void fillData(real matrix $i$, rowvector $j$, real scalar vnames, real scalar obsno [, scalar selectvar])

## Remarks and examples

Remarks are presented under the following headings:
PdfDocument class details
PdfParagraph class details
PdfText class details
PdfTable class details
Error codes
Examples

## PdfDocument class details

The PdfDocument class is the foundation for creating a PDF file. Using the PdfDocument class is required to create a PDF file. Unless otherwise noted, all sizes are measured in points where 1 point $=$ $1 / 72$ inch.

PdfDocument () is the constructor for the PdfDocument class.
save(filename) performs the final rendering of the PDF file and saves it to disk.
close() closes the PdfDocument() and releases associated memory.
setPageSize ( $s z$ ) sets the page size of the PDF file. The possible values for size are Letter, Legal, A3, A4, A5, B4, and B5.
setLandscape (landscape) sets the page orientation of the PDF file to landscape or portrait. By default, the orientation is portrait. If landscape is 1, the orientation is set to landscape.
setMargins (left, top, right, bottom) sets the page margins left, top, right, and bottom. The margins are used to define the canvas where all text, images, tables, etc., are drawn using absolute coordinates from (left, top). The canvas height is equal to the \{page height - (top + bottom $)\}$, and the canvas width is equal to \{page width $-($ left + right $)$ \}.
setHAlignment (a) sets the horizontal page alignment. The possible alignment values are left, right, center, justified, and stretch. The default is left alignment.
setLineSpace ( $s z$ ) sets the distance between lines of text for the document.
setBgColor $(r, g, b)$ sets the background color of the document with the specified RGB values in the range $[0,255]$. Note that only integer values can be used.
setColor $(r, g, b)$ sets the text color of the document using the specified RGB values in the range $[0,255]$. Note that only integer values can be used.
setFont (fontname $[$, style $]$ ) sets the font for the document. Optionally, the font style may be specified. The possible values for the style are Regular, Bold, Italic, and Bold Italic. If style is not specified, Regular is the default.
setFontSize ( $s z$ ) sets the text size in points for the document.
addImage (file $[, c x, c y]$ ) adds an image to the document. The filename of the image can be either an absolute path or a relative path from the current working directory. $c x$ and $c y$ specify the width and height of the image, which are measured in points.

If $c x$ is not specified, the width of the image is determined by the image information and the canvas width of the document. If $c x$ is larger than the canvas width, the canvas width is used.

If $c y$ is not specified, the height of the image is determined by the width and the aspect ratio of the image.

The supported image types are .jpeg and .png.
addParagraph (class PdfParagraph $p$ ) adds a new paragraph to the document.
addTable (class PdfTable $t$ ) adds a new table to the document.
addNewPage () adds a page break to the document and moves the current position to the top left corner of the new page.
addLineBreak() adds a line break to the document and moves the current position to the next line.

## PdfParagraph class details

The PdfParagraph class can be used to add a paragraph to a PdfDocument. PdfParagraph inherits basic attributes from PdfDocument such as the background color, text color, and font. Unless otherwise noted, all sizes are measured in points where 1 point $=1 / 72$ inch.

PdfParagraph() is the constructor for a PdfParagraph.
addString ( $s$ ) appends a string to the end of the paragraph.
addText (class PdfText $t$ ) appends a PdfText object to the end of the paragraph.
addLineBreak() adds a line break to the end of the paragraph. This moves the current position to the beginning of the next line.
clearContent () clears the content from the paragraph. This is useful if the paragraph object has already been added to the document and you intend to reuse its attributes with new content.
setFirstIndent ( $s z$ ) sets the indentation for the first line of the paragraph.
setLeftIndent ( $s z$ ) sets the left indentation of the paragraph, which applies to all lines.
setRightIndent ( $s z$ ) sets the right indentation of the paragraph, which applies to all lines. If the sum of the left and right indentations is greater than the canvas width, the left indentation will dominate, and the right indentation will be overridden and set to 0 .
setTopSpacing ( $s z$ ) sets the space above the paragraph.
setBottomSpacing ( $s z$ ) sets the space below the paragraph.
setBgColor $(r, g, b)$ sets the background color of the paragraph with the specified RGB values in the range [0,255]. Note that only integer values can be used.
setColor $(r, g, b)$ sets the text color of the paragraph with the specified RGB values in the range [0,255]. Note that only integer values can be used.
setFont (fontname [, style]) sets the font for the paragraph. Optionally, the font style may be specified. The possible values for the style are Regular, Bold, Italic, and Bold Italic.
setFontSize ( $s z$ ) sets the text size in points for the paragraph.
setUnderline() underlines the text in the entire paragraph.
setStrikethru() strikes through the text in the entire paragraph.
setHAlignment ( $a$ ) sets the horizontal alignment of the paragraph. The possible alignment values are left, right, center, justified, and stretch.
setVAlignment ( $a$ ) sets the vertical alignment of the paragraph. The possible alignment values are top, center, baseline, and bottom. This setting has a visible effect only when the paragraph contains characters with varying fonts.
setLineSpace ( $s z$ ) sets the distance between lines of text for the paragraph.

## PdfText class details

The PdfText class can be used to add customized text to a paragraph. The PdfText class can set properties such as the background color, text color, font, etc. Unless otherwise noted, all sizes are measured in points where 1 point $=1 / 72$ inch.

PdfText() is the constructor for a PdfText object.
setBgColor $(r, g, b)$ sets the background color of the text with the specified RGB values in the range $[0,255]$. Note that only integer values can be used.
setColor $(r, g, b)$ sets the color of the text with the specified RGB values in the range [ 0,255 ]. Note that only integer values can be used.
setFont (fontname [, style]) sets the font for the text. Optionally, the font style may be specified. The possible values for the style are Regular, Bold, Italic, and Bold Italic.
setFontSize ( $s z$ ) sets the text size in points for the text.
setUnderline() underlines the text.
setStrikethru() strikes through the text.
setSuperscript() sets the text to be a superscript.
setSubscript() sets the text to be a subscript.
addString ( $s$ ) adds the string to the text.
clearContent () clears the text content. This is useful if the text object has already been added to the paragraph and you intend to reuse its attributes with new content.

## PdfTable class details

The PdfTable class can be used to add a table to a PdfDocument. The PdfTable inherits basic attributes from the PdfDocument such as the background color, text color, and font. A PdfTable may contain up to 65,535 rows and up to 50 columns. Unless otherwise noted, all sizes are measured in points where 1 point $=1 / 72$ inch.

PdfTable() is the constructor for a PdfTable.
init (rows, cols) must be invoked before editing the table or its attributes unless fillStataMatrix(), fillMataMatrix(), or fillData() will be used.
setTotalWidth ( $s z$ ) sets the total width of the table. By default, the width of the table is equal to the canvas width.
setColumnWidths (pct_v) sets the width of each column for the table. By default, the width of each column will be equal. The column widths are specified as a fraction of the total table width. The length of the row vector must equal the number of columns in the table. The sum of the fractions must be 1 .
setWidthPercent ( $p c t$ ) sets the width as a fraction of the canvas that the table will occupy. The value may be a number in the range $(0,1]$.
setIndentation ( $s z$ ) sets the table indentation. Setting the indentation has no visible effect if the table width is equal to the canvas width.
setHAlignment (a) sets the horizontal alignment of the table. The possible alignment values are left, right, and center. left is the default. This setting has a visible effect only when the table width is less than the canvas width.
setBorderWidth (sz [, bordername $]$ ) sets the width of the table border. The possible values of bordername are top, left, bottom, right, insideH, insideV, and all, which identify, respectively, the top border, the left border, the bottom border, the right border, the inside horizontal edges border, the inside vertical edges border, and all borders. If bordername is not specified, all is used.
setBorderColor ( $r, g, b[$, bordername $]$ ) sets the color of the table border using the specified RGB values in the range $[0,255]$. Note that only integer values can be used. The possible values of bordername are top, left, bottom, right, insideH, insideV, and all, which identify, respectively, the top border, the left border, the bottom border, the right border, the inside horizontal edges border, the inside vertical edges border, and all borders. If bordername is not specified, all is used.
setTopSpacing ( $s z$ ) sets the space above the table.
setBottomSpacing ( $s z$ ) sets the space below the table.
setCellContentString $(i, j, s)$ sets the cell content of the $i$ th row and $j$ th column to be text $s$.
setCellContentParagraph ( $i, j$, class PdfParagraph $p$ ) sets the cell content of the $i$ th row and $j$ th column to be a PdfParagraph.
setCellContentImage ( $i, j$, filename) sets the cell content of the $i$ th row and $j$ th column to be the image specified by filename. The width of the image fits the column width, while the height of the image is determined by the width of the image and its aspect ratio.
setCellContentTable ( $i, j$, class PdfTable $t b l$ ) sets the cell content of the $i$ th row and $j$ th column to be a PdfTable.
setCellHAlignment $(i, j, a)$ sets the horizontal alignment for the cell of the $i$ th row and $j$ th column. The possible values for $a$ are left, right, center, justified, and stretch. left is the default.
setCellVAlignment $(i, j, a)$ sets the vertical alignment for the cell of the $i$ th row and $j$ th column. The possible values for $a$ are top, middle, and bottom. top is the default.
setCellBgColor $(i, j, r, g, b)$ sets the background color of the cell of the $i$ th row and $j$ th column to be the specified RGB values in the range [0,255]. Note that only integer values can be used.
setCellBorderWidth $(i, j, s z[$, bordername $])$ sets the width of the specified border of the cell on the $i$ th row and $j$ th column. The possible values of bordername are top, left, bottom, right, and all, which identify, respectively, the top border, the left border, the bottom border, the right border, and all the borders. If $s z$ equals 0 , the border of the cell is not shown. If bordername is not specified, all is the default.
setCellBorderColor $(i, j, r, g, b[$, bordername $]$ ) sets the color of the specified border of the cell on the $i$ th row and $j$ th column to be the specified RGB values in the range [ 0,255 ]. The possible values of bordername are top, left, bottom, right, and all, which identify, respectively, the top border, the left border, the bottom border, the right border, and all the borders. If bordername is not specified, all is the default.
$\operatorname{setCellMargin}(i, j, s z[$, marginname $])$ sets the content margin of the cell of the $i$ th row and $j$ th column. The possible values of marginname are top, left, bottom, right, and all, which identify, respectively, the top margin, the left margin, the bottom margin, the right margin, and all the margins. If marginname is not specified, all is the default.
setCellFont ( $i, j$, fontname $[$, style $]$ ) sets the font for the cell of the $i$ th row and $j$ th column. Optionally, the font style may be specified. The possible values for the style are Regular, Bold, Italic, and Bold Italic. If style is not specified, Regular is the default.
setCellFontSize $(i, j$, size $)$ sets the text size in points for the cell of the $i$ th row and $j$ th column.
setCellColor $(i, j, r, g, b)$ sets the text color for the cell of the $i$ th row and $j$ th column using the specified RGB values in the range [0,255]. Note that only integer values can be used.
setCellSpan( $i, j$, rowcount, colcount) sets the cell of the $j$ th column of the $i$ th row to span rowcount cells vertically downward and span colcount cells horizontally to the right. This is equivalent to merging all the cells in the spanning range into the original cell $(i, j)$. Any content in the spanning range other than the original cell $(i, j)$ will be discarded.
setCellRowSpan ( $i, j$, count ) sets the cell of the $j$ th column of the $i$ th row to span count cells vertically downward. This is equivalent to merging all the cells in the vertical spanning range into the original cell $(i, j)$. Any content in the spanning range other than the original cell $(i, j)$ will be discarded. This function is equivalent to $\operatorname{setCellSpan}(i, j$, count, 1$)$.
setCellColSpan ( $i, j$, count ) sets the cell of the $j$ th column of the $i$ th row to span count cells horizontally to the right. This is equivalent to merging all the cells in the horizontal spanning range into the original cell $(i, j)$. Any content in the spanning range other than the original cell $(i, j)$ will be discarded. This function is equivalent to $\operatorname{setCellSpan}(i, j, 1$, count).
setRowSplit ( $i$, split) sets the $i$ th row to split across multiple pages if split is not 0 . Otherwise, the $i$ th row will be displayed starting from the top of the next page. This setting has no effect if the $i$ th row can be displayed completely on one page.
addRow $(i)$ adds a row to the table after the $i$ th row. The range of $i$ is from 0 to rows, where rows is the number of rows of the table. If $i$ is specified as 0 , the row will be added before the first row. Otherwise, the row will be added after the $i$ th row.
delRow $(i)$ deletes the $i$ th row from the table. The range of $i$ is from 1 to rows, where rows is the number of rows of the table.
addColumn ( $j$ ) adds a column to the table on the right of the $j$ th column. The range of $j$ is from 0 to cols, where cols is the number of columns of the table. If $j$ is specified as 0 , the column will be added to the left of the first column, which is equivalent to adding a new first column. Otherwise, the column will be added to the right of the $j$ th column.
delColumn ( $j$ ) deletes the $j$ th column from the table. The range of $j$ is from 1 to cols, where cols is the number of columns of the table.
fillStataMatrix (name, colnames, rownames [,fmt]) fills the table with a Stata matrix. If colnames is not 0 , the first row of the table is filled with matrix colnames. If rownames is not 0 , the first column of the table is filled with matrix rownames. The elements of the matrix are formatted using firt, if specified. Otherwise, $\% 12.0 \mathrm{~g}$ is used. The matrix is identified by name. If the matrix does not exist, an error code will be returned.
fillMataMatrix (name $[, i, j, f m t]$ ) fills the table with a Mata matrix. The matrix is identified by name. If $i$ is specified, the matrix fills the table starting from the $i$ th row. If $j$ is specified, the matrix fills the table starting from the $j$ th column. The elements of the Mata matrix are formatted using $f m t$, if specified. Otherwise, $\% 12.0 \mathrm{~g}$ is used. If the matrix does not exist, an error code will be returned.
fillData ( $i, j$, vnames, obsno $[$, selectvar $]$ ) fills the table with the current Stata dataset in memory. If a value label is attached to the variable, the data are displayed using the value label. Otherwise, the data are displayed based on the display format. $i, j$, and selectvar are specified in the same way as st_data(). Factor variables and time-series-operated variables are not allowed. If vnames is not 0 , the first row of the table will be filled with variable names. If obsno is not 0 , the first column of the table will be filled with observation numbers.

## Error codes

Functions can abort only if one of the input parameters does not meet the specification; for example, a string scalar is used when a real scalar is required. Functions return a negative error code when there is an error:

Negative
code Meaning
-17100 an error occurred

- $17101 \quad$ PDF file not created
$-17102 \quad$ value is missing
-17103 attribute failed to set
-17104 file not saved
-17105 image not added
-17106 table not added
-17107 paragraph not added
-17108 failed to add new page
-17109 failed to add line break
- $17110 \quad$ PDF file not closed
$-17120 \quad$ table not created
-17121 table failed to set table dimensions
-17122 table not initialized
-17123 cell index out of range
-17124 table failed to set cell value
-17125 table failed to fill Stata matrix
-17126 table failed to fill Mata matrix
- 17127 table failed to fill data
-17130 paragraph not created
- 17131 paragraph failed to add text
-17132 paragraph failed to clear content


## Examples

Examples are presented under the following headings:

> Add paragraph
> Add paragraph with customized text
> Add table (simple example)
> Add table (table with header and footer)
> Add table (table with graph)

## Add paragraph

```
mata:
pdf = PdfDocument()
p = PdfParagraph()
p.addString("This is our first example of a paragraph. ")
p.addString("Let's add another sentence. ")
p.addString("Now we will conclude our first paragraph.")
pdf.addParagraph(p)
p = PdfParagraph()
p.setFirstIndent(36)
p.setFontSize(14)
p.setTopSpacing(10)
p.addString("This is our second paragraph. ")
p.addString("The first line of this paragraph has an indentation of ")
p.addString("36 points or 1/2 inch. The font size of this paragraph ")
p.addString("is 14.")
pdf.addParagraph(p)
p = PdfParagraph()
p.setTopSpacing(10)
p.setFontSize(14)
p.setHAlignment("justified")
p.addString("This is our third paragraph. ")
p.addString("Notice that we have switched back to block mode, which ")
p.addString("means that there is not any indentation. ")
p.addString("You should also notice that this paragraph sets the ")
p.addString("alignment to justified.")
pdf.addParagraph(p)
pdf.save("paragraph1.pdf")
pdf.close()
// clean up
mata drop p
mata drop pdf
end
```


## Add paragraph with customized text

```
mata:
pdf = PdfDocument()
t1 = PdfText()
t1.setSuperscript()
t1.addString("This is superscript text.")
t2 = PdfText()
t2.setSubscript()
t2.addString("This is subscript text.")
p = PdfParagraph()
p.addString("Hello, this is our paragraph's normal text. ")
p.addText(t1)
p.addText(t2)
p.addString("Here is some more text using the attributes from ")
p.addString("the paragraph. ")
t = PdfText()
t.setFont("Courier New")
t.setFontSize(18)
t.setColor(255, 0, 0)
t.addString("Here is an example of text that uses a large ")
t.addString("Courier New font. Its text color is red. ")
p.addText(t)
```

p.addString("We could insert more text here using the paragraph's ")
p.addString("normal attributes. ")
$\mathrm{t}=\operatorname{PdfText}()$
t.setFontSize (30)
t.setColor (0, 0, 255)
t.setStrikethru()
t.addString("Here is one more example of adding modified text ")
t.addString("to a paragraph. ")
p.addText ( t )
p.addString("Now we will conclude this paragraph with normal text.")
pdf.addParagraph (p)
pdf.save("text1.pdf")
pdf.close()
// clean up
mata drop $t$
mata drop t1
mata drop t2
mata drop p
mata drop pdf
end

## Add table (simple example)

mata:
pdf = PdfDocument()
$\mathrm{t}=\mathrm{PdfTable}()$
t.init(5, 4)
$\mathrm{A}=(0.2,0.5,0.15,0.15)$
t.setColumnWidths(A)
for (i=1; i<=5; i++) \{
for $(j=1 ; j<=4 ; j++)$ \{
result $=$ sprintf("This is cell (\%g, \%g)", i, j) ; t.setCellContentString(i, j, result)
\}
\}
pdf.addTable(t)
pdf.save("table1.pdf")
pdf.close()
// clean up
mata drop $t$
mata drop pdf
end

## Add table (table with header and footer)

```
mata:
pdf = PdfDocument()
t = PdfTable()
t.init(5, 4)
A = (0.2,0.5,0.15,0.15)
t.setColumnWidths(A)
t.setCellContentString(1, 1, "This is header")
t.setCellHAlignment(1, 1, "center")
t.setCellColSpan(1, 1, 4)
```

```
for(i=2; i<=4; i++) {
    for(j=1; j<=4; j++) {
        result = sprintf("This is cell(%g, %g)", i, j) ;
        t.setCellContentString(i, j, result)
    }
}
t.setCellContentString(5, 1, "This is footer")
t.setCellHAlignment(5, 1, "center")
t.setCellColSpan(5, 1, 4)
pdf.addTable(t)
pdf.save("table2.pdf")
pdf.close()
// clean up
mata drop t
mata drop pdf
end
```


## Add table (table with graph)

```
sysuse citytemp, clear
tabulate division, matcell(freq) matrow(vlabel)
local tabvar division
histogram 'tabvar', discrete frequency addlabel scheme(sj)
graph export census.png, replace width(2000)
mata:
freq = st_matrix("freq")
vlabel = st_matrix("vlabel")
svlabel = st_vlmap(st_varvaluelabel(st_local("tabvar")), vlabel)
colheader = ("Census Division","Freq.","Percent","Cum.")
nrows = rows(freq)+2
ncols = cols(colheader)
percent = 0
cum = 0
pdf = PdfDocument()
t1 = PdfTable()
t1.init(nrows, ncols)
for(i=1;i<=ncols;i++) {
    t1.setCellContentString(1,i,colheader [1,i])
}
for(i=1;i<=rows(freq);i++) {
    t1.setCellContentString(i+1,1,svlabel[i,1])
        output = sprintf("%9.0g", freq[i,1])
        t1.setCellHAlignment(i+1,2,"right")
        t1.setCellContentString(i+1,2,output)
        percent = freq[i,1]/sum(freq)*100
        output = sprintf("%9.2f", percent)
        t1.setCellHAlignment(i+1,3,"right")
        t1.setCellContentString(i+1,3,output)
        cum = cum + percent
        output = sprintf("%9.2f", cum)
        t1.setCellHAlignment(i+1,4,"right")
        t1.setCellContentString(i+1,4,output)
}
t1.setCellContentString(nrows,1, "Total")
output = sprintf("%9.0g", sum(freq))
t1.setCellHAlignment(nrows, 2,"right")
```

```
t1.setCellContentString(nrows,2,output)
t1.setCellHAlignment(nrows,3,"right")
t1.setCellContentString(nrows,3,"100.00")
t2 = PdfTable()
t2.init(1,2)
t2.setCellContentTable(1,1,t1)
t2.setCellContentImage(1,2,"census.png")
t2.setCellVAlignment(1,2,"bottom")
t2.setBorderWidth(0)
pdf.addTable(t2)
pdf.save("table3.pdf")
pdf.close()
// clean up
mata drop t1
mata drop t2
mata drop pdf
end
```


## Also see

[M-5] _docx*( ) - Generate Office Open XML (.docx) file
[M-5] $\mathbf{x l}()$ - Excel file I/O class
[M-4] IO - I/O functions
[RPT] putdocx intro - Introduction to generating Office Open XML (.docx) files
[RPT] putexcel - Export results to an Excel file
[RPT] putpdf intro - Introduction to generating PDF files

## Title

[M-5] pinv( ) - Moore-Penrose pseudoinverse

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | References | Also see |  |

## Description

$\operatorname{pinv}(A)$ returns the unique Moore-Penrose pseudoinverse of real or complex, symmetric or nonsymmetric, square or nonsquare matrix $A$.
$\operatorname{pinv}(A, r a n k)$ does the same thing, and it returns in rank the rank of $A$.
$\operatorname{pinv}(A, r a n k, t o l)$ does the same thing, and it allows you to specify the tolerance used to determine the rank of $A$, which is also used in the calculation of the pseudoinverse. See [M-5] svsolve() and [M-1] Tolerance for information on the optional tol argument.
$\quad$ _pinv $(A)$ and $\quad$ _pinv $(A, t o l)$ do the same thing as $\operatorname{pinv}()$, except that $A$ is replaced with its inverse and the rank is returned.

## Syntax

```
numeric matrix pinv(numeric matrix A)
numeric matrix pinv(numeric matrix A, rank)
numeric matrix pinv(numeric matrix A, rank, real scalar tol)
real scalar _pinv(numeric matrix A)
real scalar _pinv(numeric matrix A, real scalar tol)
```

where the type of rank is irrelevant; the rank of $A$ is returned there.
To obtain a generalized inverse of a symmetric matrix with a different normalization, see [M-5] invsym( ).

## Remarks and examples

The Moore-Penrose pseudoinverse is also known as the Moore-Penrose inverse and as the generalized inverse. Whatever you call it, the pseudoinverse $A^{*}$ of $A$ satisfies four conditions,

$$
\begin{array}{ll}
A\left(A^{*}\right) A & =A \\
\left(A^{*}\right) A\left(A^{*}\right) & =A^{*} \\
\left(A\left(A^{*}\right)\right)^{\prime} & =A\left(A^{*}\right) \\
\left(\left(A^{*}\right) A\right)^{\prime} & =\left(A^{*}\right) A
\end{array}
$$

where the transpose operator ${ }^{\prime}$ is understood to mean the conjugate transpose when $A$ is complex. Also, if $A$ is of full rank, then

$$
A^{*}=A^{-1}
$$

$\operatorname{pinv}(A)$ is logically equivalent to svsolve ( $A$, I (rows ( $A$ ) ) ) ; see [M-5] svsolve() for details and for use of the optional tol argument.

## Conformability

$\operatorname{pinv}(A$, rank, tol):
input:
A: $\quad r \times c$
tol: $1 \times 1 \quad$ (optional)
output:

| rank: |  | $1 \times 1$ | (optional) |
| ---: | :--- | ---: | :--- |
| result: | $c \times r$ |  |  |

_pinv ( $A$, tol):
input:
A: $\quad r \times c$
tol: $1 \times 1 \quad$ (optional)
output:

$$
\begin{array}{rll}
A: & c \times r & \\
\text { result: } & 1 \times 1 & \text { (containing rank) }
\end{array}
$$

## Diagnostics

The inverse returned by these functions is real if $A$ is real and is complex if $A$ is complex. $\operatorname{pinv}(A, r a n k, t o l)$ and _pinv ( $A, t o l$ ) return missing results if $A$ contains missing values. $\operatorname{pinv}()$ and _pinv() also return missing values if the algorithm for computing the SVD, [M-5] svd(), fails to converge. This is a near zero-probability event. Here rank also is returned as missing.

See [M-5] svsolve( ) and [M-1] Tolerance for information on the optional tol argument.

## References

James, I. M. 2002. Remarkable Mathematicians: From Euler to von Neumann. Cambridge: Cambridge University Press.

Moore, E. H. 1920. On the reciprocal of the general algebraic matrix. Bulletin of the American Mathematical Society 26: 394-395.

Penrose, R. 1955. A generalized inverse for matrices. Mathematical Proceedings of the Cambridge Philosophical Society 51: 406-413. https://doi.org/10.1017/S0305004100030401.

Also see<br>[M-5] cholinv( ) - Symmetric, positive-definite matrix inversion<br>[M-5] fullsvd() - Full singular value decomposition<br>[M-5] invsym( ) - Symmetric real matrix inversion<br>[M-5] luinv() - Square matrix inversion<br>[M-5] qrinv( ) - Generalized inverse of matrix via QR decomposition<br>[M-5] svd() - Singular value decomposition<br>[M-4] Matrix - Matrix functions<br>[M-4] Solvers - Functions to solve $A X=B$ and to obtain A inverse

Eliakim Hastings Moore (1862-1932) was born in Marietta, Ohio. He studied mathematics and astronomy at Yale and was awarded a PhD for a thesis on $n$-dimensional geometry. After a year studying in Germany and teaching posts at Northwestern and Yale, he settled at the University of Chicago in 1892. Moore worked on algebra, including fields and groups, the foundations of geometry and the foundations of analysis, algebraic geometry, number theory, and integral equations. He was an inspiring teacher and a great organizer in American mathematics, playing an important part in the early years of the American Mathematical Society.

Roger Penrose (1931- ) was born in Colchester in England. His father was a statistically minded medical geneticist and his mother was a doctor. Penrose studied mathematics at University College London and Cambridge and published an article on generalized matrix inverses in 1955. He taught and researched at several universities in Great Britain and the United States before being appointed Rouse Ball Professor of Mathematics at Oxford in 1973. Penrose is celebrated for research contributions ranging from cosmology and general relativity (including work with Stephen Hawking) to pure mathematics (including results on tilings of the plane) and for semipopular and wide-ranging books making controversial connections between physics, computers, mind, and consciousness. He was knighted in 1994 and jointly awarded the Nobel Prize for Physics in 2020.

## Title

[M-5] polyeval( ) — Manipulate and evaluate polynomials

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

polyeval $(c, x)$ evaluates polynomial $c$ at each value recorded in $x$, returning the results in a p-conformable-with- $x$ vector. For instance, polyeval ( $(4,2,1)$, (3\5)) returns $\left(4+2 * 3+3^{\wedge} 2 \backslash 4+2 * 5+5^{\wedge} 2\right)=(19 \backslash 39)$.
polysolve $(y, x)$ returns the minimal-degree polynomial $c$ fitting $y=\operatorname{polyeval}(c, x)$. Solution is via Lagrange's interpolation formula.
polytrim ( $c$ ) returns polynomial $c$ with trailing zeros removed. For instance, polytrim ( $(1,2,3,0)$ ) returns ( $1,2,3$ ) . polytrim ( $(0,0,0,0)$ ) returns ( 0 ). Thus if $n=$ cols(polytrim $(c)$ ), then $c$ records an $(n-1)$ th degree polynomial.
polyderiv $(c, i)$ returns the polynomial that is the $i$ th derivative of polynomial $c$. For instance, polyderiv $((4,2,1), 1)$ returns $(2,2)$ (the derivative of $4+2 x+x^{2}$ is $2+2 x$ ). The value of the first derivative of polynomial $c$ at $x$ is polyeval(polyderiv $(c, 1), x)$.
polyinteg $(c, i)$ returns the polynomial that is the $i$ th integral of polynomial $c$. For instance, polyinteg $((4,2,1), 1)$ returns $(0,4,1, .3333)$ (the integral of $4+2 x+x^{2}$ is $\left.0+4 x+x^{2}+.3333 x^{3}\right)$. The value of the integral of polynomial $c$ at $x$ is polyeval (polyinteg $(c, 1), x$ ).
polyadd $\left(c_{1}, c_{2}\right)$ returns the polynomial that is the sum of the polynomials $c_{1}$ and $c_{2}$. For instance, polyadd $((2,1),(3,5,1))$ is $(5,6,1)$ (the sum of $2+x$ and $3+5 x+x^{2}$ is $\left.5+6 x+x^{2}\right)$.
polymult $\left(c_{1}, c_{2}\right)$ returns the polynomial that is the product of the polynomials $c_{1}$ and $c_{2}$. For instance, polymult $((2,1),(3,5,1))$ is $(6,13,7,1)$ (the product of $2+x$ and $3+5 x+x^{2}$ is $\left.6+13 x+7 x^{2}+x^{3}\right)$.
polydiv $\left(c_{1}, c_{2}, c_{q}, c_{r}\right)$ calculates polynomial $c_{1} / c_{2}$, storing the quotient polynomial in $c_{q}$ and the remainder polynomial in $c_{r}$. For instance, polydiv $\left((3,5,1),(2,1), c_{q}, c_{r}\right)$ returns $c_{q}=(3,1)$ and $c_{r}=(-3)$; that is,

$$
\frac{3+5 x+x^{2}}{2+x}=3+x \text { with a remainder of }-3
$$

or

$$
3+5 x+x^{2}=(3+x)(2+x)-3
$$

polyroots (c) find the roots of polynomial $c$ and returns them in complex row vector (complex even if $c$ is real). For instance, polyroots $((3,5,1))$ returns $(-4.303+0 i,-.697+0 i)$ (the roots of $3+5 x+x^{2}$ are -4.303 and -.697 ).

## Syntax

```
numeric vector polyeval(numeric rowvector c, numeric vector x)
numeric rowvector polysolve(numeric vector y, numeric vector x)
numeric rowvector polytrim(numeric vector c)
numeric rowvector polyderiv(numeric rowvector c, real scalar i)
numeric rowvector polyinteg(numeric rowvector c, real scalar i)
numeric rowvector polyadd(numeric rowvector c}\mp@subsup{c}{1}{}\mathrm{ , numeric rowvector c c ()
numeric rowvector polymult(numeric rowvector c}\mp@subsup{c}{1}{}\mathrm{ , numeric rowvector c}\mp@subsup{c}{2}{}\mathrm{ )
void
polydiv(numeric rowvector c}\mp@subsup{c}{1}{},\mathrm{ numeric rowvector }\mp@subsup{c}{2}{},\mp@subsup{c}{q}{},\mp@subsup{c}{r}{}\mathrm{ )
complex rowvector polyroots(numeric rowvector c)
```

In the above, row vector $c$ contains the coefficients for a cols $(c)-1$ degree polynomial. For instance,

$$
c=(4,2,1)
$$

records the polynomial

$$
4+2 x+x^{2}
$$

## Remarks and examples

Given the real or complex coefficients $c$ that define an $n-1$ degree polynomial in $x$, polyroots ( $c$ ) returns the $n-1$ roots for which

$$
0=c_{1}+c_{2} x^{1}+c_{3} x^{2}+\cdots+c_{n} x^{n-1}
$$

polyroots ( $c$ ) obtains the roots by calculating the eigenvalues of the companion matrix. The $(n-1) \times(n-1)$ companion matrix for the polynomial defined by $c$ is

$$
C=\left[\begin{array}{ccccc}
-c_{n-1} s & -c_{n-2} s & \cdots & -c_{2} s & -c_{1} s \\
1 & 0 & \cdots & 0 & 0 \\
0 & 1 & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & 1 & 0 \\
0 & 0 & \cdots & 0 & 1
\end{array}\right]
$$

where $s=1 / c_{n}$ if $c$ is real and

$$
s=C\left(\frac{\operatorname{Re}\left(c_{n}\right)}{\operatorname{Re}\left(c_{n}\right)^{2}+\operatorname{Im}\left(c_{n}\right)^{2}}, \frac{-\operatorname{Im}\left(c_{n}\right)}{\operatorname{Re}\left(c_{n}\right)^{2}+\operatorname{Im}\left(c_{n}\right)^{2}}\right)
$$

otherwise.

As in all nonsymmetric eigenvalue problems, the returned roots are complex and sorted from largest to smallest, see [M-5] eigensystem().

## Conformability

```
polyeval(c, x):
```

| $c:$ |  | $1 \times n, n>0$ |  |
| ---: | :--- | :--- | :--- |
| $x:$ | $r \times 1$ | or | $1 \times c$ |
| result: |  | $r \times 1$ | or |
|  |  | $1 \times c$ |  |

polysolve $(y, x)$ :
$y: \quad n \times 1 \quad$ or $\quad 1 \times n, n \geq 1$
$x: \quad n \times 1$ or $1 \times n$
result: $\quad 1 \times k, \quad 1 \leq k \leq n$ polytrim(c):
$c: \quad 1 \times n$
result: $\quad 1 \times k, \quad 1 \leq k \leq n$
polyderiv( $c, i$ ):
$c: \quad 1 \times n, \quad n>0$
$i: \quad 1 \times 1, \quad i$ may be negative
result: $\quad 1 \times \max (1, n-i)$
polyinteg ( $c, i$ ):
$c: \quad 1 \times n, n>0$
$i$ : $\quad 1 \times 1, \quad i$ may be negative
result: $\quad 1 \times \max (1, n+i)$
polyadd $\left(c_{1}, c_{2}\right)$ :

| $c_{1}:$ | $1 \times n_{1}, \quad n_{1}>0$ |
| ---: | :--- |
| $c_{2}:$ | $1 \times n_{2}, \quad n_{2}>0$ |
| result: | $1 \times \max \left(n_{1}, \quad n_{2}\right)$ |

polymult $\left(c_{1}, c_{2}\right)$ :
$\begin{aligned} c_{1}: & 1 \times n_{1}, \quad n_{1}>0 \\ c_{2}: & 1 \times n_{2}, \quad n_{2}>0 \\ \text { result: } & 1 \times n_{1}+n_{2}-1\end{aligned}$
$\operatorname{polydiv}\left(c_{1}, c_{2}, c_{q}, c_{r}\right)$ :
input:

$$
\begin{array}{lll}
c_{1}: & 1 \times n_{1}, & n_{1}>0 \\
c_{2}: & 1 \times n_{2}, & n_{2}>0
\end{array}
$$

output:

$$
\begin{array}{lll}
c_{q}: & 1 \times k_{1}, & 1 \leq k_{1} \leq \max \left(n_{1}-n_{2}+1,1\right) \\
c_{r}: & 1 \times k_{2}, & 1 \leq k_{2} \leq \max \left(n_{1}-n_{2}, 1\right)
\end{array}
$$

polyroots (c):
$c: \quad 1 \times n_{i}, \quad n>0$
result: $\quad 1 \times k-1, \quad k=\operatorname{cols}(\operatorname{polytrim}(c))$

## Diagnostics

All functions abort with an error if a polynomial coefficient row vector is void, but they do not necessarily give indicative error messages as to the problem. Polynomial coefficient row vectors may contain missing values.
polyderiv ( $c, i$ ) returns $c$ when $i=0$. It returns polyinteg $(c,-i)$ when $i<0$. It returns ( 0 ) when $i$ is missing (think of missing as positive infinity).
polyinteg $(c, i)$ returns $c$ when $i=0$. It returns polyderiv $(c,-i)$ when $i<0$. It aborts with error if $i$ is missing (think of missing as positive infinity).
polyroots ( $c$ ) returns a vector of missing values if any element of $c$ equals missing.

## Also see

[M-4] Mathematical - Important mathematical functions

## Title

[M-5] printf() — Format output

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

printf() displays output at the terminal.
sprintf() returns a string that can then be displayed at the terminal, written to a file, or used in any other way a string might be used.

## Syntax

void $\quad$ printf(string scalar fmt $, r 1, r 2, \ldots, r N)$
string scalar $\operatorname{sprintf}($ string scalar fmt $, r 1, r 2, \ldots, r N)$
where fint may contain a mix of text and \%fmts, such as

```
printf("The result is %9.2f, adjusted\n", result)
printf("%s = %9.0g\n", name, value)
```

There must be a one-to-one correspondence between the $\%$ fimts in $f m t$ and the number of results to be displayed.

Along with the usual \% fints that Stata provides (see [D] format), also provided are

| Format | Meaning |
| :--- | :--- |
| $\% \mathrm{f}$ | \%11.0f, compressed |
| $\% \mathrm{~g}$ | $\% 11.0 \mathrm{~g}$, compressed |
| $\% \mathrm{e}$ | \%11.8e, compressed |
| $\% \mathrm{~s}$ | $\%$ \#s, \# = whatever necessary |
| $\% \mathrm{us}$ | $\%$ \#us, \# = whatever necessary |
| \%uds | \%\#uds, \# = whatever necessary |

Compressed means that, after the indicated format is applied, all leading and trailing blanks are removed.

C programmers, be warned: \%d is Stata's (old) calendar date format (equivalent to modern Stata's $\% \mathrm{td}$ format) and not an integer format; use \%f for formatting integers.

The following character sequences are given a special meaning when contained within fmt:

| Character sequence | Meaning |
| :--- | :--- |
| $\% \%$ | one $\%$ |
| $\backslash n$ | newline |
| $\backslash r$ | carriage return |
| $\backslash t$ | tab |
| $\backslash \backslash$ | one $\backslash$ |

## Remarks and examples

Both printf() and sprintf() respect the formatting specified with set dp. To convert numeric values to strings while ignoring the formatting specified with set dp, use strofreal().

Remarks are presented under the following headings:

```
printf()
sprintf()
The %us and %uds formats
```


## printf()

printf() displays output at the terminal. A program might contain the line printf("the result is \%f $\backslash \mathrm{n} "$, result)
and display the output

```
the result is 5.213
```

or it might contain the lines

```
printf("{txt}{space 13}{c |} Coefficient Std. err.\n")
printf("{hline 13}{c +}{hline 24}\n")
printf("{txt}%12s {c |} {res}%10.0g % 10.0g\n",
    varname[i], coef[i], se[i])
```

and so display the output

|  | Coefficient | Std. err. |
| :--- | :---: | ---: |
| mpg | -.0059541 | .0005921 |

Do not forget to include $\backslash \mathrm{n}$ at the end of lines. When $\backslash \mathrm{n}$ is not included, the line continues. For instance, the code

```
printf("{txt}{space 13}{c |} Coefficient Std. err.\n")
printf("{hline 13}{c +}{hline 24}\n")
printf("{txt}%12s {c |} {res}", varname[i])
printf("%10.0g", coef[i])
printf(" ")
printf("%10.0g", se[i])
printf("\n")
```

produces the same output as shown above.

Although users are unaware of it, Stata buffers output. This makes Stata faster. A side effect of the buffering, however, is that output may not appear when you want it to appear. Consider the code fragment

```
for (n=1; !converged(b, b0); n++) {
    printf("iteration %f: diff = % 12.0g\n", n, b-b0)
    bO = b
        ... new calculation of b . . .
}
```

One of the purposes of the iteration output is to keep the user informed that the code is indeed working, yet as the above code is written, the user probably will not see the iteration messages as they occur. Instead, nothing will appear for a while, and then, unexpectedly, many iteration messages will appear as Stata, buffers full, decides to send to the terminal the waiting output.

To force output to be displayed, use [M-5] displayflush():

```
for (n=1; !converged(b, b0); n++) {
    printf("iteration %f: diff = % 12.0g\n", n, b-b0)
    displayflush()
    bO = b
    ... new calculation of b ...
}
```

It is only in situations like the above that use of displayflush() is necessary. In other cases, it is better to let Stata decide when output buffers should be flushed. (Ado-file programmers: you have never had to worry about this because, at the ado-level, all output is flushed as it is created. Mata, however, is designed to be fast and so printf() does not force output to be flushed until it is efficient to do so.)

## sprintf()

The difference between sprintf() and printf() is that, whereas printf() sends the resulting string to the terminal, sprintf() returns it. Since Mata displays the results of expressions that are not assigned to variables, sprintf() used by itself also displays output:

```
: sprintf("the result is %f\n", result)
    the result is 5.2130a
```

The outcome is a little different from that produced by printf() because the output-the-unassignedexpression routine indents results by 2 and displays all the characters in the string (the $0 a$ at the end is the $\backslash \mathrm{n}$ newline character). Also, the output-the-unassigned-expression routine does not honor SMCL, electing instead to display the codes:

```
: sprintf("{txt}the result is {res}%f", result)
    {txt}the result is {res}5.213
```

The purpose of sprintf() is to create strings that will then be used with printf(), with [M-5] display(), with fput() (see [M-5] fopen()), or with some other function.

Pretend that we are creating a dynamically formatted table. One of the columns in the table contains integers, and we want to create a $\% f m t$ that is exactly the width required. That is, if the integers to appear in the table are 2,9 , and 20 , we want to create a $\% 2$. Of format for the column. We assume the integers are in the column vector dof in what follows:

```
max = 0
for (i=1; i<=rows(dof); i++) {
    len = strlen(sprintf("%f", dof[i])
    if (len>max) max = len
}
fmt = sprintf("%%%f.Of", max)
```

We used sprintf () twice in the above. We first used sprintf () to produce the string representation of the integer dof [i], and we used the \%f format so that the length would be whatever was necessary, and no more. We obtained in max the maximum length. If dof contained 2, 9 , and 20, by the end of our loop, max will contain 2 . We finally used sprintf() to create the \%\#. Of format that we wanted: $\% 2.0 f$.

The format string $\% \% \%$ f. Of in the final sprintf () is a little difficult to read. The first two percent signs amount to one real percent sign, so in the output we now have $\%$ and we are left with $\%$ f.0f. The $\% \mathrm{f}$ is a format-it is how we are to format max-and so in the output we now have $\% 2$, and we are left with . Of. . Of is just a string, so the final output is $\% 2.0 f$.

## The \%us and \%uds formats

The $\% w$ us and $\% w$ uds formats are similar to $\% w$ s. These formats display a string in a right-justified field of width $w . \%-w$ us and $\%-w$ uds display the string in a left-justified field. $\% \sim w$ us and $\% \sim w u d s$ display the string center-justified.

The difference between $\% w s, \% w u s$, and $\% w u d s$ is how the number of padding spaces is calculated. $\% w$ s pads the number of spaces to the left of $s$ to make the total number of bytes to be $w . \% w$ us pads the number of spaces to the left of $s$ to make the total number of Unicode characters to be $w . \% w u d s$ pads the number of spaces to the left of $s$ to make the total number of display columns to be $w$.

Note that $s$ is returned without change if the number of Unicode characters is greater than or equal to $w$ in $\% w$ us or if the number of display columns is greater than or equal to $w$ in $\% w$ uds.

## Conformability

```
printf(fmt, r
    fmt: }\quad1\times
    r}\mp@subsup{r}{1}{:
        r}\mp@subsup{r}{2}{}:\quad1\times
        rN: 1 < 1
        result: void
sprintf(fmt, r
    fmt: 
        r}\mp@subsup{r}{1}{:
        r
        rN: 1 < 1
        result: }1\times
```


## Diagnostics

printf() and sprintf () abort with error if a \% fmt is misspecified, if a numeric \% fmt corresponds to a string result or a string $\%$ fint to a numeric result, or there are too few or too many $\%$ fimts in $f m t$ relative to the number of results specified.

## Also see

[M-5] displayas( ) - Set display level
[M-5] displayflush( ) - Flush terminal-output buffer
[M-5] errprintf() - Format output and display as error message
[M-4] IO - I/O functions

## Title

[M-5] $\operatorname{qrd}()$ — QR decomposition

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

$\operatorname{qrd}(A, Q, R)$ calculates the QR decomposition of $A: m \times n$, returning results in $Q$ and $R$.
$\operatorname{hqrd}\left(A, H\right.$, tau,$\left.R_{1}\right)$ calculates the QR decomposition of $A: m \times n$ but, rather than returning $Q$ and $R$, returns the Householder vectors in $H$ and the scale factors tau-from which $Q$ can be formed-and returns in $R_{1}$ an upper-triangular matrix that is a submatrix of $R$ when $m \geq n$ or an upper-trapezoidal matrix when $m<n$; see Remarks and examples below for its definition. Doing this saves calculation and memory, and other routines allow you to manipulate these matrices:

1. hqrdmultq ( $H$, tau, $X$, transpose) returns $Q X$ or $Q^{\prime} X$ on the basis of the $Q$ implied by $H$ and tau. $Q X$ is returned if transpose $=0$, and $Q^{\prime} X$ is returned otherwise.
2. hqrdmultq1t ( $H, \operatorname{tau}, X$ ) returns $Q_{1}^{\prime} X$ on the basis of the $Q 1$ implied by $H$ and tau. This function requires $m \geq n$.
3. hqrdq ( $H, t a u$ ) returns the $Q$ matrix implied by $H$ and tau. This function is rarely used.
4. hqrdq1 $(H, t a u)$ returns the $Q_{1}$ matrix implied by $H$ and tau. This function requires $m \geq$ $n$ and is rarely used.
5. hqrdr ( $H$ ) returns the full $R$ matrix. This function is rarely used. (It may surprise you that hqrdr() is a function of $H$ and not $R_{1} . R_{1}$ also happens to be stored in $H$, and there is other useful information there as well.)
6. hqrdr1 ( $H$ ) returns the $R_{1}$ matrix. This function is rarely used.
7. hqrdmultq ( $H$, tau, $X$, transpose, transform) and hqrdmultq1t ( $H$, tau, $X$, transform) do the same things as hqrdmultq( $H$, tau, $X$, transpose) and hqrdmultq1t( $H$, tau, $X$ ). The difference is that the transform argument allows you to transform the input matrices. transform can be specified as 0 or 1 . When transform is specified as 1 , the functions will transform $H$, tau, and $X$ all to complex matrices if any of these matrices is complex. Not specifying transform is equivalent to specifying transform $=0$; in this case, the storage type of the input matrices does not change.
$\quad \operatorname{hqrd}\left(A, t a u, R_{1}\right)$ does the same thing as $\operatorname{hqrd}\left(A, H, \operatorname{tau}, R_{1}\right)$, except that it overwrites $H$ into $A$ and so conserves even more memory.
$\operatorname{qrdp}(A, Q, R, p)$ is similar to $\operatorname{qrd}(A, Q, R)$ : it returns the QR decomposition of $A$ in $Q$ and $R$. The difference is that this routine allows for pivoting. New argument $p$ specifies whether a column is available for pivoting, and on output, $p$ is overwritten with a permutation vector that records the pivoting actually performed. On input, $p$ can be specified as . (missing)—meaning all columns are available for pivoting-or $p$ can be specified as a $1 \times n$ row vector containing 0 s and 1 s , with 1 meaning the column is fixed and so may not be pivoted.
$\operatorname{hqrdp}\left(A, H, \operatorname{tau}, R_{1}, p\right)$ is a generalization of $\operatorname{hqrd}\left(A, H, t a u, R_{1}\right)$ just as $\operatorname{qrdp}()$ is a generalization of qrd().
_hqrdp $\left(A, \operatorname{tau}, R_{1}, p\right)$ does the same thing as hqrdp ( $A, H$, tau, $R_{1}, p$ ), except that _hqrdp() overwrites $H$ into $A$.
_hqrdp_la() is the interface to the LAPACK routine that performs the QR calculation; it is used by all the above routines. Direct use of _hqrdp_la() is not recommended.

## Syntax

| void | qrd (numeric matrix $A, Q, R$ ) |
| :---: | :---: |
| void | hqrd (numeric matrix $A, H$, tau, $R_{1}$ ) |
| void | hqrd(numeric matrix $A$, tau, $R_{1}$ ) |
| numeric matrix | hqrdmultq(numeric matrix $H$, rowvector tau, numeric matrix $X$, real scalar transpose) |
| numeric matrix | hqrdmultq(numeric matrix $H$, rowvector tau, numeric matrix $X$, real scalar transpose, real scalar transform) |
| numeric matrix | hqrdmultq1t (numeric matrix $H$, rowvector tau, numeric matrix $X$ ) |
| numeric matrix | hqrdmultq1t (numeric matrix $H$, rowvector tau, numeric matrix $X$, real scalar transform) |
| numeric matrix | hqrdq (numeric matrix $H$, numeric matrix tau) |
| numeric matrix | hqrdq1 (numeric matrix $H$, numeric matrix tau) |
| numeric matrix | hqrdr (numeric matrix $H$ ) |
| numeric matrix | hqrdr1 (numeric matrix $H$ ) |
| void | qrdp(numeric matrix $A, Q, R$, real rowvector $p$ ) |
| void | hqrdp (numeric matrix $A, H$, tau, $R_{1}$, real rowvector $p$ ) |
| void | _hqrdp (numeric matrix $A$, tau, $R_{1}$, real rowvector $p$ ) |
| void | -hqrdp_la (numeric matrix $A$, tau, real rowvector $p$ ) |

## Remarks and examples

Remarks are presented under the following headings:

```
QR decomposition
Avoiding calculation of Q
Pivoting
Least-squares solutions with omitted columns
```


## QR decomposition

The decomposition of square or nonsquare matrix $A$ can be written as

$$
\begin{equation*}
A=Q R \tag{1}
\end{equation*}
$$

where $Q$ is an orthogonal matrix ( $Q^{\prime} Q=I$ ) and $R$ is upper triangular or upper trapezoidal. A matrix is an upper-trapezoidal matrix if its nonzero elements are found only in the upper triangle of the matrix, including the main diagonal. $\operatorname{qrd}(A, Q, R)$ will make this calculation:
: A
12

|  | 1 | 4 |
| :--- | :--- | :--- |
| 2 | 8 |  |
| 3 | 4 | 6 |
| 4 | 1 | 0 |
| 4 | 2 | 4 |
| 5 | 2 | 1 |

: $\mathrm{Q}=\mathrm{R}=$.
: $\operatorname{qrd}(A, Q, R)$
Ahat $=\mathrm{Q} * \mathrm{R}$
mreldif(Ahat, A) $2.53765 \mathrm{e}-16$

## Avoiding calculation of $\mathbf{Q}$

In fact, you probably do not want to use qrd(). Calculating the necessary ingredients for $Q$ is not too difficult, but going from those necessary ingredients to form $Q$ is devilish. The necessary ingredients are usually all you need, which are the Householder vectors and their scale factors, known as $H$ and tau. For instance, one can write down a mathematical function $f(H, \operatorname{tau}, X)$ that will calculate $Q X$ or $Q^{\prime} X$ for some matrix $X$.

Also, QR decomposition is often carried out on violently nonsquare matrices $A$ : $m \times n, m \gg n$. We can write

$$
\underset{m \times n}{A}=\left[\begin{array}{cc}
Q_{1} & \underset{m \times n}{Q_{2}} \\
m \times m-n
\end{array}\right]\left[\begin{array}{c}
R_{1} \\
n \times n \\
R_{2} \\
m-n \times n
\end{array}\right]=\underset{m \times n}{Q_{1} R_{1}+\underset{m \times n}{Q_{2} R_{2}}, ~\left(\begin{array}{c} 
\\
m
\end{array}\right]}
$$

$R_{2}$ is zero, and thus

$$
\underset{m \times n}{A}=\left[\begin{array}{cc}
Q_{1} & Q_{2} \\
m \times n & m \times m-n
\end{array}\right]\left[\begin{array}{c}
R_{1} \\
n \times n \\
0 \\
m-n \times n
\end{array}\right]=\underset{m \times n}{Q_{1} R_{1}}
$$

Thus, it is enough to know $Q_{1}$ and $R_{1}$. Rather than defining QR decomposition as

$$
\begin{equation*}
A=Q R \quad Q: m \times m \quad R: m \times n \tag{1}
\end{equation*}
$$

We can better define it as

$$
A=Q_{1} R_{1} \quad Q_{1}: m \times n \quad R_{1}: n \times n
$$

To appreciate the savings, consider the reasonable case where $m=4,000$ and $n=3$,

$$
A=Q R \quad Q: 4,000 \times 4,000 \quad R: 4,000 \times 3
$$

versus

$$
A=Q_{1} R_{1} \quad Q_{1}: 4,000 \times 3 \quad R_{1}: 3 \times 3
$$

Memory consumption is reduced from 125,094 kilobytes to 94 kilobytes, a $99.92 \%$ saving!
Combining the arguments, we need not save $Q$ because $Q_{1}$ is sufficient, we need not calculate $Q_{1}$ because $H$ and tau are sufficient, and we need not store $R$ because $R_{1}$ is sufficient.

That is what hqrd ( $A, H, \operatorname{tau}, R_{1}$ ) does. Having used hqrd(), if you need to multiply the full $Q$ by some matrix $X$, you can use hqrdmultq(). Having used hqrd(), if you need the full $Q$, you can use hqrdq() to obtain it, but by that point, you will be making the devilish calculation you sought to avoid, and so you might as well have used qrd() to begin with. If you want $Q_{1}$, you can use hqrdq1(). Finally, having used hqrd(), if you need $R$ or $R_{1}$, you can use hqrdr() and hqrdr1():
: A

|  | 1 | 1 |
| :--- | :--- | :--- |
| 2 |  |  |
|  | 4 8 <br> 2 4 | 6 |
| 3 | 1 | 0 |
| 4 | 2 | 4 |
| 5 | 2 | 1 |
|  |  |  |

: $\mathrm{H}=\mathrm{tau}=\mathrm{R} 1=$.
hqrd(A, H, tau, R1)
: Ahat = hqrdq1 (H, tau) * R1 // i.e., Q1*R1
mreldif(Ahat, A)
2.53765e-16

Note that $Q_{1}$ is obtained only when $m \geq n$. When $m<n$, we can write

$$
\underset{m \times n}{A}=\underset{m \times m}{Q} \underset{m \times n}{R}
$$

where $R$, which is also called $R_{1}$ for consistency when $m \geq n$, is upper trapezoidal. For example,

```
: B
            1 2 3
1 
Q = R = .
qrd(B, Q, R)
Bhat = Q * R
mreldif(Bhat, B)
4.44089e-16
R
```

| -8.94427191 | -7.155417528 | -.4472135955 |
| ---: | ---: | ---: |
| 0 | -.894427191 | -.894427191 |

## Pivoting

The QR decomposition with column pivoting solves

$$
\begin{equation*}
A P=Q R \tag{2}
\end{equation*}
$$

or, if you prefer,

$$
A P=Q_{1} R_{1}
$$

for $m \geq n$, where $P$ is a permutation matrix; see [M-1] Permutation. We can rewrite this as

$$
\begin{equation*}
A=Q R P^{\prime} \tag{3}
\end{equation*}
$$

and

$$
A=Q_{1} R_{1} P^{\prime}
$$

for $m \geq n$.
Column pivoting can improve the numerical accuracy. The functions $\operatorname{qrdp}(A, Q, R, p)$ and $\operatorname{hqrdp}\left(A, H, \operatorname{tau}, R_{1}, p\right)$ perform pivoting and return the permutation matrix $P$ in permutation vector form:

```
: A
        1 2
    lu
:Q = R = p = .
: qrdp(A, Q, R, p)
: Ahat = (Q*R)[., invorder(p)] // i.e., QRP'
: mreldif(Ahat, A)
1.97373e-16
H = tau = R1 = p = .
hqrdp(A, H, tau, R1, p)
Ahat = (hqrdq1(H, tau)*R1)[., invorder(p)] // i.e., Q1*R1*P'
mreldif(Ahat, A)
1.97373e-16
```

Before calling qrdp() or hqrdp(), we set $p$ equal to missing, specifying that all columns could be pivoted. We could just as well have set $p$ equal to $(0,0)$, which would have stated that both columns were eligible for pivoting.

When pivoting is disallowed, and when $A$ is not of full-column rank, the order in which columns appear affects the kind of generalized solution produced; later columns are, in effect, omitted. When pivoting is allowed, the columns are reordered based on numerical accuracy considerations. In the rank-deficient case, you no longer know ahead of time which columns will be omitted, because you do not know in what order the columns will appear. Generally, you do not care, but there are occasions when you do.

In such cases, you can specify which columns are eligible for pivoting and which are not-you specify $p$ as a vector, and if $p_{i}==1$, the $i$ th column may not be pivoted. The $p_{i}==1$ columns are (conceptually) moved to appear first in the matrix, and the remaining columns are ordered optimally after that. The permutation vector that is returned in $p$ accounts for all of this.

## Least-squares solutions with omitted columns

Least-square solutions are one popular use of QR decomposition. We wish to solve for $x$

$$
\begin{equation*}
A x=b \quad(A: m \times n, \quad m \geq n) \tag{4}
\end{equation*}
$$

The problem is that there is no solution to (4) when $m>n$ because we have more equations than unknowns. Then, we want to find $x$ such that $(A x-b)^{\prime}(A x-b)$ is minimized.

If $A$ is of full-column rank, then it is well known that the least-squares solution for $x$ is given by solveupper ( $R_{1}, Q_{1}^{\prime} b$ ) where solveupper () is an upper-triangular solver; see [M-5] solvelower().

If $A$ is of less than full-column rank and we do not care which columns are omitted, then we can use the same solution: solveupper ( $R_{1}, Q_{1}^{\prime} b$ ).

Adding pivoting to the above hardly complicates the issue; the solution becomes solveupper ( $R_{1}$, $Q_{1}^{\prime} b$ ) [invorder $\left.(p)\right]$.

For both cases, the full details are

```
A
            1 2 3
    |}\begin{array}{lll}{3}&{9}&{1}\\{3}&{8}&{1}\\{3}&{7}&{1}\\{3}&{6}&{1}
b
            1
l|r|
H = tau = R1 = p = .
hqrdp(A, H, tau, R1, p)
: q1b = hqrdmultq1t(H, tau, b) // i.e., Q1'b
xhat = solveupper(R1, q1b)[invorder(p)]
: xhat
    1
        -1.166666667
            1.2
                0
```

The $A$ matrix in the above example has less than full-column rank; the first column contains a variable with no variation, and the third column contains the data for the intercept. The solution above is correct, but we might prefer a solution that included the intercept. To do that, we need to specify that the third column cannot be pivoted:

```
: p = (0, 0, 1)
: H = tau = R1 = .
hqrdp(A, H, tau, R1, p)
: q1b = hqrdmultq1t(H, tau, b)
```

```
: xhat = solveupper(R1, q1b) [invorder(p)]
xhat
        1
```



## Conformability

$\operatorname{qrd}(A, Q, R):$
input:
A: $\quad m \times n$
output:
$\begin{aligned} Q: & m \times m \\ R: & m \times n\end{aligned}$
hqrd $\left(A, H, t a u, R_{1}\right)$ :
input:
A: $\quad m \times n$
output:

| $H:$ | $m \times n$ |  |
| ---: | :--- | :--- |
| tau: |  | $1 \times n$ |
| $R_{1}:$ |  | $n \times n$ |

$\quad \operatorname{hqrd}\left(A\right.$, tau,$\left.R_{1}\right)$ :
input:
A: $\quad m \times n$
output:
A: $\quad m \times n \quad$ (contains $H$ )
tau: $\quad 1 \times n$
$R_{1}: \quad n \times n$
hqrdmultq( $H$, tau, $X$, transpose, transform):

| $H:$ | $m \times n$ |
| ---: | :--- |
| tau: | $1 \times n$ |
| $X:$ | $m \times c$ |
| transpose: | $1 \times 1$ |
| transform: | $1 \times 1$ |
| result: | $m \times c$ |

hqrdmultq1t( $H$, tau, $X$, transform):

| $H:$ | $m \times n, \quad m \geq n$ |  |
| ---: | :---: | :--- |
| tau: | $1 \times n$ |  |
| $X:$ | $m \times c$ |  |
| transform: | $1 \times 1$ | (optional) |
| result: | $n \times c$ |  |

$\operatorname{hqrdq}(H, t a u):$

$$
\begin{aligned}
H: & m \times n \\
\text { tau: } & 1 \times n \\
\text { result: } & m \times m
\end{aligned}
$$

hqrdq1 $(H, t a u):$

$$
\begin{aligned}
H: & m \times n, \quad m \geq n \\
\text { tau: } & 1 \times n \\
\text { result: } & m \times n
\end{aligned}
$$

hqrdr( $H$ ):

$$
\begin{aligned}
H: & m \times n \\
\text { result: } & m \times n
\end{aligned}
$$

hqrdr1 ( $H$ ):

$$
\begin{aligned}
& H: m \times n \\
& \text { result: } \\
& n \times n
\end{aligned}
$$

$\operatorname{qrdp}(A, Q, R, p):$
input:

| A: | $m \times n$ |  |
| :--- | :--- | :--- | :--- |
| $p:$ | $1 \times 1 \quad$ or $\quad 1 \times n$ |  |

output:

| $Q:$ | $m \times m$ |
| :--- | :--- |
| $R:$ | $m \times n$ |
| $p:$ | $1 \times n$ |

$\operatorname{hqrdp}\left(A, H, \operatorname{tau}, R_{1}, p\right)$ :
input:

$$
\begin{array}{ccc}
\text { A: } & m \times n & \\
p: & 1 \times 1 & \text { or } \\
& 1 \times n
\end{array}
$$

output:

$$
\begin{aligned}
H: & m \times n \\
\text { tau: } & 1 \times n \\
R_{1}: & n \times n \\
p: & 1 \times n
\end{aligned}
$$

$\_\operatorname{hqrdp}\left(A, t a u, R_{1}, p\right)$ :
input:

$$
\begin{array}{llll}
\text { A: } & m \times n & & \\
p: & 1 \times 1 & \text { or } & 1 \times n
\end{array}
$$

output:

$$
\begin{array}{rll}
A: & m \times n & (\text { contains } H) \\
\text { tau: } & 1 \times n & \\
R_{1}: & n \times n & \\
p: & 1 \times n &
\end{array}
$$

_hqrdp_la $(A, t a u, p)$ :
input:

$$
A: \quad m \times n
$$

$p: \quad 1 \times 1$ or $1 \times n$
output:

| A: | $m \times n \quad$ (contains $H)$ |  |
| ---: | :--- | :--- |
| tau: | $1 \times n$ |  |
| $p:$ | $1 \times n$ |  |

## Diagnostics

$\operatorname{qrd}(A, \ldots), \operatorname{hqrd}(A, \ldots), \quad \operatorname{hqrd}(A, \ldots), \operatorname{qrdp}(A, \ldots), \operatorname{hqrdp}(A, \ldots)$, and $\operatorname{hqrdp}(A$, $\ldots$ ) return missing results if $A$ contains missing values. That is, $Q$ will contain all missing values. $R$ will contain missing values on above the diagonal. $p$ will contain the integers $1,2, \ldots$.
$\quad \operatorname{hqrd}(A, \ldots)$ and $\_\operatorname{hqrdp}(A, \ldots)$ abort with error if $A$ is a view.
hqrdmultq( $H$, tau, $X$, transpose, transform) and hqrdmultq1t( $H$, tau, $X$, transform) return missing results if $X$ contains missing values.

Vera Nikolaevna Kublanovskaya (1920-2012) was born in Krokhino, Russia, a small fishing village east of St. Petersburg. After finishing her secondary studies, Vera started her training to become a primary school teacher, but her grades were so outstanding that her mentors encouraged her to pursue a career in mathematics.

After graduation, she started working on computational algorithms for the Soviet nuclear program, from which she retired in 1955. She also participated in the development of numerical linearalgebra operations in the computer language PRORAB for the first electronic computer in the Soviet Union, BESM. In 1955, she received her PhD. She then published numerous papers, in particular on the topic of numerical linear algebra. Her most acclaimed contribution is as one of the inventors of the QR algorithm for computing eigenvalues of matrices.

Alston Scott Householder (1904-1993) was born in Rockford, Illinois, and grew up in Alabama. He studied philosophy at Northwestern and Cornell, and then mathematics, earning a doctorate in the calculus of variations from the University of Chicago. Householder worked on mathematical biology for several years at Chicago, but in 1946 he moved on to Oak Ridge National Laboratory, where he became the founding director of the Mathematics Division in 1948. There he moved into numerical analysis, specializing in linear equations and eigensystems and helping to unify the field through reviews and symposia. His last post was at the University of Tennessee.

[^10]
## Title

[M-5] qrinv( ) - Generalized inverse of matrix via QR decomposition

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

qrinv $(A, \ldots)$ returns the inverse or generalized inverse of real or complex matrix $A: m \times n, m \geq$ $n$. If optional argument rank is specified, the rank of $A$ is returned there.
_qrinv $(A, \ldots)$ does the same thing except that, rather than returning the result, it overwrites the original matrix $A$ with the result. _qrinv() returns the rank of $A$.

In both cases, optional argument tol specifies the tolerance for determining singularity; see Remarks and examples below.

## Syntax

```
numeric matrix qrinv(numeric matrix A)
numeric matrix qrinv(numeric matrix A, rank)
numeric matrix qrinv(numeric matrix A, rank, real scalar tol)
real scalar _qrinv(numeric matrix A)
real scalar _qrinv(numeric matrix A, real scalar tol)
```

where the type of rank is irrelevant; the rank of $A$ is returned there.

## Remarks and examples

qrinv() and _qrinv() are most often used on square and possibly rank-deficient matrices but may be used on nonsquare matrices that have more rows than columns. Also see $[\mathrm{M}-5] \operatorname{pinv}()$ for an alternative. See [M-5] luinv() for a more efficient way to obtain the inverse of full-rank, square matrices, and see [M-5] invsym( ) for inversion of real, symmetric matrices.

When $A$ is of full rank, the inverse calculated by qrinv() is essentially the same as that computed by the faster luinv(). When $A$ is singular, qrinv() and _qrinv() compute a generalized inverse, $A^{*}$, which satisfies

$$
\begin{aligned}
A\left(A^{*}\right) A & =A \\
\left(A^{*}\right) A\left(A^{*}\right) & =A^{*}
\end{aligned}
$$

This generalized inverse is also calculated for nonsquare matrices that have more rows than columns and, then returned is a least-squares solution. If $A$ is $m \times n, m \geq n$, and if the rank of $A$ is equal to $n$, then $\left(A^{*}\right) A=I$, ignoring roundoff error.
qrinv( $A$ ) is implemented as qrsolve( $A$, I (rows ( $A$ )) ); see [M-5] qrsolve() for details and for use of the optional tol argument.

## Conformability

## qrinv( $A$, rank, tol):

input:
A: $\quad m \times n, \quad m \geq n$
tol: $\quad 1 \times 1 \quad$ (optional)
output:

$$
\begin{array}{rll}
\text { rank: } & 1 \times 1 & \text { (optional) } \\
\text { result: } & & n \times m
\end{array}
$$

-qrinv( $A$, tol):
input:

$$
\begin{array}{cll}
A: & m \times n, & m \geq n \\
\text { tol: } & & 1 \times 1
\end{array} \text { (optional) }
$$

output:
A: $\quad n \times m$
result: $\quad 1 \times 1 \quad$ (containing rank)

## Diagnostics

The inverse returned by these functions is real if $A$ is real and is complex if $A$ is complex.
qrinv $(A, \ldots)$ and _qrinv $(A, \ldots)$ return a result containing missing values if $A$ contains missing values.
-qrinv $(A, \ldots)$ aborts with error if $A$ is a view.
See [M-5] qrsolve( ) and [M-1] Tolerance for information on the optional tol argument.

## Also see

[M-5] cholinv( ) - Symmetric, positive-definite matrix inversion
[M-5] invsym( ) - Symmetric real matrix inversion
[M-5] luinv() - Square matrix inversion
[M-5] pinv() - Moore-Penrose pseudoinverse
[M-5] qrsolve( ) - Solve AX=B for X using QR decomposition
[M-5] solve_tol() - Tolerance used by solvers and inverters
[M-4] Matrix - Matrix functions
[M-4] Solvers - Functions to solve $A X=B$ and to obtain A inverse

## Title

[M-5] qrsolve() — Solve $\mathrm{AX}=\mathrm{B}$ for X using QR decomposition

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

qrsolve $(A, B, \ldots)$ uses QR decomposition to solve $A X=B$ and returns $X$. When $A$ is singular or nonsquare, qrsolve() computes a least-squares generalized solution. When rank is specified, in it is placed the rank of $A$.
_qrsolve $(A, B, \ldots)$, does the same thing, except that it destroys the contents of $A$ and it overwrites $B$ with the solution. Returned is the rank of $A$.

In both cases, tol specifies the tolerance for determining whether $A$ is of full rank. tol is interpreted in the standard way-as a multiplier for the default if $t o l>0$ is specified and as an absolute quantity to use in place of the default if $t o l \leq 0$ is specified; see [M-1] Tolerance.

## Syntax

| numeric matrix | $\operatorname{qrsolve}(A, B)$ |
| :--- | :--- |
| numeric matrix | $\operatorname{qrsolve}(A, B$, rank $)$ |
| numeric matrix | qrsolve $(A, B$, rank, tol $)$ |
| real scalar | -qrsolve $(A, B)$ |
| real scalar | -qrsolve $(A, B$, tol $)$ |

where
A: numeric matrix
B: $\quad$ numeric matrix
rank: irrelevant; real scalar returned
tol: real scalar

## Remarks and examples

qrsolve $(A, B, \ldots)$ is suitable for use with square and possibly rank-deficient matrix $A$, or when $A$ has more rows than columns. When $A$ is square and full rank, qrsolve() returns the same solution as lusolve() (see [M-5] lusolve()), up to roundoff error. When $A$ is singular, qrsolve() returns a generalized (least-squares) solution.

Remarks are presented under the following headings:

[^11]
## Derivation

We wish to solve for $X$

$$
\begin{equation*}
A X=B \tag{1}
\end{equation*}
$$

Perform QR decomposition on $A$ so that we have $A=Q R P^{\prime}$. Then (1) can be rewritten as

$$
Q R P^{\prime} X=B
$$

Premultiplying by $Q^{\prime}$ and remembering that $Q^{\prime} Q=Q Q^{\prime}=I$, we have

$$
\begin{equation*}
R P^{\prime} X=Q^{\prime} B \tag{2}
\end{equation*}
$$

Define

$$
\begin{equation*}
Z=P^{\prime} X \tag{3}
\end{equation*}
$$

Then (2) can be rewritten as

$$
\begin{equation*}
R Z=Q^{\prime} B \tag{4}
\end{equation*}
$$

It is easy to solve (4) for $Z$ because $R$ is upper triangular. Having $Z$, we can obtain $X$ via (3), because $Z=P^{\prime} X$, premultiplied by $P$ (and if we remember that $P P^{\prime}=I$ ), yields

$$
X=P Z
$$

For more information on QR decomposition, see [M-5] $\mathbf{q r d}()$.

## Relationship to inversion

For a general discussion, see Relationship to inversion in [M-5] lusolve().
For an inverse based on QR decomposition, see [M-5] qrinv(). qrinv( $A$ ) amounts to qrsolve ( $A$, I (rows (A))), although it is not actually implemented that way.

## Tolerance

The default tolerance used is

$$
\text { eta }=1 \mathrm{e}-13 * \operatorname{trace}(\operatorname{abs}(R)) / \text { rows }(R)
$$

where $R$ is the upper-triangular matrix of the QR decomposition; see Derivation above. When $A$ is less than full rank, by, say, $d$ degrees of freedom, then $R$ is also rank deficient by $d$ degrees of freedom and the bottom $d$ rows of $R$ are essentially zero. If the $i$ th diagonal element of $R$ is less than or equal to eta, then the $i$ th row of $Z$ is set to zero. Thus if the matrix is singular, qrsolve() provides a generalized solution.

If you specify tol $>0$, the value you specify is used to multiply eta. You may instead specify tol $\leq$ 0 , and then the negative of the value you specify is used in place of eta; see [M-1] Tolerance.

## Conformability

qrsolve( $A, B$, rank, tol):
input:
A: $\quad m \times n, \quad m \geq n$
B: $\quad m \times k$
tol: $\quad 1 \times 1 \quad$ (optional)
output:
rank: $1 \times 1 \quad$ (optional)
result: $\quad n \times k$
_qrsolve( $A, B$, tol):
input:

| A: | $m \times n$, | $m \geq n$ |
| ---: | :--- | :--- |
| $B:$ | $m \times k$ |  |
| tol: | $1 \times 1$ | (optional) |

output:
A: $\quad 0 \times 0$
B: $\quad n \times k$
result: $\quad 1 \times 1$

## Diagnostics

qrsolve $(A, B, \ldots)$ and _qrsolve $(A, B, \ldots)$ return a result containing missing if $A$ or $B$ contain missing values.
_qrsolve $(A, B, \ldots)$ aborts with error if $A$ or $B$ are views.

## Also see

[M-5] cholsolve () - Solve AX=B for X using Cholesky decomposition
[M-5] lusolve( ) - Solve AX=B for X using LU decomposition
[M-5] $\mathbf{q r d}()-\mathrm{QR}$ decomposition
[M-5] qrinv() - Generalized inverse of matrix via QR decomposition
[M-5] solvelower() - Solve AX=B for X, A triangular
[M-5] solve_tol( ) - Tolerance used by solvers and inverters
[M-5] svsolve( ) - Solve AX=B for X using singular value decomposition
[M-4] Matrix - Matrix functions
[M-4] Solvers - Functions to solve $A X=B$ and to obtain A inverse

## Title

[M-5] quadcross( ) — Quad-precision cross products

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

quadcross() makes calculations of the form
$X^{\prime} X$
$X^{\prime} Z$
$X^{\prime} \operatorname{diag}(w) X$
$X^{\prime} \operatorname{diag}(w) Z$
This function mirrors cross () (see [M-5] cross()), the difference being that sums are formed in quad precision rather than in double precision, so quadcross () is more accurate.
quadcrossdev() makes calculations of the form

$$
\begin{aligned}
& (X:-x)^{\prime}(X:-x) \\
& (X:-x)^{\prime}(Z:-z) \\
& (X:-x)^{\prime} \operatorname{diag}(w)(X:-x) \\
& (X:-x)^{\prime} \operatorname{diag}(w)(Z:-z)
\end{aligned}
$$

This function mirrors crossdev () (see [M-5] crossdev()), the difference being that sums are formed in quad precision rather than in double precision, so quadcrossdev() is more accurate.

## Syntax

```
real matrix quadcross(X,Z)
real matrix quadcross( }X,w,Z
real matrix quadcross(X, xc, Z, zc)
real matrix quadcross(X,xc,w,Z,zc)
real matrix quadcrossdev ( }X,x,Z,z
real matrix quadcrossdev (X, x, w, Z,z)
real matrix quadcrossdev (X, xc, x, Z, zc,z)
real matrix quadcrossdev (X,xc,x,w,Z,zc,z)
```

where
$X: \quad$ real matrix $X$
$x c: \quad$ real scalar $x c$
$x$ : real rowvector $x$
$w$ : real vector $w$
Z: real matrix $Z$
$z c: \quad$ real scalar $z c$
$z: \quad$ real rowvector $z$

## Remarks and examples

The returned result is double precision, but the sum calculations made in creating that double-precision result were made in quad precision.

## Conformability

quadcross() has the same conformability requirements as cross(); see [M-5] cross().
quadcrossdev() has the same conformability requirements as crossdev(); see [M-5] crossdev().

## Diagnostics

See Diagnostics in [M-5] cross() and Diagnostics in [M-5] crossdev().

## Also see

[M-5] cross() - Cross products
[M-5] crossdev () — Deviation cross products
[M-4] Statistical - Statistical functions
[M-4] Utility - Matrix utility functions

## Title

[M-5] Quadrature() — Numerical integration

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | References | Also see |  |

## Description

Quadrature() approximates the integral $\int_{a}^{b} f(x) d x$ by adaptive quadrature, where $f(x)$ is a real-valued function and $a$ and $b$ are lower and upper limits, respectively.

QuadratureVec() is functionally the same as Quadrature(), except that it handles a vector of integration problems more conveniently.

## Syntax

Mata's quadrature functions allow you to approximate integrals in as few as four steps-create an instance of the problem with Quadrature() or QuadratureVec (), specify the evaluator functions with setEvaluator(), set the limits with setLimits(), and perform the computation with integrate().

Quadrature() is better suited for one integration problem, whereas QuadratureVec() solves a vector of integration problems more conveniently. Example 1 demonstrates how Quadrature() approximates one integral. Example 2 demonstrates how QuadratureVec() approximates a set of integrals.

The full syntax allows you to further define the integration problem and to obtain additional results. Quadrature() and QuadratureVec() have similar syntax, except for the following difference: the arguments and returned values of Quadrature() functions are typically scalars corresponding to one integration problem, whereas the arguments and returned values of QuadratureVec () functions are a column vector corresponding to a set of problems-that is, QuadratureVec() handles a column vector of functions, limits, integration techniques, maximum iterations, tolerances, etc. The $i$ th element of the column vector corresponds to the $i$ th integration problem.

Syntax is presented under the following headings (applicable to both Quadrature() and QuadratureVec(), unless specified otherwise):

[^12]q.setAbstol(), q.getAbstol(), q.setReltol(), and q.getReltol()
q.setArgument(), q.getArgument(), q.getNarguments(), and q.getDimension()
q.setTrace() and q.getTrace()

Performing integration
q.integrate()

Functions for obtaining results
$q$.value()
q.iterations()
q.converged( )
q.errorcode(), q.errortext(), and q.returncode()

Utility function
q.query()

## Step 1: Problem initialization

$$
q=\text { Quadrature () }
$$

or

$$
q=\text { QuadratureVec }()
$$

## Step 2: Problem definition

## Quadrature(): Definition of one integration problem

| void | $q$. setEvaluator (pointer(real function) scalar fcn) |
| :---: | :---: |
| void | $q$. setLimits (real rowvector limits) |
| void | $q$. setTechnique (string scalar technique) |
| void | q.setMaxiter (real scalar maxiter) |
| void | q.setAbstol (real scalar abstol) |
| void | $q$. setReltol (real scalar reltol) |
| void | q.setArgument (real scalar i, arg) |
| void | $q$. setTrace (string scalar trace) |
| pointer(real function) scalar | q.getEvaluator () |
| real rowvector | q.getLimits() |
| string scalar | $q \cdot g e t T e c h n i q u e()$ |
| real scalar | q.getMaxiter () |
| real scalar | q.getAbstol() |
| real scalar | q.getReltol() |
| pointer scalar | $q . g$ getArgument (real scalar $i$ ) |
| real scalar | q.getNarguments() |
| string scalar | q.getTrace() |

## QuadratureVec(): Definition of a vector of integration problems

void
void
void
void
void
void
void
void
pointer(real function) colvector
real matrix
string colvector
real colvector
real colvector
real colvector
pointer colvector
real scalar
string colvector
$q$.setEvaluator (pointer(real function) colvector fcn)
$q$.setLimits (real matrix limits)
$q$.setTechnique (string colvector technique)
$q$.setMaxiter (real colvector maxiter)
$q$.setAbstol (real colvector abstol)
$q$.setReltol (real colvector reltol)
$q$.setArgument (real colvector pid, real scalar i, arg)
q. setTrace (string colvector trace)
q.getEvaluator()
q.getLimits()
$q$.getTechnique()
q.getMaxiter()
$q$. getAbstol()
$q$. getReltol()
$q \cdot$ getArgument (real colvector pid, real scalar $i$ )
$q$.getDimension()
$q$. getTrace()

## Step 3: Perform integration

## Quadrature()

real scalar
q.integrate()

## QuadratureVec()

real colvector
q.integrate()

With Quadrature(), a scalar result is returned; with QuadratureVec(), a column vector of results is returned.

## Step 4: Display or obtain results

## Quadrature()

| real scalar | $q \cdot$ value() |
| :--- | :--- |
| real scalar | $q \cdot i t e r a t i o n s()$ |
| real scalar | $q \cdot \operatorname{converged}()$ |
| real scalar | $q \cdot$ errorcode() |
| string scalar | $q \cdot$ errortext() |
| real scalar | $q \cdot r e t u r n c o d e()$ |

## QuadratureVec()

real colvector
real colvector
real colvector
real colvector
string colvector
real colvector

```
q.value()
q.iterations()
q.converged()
q.errorcode()
q.errortext()
q.returncode()
```


## Utility function for use in all steps

void
$q$.query ()

## Definition of $q$

A variable of type Quadrature is called an instance of the Quadrature() class in Mata.
You can create an instance $q$ of Quadrature() by typing

$$
\mathrm{q}=\text { Quadrature() }
$$

In a function, you would declare one instance $q$ of the Quadrature() class as a scalar.

```
void myfunc()
{
    class Quadrature scalar q
    q = Quadrature()
}
```

You can also create a vector or matrix of Quadrature() instances. For a row vector of $n$ Quadrature() instances, type

$$
\mathrm{q}=\text { Quadrature }(n)
$$

For an $m \times n$ matrix of Quadrature() instances, type

$$
\mathrm{q}=\text { Quadrature }(m, n)
$$

However, to solve several instances of integration problems, you should use QuadratureVec().
A variable of type QuadratureVec is an instance of the QuadratureVec() class in Mata. You can create an instance $q$ of QuadratureVec() by typing

$$
\mathrm{q}=\text { QuadratureVec }()
$$

In a function, you would declare one instance $q$ of the QuadratureVec() class as a scalar.

```
void myfunc()
{
    class QuadratureVec scalar q
    q = QuadratureVec()
}
```


## Functions defining the integration problem

At a minimum, you need to tell Quadrature() or QuadratureVec () about the functions you wish to integrate and the limits of integration. You can also specify the techniques used to compute the quadrature, the maximum number of iterations allowed, the convergence criteria, the arguments to be passed to the evaluators, and how to print computation details.

## q.setEvaluator() and q.getEvaluator()

With Quadrature(), q.setEvaluator (fcn) sets a pointer to the evaluator function fcn. setEvaluator () has to be called before any calculation. An evaluator has a special format-it has at least one real scalar argument (first argument, corresponding to $x$ ) and returns a real scalar value, $f(x)$.
$q \cdot$ getEvaluator () returns a pointer to the evaluator function (NULL if not specified).
With QuadratureVec(), setting evaluation functions is done the same way, except that $q$.setEvaluator () sets a column vector of pointers to evaluator functions and $q$.getEvaluator () returns a column vector of pointers to the functions. If $f c n$ is a pointer scalar, then the pointer to the evaluator specified is used for all integration problems.

## q.setLimits() and q.getLimits()

With Quadrature(), $q$.setLimits(limits) sets the integration limits as a two-dimensional row vector that contains the lower and upper limit in that order. The limits can be finite or infinite. The lower limit must be less than or equal to the upper limit. Using a missing value as the lower limit indicates $-\infty$, and using a missing value as the upper limit indicates $\infty$.

With QuadratureVec(), q.setLimits (limits) sets different limits for different integration problems, where limits is an $n \times 2$ matrix; that is, one row for each integration problem, where the first column contains lower limits and the second column contains upper limits. If limits is a two-dimensional row vector, then the pair of limits specified is used for all integration problems.
$q$. getLimits() returns the integration limits; if not specified, this will be an empty vector for Quadrature() and an empty matrix for QuadratureVec().

## q.setTechnique() and q.getTechnique()

With Quadrature(), q.setTechnique (technique) specifies the technique used to compute the quadrature. technique can be the following:
technique Description
"gauss"
"simpson"
adaptive Gauss-Kronrod method; the default adaptive Simpson method

With QuadratureVec(), q.setTechnique (technique) specifies different techniques for different integration problems, where technique is a column vector of the techniques described above. If technique is a scalar, then the technique specified is used for all integration problems.
$q$.getTechnique() returns the current technique.

## q.setMaxiter() and q.getMaxiter()

With Quadrature(), $q$.setMaxiter (maxiter) specifies the maximum number of iterations, which must be an integer greater than 0 . The default value of maxiter is 16000 .

With QuadratureVec (), q.setMaxiter (maxiter) specifies different maximum number of iterations for different integration problems, where maxiter is a column vector of maximums. If maxiter is a scalar, then the maximum specified is used for all integration problems.
$q \cdot$ getMaxiter() returns the current maximum number of iterations.

## q.setAbstol(), q.getAbstol(), q.setReltol(), and q.getReltol()

With Quadrature(), q.setAbstol (abstol) and $q$.setReltol(reltol) specify the convergence criteria with absolute and relative tolerances, which must be greater than 0 . The default values of absolute and relative tolerances are $1 \mathrm{e}-10$ and $1 \mathrm{e}-8$, respectively.

The absolute tolerance gives an upper bound for the approximate measure of the absolute difference between the computed solution and the exact solution, while the relative tolerance gives an upper bound for the approximate measure of the relative difference between the computed and the exact solution.

With QuadratureVec(), q.setAbstol(abstol) and q.setReltol(reltol) specify different tolerances for different integration problems, where abstol and reltol are column vectors of tolerances. If abstol or reltol is a scalar, then the tolerance specified is used for all integration problems.
$q \cdot$ getAbstol() and $q$. getReltol() return the current absolute and relative tolerances, respectively.

## q.setArgument(), q.getArgument(), q.getNarguments(), and q.getDimension()

With Quadrature(), the $i$ th extra argument $\arg$ of the evaluator can be specified with $q$.setArgument ( $i$, $\arg$ ), where $i$ is an integer between 1 and 9 . Here arg can be anything: a scalar, an expression, a matrix, or even a pointer to a function. If $i$ is greater than the current number of extra arguments, then the number of extra arguments will be increased to $i$.
$q \cdot$ getArgument ( $i$ ) returns a pointer to the $i$ th extra argument of the evaluator (NULL if not specified).
Example 5 demonstrates how extra arguments are specified with Quadrature().

With QuadratureVec(), extra arguments can be specified in the same way as with Quadrature(), except that the first argument is a column vector of problem identifiers (integers) for which an extra argument is set. That is, $q$.setArgument (pid, $i$, arg) will set the $i$ th extra argument to arg for integration problems in column vector pid.
$q \cdot$ getArgument (pid, $i$ ) returns a pointer to the $i$ th extra argument of the evaluator for problems pid.

Example 6 demonstrates how extra arguments are specified with QuadratureVec().
With Quadrature(), the current number of extra arguments can be obtained with $q \cdot$ getNarguments(). q.getNarguments() is not defined for QuadratureVec().

With QuadratureVec (), the number of integration problems can be obtained with $q$. getDimension(). q.getDimension() is not defined for Quadrature().

## q.setTrace() and q.getTrace()

With Quadrature(), q.setTrace (trace) sets whether to print out computation details. trace can be "on" or "off". The default value is "off".
$q \cdot$ getTrace() returns the current trace status.
With QuadratureVec(), setTrace(trace) and getTrace() work in the same way, except that a column vector of settings is input or returned. If trace is a scalar, then the trace specified is used for all integration problems.

## Performing integration

## q.integrate()

With Quadrature(), $q$.integrate() computes the numerical integration, that is, the approximation of the integral of the evaluator from the lower limit to the upper limit. $q$.integrate() returns the computed quadrature value.

With QuadratureVec(), q.integrate() does the same as with Quadrature(), except that it approximates a set of integrals and returns a column vector of computed quadrature values.

## Functions for obtaining results

After performing integration, the functions below provide results, including the value of the integrals, number of iterations, whether convergence was achieved, error messages, and return codes. The following describe the functions with Quadrature() ; the functions work the same with QuadratureVec (), except that a column vector is returned instead of a scalar.

## q.value()

$q$. value() returns the computed quadrature value or values; it returns missing if not yet computed.

## q.iterations()

$q$.iterations() returns the number of iterations; it returns 0 if not yet computed.

## q.converged()

$q$. converged () returns 1 if converged and 0 if not.

## q.errorcode( ), q.errortext(), and q.returncode()

$q$.errorcode() returns an error code generated during the computation; it returns 0 if no error is found.
$q$.errortext() returns an error message corresponding to the error code generated during the computation; it returns an empty string if no error is found.
$q$. returncode () returns the Stata return code corresponding to the error code generated during the computation.

The error codes and the corresponding Stata return codes for both Quadrature() and QuadratureVec () are as follows:

Error Return
code code Error text

1111 you must specify an evaluator function to compute numerical integration using setEvaluator()
2111 you must specify lower and upper integration limits as a rowvector with 2 columns using setLimits()
3111 you specified extra argument $n$ but did not specify extra argument $i$. When you specify extra argument $n$, you must also specify all extra arguments less than $n^{\text {a }}$
4111 code distributed with Stata has been changed so that a required subroutine cannot be found
5416 evaluator function returned a missing value at one or more quadrature points
6430 subintervals cannot be further divided to achieve the required accuracy 7430 maximum number of iterations has been reached
${ }^{\text {a }} n$ will be an actual number, and $i$ will be a number less than $n$.
The following are applicable to QuadratureVec() only:

| Error <br> code | Return <br> code | Error text |
| :---: | :---: | :--- |
| 8 | 430 | no problem is defined in the class |
| 9 | 430 | error found for some problems in the class <br> 10 |
| 430 | you must specify at least one evaluator function to compute <br> numerical integration using setEvaluator () |  |
| 11 | 430 | you must specify lower and upper integration limits as a matrix <br> with 2 columns using setLimits () |
|  |  |  |

## Utility function

At any stage of solving the integration problem, you can obtain a report of all current settings and results with

## q.query()

$q$.query() displays information stored in the class. It has no return value.

## Remarks and examples

Remarks are presented under the following headings:

```
Introduction
Examples
    A basic example of Quadrature()
    A basic example of QuadratureVec()
    Integrals with infinite limits
    Passing arguments to the evaluator function
    Singular points and setting tolerances
    Displaying settings and results at each stage
    Solving vectors and matrices of integrals
```


## Introduction

Quadrature() and QuadratureVec() are Mata classes for numerical integration. For an introduction to class programming in Mata, see [M-2] class.

Historically, quadrature means the process of determining area. The term is still used nowadays to refer to solutions in terms of integrals. Quadrature() and QuadratureVec() use the adaptive quadrature method to approximate integrals of the form $\int_{a}^{b} f(x) d x$, where $f(x)$ is a real-valued function and $a$ and $b$ are lower and upper limits, respectively.

## Examples

To approximate an integral, you first use Quadrature() or QuadratureVec() to create an instance of the class. At a minimum, you must also use setEvaluator () to specify the evaluator functions, setLimits() to specify the limits, and integrate() to perform the computations. In the examples below, we demonstrate both basic and more advanced uses of these Mata classes.

## A basic example of Quadrature()

> Example 1: Approximate an integral
We want to approximate $\int_{0}^{\pi} \sin (x) d x$ using Quadrature(). We first define an evaluator function $f()$ as a wrapper for the built-in $\sin ()$ function:

```
real scalar f(real scalar x) {
> return(sin(x))
}
```

We need this wrapper because we must use the address of the evaluator function in an instance of Quadrature(). Note that the addresses of built-in functions like sin() are not accessible.

Having defined the evaluator function, we follow the four steps that are required. First, we create an instance q of the Quadrature() class:

```
: q = Quadrature()
```

Second, we use setEvaluator () to set a pointer to the evaluator function $f()$ :

```
: q.setEvaluator(&f())
```

Third, we use setLimits() to specify the lower and upper limits:

```
: q.setLimits((0, pi()))
```

Fourth, we use integrate() to compute the approximation:

```
: q.integrate()
```

2

We find that $\int_{0}^{\pi} \sin (x) d x=2$.

## A basic example of QuadratureVec()

## $>$ Example 2: Approximate a set of integrals

We want to approximate integrals $\int_{1}^{2} 2 x d x, \int_{0}^{\pi} \sin (x) d x$, and $\int_{0}^{1} \exp (x) d x$ using QuadratureVec (). We first define the corresponding evaluator functions as $f 1(), f 2()$, and $f 3()$.

```
: real scalar f1(real scalar x) {
> return(2*x)
> }
real scalar f2(real scalar x) {
return(sin(x))
}
real scalar f3(real scalar x) {
    return(exp(x))
}
```

Having defined the evaluator functions, we follow the four steps that are required. First, we create an instance $q$ of the QuadratureVec () class:

```
: q = QuadratureVec()
```

Second, we use setEvaluator() to point to the evaluator functions, defined as the column vector evaluator.

```
: evaluator = (&f1() \ &f2() \ &f3())
: q.setEvaluator(evaluator)
```

Third, we use setLimits() to specify the lower and upper limits, defined as limits.

```
: limits = ((1, 2) \ (0,pi()) \ (0,1))
: q.setLimits(limits)
```

Fourth, we use q.integrate() to compute the approximations:


We find that $\int_{1}^{2} 2 x d x=3, \int_{0}^{\pi} \sin (x) d x=2$, and $\int_{0}^{1} \exp (x) d x=1.718281828 \ldots$

## Integrals with infinite limits

We often need to evaluate integrals where one or both limits are infinite.

## $>$ Example 3: Approximate an integral from a finite point to $\infty$

To calculate the numerical integral of $\int_{0}^{\infty} \exp (-x) d x$, we define an evaluator function $f 4$ (), create an instance q of Quadrature (), set the evaluator, and set the limits:

```
: real scalar f4(real scalar x) {
> return(exp(-x))
> }
: q = Quadrature()
: q.setEvaluator(&f4())
q.setLimits((0, .))
```

We specify . to set the upper limit to $\infty$.
Now we can compute the approximation:

```
: q.integrate()
    .9999999997
```

> Example 4: Approximate an integral from $-\infty$ to $\infty$
To calculate the numerical integral of $\int_{-\infty}^{\infty} \exp \left(-x^{2}\right) d x$, we use commands similar to those in example 3, but we now define an evaluator function $f 5()$ :

```
real scalar f5(real scalar x) {
    return(exp(-x*x))
> }
: q = Quadrature()
q.setEvaluator(&f5())
q.setLimits((., .))
```

We specify . for both the upper and the lower limits. This sets the lower limit to $-\infty$ and the upper limit to $\infty$.

Now we can compute the approximation:

```
q.integrate()
    1.772453852
```


## Passing arguments to the evaluator function

Often, statistical and mathematical functions that we wish to integrate need additional arguments. For instance, we may want to integrate with respect to one variable while setting other variables in the function to specific values. We can do this by passing arguments to the function evaluator.
$>$ Example 5: Add an extra argument to the evaluator function with Quadrature()
We want to calculate the numerical integral of $\int_{0}^{1}\left(x^{2}+z^{2}\right) d x$, where $z=5$. The code is as follows:

```
: real scalar f6(real scalar x, real scalar z) {
> return(x*x + z*z)
> }
: q = Quadrature()
: q.setEvaluator(&f6())
: q.setLimits((0, 1))
```

We use setArgument () to set the value of the extra argument $z$ into q . Typing

```
: q.setArgument(1, 5)
```

specifies that the first extra argument has a value of 5 . There is only one extra argument, so we are now ready to compute the approximation.

```
: q.integrate()
25.33333333
```

The value of the first extra argument can be obtained by typing

```
: *(q.getArgument(1))
    5
```

Note that getArgument () returns a pointer, which is why the monadic operator $*$ is used. For more information on pointers in Mata, see [M-2] pointers.

## > Example 6: Add extra arguments to the evaluator functions with QuadratureVec()

We want to calculate the numerical integral of four functions, all with extra arguments. In the code below, the extra argument in f 7() and f 8() is the vector $y$; in f 9() , it is the string $s$; and in f 10() , it is a pointer to a function $f()$. We also define a function called $f \mathrm{~cm}^{()}$.

```
real scalar f7(real scalar x, real vector y) {
    return(x + norm(y))
}
real scalar f8(real scalar x, real vector y) {
    return(x*x + sum(y))
}
real scalar f9(real scalar x, string scalar s) {
    return(sin(x) + strlen(s))
}
real scalar f10(real scalar x, pointer(function) scalar f) {
    return(x + (*f)(0))
}
y = (1, 1.5, 10)
```

```
s = "abc"
real scalar fcn(real scalar x) {
    return(-cos(x))
}
```

We define a new instance $S$ of QuadratureVec () and set the four evaluator functions and the limits from 0 to 1 for all integration problems.

```
: S = QuadratureVec()
: S.setEvaluator((&f7() \ &f8() \ &f9() \ &f10()))
S.setLimits((0, 1))
```

Now we use setArgument () to set the extra arguments. For the first two problems, with evaluator functions f 7() and f 8() , extra argument 1 is set to vector $y$. For the third problem, which involves evaluator function f 9() , extra argument 1 is set to string $s$. And, for the fourth integration problem, which involves function $f 10()$, extra argument 1 is set to a pointer to the function $f \mathrm{cn}()$.

```
: S.setArgument((1 \ 2), 1, y)
: S.setArgument(3, 1, s)
: S.setArgument(4, 1, &fcn())
```

We can get the extra arguments as follows. We use getArgument () to get four pointers (one for each integration problem) for extra argument 1 . To get the actual value of the extra argument, we use the monadic operator *. For more information on pointers in Mata, see [M-2] pointers.

```
: ptr = S.getArgument((1 \ 2 \ 3 \ 4), 1)
: (*ptr[1])
            1 2 3
    1 1 1.5 10
: (*ptr[2])
            1 2 3
    1
        1 1.5 10
: (*ptr[3])
    abc
: (*(*ptr [4]))(0)
    -1
```

Now we can compute the approximations:

```
S.integrate()
```

            1
    ```
10.66120072
12.83333333
3.459697694
-. }
```


## Singular points and setting tolerances

Problematic points of a function are known as "singular points". There are two common types of singular points. In the first type, $x_{0}$ is a singular point because $f\left(x_{0}\right)$ is $\infty$ or $-\infty$. In the second type, $f\left(x_{0}\right)$ is not continuous at $x_{0}$.

The methods implemented in the Quadrature() and QuadratureVec () classes are robust to singular points. Sometimes, a function with singular points might be better approximated by specifying a smaller convergence tolerance, as in the example below.

## > Example 7: Approximate an integral with a singular point

To calculate the numerical integral of $\int_{0}^{1} \log (x) d x$, we define q 2 as a new instance of the Quadrature () class, and we initialize the new evaluator function and limits:

```
: q2 = Quadrature()
: real scalar f11(real scalar x) {
> return(log(x))
> }
: q2.setEvaluator(&f11())
: q2.setLimits((0,1))
```

The $\log ()$ function has a singular point at 0 . We can compute the approximation by using the default absolute tolerance of $1 \mathrm{e}-10$ and the default relative tolerance of $1 \mathrm{e}-8$.

```
: q2.integrate()
    -1.000000004
```

This is close to the value of -1 that we expect. However, we can obtain a more accurate result if we set stricter absolute and relative tolerances.

```
: q2.setAbstol(1e-15)
q2.setReltol(1e-12)
: q2.integrate()
-1
```

The absolute tolerance is an upper bound on the estimated absolute difference between the computed approximation and the exact solution. The relative tolerance is an upper bound on the estimated relative difference between the computed approximation and the exact solution.

## Displaying settings and results at each stage

Each instance of the Quadrature() or QuadratureVec () class contains a lot of information about the problem at hand. You can use the utility function query () to display this information. Several other utility functions display specific pieces of information. The example below illustrates how to use these functions.
$>$ Example 8: Display information about the integration problem
To calculate the numerical integral $\int_{-1}^{1}|x| d x$, we define the class instance q 3 .

```
: q3 = Quadrature()
```

We can show all the default values by using query() before we perform any initialization or computation.

```
: q3.query()
Settings for Quadrature
Version:
1.00
Evaluator
    Function: unknown
User-defined arguments: <none>
Integration limits
    Limits: unknown
Quadrature technique: gauss
Trace:
off
Convergence
    Maximum iterations: 16000
    Absolute tolerance: 1.0000e-10
    Relative tolerance: 1.0000e-08
Current status
    Quadrature value: .
    Converged: no
```

Note: The evaluator function has not been specified.
Note: Integration limits have not been specified.

Now we initialize the evaluator function and limits.

```
real scalar f12(real scalar x) {
    return(abs(x))
}
q3.setEvaluator(&f12())
q3.setLimits((-1, 1))
```

We then compute the approximation.

```
: q3.integrate()
1
```

We can set the trace to "on" to see the computation details.

```
: q3.setTrace("on")
: q3.integrate()
Quadrature trace:
    Iteration Current value Current error estimate
    0 1.017294655 2.57082e-02
    1 1 4.18554e-14
1
```

We turn off the trace by typing

```
: q3.setTrace("off")
```

For illustrative purposes, we switch the technique to the adaptive Simpson's method.

```
: q3.setTechnique("simpson")
```

The default method, adaptive Gauss-Kronrod quadrature (gauss), is almost always better than Simpson's method (simpson). Simpson's method is a benchmark method included for completeness.

The default maximum number of iterations is 16,000 , and a warning message is printed when the maximum number of iterations is reached. For illustrative purposes, we set the maximum to 2 and compute the approximation.

```
: q3.setMaxiter(2)
: q3.integrate()
Warning: Convergence not achieved; maximum number of iterations has been
    reached.
```

The integration could not be performed using Simpson's method and allowing only 2 iterations, so we see a warning message and receive a missing value (.) as the value of the integral.

We switch back to a maximum of 16,000 iterations and compute the approximation.

```
: q3.setMaxiter(16000)
: q3.integrate()
1
```

We can check the result after computation.

```
: q3.value()
    1
```

We can also check the number of iterations needed to converge.

```
: q3.iterations()
46
```

No error was found, so the error code is 0 , and the error text is an empty string.

```
: q3.errorcode()
    0
: q3.errortext()
```

We can show all the information with query () after the computation.

```
: q3.query()
Settings for Quadrature
Version: 1.00
Evaluator
    Function: f12()
User-defined arguments: <none>
Integration limits
    Limits
        1: -1
        2: 1
Quadrature technique: simpson
Trace: off
Convergence
    Maximum iterations: 16000
    Absolute tolerance: 1.0000e-10
    Relative tolerance: 1.0000e-08
Current status
    Quadrature value: 1
    Converged: yes
    Iterations: 46
```


## Solving vectors and matrices of integrals

If you have more than one integral to approximate, you can create a vector or matrix of instances of the Quadrature () class. However, we recommend using QuadratureVec() because the latter makes it more convenient to define and solve a vector of integration problems.

## > Example 9: Solve a vector of integrals

In this example, we show how both Quadrature() and QuadratureVec() can be used to solve the same set of problems.

First, we use the Quadrature () class to approximate the three integrals $\int_{0}^{1} \sqrt{x} d x, \int_{1}^{2} \exp (x) d x$, and $\int_{0}^{0.5}(1 / \sqrt{x}) d x$. We begin by defining the three evaluator functions.

```
real scalar f13(real scalar x) {
return(sqrt(x))
}
real scalar f14(real scalar x) {
return(exp(x))
}
real scalar f15(real scalar x) {
    return(1/sqrt(x))
}
```

Now, we create qv, a vector of three instances of the Quadrature () class.

```
: qv = Quadrature(3)
```

Next, we set the evaluator function and the limits for each element of the vector of instances.

```
: qv[1].setEvaluator(&f13())
: qv[1].setLimits((0, 1))
: qv[2].setEvaluator(&f14())
: qv[2].setLimits((1, 2))
: qv[3].setEvaluator(&f15())
: qv[3].setLimits((0, 0.5))
```

We use integrate() to compute each of the approximations.

```
: qv[1].integrate()
    .6666666667
: qv[2].integrate()
    4.67077427
: qv[3].integrate()
    1.414213562
```

QuadratureVec() provides a more concise way to define and solve such a set of integration problems.

We first create QV, an instance of the QuadratureVec() class with

```
: QV = QuadratureVec()
```

The evaluator functions have to be defined as above and then can be set with

```
: evaluator = (&f13() \ &f14() \ &f15())
: QV.setEvaluator(evaluator)
```

The limits can be set with
$:$ limits $=((0,1) \backslash(1,2) \backslash(0,0.5))$
: QV.setLimits(limits)
We use integrate() to compute all the approximations as follows:

```
    : QV.integrate()
```

        1
        .6666666667
        4.67077427
        1.414213562
    
## Conformability

In the descriptions below, if the input or output dimensions differ for Quadrature() and QuadratureVec (), both dimensions are specified-first for Quadrature(), followed by "or", and then for QuadratureVec().

Quadrature():
input:
void
output:

$$
\text { result: } \quad 1 \times 1
$$

Quadrature(n):
input:

$$
n: \quad 1 \times 1
$$

output:
result: $\quad 1 \times n$
Quadrature $(m, n)$ :
input:

$$
\begin{aligned}
& m: \\
& n: \\
& 1 \times 1 \times 1
\end{aligned}
$$

output:

$$
\text { result: } \quad m \times n
$$

QuadratureVec():
input:
void
output:
result: $\quad 1 \times 1$
setEvaluator (fcn):
input:
fcn: $\quad 1 \times 1$ or $n \times 1$
output:
result: void
getEvaluator():
input:
void
output:

$$
\text { result: } \quad 1 \times 1 \text { or } n \times 1
$$

setLimits(limits):
input:
limits: $\quad 1 \times 2$ or $n \times 2$
output:
result: void
getLimits():
input:
void
output:

$$
\text { result: } \quad 1 \times 2 \text { or } n \times 2
$$

setTechnique(technique):
input:

$$
\text { technique: } \quad 1 \times 1 \text { or } n \times 1
$$

output:
result: void
getTechnique():
input:
void
output:

$$
\text { result: } \quad 1 \times 1 \text { or } n \times 1
$$

setMaxiter(maxiter):
input:

$$
\text { maxiter: } \quad 1 \times 1 \text { or } n \times 1
$$

output:
result: void
getMaxiter():
input:
void
output:
result: $\quad 1 \times 1$ or $n \times 1$
setAbstol(abstol):
input:
abstol: $\quad 1 \times 1$ or $n \times 1$
output:

$$
\text { result: } \quad \text { void }
$$

getAbstol():
input:
void
output:

$$
\text { result: } \quad 1 \times 1 \text { or } n \times 1
$$

setReltol(reltol):
input:

$$
\text { reltol: } \quad 1 \times 1 \text { or } n \times 1
$$

output:
result: void
getReltol():
input:
void
output:

$$
\text { result: } \quad 1 \times 1 \text { or } n \times 1
$$

setArgument ( $i, \arg$ ) (Quadrature() only):
input:

$$
\begin{aligned}
& i: \\
& \text { arg: } \\
& \text { anything }
\end{aligned}
$$

output:
result: void
getArgument (i) (Quadrature() only):
input:

$$
i: \quad 1 \times 1
$$

output:

$$
\text { result: } \quad 1 \times 1
$$

setArgument (pid, $i$, arg) (QuadratureVec() only)
input:

| pid: |  |
| ---: | :--- |
| $i:$ |  |
| $i \times 1$ |  |
| arg: |  |
|  |  |
|  | anything |

output:
result: void
getArgument (pid, i) (QuadratureVec() only):
input:

$$
\text { pid: } \quad n \times 1
$$

$i: \quad 1 \times 1$
output:
result: $\quad n \times 1$
getNarguments() (Quadrature() only):
input:
void
output:
result: $\quad 1 \times 1$
getDimension() (QuadratureVec() only):
input:
void
output:
result: $\quad 1 \times 1$
setTrace (trace):
input:
trace: $\quad 1 \times 1$ or $n \times 1$
output:
result: void
getTrace():
input:
void
output:
result: $\quad 1 \times 1$ or $n \times 1$
integrate():
input:
void
output:

$$
\text { result: } \quad 1 \times 1 \text { or } n \times 1
$$

value():
input:
void
output:
result: $\quad 1 \times 1$ or $n \times 1$
iterations():
input:
void
output:
result: $\quad 1 \times 1$ or $n \times 1$
converged():
input:
void
output:

$$
\text { result: } \quad 1 \times 1 \text { or } n \times 1
$$

errorcode():
input:
void
output:

$$
\text { result: } \quad 1 \times 1 \text { or } n \times 1
$$

errortext():
input:
void
output:

$$
\text { result: } \quad 1 \times 1 \text { or } n \times 1
$$

returncode():
input:
void
output:

$$
\text { result: } \quad 1 \times 1 \text { or } n \times 1
$$

query ():
input:
void
output:
void

## Diagnostics

When used incorrectly, all functions abort with an error. If integrate() runs into numerical difficulties, it returns a missing value and displays a warning message with details about the problem encountered.

## References

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## Also see

[M-2] class - Object-oriented programming (classes)
[M-2] pointers - Pointers
[M-5] deriv( ) - Numerical derivatives
[M-5] moptimize( ) - Model optimization
[M-5] optimize( ) - Function optimization
[M-4] Mathematical - Important mathematical functions

## Title

[M-5] range() — Vector over specified range

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

range ( $a, b$, delta) returns a column vector going from $a$ to $b$ in steps of abs (delta) ( $b \geq a$ ) or $-\mathrm{abs}($ delta $)(b<a)$.
rangen $(a, b, n)$ returns a round $(n) \times 1$ column vector going from $a$ to $b$ in round ( $n$ ) -1 steps. $a$ may be less than, equal to, or greater than $b$.

## Syntax

```
numeric colvector range( }a,b,numeric scalar delta
numeric colvector rangen(a,b, real scalar n)
```

where $a$ and $b$ are numeric scalars.

## Remarks and examples

range ( $0,1, .25$ ) returns ( $0 \backslash .25 \backslash .5 \backslash .75 \backslash 1$ ). The sign of the third argument does not matter; range ( $0,1,-.25$ ) returns the same thing. range ( $1,0, .25$ ) and range ( $1,0,-.25$ ) return $(1 \backslash .75 \backslash .5 \backslash .25 \backslash 0)$.

```
rangen(0, .5, 6) returns (0\.1\.2\.3\.4\.5). rangen(.5, 0, 6) returns (.5\.4\.3\.2\.1\0).
```

range( ) and rangen( ) may be used with complex arguments. range (1, 1i, .4) returns ( $1 \backslash .75+.25 i \backslash .5+.5 i \backslash .25+.75 i \backslash 1 i$ ). rangen $(1,1 i, 5)$ returns the same thing. For range (), only the distance of delta from zero matters, so range (1, 1i, .4i) would produce the same result, as would range (1, 1i, . 25+.312i).

## Conformability

range ( $a, b$, delta):

```
            a: }1\times
            b: 
delta: }1\times
result: }\quad1\times1,\quad\mathrm{ if }a=
    max(1+abs (b-a)/abs (delta),2) }\times1\mathrm{ , otherwise
```

rangen $(a, b, n)$ :

```
    a: 
    b: 
    n: n}<n=
    result: round(n) }\times
```


## Diagnostics

range ( $a, b$, delta) aborts with error if $a, b$, or delta contains missing, if abs $(b-a) / \mathrm{abs}$ (delta) results in overflow, or if $1+\mathrm{abs}(b-a) / \mathrm{abs}$ (delta) results in a vector that is too big given the amount of memory available.
range ( $a, b$, delta) returns a $1 \times 1$ result when $a=b$. In all other cases, the result is $2 \times 1$ or longer.
rangen $(a, b, n)$ aborts with error if round ( $n$ ) is less than 0 or missing.

## Also see

[M-4] Standard - Functions to create standard matrices

## Title

[M-5] rank() — Rank of matrix

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

$\operatorname{rank}(A)$ and $\operatorname{rank}(A$, tol $)$ return the rank of $A: m \times n$.

## Syntax

real scalar $\operatorname{rank}($ numeric matrix $A)$
real scalar rank(numeric matrix $A$, real scalar tol)

## Remarks and examples

The row rank of a matrix $A$ : $m \times n$ is the number of rows of $A$ that are linearly independent. The column rank is the number of columns that are linearly independent. The terms row rank and column rank, however, are used merely for emphasis. The ranks are equal, and the result is simply called the rank of $A$.
rank() calculates the rank by counting the number of nonzero singular values of the SVD of $A$, where nonzero is interpreted relative to a tolerance. rank() uses the same tolerance as pinv() (see [M-5] pinv()) and as svsolve() (see [M-5] svsolve()), and optional argument tol is specified in the same way as with those functions.

Thus if you were going to use rank() before calculating an inverse using pinv(), it would be better to skip rank() altogether and proceed to the pinv() step, because pinv() will return the rank, calculated as a by-product of calculating the inverse. Using rank() ahead of time, the SVD would be calculated twice.
rank() in general duplicates calculations; and, worse, if you are not planning on using pinv() or svsolve() but rather are planning on using some other function, the rank returned by rank() may disagree with the implied rank of whatever numerical method you subsequently use because each numerical method has its own precision and tolerances.

All that said, $\operatorname{rank}()$ is useful in interactive and pedagogical situations.

## Conformability

rank ( $A$, tol):

| A: | $m \times n$ |  |
| ---: | :--- | :--- |
| tol: | $1 \times 1 \quad$ (optional) |  |
| result: | $1 \times 1$ |  |

## Diagnostics

rank ( $A$ ) returns missing if $A$ contains missing values.

## Also see

[M-5] fullsvd() - Full singular value decomposition
[M-5] pinv() - Moore-Penrose pseudoinverse
[M-5] svd( ) - Singular value decomposition
[M-4] Matrix - Matrix functions

## Title

[M-5] $\mathbf{R e}()$ - Extract real or imaginary part
Description Syntax Conformability Diagnostics Also see

## Description

$\operatorname{Re}(Z)$ returns a real matrix containing the real part of $Z . Z$ may be real or complex.
$\operatorname{Im}(Z)$ returns a real matrix containing the imaginary part of $Z . Z$ may be a real or complex. If $Z$ is real, $\operatorname{Im}(Z)$ returns a matrix of zeros.

## Syntax

```
real matrix }\operatorname{Re}(\mathrm{ numeric matrix Z)
real matrix Im(numeric matrix Z)
```


## Conformability

$\operatorname{Re}(Z), \operatorname{Im}(Z):$

| $Z:$ | $r \times c$ |
| ---: | :--- |
| result: |  |
| $r \times c$ |  |

## Diagnostics

$\operatorname{Re}(Z)$, if $Z$ is real, literally returns $Z$ and not a copy of $Z$. This makes execution of $\operatorname{Re}()$ applied to real arguments instant.

## Also see

[M-5] C( ) - Make complex
[M-4] Scalar - Scalar mathematical functions
[M-4] Utility - Matrix utility functions

## Title

[M-5] reldif( ) — Relative/absolute difference

## Description Syntax Conformability Diagnostics Also see

## Description

reldif $(X, Y)$ returns the relative difference defined by

$$
r=\frac{|X-Y|}{|Y|+1}
$$

calculated element by element.
mreldif $(X, Y)$ returns the maximum relative difference and is equivalent to max (reldif $(X, Y)$ ). mreldifsym $(X)$ is equivalent to mreldif $\left(X^{\prime}, X\right)$ and so is a measure of how far the matrix is from being symmetric (Hermitian).
mreldifre $(X)$ is equivalent to mreldif $(\operatorname{Re}(X), X)$ and so is a measure of how far the matrix is from being real.

## Syntax

```
real matrix reldif(numeric matrix X, numeric matrix }Y\mathrm{ )
real scalar mreldif(numeric matrix X, numeric matrix }Y\mathrm{ )
real scalar mreldifsym(numeric matrix X)
real scalar mreldifre(numeric matrix X)
```


## Conformability

```
reldif(X,Y):
        X: r
        Y: r
    result: }\quadr\times
```

mreldif $(X, Y)$ :
$X: \quad r \times c$
$Y: \quad r \times c$
result: $\quad 1 \times 1$
mreldifsym $(X)$ :
$\begin{array}{rr}X: & n \times n \\ \text { result: } & \\ 1 \times 1\end{array}$
mreldifre ( $X$ ):

$$
\begin{aligned}
X: & r \times c \\
\text { result: } & 1 \times 1
\end{aligned}
$$

## Diagnostics

The relative difference function treats equal missing values as having a difference of 0 and different missing values as having a difference of missing (.):

```
reldif(., .) == reldif(.a, .a) == ... == reldif(.z, .z) == 0
reldif(., .a) == reldif(., .z) == ... == reldif(.y, .z) == .
```


## Also see

[M-4] Utility - Matrix utility functions

## Title

[M-5] rows( ) - Number of rows and number of columns

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

rows $(P)$ returns the number of rows of $P$.
$\operatorname{cols}(P)$ returns the number of columns of $P$.
length $(P)$ returns rows $(P) * \operatorname{cols}(P)$.

## Syntax

real scalar rows (transmorphic matrix $P$ )
real scalar cols (transmorphic matrix $P$ )
real scalar length (transmorphic matrix $P$ )

## Remarks and examples

length $(P)$ is typically used with vectors, as in
for (i=1; i<=length(x); i++) \{
... x[i] ...
\}

## Conformability

```
rows(P), cols(P), length(P):
    P: r
    result: }1\times
```


## Diagnostics

rows $(P)$, cols $(P)$, and length $(P)$ return a result that is greater than or equal to zero.

## Also see

[M-4] Utility - Matrix utility functions

## Title

[M-5] rowshape() - Reshape matrix

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

rowshape ( $T, r$ ) returns $T$ transformed into a matrix with trunc $(r)$ rows.
colshape ( $T, c$ ) returns $T$ having trunc ( $c$ ) columns.
In both cases, elements are assigned sequentially with the column index varying more rapidly. See [M-5] vec() for a function that varies the row index more rapidly.

## Syntax

$$
\begin{array}{ll}
\text { transmorphic matrix } & \text { rowshape (transmorphic matrix } T \text {, real scalar } r \text { ) } \\
\text { transmorphic matrix } & \text { colshape (transmorphic matrix } T, \text { real scalar } c \text { ) }
\end{array}
$$

## Remarks and examples

Remarks are presented under the following headings:
Example of rowshape()
Example of colshape( )

## Example of rowshape()

```
: A
        1 2 3 4
        11
        21 22 23 24
        31
        41 42 43 44
: rowshape(A,2)
    1 
\begin{tabular}{l|llllllll|}
1 & 11 & 12 & 13 & 14 & 21 & 22 & 23 & 24 \\
2 & 31 & 32 & 33 & 34 & 41 & 42 & 43 & 44
\end{tabular}
```


## Example of colshape()

: colshape (A, 2)

| 1 | 2 |  |
| :--- | :---: | :---: |
|  | 11 | 12 |
| 2 | 13 | 14 |
| 3 | 21 | 22 |
| 4 | 23 | 24 |
| 5 | 31 | 32 |
| 6 | 33 | 34 |
| 7 | 41 | 42 |
| 8 | 43 | 44 |
|  |  |  |

## Conformability

```
rowshape(T,r):
    T: }\quad\mp@subsup{r}{0}{}\times\mp@subsup{c}{0}{
    r: }1\times
    result: }\quadr\times\mp@subsup{r}{0}{}\mp@subsup{c}{0}{}/
colshape(T, c):
        T: }\quad\mp@subsup{r}{0}{}\times\mp@subsup{c}{0}{
        c: 
    result: }\quad\mp@subsup{r}{0}{}\mp@subsup{c}{0}{}/c\times
```


## Diagnostics

Let $r_{0}$ and $c_{0}$ be the number of rows and columns of $T$.
rowshape () aborts with error if $r_{0} \times c_{0}$ is not evenly divisible by trunc $(r)$. colshape() aborts with error if $r_{0} \times c_{0}$ is not evenly divisible by trunc (c).

## Also see

[M-4] Manipulation - Matrix manipulation

## Title

[M-5] runiform( ) - Uniform and nonuniform pseudorandom variates

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | References | Also see |  |

## Description

runiform $(r, c)$ returns an $r \times c$ real matrix containing uniformly distributed random variates over $(0,1)$. runiform() is the same function as Stata's runiform() function.
runiform ( $r, c, a, b$ ) returns an $i r \times j c$ real matrix containing uniformly distributed random variates over $(a, b)$. The matrices $a$ and $b$ must be r-conformable, where $i=\max$ (rows ( $a$ ), rows ( $b$ ) ) and $j=\max (\operatorname{cols}(a), \operatorname{cols}(b))$.
runiformint ( $r, c, a, b$ ) returns an $i r \times j c$ real matrix containing uniformly distributed random integer variates over $[a, b]$. The matrices $a$ and $b$ must be r-conformable, where $i=$ $\max (\operatorname{rows}(a)$, rows $(b))$ and $j=\max (\operatorname{cols}(a)$, cols $(b)$ ).
rseed() returns the current random-variate seed in an encrypted string form.
rseed(newseed) sets the seed: an integer can be specified. rseed(newseed) has the same effect as Stata's set seed command; see [R] set seed.
rngstate() returns the current state of the random-number generator. rngstate() returns the same thing as Stata's c (rngstate); see [R] set seed and [P] creturn.
rngstate (newstate) sets the state of the random-number generator using a string previously obtained from rngstate(). rngstate(newstate) has the same effect as Stata's set rngstate newstate; see $[R]$ set seed.
rbeta $(r, c, a, b)$ returns an $i r \times j c$ real matrix containing beta random variates. The real-valued matrices $a$ and $b$ contain the beta shape parameters. The matrices $a$ and $b$ must be r-conformable, where $i=\max (\operatorname{rows}(a)$, rows $(b))$ and $j=\max (\operatorname{cols}(a), \operatorname{cols}(b))$.
rbinomial $(r, c, n, p)$ returns an $i r \times j c$ real matrix containing binomial random variates. The real-valued matrices $n$ and $p$ contain the number of trials and the probability parameters, respectively. The matrices $n$ and $p$ must be r-conformable, where $i=\max (\operatorname{rows}(n)$, $\operatorname{rows}(p)$ ) and $j=\max (\operatorname{cols}(n), \operatorname{cols}(p))$.
rcauchy ( $r, c, a, b$ ) returns an $i r \times j c$ real matrix containing Cauchy random variates. The real-valued matrices $a$ and $b$ contain the Cauchy location and scale parameters, respectively. The matrices $a$ and $b$ must be r-conformable, where $i=\max (\operatorname{rows}(a)$, rows $(b))$ and $j=\max (\operatorname{cols}(a)$, cols (b)). $\operatorname{rchi2}(r, c, d f)$ returns an $i r \times j c$ real matrix containing $\chi^{2}$ random variates. The real-valued matrix $d f$ contains the degrees of freedom parameters, where $i=\operatorname{rows}(d f)$ and $j=\operatorname{cols}(d f)$.
rdiscrete $(r, c, p)$ returns an $r \times c$ real matrix containing random variates from the discrete distribution specified by the probabilities in the vector $p$ of length $k$. The range of the discrete variates is $1,2, \ldots, k$, where $2 \leq k \leq 10000$. The alias method of Walker (1977) is used to sample from the discrete distribution.
rexponential ( $r, c, b$ ) returns an $i r \times j c$ real matrix containing exponential random variates. The real-valued matrix $b$ contains the scale parameters, where $i=\operatorname{rows}(b)$ and $j=\operatorname{cols}(b)$.
rgamma ( $r, c, a, b$ ) returns an $i r \times j c$ real matrix containing gamma random variates. The real-valued matrices $a$ and $b$ contain the gamma shape and scale parameters, respectively. The matrices $a$ and $b$ must be r-conformable, where $i=\max (\operatorname{rows}(a)$, rows $(b)$ ) and $j=\max (\operatorname{cols}(a)$, cols $(b)$ ).
rhypergeometric $(r, c, N, K, n)$ returns an $i r \times j c$ real matrix containing hypergeometric random variates. The integer-valued matrix $N$ contains the population sizes, the integer-valued matrix $K$ contains the number of elements in each population that have the attribute of interest, and the integer-valued matrix $n$ contains the sample size. The matrices $N, K$, and $n$ must be r-conformable, where $i=\max (\operatorname{rows}(N)$, $\operatorname{rows}(K)$, rows $(n))$ and $j=\max (\operatorname{cols}(N), \operatorname{cols}(K), \operatorname{cols}(n))$.
rigaussian $(r, c, m, a)$ returns an $i r \times j c$ real matrix containing inverse Gaussian random variates. The real-valued matrices $m$ and $a$ contain the mean and shape parameters, respectively. The matrices $m$ and $a$ must be r-conformable, where $i=\max$ (rows $(m)$, rows $(a)$ ) and $j=$ $\max (\operatorname{cols}(m), \operatorname{cols}(a))$.
rlaplace $(r, c, m, b)$ returns an $i r \times j c$ real matrix containing Laplace random variates. The real-valued matrices $m$ and $b$ contain the mean and scale parameters, respectively. The matrices $m$ and $b$ must be r-conformable, where $i=\max (\operatorname{rows}(m)$, rows $(b)$ ) and $j=\max (\operatorname{cols}(m)$, cols $(b)$ ).
rlogistic $(r, c)$ returns an $r \times c$ real matrix containing logistic random variates with mean zero and standard deviation $\pi / \sqrt{3}$.
rlogistic ( $r, c, s$ ) returns an $i r \times j c$ real matrix containing mean-zero logistic random variates. The real-valued matrix $s$ contains scale parameters, where $i=\operatorname{rows}(s)$ and $j=\operatorname{cols}(s)$.
rlogistic $(r, c, m, s)$ returns an $i r \times j c$ real matrix containing logistic random variates. The real-valued matrices $m$ and $s$ contain the mean and scale parameters, respectively. The matrices $m$ and $s$ must be r-conformable, where $i=\max (\operatorname{rows}(m), \operatorname{rows}(s))$ and $j=\max (\operatorname{cols}(m), \operatorname{cols}(s))$.
rnbinomial ( $r, c, n, p$ ) returns an $i r \times j c$ real matrix containing negative binomial random variates. When the elements of the matrix $n$ are integer-valued, rnbinomial () returns the number of failures before the $n$th success, where the probability of success on a single draw is contained in the real-valued matrix $p$. The elements of $n$ can also be nonintegral but must be positive. The matrices $n$ and $p$ must be r-conformable, where $i=\max (\operatorname{rows}(n), \operatorname{rows}(p))$ and $j=\max (\operatorname{cols}(n), \operatorname{cols}(p))$.
rnormal ( $r, c, m, s$ ) returns an $i r \times j c$ real matrix containing normal (Gaussian) random variates. The real-valued matrices $m$ and $s$ contain the mean and standard deviation parameters, respectively. The matrices $m$ and $s$ must be r-conformable, where $i=\max$ (rows $(m)$, rows $(s)$ ) and $j=$ $\max (\operatorname{cols}(m), \operatorname{cols}(s))$.
rpoisson ( $r, c, m$ ) returns an $i r \times j c$ real matrix containing Poisson random variates. The real-valued matrix $m$ contains the Poisson mean parameters, where $i=\operatorname{rows}(m)$ and $j=\operatorname{cols}(m)$.
$\mathrm{rt}(r, c, d f)$ returns an $i r \times j c$ real matrix containing Student's $t$ random variates. The real-valued matrix $d f$ contains the degrees-of-freedom parameters, where $i=\operatorname{rows}(d f)$ and $j=\operatorname{cols}(d f)$.
rweibull ( $r, c, a, b$ ) returns an $i r \times j c$ real matrix containing Weibull random variates. The real-valued matrices $a$ and $b$ contain the shape and scale parameters, respectively. The matrices $a$ and $b$ must be r-conformable, where $i=\max (\operatorname{rows}(a)$, rows $(b))$ and $j=\max (\operatorname{cols}(a)$, cols $(b))$.
rweibull ( $r, c, a, b, g$ ) returns an $i r \times j c$ real matrix containing Weibull random variates. The real-valued matrices $a, b$, and $g$ contain the shape, scale, and location parameters, respectively. The matrices $a, b$, and $g$ must be r-conformable, where $i=\max$ (rows $(a)$, rows $(b)$, rows $(g)$ ) and $j=\max (\operatorname{cols}(a), \operatorname{cols}(b), \operatorname{cols}(g))$.
rweibullph ( $r, c, a, b$ ) returns an $i r \times j c$ real matrix containing Weibull (proportional hazards) random variates. The real-valued matrices $a$ and $b$ contain the shape and scale parameters, respectively. The matrices $a$ and $b$ must be r-conformable, where $i=\max$ (rows ( $a$ ), rows(b)) and $j=$ $\max (\operatorname{cols}(a), \operatorname{cols}(b))$.
rweibullph $(r, c, a, b, g)$ returns an $i r \times j c$ real matrix containing Weibull (proportional hazards) random variates. The real-valued matrices $a$, $b$, and $g$ contain the shape, scale, and location parameters, respectively. The matrices $a$, $b$, and $g$ must be r-conformable, where $i=\max (\operatorname{rows}(a)$, rows $(b)$, rows $(g))$ and $j=\max (\operatorname{cols}(a), \operatorname{cols}(b), \operatorname{cols}(g))$.

## Syntax

| real matrix | runiform (real scalar $r$, real scalar $c$ ) |
| :---: | :---: |
| real matrix | runiform(real scalar $r$, real scalar $c$, real matrix $a$, real matrix $b$ ) |
| real matrix | runiformint (real scalar $r$, real scalar $c$, real matrix $a$, real matrix $b$ ) |
| string scalar | rseed() |
| void | rseed (real scalar newseed) |
| string scalar | rngstate() |
| void | rngstate (string scalar newstate) |
| real matrix | rbeta (real scalar $r$, real scalar $c$, real matrix $a$, real matrix $b$ ) |
| real matrix | rbinomial (real scalar r, real scalar $c$, real matrix $n$, real matrix $p$ ) |
| real matrix | rcauchy (real scalar $r$, real scalar $c$, real matrix $a$, real matrix $b$ ) |
| real matrix | rchi2 (real scalar r, real scalar $c$, real matrix df) |
| real matrix | rdiscrete(real scalar r, real scalar c, real colvector $p$ ) |
| real matrix | rexponential (real scalar $r$, real scalar $c$, real matrix $b$ ) |
| real matrix | rgamma (real scalar $r$, real scalar $c$, real matrix $a$, real matrix $b$ ) |

real matrix rhypergeometric (real scalar r, real scalar $c$, real matrix $N$, real matrix $K$, real matrix $n$ )
real matrix rigaussian(real scalar $r$, real scalar $c$, real matrix $m$, real matrix a)
real matrix rlaplace (real scalar real scalar $c$, real matrix $m$, real matrix $b$ )
real matrix rlogistic (real scalar r, real scalar $c$ )
real matrix rlogistic (real scalar real scalar $c$, real matrix $s$ ) real matrix rlogistic (real scalar $r$, real scalar $c$, real matrix $m$, real matrix $s$ ) real matrix rnbinomial (real scalar real scalar $c$, real matrix $n$, real matrix $p$ ) real matrix rnormal(real scalar $r$, real scalar $c$, real matrix $m$, real matrix $s$ ) real matrix rpoisson(real scalar $r$, real scalar $c$, real matrix $m$ ) real matrix $\mathrm{rt}($ real scalar $r$, real scalar $c$, real matrix $d f$ )
real matrix rweibull (real scalar $r$, real scalar $c$, real matrix $a$, real matrix $b$ ) real matrix rweibull (real scalar r, real scalar $c$, real matrix $a$, real matrix $b$, real matrix $g$ )
real matrix rweibullph (real scalar $r$, real scalar $c$, real matrix $a$, real matrix $b$ ) real matrix rweibullph (real scalar $r$, real scalar $c$, real matrix $a$, real matrix $b$, real matrix $g$ )

## Remarks and examples

The functions described here generate random variates. The parameter limits for each generator are the same as those documented for Stata's random-number functions, except for rdiscrete(), which has no Stata equivalent.

In the example below, we generate and summarize 1,000 random normal deviates with a mean of 3 and standard deviation of 1 .

```
: rseed(13579)
: x = rnormal(1000, 1, 3, 1)
: meanvariance(x)
    1
    2.99162691
    1.056033182
```

The next example uses a $1 \times 3$ vector of gamma shape parameters to generate a $1000 \times 3$ matrix of gamma random variates, X .

```
a = (0.5, 1.5, 2.5)
rseed(13579)
X = rgamma(1000, 1, a, 1)
mean(X)
```

1

diagonal(variance(X))'
$\begin{array}{lll}1 & 2 & 3\end{array}$
1

| .5082196561 | 1.434504411 | 2.512575559 |
| :--- | :--- | :--- |

The first column of $X$ contains gamma variates with shape parameter 0.5 , the second column contains gamma variates with shape parameter 1.5 , and the third column contains gamma variates with shape parameter 2.5.

Below we generate a $4 \times 3$ matrix of beta variates where we demonstrate the use of two r-conformable parameter matrices, a and b .

```
: a = (0.5, 1.5, 2.5)
: b = (0.5, 0.75, 1.0\1.25, 1.5, 1.75)
: rseed(13579)
: rbeta(2, 1, a, b)
```

    \(\begin{array}{lll}1 & 2 & 3\end{array}\)
    | .8389820448 | .9707672865 | .2122592494 |
| :--- | :--- | :--- |
| .5997013245 | .6617211509 | .8775212495 |
| .9552933701 | .1133821372 | .8006242906 |
| .2279075363 | .4298247049 | .6683477165 |

The $4 \times 3$ shape-parameter matrices used to generate these beta variates are given below:

```
: J (2, 1, J J (rows (b), 1, a))
\begin{tabular}{l|lll|}
\cline { 2 - 4 } & \begin{tabular}{lll}
.5 & 1.5 & 2.5 \\
2 & .5 & 1.5 \\
3 & 2.5 \\
4 & .5 & 1.5 \\
\hline
\end{tabular} & 2.5 \\
& & 1.5 & 2.5 \\
\hline
\end{tabular}
```

$$
: J(2,1, b)
$$

| 1 | 2 | 3 |  |
| :--- | ---: | ---: | ---: |
| 1 | .5 | .75 | 1 |
| 2 | 1.25 | 1.5 | 1.75 |
| 3 | .5 | .75 | 1 |
| 4 | 1.25 | 1.5 | 1.75 |
|  |  |  |  |

This example illustrates how to restart a random-number generator from a particular point in its sequence. We begin by setting the seed and drawing some uniform variates.

```
rseed(12345)
x = runiform(1,3)
x
    1 2 3
1
    . 3576297229 . 400442617 . }68938331
```

We save off the current state of the random-number generator, so that we can subsequently return to this point in the sequence.

```
: rngstate = rngstate()
```

Having saved off the state, we draw some more numbers from the sequence.

```
x = runiform(1,3)
x
```

    \(\begin{array}{lll}1 & 2 & 3\end{array}\)
    $1.5597355706 \quad .574451294 \quad .2076905269$

Now we restore the state of the random-number generator to where it was and obtain the same numbers from the sequence.

```
rngstate(rngstate)
x = runiform(1,3)
x
    1 2 3
1
    .5597355706 . 574451294 . }207690526
```


## Conformability

```
runiform(r, c):
    r: 
    c: 
    result: }\quadr\times
runiform(r, c, a, b):
    r: 
    c: 
    a: }\quad1\times1\mathrm{ or }i\times1\mathrm{ or }1\timesj\mathrm{ or }i\times
    b: }\quad1\times1\mathrm{ or }i\times1\mathrm{ or }1\timesj\mathrm{ or }i\times
    result: }\quadr\timesc\mathrm{ or ir }\timesc\mathrm{ or }r\timesjc\mathrm{ or ir }\timesj
runiformint (r, c, a,b):
    r: }1\times
    c: 
    a: }\quad1\times1\mathrm{ or }i\times1\mathrm{ or }1\timesj\mathrm{ or }i\times
    b: }\quad1\times1\mathrm{ or }i\times1\mathrm{ or }1\timesj\mathrm{ or }i\times
    result: }\quadr\timesc\mathrm{ or ir }\timesc\mathrm{ or }r\timesjc\mathrm{ or ir }\timesj
rseed():
    result: }1\times
rseed(newseed):
newseed: }1\times
    result: void
rngstate():
    result: }1\times
rngstate(newstate):
newstate: }1\times
    result: void
rbeta(r, c, a, b):
    r: }1\times
    c: 
    a: }\quad1\times1\mathrm{ or }i\times1\mathrm{ or }1\timesj\mathrm{ or }i\times
    b: }\quad1\times1\mathrm{ or }i\times1\mathrm{ or }1\timesj\mathrm{ or }i\times
    result: }\quadr\timesc\mathrm{ or ir }\timesc\mathrm{ or }r\timesjc\mathrm{ or ir }\timesj
```

rbinomial $(r, c, n, p)$ :
$r: \quad 1 \times 1$
$c: \quad 1 \times 1$
$n$ : $\quad 1 \times 1$ or $i \times 1$ or $1 \times j$ or $i \times j$
p: $\quad 1 \times 1$ or $i \times 1$ or $1 \times j$ or $i \times j$
result: $\quad r \times c$ or $i r \times c$ or $r \times j c$ or $i r \times j c$
rcauchy $(r, c, a, b)$ :
$r: \quad 1 \times 1$
c: $\quad 1 \times 1$
a: $\quad 1 \times 1$ or $i \times 1$ or $1 \times j$ or $i \times j$
b: $\quad 1 \times 1$ or $i \times 1$ or $1 \times j$ or $i \times j$
result: $\quad r \times c$ or $i r \times c$ or $r \times j c$ or $i r \times j c$
$\operatorname{rchi2}(r, c, d f)$ :
$r: \quad 1 \times 1$
$c: \quad 1 \times 1$
$d f: \quad i \times j$
result: $\quad i r \times j c$
rdiscrete ( $r, c, p$ ):
$r: \quad 1 \times 1$
c: $\quad 1 \times 1$
$p: \quad k \times 1$
result: $\quad r \times c$
rexponential $(r, c, b)$ :
$r: \quad 1 \times 1$
$c: \quad 1 \times 1$
b: $\quad 1 \times 1$ or $i \times 1$ or $1 \times j$ or $i \times j$
result: $\quad r \times c$ or $i r \times c$ or $r \times j c$ or $i r \times j c$
$\operatorname{rgamma}(r, c, a, b)$ :
$r: \quad 1 \times 1$
c: $\quad 1 \times 1$
a: $\quad 1 \times 1$ or $i \times 1$ or $1 \times j$ or $i \times j$
b: $\quad 1 \times 1$ or $i \times 1$ or $1 \times j$ or $i \times j$
result: $\quad r \times c$ or $i r \times c$ or $r \times j c$ or $i r \times j c$
rhypergeometric $(r, c, N, K, n)$ :
$r: \quad 1 \times 1$
c: $\quad 1 \times 1$
$N: \quad 1 \times 1$
K: $\quad 1 \times 1$ or $i \times 1$ or $1 \times j$ or $i \times j$
$n$ : $\quad 1 \times 1$ or $i \times 1$ or $1 \times j$ or $i \times j$
result: $\quad r \times c$ or $i r \times c$ or $r \times j c$ or $i r \times j c$
rigaussian $(r, c, m, a)$ :
$r: \quad 1 \times 1$
c: $\quad 1 \times 1$
m: $\quad 1 \times 1$ or $i \times 1$ or $1 \times j$ or $i \times j$
a: $\quad 1 \times 1$ or $i \times 1$ or $1 \times j$ or $i \times j$
result: $\quad r \times c$ or $i r \times c$ or $r \times j c$ or $i r \times j c$
rlaplace $(r, c, m, b)$ :
$r: \quad 1 \times 1$
$c: \quad 1 \times 1$
$m: \quad 1 \times 1$ or $i \times 1$ or $1 \times j$ or $i \times j$
b: $\quad 1 \times 1$ or $i \times 1$ or $1 \times j$ or $i \times j$
result: $\quad r \times c$ or $i r \times c$ or $r \times j c$ or $i r \times j c$
rlogistic $(r, c)$ :
$r: \quad 1 \times 1$
c: $\quad 1 \times 1$
result: $\quad r \times c$
rlogistic $(r, c, s)$ :
$r: \quad 1 \times 1$
$c: \quad 1 \times 1$
$s: \quad 1 \times 1$ or $i \times 1$ or $1 \times j$ or $i \times j$
result: $\quad r \times c$ or $i r \times c$ or $r \times j c$ or $i r \times j c$
rlogistic $(r, c, m, s)$ :
$r: \quad 1 \times 1$
$c: \quad 1 \times 1$
m: $\quad 1 \times 1$ or $i \times 1$ or $1 \times j$ or $i \times j$
$s: \quad 1 \times 1$ or $i \times 1$ or $1 \times j$ or $i \times j$
result: $\quad r \times c$ or $i r \times c$ or $r \times j c$ or $i r \times j c$
rnbinomial $(r, c, n, p)$ :
$r: \quad 1 \times 1$
c: $\quad 1 \times 1$
$n: \quad 1 \times 1$ or $i \times 1$ or $1 \times j$ or $i \times j$
$p: \quad 1 \times 1$ or $i \times 1$ or $1 \times j$ or $i \times j$
result: $\quad r \times c$ or $i r \times c$ or $r \times j c$ or $i r \times j c$
rnormal ( $r, c, m, s$ ):
$r: \quad 1 \times 1$
$c: \quad 1 \times 1$
m: $\quad 1 \times 1$ or $i \times 1$ or $1 \times j$ or $i \times j$
s: $\quad 1 \times 1$ or $i \times 1$ or $1 \times j$ or $i \times j$
result: $\quad r \times c$ or $i r \times c$ or $r \times j c$ or $i r \times j c$
rpoisson $(r, c, m)$ :
$r: \quad 1 \times 1$
$c: \quad 1 \times 1$
$m: \quad i \times j$
result: $\quad i r \times j c$
$\operatorname{rt}(r, c, d f)$ :
$r: \quad 1 \times 1$
c: $\quad 1 \times 1$
$d f: \quad 1 \times 1$ or $i \times 1$ or $1 \times j$ or $i \times j$
result: $\quad r \times c$ or $i r \times c$ or $r \times j c$ or $i r \times j c$

```
rweibull (r, c,a,b):
    r: 1 < 1
    c: 
    a: }\quad1\times1\mathrm{ or }i\times1\mathrm{ or }1\timesj\mathrm{ or }i\times
    b: }\quad1\times1\mathrm{ or }i\times1\mathrm{ or }1\timesj\mathrm{ or }i\times
result: }\quadr\timesc\mathrm{ or ir }\timesc\mathrm{ or }r\timesjc\mathrm{ or ir }\timesj
rweibull(r, c,a,b,g):
    r: 1 < 1
    c: }\quad1\times
    a: }\quad1\times1\mathrm{ or }i\times1\mathrm{ or }1\timesj\mathrm{ or }i\times
    b: }\quad1\times1\mathrm{ or }i\times1\mathrm{ or }1\timesj\mathrm{ or }i\times
    g: 1}1\times1\mathrm{ or }i\times1\mathrm{ or }1\timesj\mathrm{ or }i\times
result: }\quadr\timesc\mathrm{ or ir }\timesc\mathrm{ or }r\timesjc\mathrm{ or ir }\timesj
rweibullph(r, c, a, b):
    r: }1\times
    c: 
    a: }\quad1\times1\mathrm{ or }i\times1\mathrm{ or }1\timesj\mathrm{ or }i\times
    b: }\quad1\times1\mathrm{ or }i\times1\mathrm{ or }1\timesj\mathrm{ or }i\times
result: }\quadr\timesc\mathrm{ or ir }\timesc\mathrm{ or }r\timesjc\mathrm{ or ir }\timesj
rweibullph(r, c,a,b,g):
    r: 1 < 1
    c: 
    a: }\quad1\times1\mathrm{ or }i\times1\mathrm{ or }1\timesj\mathrm{ or }i\times
    b: 1 1 1 or }i\times1\mathrm{ or }1\timesj\mathrm{ or }i\times
    g: 1 1 1 or }i\times1\mathrm{ or }1\timesj\mathrm{ or }i\times
result: }\quadr\timesc\mathrm{ or ir }\timesc\mathrm{ or }r\timesjc\mathrm{ or ir }\timesj
```


## Diagnostics

All random-variate generators abort with an error if $r<0$ or $c<0$.
rseed (seed) aborts with error if a string seed is specified and it is malformed.
rngstate (newstate) aborts with error if the specified newstate is malformed, which almost certainly is the case if newstate was not previously obtained from rngstate().
runiform $(r, c, a, b)$, runiformint $(r, c, a, b), \operatorname{rnormal}(r, c, m, s), \operatorname{rbeta}(r, c, a, b)$, rbinomial $(r, c, n, p)$, rcauchy $(r, c, a, b)$, rgamma $(r, c, a, b)$, rhypergeometric $(r$, $c, N, K, n$ ), rigaussian $(r, c, m, a)$, rlaplace $(r, c, m, b)$, rlogistic $(r, c, m, s)$, rnbinomial ( $r, c, n, p$ ), rweibull ( $r, c, a, b$ ), rweibull ( $r, c, a, b, g$ ), rweibullph ( $r$, $c, a, b)$, and rweibullph $(r, c, a, b, g)$ abort with an error if the parameter matrices do not conform. See $r$-conformability in [M-6] Glossary for rules on matrix conformability.
rdiscrete() aborts with error if the probabilities in $p$ are not in $[0,1]$ or do not sum to 1 .

## References

Gould, W. W. 2012a. Using Stata's random-number generators, part 1. The Stata Blog: Not Elsewhere Classified. http://blog.stata.com/2012/07/18/using-statas-random-number-generators-part-1/.
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Walker, A. J. 1977. An efficient method for generating discrete random variables with general distributions. ACM Transactions on Mathematical Software 3: 253-256. https://doi.org/10.1145/355744.355749.

## Also see

[M-4] Standard - Functions to create standard matrices
[M-4] Statistical - Statistical functions

## Title

[M-5] runningsum() — Running sum of vector

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

runningsum ( $x$ ) returns a vector of the same dimension as $x$ containing the running sum of $x$. Missing values are treated as contributing zero to the sum.
runningsum ( $x$, missing) does the same but lets you specify how missing values are treated. runningsum ( $x, 0$ ) is the same as runningsum $(x)$. runningsum ( $x, 1$ ) specifies that missing values are to turn the sum to missing where they occur.
quadrunningsum ( $x$ ) and quadrunningsum ( $x$, missing) do the same but perform the accumulation in quad precision.
_runningsum ( $y, x[$, missing $]$ ) and _quadrunningsum ( $y, x[$, missing $]$ ) work the same way, except that rather than returning the running-sum vector, they store the result in $y$. This method is slightly more efficient when $y$ is a view.

## Syntax

| numeric vector | runningsum (numeric vector $x[$, missing $])$ |
| :--- | :--- |
| numeric vector | quadrunningsum (numeric vector $x[$, missing $])$ |
| void | _runningsum $(y$, numeric vector $x[$, missing $])$ |
| void | -quadrunningsum $(y$, numeric vector $x[$, missing $])$ |

where optional argument missing is a real scalar that determines how missing values in $x$ are treated:

1. Specifying missing as 0 is equivalent to not specifying the argument; missing values in $x$ are treated as contributing 0 to the sum.
2. Specifying missing as 1 specifies that missing values in $x$ are to be treated as missing values and turn the sum to missing.

## Remarks and examples

The running sum of $(1,2,3)$ is $(1,3,6)$.
All functions return the same type as the argument, real if argument is real, complex if complex.

## Conformability

runningsum ( $x$, missing), quadrunningsum ( $x$, missing):

| $x:$ | $r \times 1$ | or | $1 \times c$ |  |
| ---: | :--- | :--- | :--- | :--- |
| missing: | $1 \times 1$ |  |  | (optional) |
| result: | $r \times 1$ | or | $1 \times c$ |  |

_runningsum( $y, x$, missing), _quadrunningsum ( $y, x$, missing):
input:

$$
\begin{array}{ccccl}
x: & r \times 1 & \text { or } & 1 \times \mathrm{c} & \\
y: & r \times 1 & \text { or } & 1 \times \mathrm{c} & \text { (contents irrelevant) } \\
\text { missing: } & 1 \times 1 & & & \text { (optional) }
\end{array}
$$

output:

$$
y: \quad r \times 1 \quad \text { or } \quad 1 \times \mathrm{c}
$$

## Diagnostics

If missing $=0$, missing values are treated as contributing zero to the sum; they do not turn the sum to missing. Otherwise, missing values turn the sum to missing.
_runningsum ( $y, x$, missing) and _quadrunningsum ( $y, x$, missing) abort with error if $y$ is not p-conformable with $x$ and of the same eltype. The contents of $y$ are irrelevant.

## Also see

[M-5] sum () - Sums
[M-4] Mathematical - Important mathematical functions
[M-4] Utility - Matrix utility functions

## Title

[M-5] schurd() — Schur decomposition

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Reference | Also see |  |

## Description

$\operatorname{sch} u r d(X, T, Q)$ computes the Schur decomposition of a square, numeric matrix, $X$, returning the Schur-form matrix, $T$, and the matrix of Schur vectors, $Q . Q$ is orthogonal if $X$ is real and unitary if $X$ is complex.
$\quad \operatorname{sch} \operatorname{cor}(X, Q)$ does the same thing as schurd(), except that it returns $T$ in $X$.
schurdgroupby ( $X, f, T, Q, w, m$ ) computes the Schur decomposition and the eigenvalues of a square, numeric matrix, $X$, and groups the results according to whether a condition on each eigenvalue is satisfied. schurdgroupby () returns the Schur-form matrix in $T$, the matrix of Schur vectors in $Q$, the eigenvalues in $w$, and the number of eigenvalues for which the condition is true in $m . f$ is a pointer of the function that implements the condition on each eigenvalue, as discussed below.
_schurdgroupby $(X, f, Q, w, m)$ does the same thing as schurdgroupby() except that it returns $T$ in $X$.
_schurd_la() and _schurdgroupby_la() are the interfaces into the LAPACK routines used to implement the above functions; see [M-1] LAPACK. Their direct use is not recommended.

## Syntax

```
void schurd(X,T,Q)
void _schurd(X,Q)
void schurdgroupby(X,f,T,Q,w,m)
void _schurdgroupby (X,f, Q,w,m)
```

where inputs are
$X$ : numeric matrix
$f$ : pointer scalar (points to a function used to group eigenvalues)
and outputs are

| $T:$ | numeric matrix | (Schur-form matrix) |
| :--- | :--- | :--- |
| $Q:$ | numeric matrix | (orthogonal or unitary) |
| $w:$ | numeric vector of eigenvalues |  |
| $m:$ | real scalar $\quad$ (the number of eigenvalues satisfy the grouping condition) |  |

## Remarks and examples

Remarks are presented under the following headings:
Schur decomposition
Grouping the results

## Schur decomposition

Many algorithms begin by obtaining the Schur decomposition of a square matrix.
The Schur decomposition of matrix $\mathbf{X}$ can be written as

$$
\mathbf{Q}^{\prime} \times \mathbf{X} \times \mathbf{Q}=\mathbf{T}
$$

where $\mathbf{T}$ is in Schur form, $\mathbf{Q}$, the matrix of Schur vectors, is orthogonal if $\mathbf{X}$ is real or unitary if $\mathbf{X}$ is complex.

A real, square matrix is in Schur form if it is block upper triangular with $1 \times 1$ and $2 \times 2$ diagonal blocks. Each $2 \times 2$ diagonal block has equal diagonal elements and opposite sign off-diagonal elements. A complex, square matrix is in Schur form if it is upper triangular. The eigenvalues of $\mathbf{X}$ are obtained from the Schur form by a few quick computations.

In the example below, we define X , obtain the Schur decomposition, and list T .

```
X=(.31,.69,.13,.56\.31,.5,.72,.42\.68,.37,.71,.8\.09,.16,.83,.9)
: schurd(X, T=., Q=.)
T
    1 2 3 4
l 1 rrrre
l 1 rrrre
l 1 rrrre
```


## Grouping the results

In many applications, there is a stable solution if the modulus of an eigenvalue is less than one and an explosive solution if the modulus is greater than or equal to one. One frequently handles these cases differently and would group the Schur decomposition results into a block corresponding to stable solutions and a block corresponding to explosive solutions.

In the following example, we use schurdgroupby () to put the stable solutions first. One of the arguments to schurdgroupby () is a pointer to a function that accepts a complex scalar argument, an eigenvalue, and returns 1 to select the eigenvalue and 0 otherwise. Here isstable() returns 1 if the eigenvalue is less than 1 :

```
real scalar isstable(scalar p)
{
return((abs(p)<1))
}
```

Using this function to group the results, we see that the Schur-form matrix has been reordered.

```
: schurdgroupby(X, &isstable(), T=., Q=., W=., m=.)
T
```

|  | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| 1 | -. 0766307549 | . 445046622 | . 3029641608 | -. 0341867415 |
| 2 | -. 3473539401 | -. 0766307549 | -. 1036266286 | . 0799058566 |
| 3 | 0 | 0 | . 4658398402 | -. 3475944606 |
| 4 | 0 | 0 | 0 | 2.10742167 |

Listing the moduli of the eigenvalues reveals that they are grouped into stable and explosive groups.

```
abs(w)
            1 2 % 3 4
1 .4005757984 .4005757984 .4658398402 2.10742167
```

m contains the number of stable solutions

## Conformability

$\operatorname{schurd}(X, T, Q):$
input:

$$
X: \quad n \times n
$$

output:
T: $\quad n \times n$
$Q: \quad n \times n$
$-\operatorname{schurd}(X, Q)$ :
input:

$$
X: \quad n \times n
$$

output:
$X: \quad n \times n$
$Q: \quad n \times n$
schurdgroupby $(X, f, T, Q, w, m)$ :
input:

$$
\begin{array}{rll}
X: & & n \times n \\
f: & & 1 \times 1
\end{array}
$$

output:
T: $\quad n \times n$
Q: $\quad n \times n$
$w: \quad 1 \times n$
$m: \quad 1 \times 1$
_schurdgroupby $(X, f, Q, w, m)$ :
input:

$$
X: \quad n \times n
$$

$$
f: \quad 1 \times 1
$$

output:
$X: \quad n \times n$
$Q: \quad n \times n$
$w: \quad 1 \times n$
$m: \quad 1 \times 1$

## Diagnostics

_schurd() and _schurdgroupby() abort with error if $X$ is a view.
schurd(), _schurd(), schurdgroupby(), and _schurdgroupby() return missing results if $X$ contains missing values.
schurdgroupby () groups the results via a matrix transform. If the problem is very ill conditioned, applying this matrix transform can cause changes in the eigenvalues. In extreme cases, the grouped eigenvalues may no longer satisfy the condition used to perform the grouping.

Issai Schur (1875-1941) was born in Mogilev, which is now in Belarus. He studied mathematics and physics at the University of Berlin, where Frobenius was one of his teachers. Schur obtained his doctorate with a thesis on rational representations of the general linear group over the complex field. Thereafter, he taught and researched at Berlin University (with an interval in Bonn) until he was dismissed by the Nazis in 1935. He was a superb lecturer. Schur is now known mainly for his fundamental contributions to the representation theory of groups, but he also worked in other areas of algebra, including matrices, number theory, and analysis. In 1939, he emigrated to Palestine, where he later died in poverty.

## Reference

Ledermann, W. 1983. Issai Schur and his school in Berlin. Bulletin of the London Mathematical Society 15: 97-106. https://doi.org/10.1112/blms/15.2.97.

## Also see

[M-1] LAPACK - Linear algebra package (LAPACK) routines
[M-5] hessenbergd( ) - Hessenberg decomposition
[M-4] Matrix - Matrix functions

## Title

[M-5] select() — Select rows, columns, or indices

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

select $(X, v)$ returns $X$

1. omitting the rows for which $v[i]==0$ ( $v$ a column vector) or
2. omitting the columns for which $v[j]==0$ ( $v$ a row vector).
st_select $(A, X, v)$ does the same thing, except that the result is placed in $A$ and, if $X$ is a view, $A$ will be a view.
selectindex (v) returns
3. a row vector of column indices $j$ for which $v[j]!=0$ ( $v$ a row vector) or
4. a column vector of row indices $i$ for which $v[i]!=0$ ( $v$ a column vector).

## Syntax

transmorphic matrix select (transmorphic matrix $X$, real vector $v$ )
void $\quad$ st_select $(A$, transmorphic matrix $X$, real vector $v)$
real vector selectindex (real vector $v$ )

## Remarks and examples

Remarks are presented under the following headings:

```
Examples
Using st_select()
```


## Examples

1. To select rows 1,2 , and 4 of $5 \times c$ matrix $X$,

$$
\text { submat }=\operatorname{select}(X, \quad(1 \backslash 1 \backslash 0 \backslash 1 \backslash 0))
$$

See [M-2] Subscripts for another solution, submat $=X[(1 \backslash 2 \backslash 4),$.$] .$
2. To select columns 1,2 , and 4 of $r \times 5$ matrix X ,

```
submat = select(X, (1,1,0,1,0))
```

See $[\mathrm{M}-2]$ Subscripts for another solution, submat $=X[.,(1,2,4)]$.
3. To select rows of $X$ for which the first element is positive,

$$
\text { submat }=\operatorname{select}(X, X[., 1]:>0)
$$

4. To select columns of $X$ for which the first element is positive,

$$
\text { submat }=\operatorname{select}(X, X[1, .]:>0)
$$

5. To select rows of $X$ for which there are no missing values,

$$
\text { submat }=\operatorname{select}(X, \text { rowmissing }(X):==0)
$$

6. To select rows and columns of square matrix $X$ for which the diagonal elements are positive,
```
pos = diagonal(X):>0
submat = select(X, pos)
submat = select(submat, pos')
```

or, equivalently,

```
pos = diagonal(X):>0
submat = select(select(X, pos), pos')
```

7. To select column indices for which $v[j]!=0$,
: v

| 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- |

1
$\begin{array}{lllll}6 & 0 & 7 & 0 & 8\end{array}$
: selectindex(v)
123
$1 \begin{array}{lll}1 & 3 & 5\end{array}$
8. To select row indices for which $v[i]!=0$,

| : w | 1 |
| :---: | :---: |
|  |  |
| 1 | 0 |
| 2 | 3 |
| 3 | 0 |
| 4 | 2 |
| 5 | 1 |
| : selectindex(w) |  |
| 1 |  |
| 1 | 2 |
| 2 | 4 |
| 3 | 5 |

## Using st_select()

Coding
st_select(submat, X, v)
produces the same result as coding

$$
\begin{equation*}
\text { submat }=\text { st_select(X, v) } \tag{2}
\end{equation*}
$$

The difference is in how the result is stored. If $X$ is a view (it need not be), then (1) will produce submat as a view or, if you will, a subview, whereas in (2), submat will always be a regular (nonview) matrix.

When $X$ is a view, (1) executes more quickly than (2) and produces a result that consumes less memory.

See [M-5] st_view() for a description of views.

## Conformability

select $(X, v)$ :

$$
\begin{array}{llll}
X: & r_{1} \times c_{1} & & \\
v: & r_{1} \times 1 & \text { or } & 1 \times c_{1} \\
\text { result: } & r_{2} \times c_{1} & \text { or } & r_{1} \times c_{2}, \quad r_{2} \leq r_{1}, c_{2} \leq c_{1}
\end{array}
$$

st_select $(A, X, v)$ :
input:

$$
\begin{array}{llll}
X: & & r_{1} \times c_{1} & \\
v: & & r_{1} \times 1
\end{array} \quad \text { or } \quad 1 \times c_{1}
$$

output:

$$
A: \quad r_{2} \times c_{1} \quad \text { or } \quad r_{1} \times c_{2}, \quad r_{2} \leq r_{1}, c_{2} \leq c_{1}
$$

selectindex $(v)$ :

$$
\begin{array}{clll}
v: & r_{1} \times 1 & \text { or } & 1 \times c_{1} \\
\text { result: } & r_{2} \times 1 & \text { or } \quad 1 \times c_{2},
\end{array} \quad r_{2} \leq r_{1}, c_{2} \leq c_{1}
$$

## Diagnostics

None.

## Also see

[M-5] st_subview() - Make view from view
[M-2] op_colon - Colon operators
[M-2] Subscripts - Use of subscripts
[M-4] Utility - Matrix utility functions

## Title

[M-5] setbreakintr() — Break-key processing

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

setbreakintr (val) turns the break-key interrupt off (val==0) or on (val!=0) and returns the value of the previous break-key mode, 1 , it was on, or 0 , it was off.
querybreakintr () returns 1 if the break-key interrupt is on and 0 otherwise.
breakkey () (for use in setbreakintr (0) mode) returns 1 if the break key has been pressed since it was last reset.
breakkeyreset() (for use in setbreakintr(0) mode) resets the break key.

## Syntax

```
real scalar setbreakintr(real scalar val)
real scalar querybreakintr()
real scalar breakkey()
void breakkeyreset()
```


## Remarks and examples

Remarks are presented under the following headings:
Default break-key processing
Suspending the break-key interrupt
Break-key polling

## Default break-key processing

By default, if the user presses Break, Mata stops execution and returns control to the console, setting the return code to 1 .

To obtain this behavior, there is nothing you need do. You do not need to use these functions.

## Suspending the break-key interrupt

The default behavior is known as interrupt-on-break and is also known as setbreakintr(1) mode. The alternative is break-key suspension, also known as setbreakintr(0) mode.

For instance, you have several steps that must be performed in their entirety or not at all. The way to do this is

```
val = setbreakintr(0)
... (critical code) ...
(void) setbreakintr(val)
```

The first line stores in val the current break-key processing mode and then sets the mode to break-key suspension. The critical code then runs. If the user presses Break during the execution of the critical code, that will be ignored. Finally, the code restores the previous break-key processing mode.

## Break-key polling

In coding large, interactive systems, you may wish to adopt the break-key polling style of coding rather than interrupt-on-break. In this alternative style of coding, you turn off interrupt-on-break:

```
val = setbreakintr(0)
```

and, from then on in your code, wherever you are willing to interrupt your code, you ask (poll whether) the break key has been pressed:

```
if (breakkey()) {
}
```

In this style of coding, you must decide where and when you are going to reset the break key, because once the break key has been pressed, breakkey () will continue to return 1 every time it is called. To reset the break key, code,

```
breakkeyreset()
```

You can also adopt a mixed style of coding, using interrupt-on-break in some places and polling in others. Function querybreakintr () can then be used to determine the current mode.

## Conformability

```
setbreakintr(val):
        val: }1\times
    result: }1\times
querybreakintr(), breakkey():
    result: }1\times
breakkeyreset():
    result: void
```


## Diagnostics

setbreakintr(1) aborts with break if the break key has been pressed since the last setbreakintr(0) or breakkeyreset(). Code breakkeyreset() before setbreakintr(1) if you do not want this behavior.

After coding setbreakintr(1), remember to restore setbreakintr(0) mode. It is not, however, necessary, to restore the original mode if exit() or _error() is about to be executed.
breakkey(), once the break key has been pressed, continues to return 1 until breakkeyreset() is executed.

There is absolutely no reason to use breakkey() in setbreakintr (0) mode, because the only value it could return is 0 .

## Also see

[M-5] error() - Issue error message
[M-4] Programming - Programming functions

## Title

[M-5] sign( ) - Sign and complex quadrant functions
Description Syntax Conformability Diagnostics Also see

## Description

$\operatorname{sign}(R)$ returns the elementwise $\operatorname{sign}$ of $R . \operatorname{sign}()$ is defined
Argument range sign(arg)

$$
\begin{array}{lr}
\arg \geq 0 & -1 \\
\arg <0 & 0 \\
\arg =0 & 1 \\
\arg >0 &
\end{array}
$$

quadrant ( $Z$ ) returns a real matrix recording the quadrant of each complex entry in $Z$. quadrant () is defined

| Argument range |  |  |
| :--- | :---: | :---: |
| $\operatorname{Re}($ arg $)$ | $\operatorname{Im}(\arg )$ | quadrant (arg) |
|  |  |  |
| $\operatorname{Re} \geq$. |  | . |
| $\operatorname{Re}=0$ | $\operatorname{Im}=0$ | 1 |
| $\operatorname{Re}>0$ | $\mathrm{Im} \geq 0$ | 2 |
| $\operatorname{Re} \leq 0$ | $\mathrm{Im}>0$ | 3 |
| $\operatorname{Re}<0$ | $\mathrm{Im} \leq 0$ | 4 |
| $\operatorname{Re} \geq 0$ | $\mathrm{Im}<0$ |  |
| quadrant $(1+0 i)==1$, quadrant $(-1+0 i)==3$ |  |  |
| quadrant $(0+1 i)==2$, quadrant $(0-1 i)==4$ |  |  |

## Syntax

real matrix $\quad$ sign (real matrix $R$ )
real matrix quadrant (complex matrix $Z$ )

## Conformability

sign $(R)$ :

$$
\begin{aligned}
& R: \\
& \text { result: } \\
& r \times c
\end{aligned}
$$

quadrant $(Z)$ :
$\begin{array}{rlr}Z: & & r \times c \\ \text { result: } & & r \times c\end{array}$

## Diagnostics

$\operatorname{sign}(R)$ returns missing when $R$ is missing.
quadrant ( $Z$ ) returns missing when $Z$ is missing.

## Also see

[M-5] dsign( ) - FORTRAN-like DSIGN( ) function
[M-4] Scalar - Scalar mathematical functions

## Title

[M-5] $\sin ()$ - Trigonometric and hyperbolic functions

## Description Syntax Conformability Diagnostics Also see

## Description

$\sin (Z), \cos (Z)$, and $\tan (Z)$ return the appropriate trigonometric functions. Angles are measured in radians. All return real if the argument is real and complex if the argument is complex.
$\sin (x), x$ real, returns the sine of $x . \sin ()$ returns a value between -1 and 1.
$\sin (z), z$ complex, returns the complex sine of $z$, mathematically defined as
$\{\exp (i * z)-\exp (-i * z)\} / 2 i$.
$\cos (x), x$ real, returns the cosine of $x . \cos ()$ returns a value between -1 and 1 .
$\cos (z), z$ complex, returns the complex cosine of $z$, mathematically defined as $\{\exp (i * z)+\exp (-i * z)\} / 2$.
$\tan (x), x$ real, returns the tangent of $x$.
$\tan (z), z$ complex, returns the complex tangent of $z$, mathematically defined as $\sin (z) / \cos (z)$. $\operatorname{asin}(Z), \operatorname{acos}(Z)$, and $\operatorname{atan}(Z)$ return the appropriate inverse trigonometric functions. Returned results are in radians. All return real if the argument is real and complex if the argument is complex.
$\operatorname{asin}(x), x$ real, returns arcsine in the range $[-\pi / 2, \pi / 2]$. If $x<-1$ or $x>1$, missing (.) is returned.
$\operatorname{asin}(z), z$ complex, returns the complex arcsine, mathematically defined as $-i * \ln \{i * z+\operatorname{sqrt}(1-z * z)\} . \operatorname{Re}(\operatorname{asin}())$ is chosen to be in the interval $[-\pi / 2, \pi / 2]$.
$\operatorname{acos}(x), x$ real, returns arccosine in the range $[0, \pi]$. If $x<-1$ or $x>1$, missing (.) is returned.
$\operatorname{acos}(z), z$ complex, returns the complex arccosine, mathematically defined as
$-i * \ln \{z+\operatorname{sqrt}(z * z-1)\} . \operatorname{Re}(\operatorname{acos}())$ is chosen to be in the interval $[0, \pi]$.
$\operatorname{atan}(x), x$ real, returns arctangent in the range $(-\pi / 2, \pi / 2)$.
$\operatorname{atan}(z), z$ complex, returns the complex arctangent, mathematically defined as
$\ln \{(1+i z) /(1-i z)\} /(2 i) . \operatorname{Re}(\operatorname{atan}())$ is chosen to be in the interval $[0, \pi]$.
atan2 $(X, Y)$ returns the radian value in the range $(-\pi, \pi]$ of the angle of the vector determined by $(X, Y)$, the result being in the range $[0, \pi]$ for quadrants 1 and 2 and $[0,-\pi)$ for quadrants 4 and 3. $X$ and $Y$ must be real. atan2 $(X, Y)$ is equivalent to $\arg (\mathrm{C}(X, Y))$.
$\arg (Z)$ returns the arctangent of $\operatorname{Im}(Z) / \operatorname{Re}(Z)$ in the correct quadrant, the result being in the range $(-\pi, \pi] ;[0, \pi]$ in quadrants 1 and 2 and $[0,-\pi)$ in quadrants 4 and $3 . \arg (Z)$ is equivalent to atan2 $(\operatorname{Re}(Z), \operatorname{Im}(Z))$.
$\sinh (Z), \cosh (Z)$, and $\tanh (Z)$ return the hyperbolic sine, cosine, and tangent, respectively. The returned value is real if the argument is real and complex if the argument is complex.
$\sinh (x), x$ real, returns the inverse hyperbolic sine of $x$, mathematically defined as $\{\exp (x)-\exp (-x)\} / 2$.
$\sinh (z), z$ complex, returns the complex hyperbolic sine of $z$, mathematically defined as $\{\exp (z)-\exp (-z)\} / 2$.
$\cosh (x), x$ real, returns the inverse hyperbolic cosine of $x$, mathematically defined as
$\{\exp (x)+\exp (-x)\} / 2$.
$\cosh (z), z$ complex, returns the complex hyperbolic cosine of $z$, mathematically defined as $\{\exp (z)+\exp (-z)\} / 2$.
$\tanh (x), x$ real, returns the inverse hyperbolic tangent of $x$, mathematically defined as $\sinh (x) / \cosh (x)$.
$\tanh (z), z$ complex, returns the complex hyperbolic tangent of $z$, mathematically defined as $\sinh (z) / \cosh (z)$.
$\operatorname{asinh}(Z), \operatorname{acosh}(Z)$, and $\operatorname{atanh}(Z)$ return the inverse hyperbolic sine, cosine, and tangent, respectively. The returned value is real if the argument is real and complex if the argument is complex.
$\operatorname{asinh}(x), x$ real, returns the inverse hyperbolic sine.
$\operatorname{asinh}(z), z$ complex, returns the complex inverse hyperbolic sine, mathematically defined as $\ln \{z+\operatorname{sqrt}(z * z+1)\} . \operatorname{Im}(\operatorname{asinh}())$ is chosen to be in the interval $[-\pi / 2, \pi / 2]$.
$\operatorname{acosh}(x), x$ real, returns the inverse hyperbolic cosine. If $x<1$, missing (.) is returned.
$\operatorname{acosh}(z), z$ complex, returns the complex inverse hyperbolic cosine, mathematically defined as $\ln \{z+\operatorname{sqrt}(z * z-1)\} . \operatorname{Im}(\operatorname{acosh}())$ is chosen to be in the interval $[-\pi, \pi] ; \operatorname{Re}(\operatorname{acosh}())$ is chosen to be nonnegative.
$\operatorname{atanh}(x), x$ real, returns the inverse hyperbolic tangent. If $|x|>1$, missing (.) is returned.
$\operatorname{atanh}(z), z$ complex, returns the complex inverse hyperbolic tangent, mathematically defined as $\ln \{(1+z) /(1-z)\} / 2 \operatorname{Im}(\operatorname{atanh}())$ is chosen to be in the interval $[-\pi / 2, \pi / 2]$.
pi() returns the value of $\pi$.

## Syntax

```
numeric matrix sin(numeric matrix Z)
numeric matrix cos(numeric matrix Z)
numeric matrix tan(numeric matrix Z)
numeric matrix asin(numeric matrix Z)
numeric matrix acos(numeric matrix Z)
numeric matrix atan(numeric matrix Z)
real matrix atan2(real matrix }X\mathrm{ , real matrix }Y\mathrm{ )
real matrix }\quad\operatorname{arg}(\mathrm{ complex matrix Z)
numeric matrix sinh(numeric matrix Z)
numeric matrix cosh(numeric matrix Z)
numeric matrix tanh(numeric matrix Z)
numeric matrix asinh(numeric matrix Z)
numeric matrix acosh(numeric matrix Z)
numeric matrix atanh(numeric matrix Z)
real scalar pi()
```


## Conformability

```
atan2(X,Y):
X:
```

pi() returns a $1 \times 1$ scalar.

All other functions return a matrix of the same dimension as input containing element-by-element calculated results.

## Diagnostics

All functions return missing for real arguments when the result would be complex. For instance, $\operatorname{acos}(2)=.$, whereas $\operatorname{acos}(2+0 i)=-1.317 i$.

## Also see <br> [M-4] Scalar - Scalar mathematical functions

## Title

[M-5] sizeof( ) - Number of bytes consumed by object

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

sizeof $(A)$ returns the number of bytes consumed by $A$.

## Syntax

real scalar sizeof (transmorphic matrix $A$ )

## Remarks and examples

sizeof $(A)$ returns the same number as shown by mata describe; see [M-3] mata describe.
A $500 \times 5$ real matrix consumes 20,000 bytes:

```
: sizeof(mymatrix)
    20000
```

A $500 \times 5$ view matrix, however, consumes only 24 bytes:
: sizeof(myview) 24

To obtain the number of bytes consumed by a function, pass a dereferenced function pointer:

```
: sizeof(*&myfcn())
    320
```


## Conformability

```
sizeof(A):
```

| A: | $r \times c$ |
| ---: | :--- |
| result: | $1 \times 1$ |

## Diagnostics

The number returned by sizeof $(A)$ does not include any overhead, which usually amounts to 64 bytes, but can be less (as small as zero in the case of recently used scalars).

If $A$ is a pointer matrix, the number returned reflects the amount of memory required to store $A$ itself and does not include the memory consumed by its siblings.

## Also see

[M-4] Programming - Programming functions

## Title

[M-5] solve_tol( ) - Tolerance used by solvers and inverters

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

solve_tol ( $Z$, usertol) returns the tolerance used by many Mata solvers to solve $A X=B$ and by many Mata inverters to obtain $A^{-1}$. usertol is the tolerance specified by the user or is missing value if the user did not specify a tolerance.

## Syntax

real scalar solve_tol(numeric matrix $Z$, real scalar usertol)

## Remarks and examples

The tolerance used by many Mata solvers to solve $A X=B$ and by many Mata inverters to obtain $A^{-1}$ is

$$
\begin{array}{ll}
\text { eta }=s * \frac{\operatorname{trace}(\operatorname{abs}(Z))}{n} & \text { when } s>0  \tag{1}\\
\text { eta }=-s & \text { when } s \leq 0
\end{array}
$$

where $s=1 \mathrm{e}-13$ or a value specified by the user, $n$ is the $\min (r o w s(Z), \operatorname{cols}(Z)$ ), and $Z$ is a matrix related to $A$, usually by some form of decomposition, but could be $A$ itself (for instance, if $A$ were triangular). See, for instance, [M-5] solvelower() and [M-5] cholsolve().

When usertol $>0$ and usertol $<$. is specified, solvetol() returns eta calculated with $s=$ usertol.
When usertol $\leq 0$ is specified, solvetol () returns -usertol.
When usertol $\geq$. is specified, solvetol() returns a default result, calculated as

1. If the matasolvetol setting is set to . (missing), the value of eta is computed using $s=1 \mathrm{e}-13$.
2. If the matasolvetol setting is set to positive, the value of eta is computed using $s=$ st_numscalar("c(matasolvetol)").
3. If the matasolvetol setting is set to 0 or negative, the value of eta is -st_numscalar("c(matasolvetol)").

## Conformability

solve_tol ( $Z$, usertol):

| $Z:$ | $r \times c$ |  |
| ---: | :--- | ---: |
| usertol: |  | $1 \times 1$ |
| result: |  | $1 \times 1$ |

## Diagnostics

solve_tol ( $Z$, usertol) skips over missing values in $Z$ in calculating (1); $n$ is defined as the number of nonmissing elements on the diagonal.

## Also see

[M-4] Utility - Matrix utility functions

## Title

[M-5] solvelower( ) - Solve $\mathrm{AX}=\mathrm{B}$ for X , A triangular

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

These functions are used in the implementation of the other solve functions; see [M-5] lusolve(), [M-5] qrsolve( ), and [M-5] svsolve( ).
solvelower $(A, B, \ldots)$ and _solvelower $(A, B, \ldots)$ solve lower-triangular systems.
solveupper $(A, B, \ldots)$ and _solveupper $(A, B, \ldots)$ solve upper-triangular systems.
Functions without a leading underscore—solvelower() and solveupper()—return the solution; $A$ and $B$ are unchanged.

Functions with a leading underscore—_solvelower() and _solveupper()—return the solution in $B$.

All four functions produce a generalized solution if $A$ is singular. The functions without an underscore place the rank of $A$ in rank, if the argument is specified. The underscore functions return the rank.

Determination of singularity is made via tol. tol is interpreted in the standard way-as a multiplier for the default if $t o l>0$ is specified and as an absolute quantity to use in place of the default if tol $\leq 0$ is specified.

All four functions allow $d$ to be optionally specified. Specifying $d=$. is equivalent to not specifying $d$.

If $d \neq$. is specified, that value is used as if it appeared on the diagonal of $A$. The four functions do not in fact require that $A$ be triangular; they merely look at the lower or upper triangle and pretend that the opposite triangle contains zeros. This feature is useful when a decomposition utility has stored both the lower and upper triangles in one matrix, because one need not take apart the combined matrix. In such cases, it sometimes happens that the diagonal of the matrix corresponds to one matrix but not the other, and that for the other matrix, one merely knows that the diagonal elements are, say, 1 . Then you can specify $d=1$.
solvelowerlapacke $(A, B, \ldots)$ and _solvelowerlapacke $(A, B, \ldots)$ solve lower-triangular systems using LAPACK routines. If $A$ is not full rank, these functions produce a solution filled with missing values.
solveupperlapacke $(A, B, \ldots)$ and _solveupperlapacke $(A, B, \ldots)$ solve upper-triangular systems using LAPACK routines. If $A$ is not full rank, these functions produce a solution filled with missing values.

Because these functions produce solutions filled with missing values when $A$ is not full rank, they do not need the rank argument.

## Syntax

| numeric matrix | solvelower ( $A, B[, \operatorname{rank}[, \operatorname{tol}[, d]]])$ |
| :---: | :---: |
| numeric matrix | solveupper $(A, B[, \operatorname{rank}[, \operatorname{tol}[, d]]])$ |
| real scalar | _solvelower ( $A, B[$ tol $[, d]])$ |
| real scalar | _solveupper ( $A, B[$, tol $[, d]])$ |
| numeric matrix | solvelowerlapacke ( $A, B[$, tol $[, d]])$ |
| numeric matrix | solveupperlapacke ( $A, B[$ tol $[, d]])$ |
| void | _solvelowerlapacke ( $A, B[$, tol $[, d]])$ |
| void | _solveupperlapacke ( $A, B[$, tol $[, d]])$ |

where

$$
\begin{aligned}
\text { A: } & \text { numeric matrix } \\
B: & \text { numeric matrix } \\
\text { rank: } & \text { irrelevant; real scalar returned } \\
\text { tol: } & \text { real scalar } \\
d: & \text { numeric scalar }
\end{aligned}
$$

## Remarks and examples

The triangular-solve functions solvelower(), _solvelower(), solveupper(), and _solveupper() exploit the triangular structure in $A$ and solve for $X$ by recursive substitution.

The solvelowerlapacke(), _solvelowerlapacke(), solveupperlapacke(), and _solveupperlapacke() functions solve full-rank triangular matrix systems using underlying built-in LAPACK routines.

When $A$ is of full rank, these functions provide the same solution as the other solve functions, such as [M-5] lusolve(), [M-5] qrsolve(), and [M-5] svsolve(). The solvelower() and solveupper() functions, however, will produce the answer more quickly because of the large computational savings.

When $A$ is singular, however, you may wish to consider whether you want to use these triangular-solve functions. The triangular-solve functions documented here reach a generalized solution by setting $B_{i j}=0$, for all $j$, when $A_{i j}$ is zero or too small (as determined by tol). The method produces a generalized inverse, but there are many generalized inverses, and this one may not have the other properties you want.

Remarks are presented under the following headings:

[^13]
## Derivation

We wish to solve

$$
\begin{equation*}
A X=B \tag{1}
\end{equation*}
$$

when $A$ is triangular. Let us consider the lower-triangular case first. solvelower () is up to handling full matrices for $B$ and $X$, but let us assume $X: n \times 1$ and $B: m \times 1$ :

$$
\left[\begin{array}{cccc}
a_{11} & 0 & 0 \ldots & 0 \\
a_{21} & 0 & 0 \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
a_{m 1} & a_{m 2} & a_{m 3} & a_{m n}
\end{array}\right]\left[\begin{array}{c}
x_{1} \\
x_{2} \\
\vdots \\
x_{n}
\end{array}\right]=\left[\begin{array}{c}
b_{1} \\
b_{2} \\
\vdots \\
b_{m}
\end{array}\right]
$$

The first equation to be solved is

$$
a_{11} x_{1}=b_{1}
$$

and the solution is simply

$$
\begin{equation*}
x_{1}=\frac{b_{1}}{a_{11}} \tag{2}
\end{equation*}
$$

The second equation to be solved is

$$
a_{21} x_{1}+a_{22} x_{2}=b_{2}
$$

and because we have already solved for $x_{1}$, the solution is simply

$$
\begin{equation*}
x_{2}=\frac{b_{2}-a_{21} x_{1}}{a_{22}} \tag{3}
\end{equation*}
$$

We proceed similarly for the remaining rows of $A$. If there are additional columns in $B$ and $X$, we can then proceed to handling each remaining column just as we handled the first column above.

In the upper-triangular case, the formulas are similar except that you start with the last row of $A$.
These formulas apply only to the solvelower(), _solvelower(), solveupper(), and _solveupper() functions.

## Tolerance

In (2) and (3), we divide by the diagonal elements of $A$. If element $a_{i i}$ is less than eta in absolute value, the corresponding $x_{i}$ is set to zero. eta is given by

$$
\text { eta }=1 \mathrm{e}-13 * \operatorname{trace}(\operatorname{abs}(A)) / \operatorname{rows}(A)
$$

If you specify tol $>0$, the value you specify is used to multiply eta. You may instead specify tol $\leq$ 0 , and then the negative of the value you specify is used in place of eta; see [M-1] Tolerance.
solvelowerlapacke(), _solvelowerlapacke(), solveupperlapacke(), and _solveupperlapacke() share the same definitions of eta and tol. If element $a_{i i}$ is less than eta in absolute value, these functions produce a solution filled with missing values.

## Conformability

solvelower ( $A, B$, rank, tol, $d$ ), solveupper $(A, B, r a n k, t o l, d)$ :
input:

|  | $A:$ | $n \times n$ |  |
| ---: | ---: | ---: | :--- |
|  | $B:$ | $n \times k$ |  |
|  | tol: | $1 \times 1$ | (optional) |
| output: | $d:$ | $1 \times 1$ | (optional) |
|  |  |  |  |
|  | rank: | $1 \times 1$ | (optional) |
|  | result: | $n \times k$ |  |

_solvelower $(A, B, t o l, d)$, _solveupper $(A, B, t o l, d)$ :
input:

| A: | $n \times n$ |  |
| ---: | :--- | :--- |
| $B:$ | $n \times k$ |  |
| tol $:$ | $1 \times 1$ | (optional) |
| $d:$ | $1 \times 1$ | (optional) |

output:

$$
\begin{array}{rll}
B: & n \times k & \\
\text { result: } & 1 \times 1 & \text { (contains rank) }
\end{array}
$$

solvelowerlapacke $(A, B, t o l, d)$, solveupperlapacke $(A, B, t o l, d)$ :
input:

| $A:$ | $n \times n$ |  |
| ---: | :--- | :--- |
| $B:$ | $n \times k$ |  |
| tol: $:$ | $1 \times 1$ | (optional) |
| $d:$ | $1 \times 1$ | (optional) |

output:

$$
\text { result: } \quad n \times k
$$

_solvelowerlapacke ( $A, B$, tol, $d$ ), _solveupperlapacke $(A, B$, tol, $d$ ):
input:

| A: | $n \times n$ |  |
| ---: | :--- | :--- |
| $B:$ | $n \times k$ |  |
| tol: | $1 \times 1$ | (optional) |
| $d:$ | $1 \times 1$ | (optional) |

output:

$$
B: \quad n \times k
$$

## Diagnostics

solvelower ( $A, B, \ldots$ ), _solvelower ( $A, B, \ldots$ ), solveupper $(A, B, \ldots)$, and _solveupper $(A, B, \ldots)$ do not verify that the upper (lower) triangle of $A$ contains zeros; they just use the lower (upper) triangle of $A$.
_solvelower $(A, B, \ldots)$ and _solveupper $(A, B, \ldots)$ do not abort with error if $B$ is a view but can produce results subject to considerable roundoff error.
solvelowerlapacke( $A, B, \ldots$ ), _solvelowerlapacke ( $A, B, \ldots$ ), solveupperlapacke ( $A, B, \ldots$ ), and _solveupperlapacke ( $A, B, \ldots$ ) do not verify that the upper (lower) triangle of $A$ contains zeros; they just use the lower (upper) triangle of $A$.

## Also see

[M-5] cholsolve( ) - Solve AX=B for X using Cholesky decomposition
[M-5] lusolve( ) - Solve AX=B for X using LU decomposition
[M-5] qrsolve( ) - Solve AX=B for X using QR decomposition
[M-5] solve_tol( ) - Tolerance used by solvers and inverters
[M-5] svsolve( ) - Solve AX=B for X using singular value decomposition
[M-4] Matrix - Matrix functions

## Title

[M-5] solvenl( ) - Solve systems of nonlinear equations

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | References | Also see |  |

## Description

The solvenl() suite of functions finds solutions to systems of nonlinear equations.
solvenl_init() initializes the problem and returns $S$, a structure that contains information regarding the problem, including default values. If you declare a storage type for $S$, declare it to be a transmorphic scalar.

The solvenl_init_* $(S, \ldots)$ functions allow you to modify those default values and specify other aspects of your problem, including whether your problem refers to finding a fixed point or a zero starting value to use, etc.
solvenl_solve ( $S$ ) solves the problem. solvenl_solve() returns a vector that represents either a fixed point of your function or a vector at which your function is equal to a vector of zeros.

The solvenl_result_*( $S$ ) functions let you access other information associated with the solution to your problem, including whether a solution was achieved, the final Jacobian matrix, and diagnostics.

Aside: The solvenl_init_* $(S, \ldots)$ functions have two modes of operation. Each has an optional argument that you specify to set the value and that you omit to query the value. For instance, the full syntax of solvenl_init_startingvals() is
void solvenl_init_startingvals( $S$, real colvector ivals)
real colvector solvenl_init_startingvals ( $S$ )
The first syntax sets the parameter values and returns nothing. The second syntax returns the previously set (or default, if not set) parameter values.

All the solvenl_init_*( $S, \ldots$ ) functions work the same way.

## Syntax

$$
S=\text { solvenl_init() }
$$

(varies)

```
solvenl_init_type(S [, { "fixedpoint"|"zero" }])
```

(varies) solvenl_init_startingvals (S [, real colvector ivals])
(varies) solvenl_init_numeq (S [, real scalar nvars])
(varies) solvenl_init_technique(S [, "technique" ])
(varies) solvenl_init_conv_iterchng(S [, real scalar itol])
(varies) solvenl_init_conv_nearzero (S [, real scalar ztol])
(varies) solvenl_init_conv_maxiter ( $S$ [, real scalar maxiter $]$ )
(varies) solvenl_init_evaluator $(S[$, \&evaluator ()$])$
(varies) solvenl_init_argument ( $S$, real scalar $k[, X]$ )
(varies) solvenl_init_narguments ( $S$ [, real scalar $K]$ )
(varies) solvenl_init_damping(S [, real scalar damp])
(varies) solvenl_init_iter_log (S [, \{"on"|"off" \}])
(varies) solvenl_init_iter_dot(S [, \{"on"|"off" \}])
(varies) solvenl_init_iter_dot_indent (S [, real scalar indent])
(varies) solvenl_init_deriv_usemin(S [, \{"off"|"on" \}])
(varies)
solvenl_init_deriv_min(S [, real rowvector min $])$
real colvector solvenl_solve ( $S$ )
real scalar _solvenl_solve (S)
real scalar solvenl_result_converged ( $S$ )
real scalar solvenl_result_conv_iter ( $S$ )
real scalar solvenl_result_conv_iterchng ( $S$ )
real scalar solvenl_result_conv_nearzero ( $S$ )
real colvector solvenl_result_values ( $S$ )
real matrix solvenl_result_Jacobian ( $S$ )
real scalar solvenl_result_error_code ( $S$ )
real scalar solvenl_result_return_code ( $S$ )
string scalar solvenl_result_error_text ( $S$ )
void solvenl_dump ( $S$ )
$S$, if it is declared, should be declared as

```
transmorphic S
```

and technique optionally specified in solvenl_init_technique() is one of the following:

| technique | Description |
| :--- | :--- |
| gaussseidel | Gauss-Seidel |
| dampedgaussseidel | Damped Gauss-Seidel |
| *roydenpowell | Broyden-Powell |
| newtonraphson | Newton-Raphson |
| * newton may also be abbreviated as nr. |  |

For fixed-point problems, allowed techniques are gaussseidel and dampedgaussseidel. For zerofinding problems, allowed techniques are broydenpowell and newtonraphson. solvenl_*() exits with an error message if you specify a technique that is incompatible with the type of evaluator you declared by using solvenl_init_type(). The default technique for fixed-point problems is dampedgaussseidel with a damping parameter of 0.1 . The default technique for zero-finding problems is broydenpowell.

## Remarks and examples

Remarks are presented under the following headings:

```
Introduction
A fixed-point example
A zero-finding example
Writing a fixed-point problem as a zero-finding problem and vice versa
Gauss-Seidel methods
Newton-type methods
Convergence criteria
Exiting early
Functions
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    solvenl_init_startingvals(()
    solvenl_init_numeq()
    solvenl_init_technique()
    solvenl_init_conv_iterchng()
    solvenl_init_conv_nearzero()
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    solvenl_init_iter_dot()
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    solvenl_init_deriv_usemin() and solvenl_init_deriv_min()
    solvenl_solve() and _solvenl_solve()
    solvenl_result_converged()
    solvenl_result_conv_iter()
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    solvenl_result_conv_nearzero()
    solvenl_result_values()
    solvenl_result_Jacobian()
    solvenl_result_error_code( ), . . _return_code(), and . . . _error_text()
    solvenl_dump()
```


## Introduction

Let $\mathbf{x}$ denote a $k \times 1$ vector and let $\mathrm{F}: R^{k} \rightarrow R^{k}$ denote a function that represents a system of equations. The solvenl() suite of functions can be used to find fixed-point solutions $\mathbf{x}^{*}=\mathbf{F}\left(\mathbf{x}^{*}\right)$, and it can be used to find a zero of the function, that is, a vector $\mathbf{x}^{*}$ such that $\mathbf{F}\left(\mathbf{x}^{*}\right)=\mathbf{0}$.

Four solution methods are available: Gauss-Seidel (GS), damped Gauss-Seidel (dGS), Newton's method (also known as the Newton-Raphson method), and the Broyden-Powell (BP) method. The first two methods are used to find fixed points, and the latter two are used to find zeros. However, as we discuss below, fixed-point problems can be rewritten as zero-finding problems, and many zero-finding problems can be rewritten as fixed-point problems.

Solving systems of nonlinear equations is inherently more difficult than minimizing or maximizing a function. The set of first-order conditions associated with an optimization problem satisfies a set of integrability conditions, while solvenl_*() works with arbitrary systems of nonlinear equations. Moreover, while one may be tempted to approach a zero-finding problem by defining a function

$$
f(\mathbf{x})=\mathbf{F}(\mathbf{x})^{\prime} \mathbf{F}(\mathbf{x})
$$

and minimizing $f(\mathbf{x})$, there is a high probability that the minimizer will find a local minimum for which $\mathbf{F}(\mathbf{x}) \neq \mathbf{0}$ (Press et al. 2007, 476). Some problems may have multiple solutions.

## A fixed-point example

We want to solve the system of equations

$$
\begin{aligned}
& x=\frac{5}{3}-\frac{2}{3} y \\
& y=\frac{10}{3}-\frac{2}{3} x
\end{aligned}
$$

First, we write a program that takes two arguments: a column vector representing the values at which we are to evaluate our function and a column vector into which we are to place the function values.

```
: void function myfun(real colvector from, real colvector values)
> {
values[1] = 5/3 - 2/3*from[2]
values[2] = 10/3 - 2/3*from[1]
}
```

Our invocation of solvenl_*() proceeds as follows:

```
: S = solvenl_init()
: solvenl_init_evaluator(S, &myfun())
: solvenl_init_type(S, "fixedpoint")
: solvenl_init_technique(S, "gaussseidel")
: solvenl_init_numeq(S, 2)
: solvenl_init_iter_log(S, "on")
x = solvenl_solve(S)
Iteration 1: 3.3333333
Iteration 2: . }8333333
    (output omitted)
x
                            1
        -.9999999981
        4
```

In our equation with $x$ on the left-hand side, $x$ did not appear on the right-hand side, and similarly for the equation with $y$.However, that is not required. Fixed-point problems with left-hand-side variables appearing on the right-hand side of the same equation can be solved, though they typically require more iterations to reach convergence.

## A zero-finding example

We wish to solve the following system of equations (Burden, Faires, and Burden 2016, 657) for the three unknowns $x, y$, and $z$ :

$$
\begin{aligned}
& 10-x e^{y}-z=0 \\
& 12-x e^{2 y}-2 z=0 \\
& 15-x e^{3 y}-3 z=0
\end{aligned}
$$

We will use Newton's method. We cannot use $x=y=z=0$ as initial values because the Jacobian matrix is singular at that point; we will instead use $x=y=z=0.2$. Our program is

```
: void function myfun2(real colvector x, real colvector values)
> {
> values[1] = 10-x[1]*exp(x[2]*1) - x[3]*1
> values[2] = 12 - x[1]*exp(x[2]*2) - x[3]*2
> values[3] = 15 - x[1]*exp(x[2]*3) - x[3]*3
}
: S = solvenl_init()
: solvenl_init_evaluator(S, &myfun2())
: solvenl_init_type(S, "zero")
: solvenl_init_technique(S, "newton")
: solvenl_init_numeq(S, 3)
: solvenl_init_startingvals(S, J(3,1,.2))
solvenl_init_iter_log(S, "on")
x = solvenl_solve(S)
Iteration 0: function = 416.03613
Iteration 1: function = 63.014451 delta X = 1.2538445
Iteration 2: function = 56.331397 delta X = .70226488
Iteration 3: function = 48.572941 delta X = . 35269647
Iteration 4: function = 37.434106 delta X = . 30727054
Iteration 5: function = 19.737501 delta X = . 38136739
Iteration 6: function = . 49995202 delta X = . 2299557
Iteration 7: function = 1.164e-08 delta X = . 09321045
Iteration 8: function = 4.154e-16 delta X = .00011039
x
    1
        8.771286448
        . }259695449
        -1.372281335
```


## Writing a fixed-point problem as a zero-finding problem and vice versa

Earlier, we solved the system of equations

$$
\begin{aligned}
& x=\frac{5}{3}-\frac{2}{3} y \\
& y=\frac{10}{3}-\frac{2}{3} x
\end{aligned}
$$

by searching for a fixed point. We can rewrite this system as

$$
\begin{aligned}
& x-\frac{5}{3}+\frac{2}{3} y=0 \\
& y-\frac{10}{3}+\frac{2}{3} x=0
\end{aligned}
$$

and then use BP or Newton's method to find the solution. In general, we simply rewrite $\mathbf{x}^{*}=\mathbf{F}\left(\mathbf{x}^{*}\right)$ as $\mathbf{x}^{*}-\mathbf{F}\left(\mathbf{x}^{*}\right)=\mathbf{0}$.

Similarly, we may be able to rearrange the constituent equations of a system of the form $\mathbf{F}(\mathbf{x})=\mathbf{0}$ so that each variable is an explicit function of the other variables in the system. If that is the case, then GS or dGS can be used to find the solution.

## Gauss-Seidel methods

Let $\mathbf{x}_{i-1}$ denote the previous iteration's values or the initial values, and let $\mathbf{x}_{i}$ denote the current iteration's values. The Gauss-Jacobi method simply iterates on $\mathbf{x}_{i}=\mathbf{F}\left(\mathbf{x}_{i-1}\right)$ by evaluating each equation in order. The Gauss-Seidel method implemented in solvenl_*() instead uses the new, updated values of $\mathbf{x}_{i}$ that are available for equations 1 through $j-1$ when evaluating equation $j$ at iteration $i$.

For damped Gauss-Seidel, again let $\mathbf{x}_{i}$ denote the values obtained from evaluating $\mathbf{F}\left(\mathbf{x}_{i-1}\right)$. However, after evaluating $\mathbf{F}$, dGS calculates the new parameter vector that is carried over to the next iteration as

$$
\mathbf{x}_{i}^{\#}=(1-\delta) \mathbf{x}_{i}+\delta \mathbf{x}_{i-1}
$$

where $\delta$ is the damping factor. Not fully updating the parameter vector at each iteration helps facilitate convergence in many problems. The default value of $\delta$ for method dGS is 0.1 , representing just a small amount of damping, which is often enough to achieve convergence. You can use solvenl_init_damping () to change $\delta$; the current implementation uses the same value of $\delta$ for all iterations. Increasing the damping factor generally slows convergence by requiring more iterations.

## Newton-type methods

Newton's method for solving $\mathbf{F}(\mathbf{x})=\mathbf{0}$ is based on the approximation

$$
\mathbf{F}\left(\mathbf{x}_{i}\right) \approx \mathbf{F}\left(\mathbf{x}_{i-1}\right)+\mathbf{J}\left(\mathbf{x}_{i-1}\right) \times\left(\mathbf{x}_{i}-\mathbf{x}_{i-1}\right)
$$

where $\mathbf{J}\left(\mathbf{x}_{i-1}\right)$ is the Jacobian matrix of $\mathbf{F}\left(\mathbf{x}_{i-1}\right)$. Rearranging and incorporating a step-length parameter $\alpha$, we have the iteration

$$
\mathbf{x}_{i}=\mathbf{x}_{i-1}-\alpha \mathbf{J}^{-1}\left(\mathbf{x}_{i-1}\right) \times \mathbf{F}\left(\mathbf{x}_{i-1}\right)
$$

We calculate $\mathbf{J}$ numerically by using the deriv() (see [M-5] deriv()) suite of functions. In fact, we do not calculate the inverse of $\mathbf{J}$; we instead use LU decomposition to solve for $\mathbf{x}_{i}-\mathbf{x}_{i-1}$.

To speed up convergence, we define the function $f(\mathbf{x})=\mathbf{F}(\mathbf{x})^{\prime} \mathbf{F}(\mathbf{x})$ and then choose $\alpha$ between 0 and 1 such that $f\left(\mathbf{x}_{i}\right)$ is minimized. We use a golden-section line search with a maximum of 20 iterations to find $\alpha$.

Because we must compute a $k \times k$ Jacobian matrix at each iteration, Newton's method can be slow. The BP method, similar to quasi-Newton methods for optimization, instead builds and updates an approximation B to the Jacobian matrix at each iteration. The BP update is

$$
\mathbf{B}_{i}=\mathbf{B}_{i-1}+\frac{\mathbf{y}_{i}-\mathbf{B}_{i-1} \mathbf{d}_{i}}{\mathbf{d}_{i}^{\prime} \mathbf{d}_{i}} \mathbf{d}_{i}^{\prime}
$$

where $\mathbf{d}_{i}=\mathbf{x}_{i}-\mathbf{x}_{i-1}$ and $\mathbf{y}_{i}=\mathbf{F}\left(\mathbf{x}_{i}\right)-\mathbf{F}\left(\mathbf{x}_{i-1}\right)$. Our initial estimate of the Jacobian matrix is calculated numerically at the initial values by using deriv(). Other than how the Jacobian matrix is updated, the BP method is identical to Newton's method, including the use of a step-length parameter determined by using a golden-section line search at each iteration.

## Convergence criteria

solvenl_*() stops if more than maxiter iterations are performed, where maxiter is c (maxiter) by default and can be changed by using solvenl_init_conv_maxiter(). Convergence is not declared after maxiter iterations unless one of the following convergence criteria is also met.

Let $\mathbf{x}_{i}$ denote the proposed solution at iteration $i$, and let $\mathbf{x}_{i-1}$ denote the proposed solution at the previous iteration. Then the parameters have converged when mreldif ( $\mathbf{x}_{i}, \mathbf{x}_{i-1}$ ) <itol, where itol is $1 e-9$ by default and can be changed by using solvenl_init_conv_iterchng(). Techniques GS and dGS use only this convergence criterion.

For BP and Newton's method, let $f\left(\mathbf{x}_{i}\right)=\mathbf{F}\left(\mathbf{x}_{i}\right)^{\prime} \mathbf{F}\left(\mathbf{x}_{i}\right)$. Then convergence is declared if mreldif $\left(\mathbf{x}_{i}, \mathbf{x}_{i-1}\right)<$ itol or $f\left(\mathbf{x}_{i}\right)<z$ tol, where ztol is $1 \mathrm{e}-9$ by default and can be changed by using solvenl_init_conv_nearzero().

## Exiting early

In some applications, you might have a condition that indicates your problem either has no solution or has a solution that you know to be irrelevant. In these cases, you can return a column vector with zero rows. solvenl() will then exit immediately and return an error code indicating you have requested an early exit.

To obtain this behavior, include the following code in your evaluator:

```
: void function myfun(real colvector from, real colvector values)
>
> ...
> if (condition)
> values = J(0, 1, .)
> return
>
> values[1] = 5/3 - 2/3*from[2]
> values[2] = 10/3 - 2/3*from[1]
> ...
>
```

Then if condition is true, solvenl () exits, solvenl_result_error_code() returns error code 27, and solvenl_result_converged() returns 0 (indicating a solution has not been found).

## Functions

## solvenl_init()

```
solvenl_init()
```

solvenl_init() is used to initialize the solver. Store the returned result in a variable name of your choosing; we use the letter $S$. You pass $S$ as the first argument to the other solvenl() suite of functions.
solvenl_init() sets all solvenl_init_*() values to their defaults. You can use the query form of the solvenl_init_*() functions to determine their values. Use solvenl_dump() to see the current state of the solver, including the current values of the solvenl_init_*() parameters.

## solvenl_init_type()

void

```
solvenl_init_type(S, { "fixedpoint" | "zero" })
```

string scalar solvenl_init_type (S)
solvenl_init_type ( $S$, type) specifies whether to find a fixed point or a zero of the function. type may be fixedpoint or zero.

If you specify solvenl_init_type( $S$, "fixedpoint") but have not yet specified a technique, then technique is set to dampedgaussseidel.

If you specify solvenl_init_type( $S$, "zero") but have not yet specified a technique, then technique is set to broydenpowell.
solvenl_init_type ( $S$ ) returns "fixedpoint" or "zero" depending on how the solver is currently set.

## solvenl_init_startingvals( )

```
void solvenl_init_startingvals ( \(S\), real colvector ivals)
real colvector solvenl_init_startingvals(S)
```

solvenl_init_startingvals ( $S$, ivals) sets the initial values for the solver to ivals. By default, ivals is set to the zero vector.
solvenl_init_startingvals $(S)$ returns the currently set initial values.

## solvenl_init_numeq()

void solvenl_init_numeq ( $S$, real scalar $k$ )
real scalar solvenl_init_numeq ( $S$ )
solvenl_init_numeq $(S, k)$ sets the number of equations in the system to $k$.
solvenl_init_numeq $(S)$ returns the currently specified number of equations.

## solvenl_init_technique()

```
void solvenl_init_technique(S, technique)
string scalar solvenl_init_technique(S)
```

solvenl_init_technique ( $S$, technique) specifies the solver technique to use. For more information, see technique above.

If you specify techniques gaussseidel or dampedgaussseidel but have not yet called solvenl_init_type(), solvenl_*() assumes you are solving a fixed-point problem until you specify otherwise.

If you specify techniques broydenpowell or newtonraphson but have not yet called solvenl_init_type(), solvenl_*() assumes you have a zero-finding problem until you specify otherwise.
solvenl_init_technique $(S)$ returns the currently set solver technique.

## solvenl_init_conv_iterchng()

```
void solvenl_init_conv_iterchng(S,itol)
real scalar solvenl_init_conv_iterchng(S)
```

solvenl_init_conv_iterchng ( $S$, itol) specifies the tolerance used to determine whether successive estimates of the solution have converged. Convergence is declared when mreldif $(\mathbf{x}(i), \mathbf{x}(i-1))<$ itol. For more information, see Convergence criteria above. The default is $1 \mathrm{e}-9$.
solvenl_init_conv_iterchng $(S)$ returns the currently set value of itol.

## solvenl_init_conv_nearzero()

void solvenl_init_conv_nearzero ( $S$, ztol)
real scalar solvenl_init_conv_nearzero $(S)$
solvenl_init_conv_nearzero ( $S$, ztol) specifies the tolerance used to determine whether the proposed solution to a zero-finding problem is sufficiently close to 0 based on the squared Euclidean distance. For more information, see Convergence criteria above. The default is 1e-9.
solvenl_init_conv_nearzero ( $S$ ) returns the currently set value of ztol.
solvenl_init_conv_nearzero() only applies to zero-finding problems. solvenl_*() simply ignores this criterion when solving fixed-point problems.

## solvenl_init_conv_maxiter()

```
void solvenl_init_conv_maxiter (S, maxiter)
real scalar solvenl_init_conv_maxiter \((S)\)
```

solvenl_init_conv_maxiter ( $S$, maxiter) specifies the maximum number of iterations to perform. Even if maxiter iterations are performed, convergence is not declared unless one of the other convergence criteria is also met. For more information, see Convergence criteria above. The default maxiter is the number set using set maxiter, which is 300 by default.
solvenl_init_conv_maxiter $(S)$ returns the currently set value of maxiter.

## solvenl_init_evaluator()

```
void
solvenl_init_evaluator ( \(S\), pointer(real function)
    scalar fptr)
```

pointer(real function) scalar solvenl_init_evaluator $(S)$
solvenl_init_evaluator ( $S$, fptr) specifies the function to be called to evaluate $\mathbf{F}(\mathbf{x})$. You must use this function. If your function is named myfen(), then you specify solvenl_init_evaluator ( $S$, \&myfcn()).
solvenl_init_evaluator $(S)$ returns a pointer to the function that has been set.

## solvenl_init_argument( ) and solvenl_init_narguments()

```
void
void solvenl_init_narguments(S, real scalar K)
pointer scalar solvenl_init_argument(S, real scalar k)
real scalar solvenl_init_narguments(S)
```

solvenl_init_argument $(S, k, X)$ sets the $k$ th extra argument of the evaluator function as $X$, where $k$ can be 1,2 , or 3 . If you need to pass more items to your evaluator, collect them into a structure and pass the structure. $X$ can be anything, including a pointer, a view matrix, or simply a scalar. No copy of $X$ is made; it is passed by reference. Any changes you make to $X$ elsewhere in your program will be reflected in what is passed to your evaluator function.
solvenl_init_narguments $(S, K)$ sets the number of extra arguments to be passed to your evaluator function. Use of this function is optional; initializing an additional argument by using solvenl_init_argument() automatically sets the number of arguments.
solvenl_init_argument $(S, k)$ returns a pointer to the previously set $k$ th additional argument.
solvenl_init_narguments $(S)$ returns the number of extra arguments that are passed to the evaluator function.

## solvenl_init_damping()

```
void solvenl_init_damping(S, real scalar d)
real scalar solvenl_init_damping(S)
```

solvenl_init_damping $(S, d)$ sets the damping parameter used by the damped Gauss-Seidel technique to $d$, where $0 \leq d<1$. That is, $d=0$ corresponds to no damping, which is equivalent to plain Gauss-Seidel. As $d$ approaches 1 , more damping is used. The default is $d=0.1$. If the dGS technique is not being used, this parameter is ignored.
solvenl_init_damping $(S)$ returns the currently set damping parameter.

## solvenl_init_iter_log()

```
void solvenl_init_iter_log(S, {"on"|"off"})
string scalar solvenl_init_iter_log(S)
```

solvenl_init_iter_log ( $S$, onoff ) specifies whether an iteration $\log$ should or should not be displayed. onoff may be "on" or "off". By default, an iteration log is displayed unless set iterlog is set to "off"; see [R] set iter.
solvenl_init_iter_log $(S)$ returns the current status of the iteration $\log$ indicator.

## solvenl_init_iter_dot()

```
void solvenl_init_iter_dot(S, {"on"|"off"})
string scalar solvenl_init_iter_dot(S)
```

solvenl_init_iter_dot ( $S$, onoff) specifies whether an iteration dot should or should not be displayed. onoff may be "on" or "off". By default, an iteration dot is not displayed.

Specifying solvenl_init_iter_dot( $S$, on) results in the display of a single dot without a new line after each iteration is completed. This option can be used to create a compact status report when a full iteration $\log$ is too detailed but some indication of activity is warranted.
solvenl_init_iter_dot $(S)$ returns the current status of the iteration dot indicator.

## solvenl_init_iter_dot_indent()

```
void solvenl_init_iter_dot_indent(S, real scalar indent)
string scalar solvenl_init_iter_dot_indent(S)
```

solvenl_init_iter_dot_indent ( $S$, indent) specifies how many spaces from the left edge iteration dots should begin. This option is useful if you are writing a program that calls solvenl() and if you want to control how the iteration dots appear to the user. By default, the dots start at the left edge (indent $=0$ ). If you do not turn on iteration dots with solvenl_init_iter_dot(), this option is ignored.
solvenl_init_iter_dot_indent $(S)$ returns the current amount of indentation.

## solvenl_init_deriv_usemin( ) and solvenl_init_deriv_min( )

```
void solvenl_init_deriv_usemin(S, \{"off"|"on" \})
void solvenl_init_deriv_min( \(S\), real rowvector min)
string scalar solvenl_init_deriv_usemin(S)
real rowvector solvenl_init_deriv_min( \(S\) )
```

solvenl_init_deriv_usemin(S, \{ "off"|"on" \}) specifies whether to use the minimum values of step sizes for computing numerical derivatives. The default is "off", meaning minimum values will not be used. If you specify "on", minimum values will be used; when minimum values are not specified by solvenl_init_deriv_min(), default minimum values are $1 e-6$ and $1 \mathrm{e}-4$ for firstand second-order derivatives, respectively.
solvenl_init_deriv_min ( $S$, min) sets the minimum values of step sizes used for computing numerical derivatives. min is a row vector with two columns; the first column specifies the minimum for first-order derivatives, and the second column specifies the minimum for second-order derivatives. If there is a missing value in the row vector, the default values of $1 e-6$ for first-order derivatives and $1 e-4$ for second-order derivatives are used. If this function is used, solvenl_init_deriv_usemin() is automatically set to "on".
solvenl_init_deriv_usemin( $S$ ) returns the current value "off" or "on", indicating whether to use minimum values of the step size.
solvenl_init_deriv_min(S) returns the current value min.
These functions only work with techniques broydenpowell or newtonraphson.

## solvenl_solve() and _solvenl_solve()

real colvector solvenl_solve ( $S$ )
void _solvenl_solve ( $S$ )
solvenl_solve ( $S$ ) invokes the solver and returns the resulting solution. If an error occurs, solvenl_solve() aborts with error.
_solvenl_solve ( $S$ ) also invokes the solver. Rather than returning the solution, this function returns an error code if something went awry. If the solver did find a solution, this function returns 0 . See below for a list of the possible error codes.

Before calling either of these functions, you must have defined your problem. At a minimum, this involves calling the following functions:

```
solvenl_init()
solvenl_init_numeq()
solvenl_init_evaluator()
solvenl_init_type() or solvenl_init_technique()
```


## solvenl_result_converged()

real scalar solvenl_result_converged ( $S$ )
solvenl_result_converged $(S)$ returns 1 if the solver found a solution to the problem and 0 otherwise.

```
solvenl_result_conv_iter()
```

```
real scalar solvenl_result_conv_iter(S)
```

solvenl_result_conv_iter $(S)$ returns the number of iterations required to obtain the solution. If a solution was not found or the solver has not yet been called, this function returns missing.

## solvenl_result_conv_iterchng()

real scalar solvenl_result_conv_iterchng ( $S$ )
solvenl_result_conv_iterchng $(S)$ returns the final tolerance achieved for the parameters if a solution has been reached. Otherwise, this function returns missing. For more information, see Convergence criteria above.

## solvenl_result_conv_nearzero()

real scalar solvenl_result_conv_nearzero $(S)$
solvenl_result_conv_nearzero ( $S$ ) returns the final distance the solution lies from zero if a solution has been reached. Otherwise, this function returns missing. This function also returns missing if called after either GS or dGS was used because this criterion does not apply. For more information, see Convergence criteria above.

## solvenl_result_values()

real colvector solvenl_result_values ( $S$ )
solvenl_result_values $(S)$ returns the column vector representing the fixed- or zero-point of the function if a solution was found. Otherwise, it returns a $0 \times 1$ vector of missing values.

## solvenl_result_Jacobian()

real matrix solvenl_result_Jacobian (S)
solvenl_result_Jacobian ( $S$ ) returns the last-calculated Jacobian matrix if BP or Newton's method was used to find a solution. The Jacobian matrix is returned even if a solution was not found because we have found the Jacobian matrix to be useful in pinpointing problems. This function returns a $1 \times 1$ matrix of missing values if called after either GS or dGS was used.
solvenl_result_error_code( ), . . __return_code( ), and . . ._error_text( )

```
real scalar solvenl_result_error_code(S)
real scalar solvenl_result_return_code(S)
string scalar solvenl_result_error_text(S)
```

solvenl_result_error_code ( $S$ ) returns the unique solvenl_*() error code generated or zero if there was no error. Each error that can be produced by the system is assigned its own unique code.
solvenl_result_return_code $(S)$ returns the appropriate return code to be returned to the user if an error was produced.
solvenl_result_error_text ( $S$ ) returns an appropriate textual description to be displayed if an error was produced.

The error codes, return codes, and error text are listed below.

| Error <br> code | Return <br> code | Error text |
| :---: | :---: | :--- |
| 0 | 0 | (no error encountered) |
| 1 | 0 | (problem not yet solved) |
| 2 | 111 | did not specify function |
| 3 | 198 | invalid number of equations specified |
| 4 | 504 | initial value vector has missing values |
| 5 | 503 | initial value vector length does not equal number of equations declared |
| 6 | 430 | maximum iterations reached; convergence not achieved |
| 7 | 416 | missing values encountered when evaluating function |
| 8 | 3498 | invalid function type |
| 9 | 3498 | function type ... cannot be used with technique ... |
| 10 | 3498 | invalid log option |
| 11 | 3498 | invalid solution technique |
| 12 | 3498 | solution technique technique cannot be used with function |
|  |  | type \{ "fixedpoint" \|"zero" \} |
| 13 | 3498 | invalid iteration change criterion |
| 14 | 3498 | invalid near-zeroness criterion |
| 15 | 3498 | invalid maximum number of iterations criterion |
| 16 | 3498 | invalid function pointer |
| 17 | 3498 | invalid number of arguments |
| 18 | 3498 | optional argument out of range |
| 19 | 3498 | could not evaluate function at initial values |
| 20 | 3498 | could not calculate Jacobian at initial values |
| 21 | 3498 | iterations found local minimum of $\mathbf{F}^{\prime} \mathbf{F}$; convergence not achieved |
| 22 | 3498 | could not calculate Jacobian matrix |
| 23 | 198 | damping factor must be in [0, 1) |
| 24 | 198 | must specify a function type, technique, or both |
| 25 | 3498 | invalid solvenl_init_iter_dot () option |

## solvenl_dump()

void solvenl_dump ( $S$ )
solvenl_dump ( $S$ ) displays the current status of the solver, including initial values, convergence criteria, results, and error messages. This function is particularly useful while debugging.

## Conformability

```
All functions' inputs are 1\times1 and return 1\times1 or void results except as noted below:
solvenl_init_startingvals(S, ivals):
            S: transmorphic
    ivals: 
    result: void
solvenl_init_startingvals(S):
            S: transmorphic
    result: }k\times
solvenl_init_argument(S,k,X):
            S: transmorphic
            k: 1 < 1
            X: anything
    result: void
solvenl_init_argument(S,k):
            S: transmorphic
            k: 
    result: anything
solvenl_init_deriv_min(S, min):
            S: transmorphic
        min: }1\times
    result: void
solvenl_init_deriv_min(S):
    S: transmorphic
    result: }\quad1\times
solvenl_solve(S):
            S: transmorphic
    result: }\quadk\times
solvenl_result_values(S):
            S: transmorphic
    result: }k\times
solvenl_result_Jacobian(S):
            S: transmorphic
    result: }\quadk\times
```


## Diagnostics

All functions abort with an error if used incorrectly.
solvenl_solve() aborts with an error if it encounters difficulties. _solvenl_solve() does not; instead, it returns a nonzero error code.

The solvenl_result_*() functions return missing values if the solver encountered difficulties or else has not yet been invoked.

## References

Burden, R. L., D. J. Faires, and A. M. Burden. 2016. Numerical Analysis. 10th ed. Boston: Cengage.
Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery. 2007. Numerical Recipes: The Art of Scientific Computing. 3rd ed. New York: Cambridge University Press.

## Also see

[M-4] Mathematical - Important mathematical functions
[R] set iter - Control iteration settings

## Title

[M-5] sort() — Reorder rows of matrix

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

$\operatorname{sort}(X, i d x)$ returns $X$ with rows in ascending or descending order of the columns specified by $i d x$. For instance, $\operatorname{sort}(X, 1)$ sorts $X$ on its first column; $\operatorname{sort}(X,(1,2))$ sorts $X$ on its first and second columns (meaning rows with equal values in their first column are ordered on their second column). In general, the $i$ th sort key is column abs ( $i d x[i]$ ). Order is ascending if $i d x[i]>$ 0 and descending otherwise. Ascending and descending are defined in terms of [M-5] abs() (length of elements) for complex.
$-\operatorname{sort}(X, i d x)$ does the same as $\operatorname{sort}(X, i d x)$, except that $X$ is sorted in place.
jumble $(X)$ returns $X$ with rows in random order. For instance, to shuffle a deck of cards numbered 1 to 52 , one could code jumble(1::52). See rseed() in [M-5] runiform() for information on setting the random-number seed.
_jumble $(X)$ does the same as jumble $(X)$, except that $X$ is jumbled in place.
order ( $X, i d x$ ) returns the permutation vector-see [M-1] Permutation-that would put $X$ in ascending (descending) order of the columns specified by $i d x$. A row-permutation vector is a $1 \times c$ column vector containing the integers $1,2, \ldots, c$ in some order. Vectors $(1 \backslash 2 \backslash 3),(1 \backslash 3 \backslash 2),(2 \backslash 1 \backslash 3)$, $(2 \backslash 3 \backslash 1),(3 \backslash 1 \backslash 2)$, and $(3 \backslash 2 \backslash 1)$ are examples. Row-permutation vectors are used to specify the order in which the rows of a matrix $X$ are to appear. If $p$ is a row-permutation vector, $X[p,$. returns $X$ with its rows in the order of $p ; p=(3 \backslash 2 \backslash 1)$ would reverse the rows of $X$. order $(X$, $i d x$ ) returns the row-permutation vector that would sort $X$ and, as a matter of fact, sort ( $X, i d x$ ) is implemented as $X[\operatorname{order}(X, i d x),$.$] .$
unorder ( $n$ ) returns a $1 \times n$ permutation vector for placing the rows in random order. Random numbers are calculated by runiform(); see rseed() in [M-5] runiform() for information on setting the random-number seed. jumble() is implemented in terms of unorder(): jumble ( $X$ ) is equivalent to $X$ [unorder (rows $(X)$ ), .].
_collate $(X, p)$ is equivalent to $X=X[p,$.$] ; it changes the order of the rows of X$. _collate () is used by _sort () and _jumble() and has the advantage over subscripting in that no extra memory is required when the result is to be assigned back to itself. Consider

$$
X=X[p, .]
$$

There will be an instant after $X[p,$.$] has been calculated but before the result has been assigned$ back to $X$ when two copies of $X$ exist. _collate ( $X, p$ ) avoids that. _collate() is not a substitute for subscripting in all cases; _collate() requires $p$ be a permutation vector.

## Syntax

| transmorphic matrix | sort(transmorphic matrix $X$, real rowvector idx) |
| :--- | :--- |
| void | _sort(transmorphic matrix $X$, real rowvector idx) |
| transmorphic matrix | jumble (transmorphic matrix $X$ ) |
| void | -jumble (transmorphic matrix $X$ ) |
| real colvector | order (transmorphic matrix $X$, real rowvector idx) |
| real colvector | unorder (real scalar $n)$ |
| void |  |

where

1. $X$ may not be a pointer matrix.
2. $p$ must be a permutation column vector, a $c \times 1$ vector containing the integers $1,2, \ldots, c$ in some order.

## Remarks and examples

If $X$ is complex, the ordering is defined in terms of [M-5] abs() of its elements.
Also see invorder () and revorder () in [M-5] invorder( ). Let $p$ be the permutation vector returned by order():

$$
p=\operatorname{order}(X, \ldots)
$$

Then $X[p,$.$] are the sorted rows of X$. revorder() can be used to reverse sort order: $X$ [revorder $(p),$.$] are the rows of X$ in the reverse of the order of $X[p,$.$] . invorder () provides$ the inverse transform: If $Y=X[p,$.$] , then X=Y[\operatorname{invorder}(p),$.$] .$

## Conformability

```
sort(X,idx), jumble(X):
            X: }\quad\mp@subsup{r}{1}{}\times\mp@subsup{c}{1}{
        idx: }\quad1\times\mp@subsup{c}{2}{},\mp@subsup{c}{2}{}\leq\mp@subsup{c}{1}{
    result: }\quad\mp@subsup{r}{1}{}\times\mp@subsup{c}{1}{
_sort(X,idx), _jumble(X):
            X: }\quad\mp@subsup{r}{1}{}\times\mp@subsup{c}{1}{
        idx:
    result: void; }X\mathrm{ row order modified
order (X,idx):
\[
\begin{aligned}
X: & r_{1} \times c_{1} \\
\text { idx: } & 1 \times c_{2}, c_{2} \leq c_{1} \\
\text { result: } & r_{1} \times 1
\end{aligned}
\]
```

unorder ( $n$ ):

| $n:$ | $1 \times 1$ |
| :---: | :---: |
| result: | $n \times 1$ |
| ( $(X, p):$ |  |

$X: \quad r \times c$
$p: \quad r \times 1$
result: $\quad$ void; $\quad X$ row order modified

## Diagnostics

$\operatorname{sort}(X, i d x)$ aborts with error if any element of abs (idx) is less than 1 or greater than rows $(X)$. $\quad \operatorname{sort}(X, i d x)$ aborts with error if any element of abs (idx) is less than 1 or greater than rows $(X)$, or if $X$ is a view.
-jumble ( $X$ ) aborts with error if $X$ is a view.
$\operatorname{order}(X, i d x)$ aborts with error if any element of abs (idx) is less than 1 or greater than rows $(X)$. unorder ( $n$ ) aborts with error if $n<1$.
_collate $(X, p)$ aborts with error if $p$ is not a permutation vector or if $X$ is a view.

## Also see

[M-5] invorder() - Permutation vector manipulation
[M-5] uniqrows() - Obtain sorted, unique values
[M-5] ustrcompare( ) - Compare or sort Unicode strings
[M-4] Manipulation - Matrix manipulation

## Title

[M-5] soundex( ) - Convert string to soundex code

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

soundex $(s)$ returns the soundex code for a string, $s$. The soundex code consists of a letter followed by three numbers: the letter is the first letter of the name and the numbers encode the remaining consonants. Similar sounding consonants are encoded by the same number.
soundex_nara ( $s$ ) returns the U.S. Census soundex code for a string, $s$. The soundex code consists of a letter followed by three numbers: the letter is the first letter of the name and the numbers encode the remaining consonants. Similar sounding consonants are encoded by the same number.

When $s$ is not a scalar, these functions return element-by-element results.

## Syntax

```
string matrix soundex(string matrix s)
string matrix soundex_nara(string matrix s)
```


## Remarks and examples

```
soundex("Ashcraft") returns "A226".
soundex_nara("Ashcraft") returns "A261".
```


## Conformability

```
soundex (s), soundex_nara(s):
    s: }\quadr\times
    result: }\quadr\times
```


## Diagnostics

None.

## Also see <br> [M-4] String - String manipulation functions

## Title

[M-5] spline3() - Cubic spline interpolation

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Reference | Also see |  |

## Description

spline3 $(x, y)$ returns the coefficients of a cubic natural spline $S(x)$. The elements of $x$ must be strictly monotone increasing.
spline3eval (spline_info, $x$ ) uses the information returned by spline3() to evaluate and return the spline at the abscissas $x$. Elements of the returned result are set to missing if outside the range of the spline. $x$ is assumed to be monotonically increasing.

## Syntax

```
real matrix spline3(real vector x, real vector y)
real vector spline3eval(real matrix spline_info, real vector }x\mathrm{ )
```


## Remarks and examples

spline3() and spline3eval() is a translation into Mata of Herriot and Reinsch (CUBNATSPLINE) (1973).

For $x x$ in $\left[x_{i}, x_{i+1}\right)$ :

$$
\mathbf{S}(x x)=\left\{\left(d_{i} t+c_{i}\right) t+b_{i}\right\} t+y_{i}
$$

with $t=x x-x_{i}$.
spline3() returns $(b, c, d, x, y)$ or, if x and y are row vectors, $\left(b, c, d, x^{\prime}, y^{\prime}\right)$.

## Conformability

```
spline3(x, y):
\begin{tabular}{rlll}
\(x:\) & \(n \times 1\) & or & \(1 \times n\) \\
\(y:\) & \(n \times 1\) & or & \(1 \times n\) \\
result: & \(n \times 5\) & &
\end{tabular}
```

spline3eval (spline_info, $x$ ):
spline_info: $\quad n \times 5$

| $x:$ | $m \times 1$ | or | $1 \times m$ |
| ---: | :--- | :--- | :--- |
| result: | $m \times 1$ | or | $1 \times m$ |

## Diagnostics

spline3 $(x, y)$ requires that $x$ be in ascending order.
spline3eval (spline_info, $x$ ) requires that $x$ be in ascending order.

## Reference

Herriot, J. G., and C. H. Reinsch. 1973. Algorithm 472: Procedures for natural spline interpolation [E1]. Communications of the ACM 16: 763-768. https://doi.org/10.1145/362552.362558.

## Also see

[M-4] Mathematical - Important mathematical functions

## Title

[M-5] sqrt() - Square root
Description Syntax Conformability Diagnostics Also see

## Description

sqrt ( $Z$ ) returns the elementwise square root of $Z$.

## Syntax

numeric matrix sqrt (numeric matrix $Z$ )

## Conformability

```
sqrt(Z)
Z: 
```


## Diagnostics

sqrt $(Z)$ returns missing when $Z$ is real and $Z<0$; that is, sqrt $(-4)=$. but sqrt $(-4+0$ i) $=$ 2 i .

## Also see

[M-5] cholesky () - Cholesky square-root decomposition
[M-4] Scalar - Scalar mathematical functions

## Title

[M-5] st_addalias() - Add alias variable to current Stata dataset

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

st_addalias (name, linkname, varname) adds new variable name to the Stata dataset as an alias to variable varname in another frame linked with the current frame by linkname. Returned is the scalar variable index of the new variable. st_addalias () aborts with error (and adds no variable) if the variable already exists or cannot be added for other reasons.
_st_addalias() does the same thing as st_addalias() except that, rather than aborting with error if the new variable cannot be added, returned is a scalar containing the negative of the appropriate Stata return code.

If you do not know what a frame is, see [D] frames intro.

## Syntax

```
real scalar st_addalias(name, linkname, varname)
real scalar _st_addalias(name, linkname, varname)
```

where
name: $\quad$ string scalar containing a new variable name
linkname: string scalar containing the name of a Stata variable in the current frame that identifies a linked frame
varname: string scalar containing the name of a Stata variable in the linked frame

## Remarks and examples

Remarks are presented under the following headings:
Creating a new alias variable
Handling errors

## Creating a new alias variable

To create an alias variable, you must have already established a link between two frames. If you do not know about linking frames, see [D] frlink. Suppose the linkage variable created by frlink is named linkage and a variable in the linked frame is named targvar. To create an alias variable named myvar linked to targvar, code

```
idx = st_addalias("myvar", "linkage", "targvar")
```

or

```
(void) st_addalias("myvar", "linkage", "targvar")
```

You use the first form if you will subsequently need the variable's index number, or you use the second form otherwise.

## Handling errors

There are many common reasons why st_addalias() might fail: the variable name is invalid, a variable under that name already exists, the linkage variable is invalid or does not exist, or the target variable is invalid or does not exist. If there is a problem adding a variable, st_addalias() will abort with error. If you wish to avoid the traceback $\log$ and just have Stata issue an error, use _st_addalias() and code

```
idx = _st_addalias("myvar", "linkage", "targvar")
if (idx<0) exit(-idx)
```


## Conformability

```
st_addalias(name, linkname, varname):
    name: }1\times
    linkname: 
    varname: }1\times
    result: }1\times
_st_addalias(name, linkname, varname):
    name: 
    linkname: 
    varname: }1\times
    result: }\quad1\times
```


## Diagnostics

st_addalias (name, linkname, varname) aborts with error if

1. name is not a valid variable name;
2. a variable named name already exists;
3. linkname is not a valid link variable in the current dataset;
4. varname is not a variable in the linked dataset.
_st_addalias(name, linkname, varname) returns the negative value of the appropriate Stata return code.

If a variable is created, st_updata() (see [M-5] st_updata()) is set unless name is temporary; see [M-5] st_tempname().

## Also see

[M-5] st_addvar( ) - Add variable to current Stata dataset
[M-5] st_isalias( ) - Properties of alias variable
[M-4] Stata - Stata interface functions
[D] fralias - Alias variables from linked frames
[D] frlink — Link frames
[D] frunalias - Change storage type of alias variables

## Title

[M-5] st_addobs( ) — Add observations to current Stata dataset

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

st_addobs ( $n$ ) adds $n$ observations to the current Stata dataset.
st_addobs ( $n$, nofill) does the same thing but saves computer time by not filling in the additional observations with the appropriate missing-value code if nofill $\neq 0$. st_addobs ( $n, 0$ ) is equivalent to st_addobs ( $n$ ). Use of st_addobs () with nofill $\neq 0$ is not recommended. If you specify nofill $\neq 0$, it is your responsibility to ensure that the added observations ultimately are filled in or removed before control is returned to Stata.
_st_addobs ( $n$ ) and _st_addobs ( $n$, nofill) perform the same action as st_addobs ( $n$ ) and st_addobs ( $n$, nofill), except that they return 0 if successful and the appropriate Stata return code otherwise (otherwise usually being caused by insufficient memory). Where _st_addobs() would return nonzero, st_addobs () aborts with error.

## Syntax

| void | st_addobs (real scalar $n$ ) |
| :--- | :--- |
| void | st_addobs (real scalar n, real scalar nofill) |
| real scalar | _st_addobs (real scalar $n$ ) |
| real scalar | _st_addobs (real scalar $n$, real scalar nofill) |

## Remarks and examples

There need not be any variables defined to add observations. If you are attempting to create a dataset from nothing, you can add the observations first and then add the variables, or you can add the variables and then add the observations. Use st_addvar () (see [M-5] st_addvar()) to add variables.

## Conformability

```
st_addobs(n, nofill):
    n:
        nofill: }1\times1\quad\mathrm{ (optional)
        result: void
_st_addobs(n, nofill):
            n:
    nofill: }1\times1\quad\mathrm{ (optional)
    result: }1\times
```


## Diagnostics

st_addobs ( $n$ [, nofill $]$ ) and _st_addobs ( $n[$, nofill $]$ ) abort with error if $n<0$. They do nothing if $n=0$.
st_addobs() aborts with error if there is insufficient memory to add the requested number of observations.
_st_addobs () aborts with error if $n<0$ but otherwise returns the appropriate Stata return code if the observations cannot be added. If they are added, 0 is returned.
st_addobs() and _st_addobs() do not set st_updata() (see [M-5] st_updata()); you must set it if you want it set.

## Also see

[M-4] Stata - Stata interface functions

## Title

[M-5] st_addvar( ) — Add variable to current Stata dataset

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Reference | Also see |  |

## Description

st_addvar (type, name) adds new variable name(s) of type type to the Stata dataset. Returned are the variable indices of the new variables. st_addvar () aborts with error (and adds no variables) if any of the variables already exist or cannot be added for other reasons.
st_addvar (type, name, nofill) does the same thing. nofill $\neq 0$ specifies that the variables' values are not to be filled in with missing values. st_addvar (type, name, 0 ) is the same as st_addvar (type, name). Use of nofill $\neq 0$ is not, in general, recommended. See Using nofill in Remarks and examples below.
_st_addvar () does the same thing as st_addvar() except that, rather than aborting with error if the new variable cannot be added, returned is a $1 \times 1$ scalar containing the negative of the appropriate Stata return code.

To create alias variables, see [M-5] st_addalias().

## Syntax

```
real rowvector st_addvar(type, name)
real rowvector st_addvar(type, name, nofill)
real rowvector _st_addvar(type, name)
real rowvector _st_addvar(type, name, nofill)
```

where
type: string scalar or rowvector containing "byte", "int", "long", "float", "double", "str\#", or "strL"
or
real scalar or rowvector containing \# (interpreted as str\#)
name: string rowvector containing new variable names
nofill: real scalar containing 0 or non-0

## Remarks and examples

Remarks are presented under the following headings:

> Creating a new variable
> Creating new variables
> Creating new string variables
> Creating a new temporary variable
> Creating temporary variables
> Handling errors
> Using nofill

## Creating a new variable

To create new variable myvar as a double, code

```
idx = st_addvar("double", "myvar")
```

or

```
(void) st_addvar("double", "myvar")
```

You use the first form if you will subsequently need the variable's index number, or you use the second form otherwise.

## Creating new variables

You can add more than one variable. For instance,

```
idx = st_addvar("double", ("myvar1","myvar2"))
```

adds two new variables, both of type double.

```
idx = st_addvar(("double","float"), ("myvar1","myvar2"))
```

also adds two new variables, but this time, myvar1 is double and myvar2 is float.

## Creating new string variables

Creating string variables is no different from any other type:

```
idx = st_addvar(("str10","str5"), ("myvar1","myvar2"))
```

creates myvar1 as a str10 and myvar2 as a str5.
There is, however, another way to specify the types.

```
idx = st_addvar((10,5), ("myvar1","myvar2"))
```

also creates myvar1 as a str10 and myvar2 as a str5.

```
idx = st_addvar(10, ("myvar1","myvar2"))
```

creates both variables as str10s.

## Creating a new temporary variable

Function st_tempname() (see [M-5] st_tempname()) returns temporary variable names. To create a temporary variable as a double, code

```
idx = st_addvar("double", st_tempname())
```

or code
(void) st_addvar("double", name=st_tempname())

You use the first form if you will subsequently need the variable's index, or you use the second form if you will subsequently need the variable's name. You will certainly need one or the other. If you will need both, code

```
idx = st_addvar("double", name=st_tempname())
```


## Creating temporary variables

st_tempname() can return a vector of temporary variable names.

```
idx = st_addvar("double", st_tempname(5))
```

creates five temporary variables, each of type double.

## Handling errors

There are three common reasons why st_addvar() might fail: the variable name is invalid or a variable under that name already exists or there is insufficient memory to add another variable. If there is a problem adding a variable, st_addvar() will abort with error. If you wish to avoid the traceback $\log$ and just have Stata issue an error, use _st_addvar () and code
if ((idx = _st_addvar("double", "myvar"))<0) exit(error(-idx))

If you are adding multiple variables, look at the first element of what _st_addvar() returns:
if ((idx = _st_addvar(types, names)) [1]<0) exit(error(-idx))

## Using nofill

The three-argument versions of st_addvar() and _st_addvar() allow you to avoid filling in the values of the newly created variable. Filling in those values with missing really is a waste of time if the next thing you are going to do is fill in the values with something else. On the other hand, it is important that all the observations be filled in on the new variable before control is returned to Stata, and this includes returning to Stata because of subsequent error or the user pressing Break. Thus use of nofill $\neq 0$ is not, in general, recommended. Filling in values really does not take that long.

If you are determined to save the computer time, however, see [M-5] setbreakintr(). To do things right, you need to set the break key off, create your variable, fill it in, and turn break-key processing back on.

There is, however, a case in which use of nofill $\neq 0$ is acceptable and such effort is not required: when you are creating a temporary variable. Temporary variables vanish in any case, and it does not matter whether they are filled in before they vanish.

Temporary variables in fact vanish not when Mata ends but when the ado-file calling Mata ends, if there is an ado-file. We will assume there is an ado-file because that is the only case in which you would be creating a temporary variable anyway. Because they do not disappear until later, there is the possibility of there being an issue if the variable is not filled in. If we assume, however, that your Mata program is correctly written and does fill in the variable ultimately, then the chances of a problem are minimal. If the user presses Break or there is some other problem in your program that causes Mata to abort, the ado-file will be aborted, too, and the variable will vanish.

Let us add that Stata will not crash if a variable is not filled in, even if it regains control. The danger is that the user will look at the variable or, worse, use it and be baffled by what he or she sees, which might concern not only odd values but also NaNs and worse.

## Conformability

```
st_addvar(type, name, nofill):
        type: }1\times1\mathrm{ or }1\times
        name: }1\times
        nofill: }1\times1\quad\mathrm{ (optional)
        result: }1\times
_st_addvar(type, name, nofill):
        type: }1\times1\quad\mathrm{ or }1\times
        name: }1\times
        nofill: }1\times1\quad\mathrm{ (optional)
        result: }1\timesk\quad\mathrm{ or, if error, }1\times
```


## Diagnostics

st_addvar (type, name, nofill) aborts with error if

1. type is not equal to a valid Stata variable type and it is not a number that would form a valid str\# variable type;
2. name is not a valid variable name;
3. a variable named name already exists;
4. there is insufficient memory to add another variable.
_st_addvar (type, name, nofill) aborts with error for reason 1 above, but otherwise, it returns the negative value of the appropriate Stata return code.

Both functions, when creating multiple variables, create either all the variables or none of them. Whether creating one variable or many, if variables are created, st_updata() (see [M-5] st_updata()) is set unless all variables are temporary; see [M-5] st_tempname().

## Reference

Gould, W. W. 2006. Mata Matters: Creating new variables—sounds boring, isn't. Stata Journal 6: 112-123.

## Also see

[M-5] st_addalias( ) - Add alias variable to current Stata dataset
[M-5] st_store( ) - Modify values stored in current Stata dataset
[M-5] st_tempname( ) - Temporary Stata names
[M-4] Stata - Stata interface functions

## Title

[M-5] st_data( ) - Load copy of current Stata dataset

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

_st_data $(i, j)$ returns the numeric value of the $i$ th observation of the $j$ th Stata variable. Observations are numbered 1 through st_nobs(). Variables are numbered 1 through st_nvar().
st_data $(i, j)$ is similar to _st_data $(i, j)$ except

1. $i$ may be specified as a vector or matrix to obtain multiple observations simultaneously,
2. $j$ may be specified using names or indices (indices are faster), and
3. $j$ may be specified to obtain multiple variables simultaneously.

The net effect is that st_data() can return a scalar (the value of one variable in one observation), a row vector (the value of many variables in an observation), a column vector (the value of a variable in many observations), or a matrix (the value of many variables in many observations).
st_data ( $i, j$, selectvar) works like st_data ( $i, j$ ) except that only observations for which selectvar $\neq 0$ are returned.
_st_sdata() and st_sdata() are the string variants of _st_data() and st_data(). _st_data() and st_data() are for use with numeric variables; they return missing (.) when used with string variables. _st_sdata() and st_sdata() are for use with string variables; they return empty string (" ") when used with numeric variables.

## Syntax

```
real scalar _st_data(real scalar i, real scalar j)
real matrix st_data(real matrix i, rowvector j)
real matrix st_data(real matrix i, rowvector j, scalar selectvar)
string scalar _st_sdata(real scalar i, real scalar j)
string matrix st_sdata(real matrix i, rowvector j)
string matrix st_sdata(real matrix i, rowvector j, scalar selectvar)
```

where

1. $i$ may be specified as a $1 \times 1$ scalar, as a $1 \times 1$ scalar containing missing, as a column vector of observation numbers, as a row vector specifying an observation range, or as a $k \times 2$ matrix specifying both.
a. st_data $(1,2)$ returns the first observation on the second variable.
b. st_data(., 2) returns all observations on the second variable.
c. st_data $((1 \backslash 2 \backslash 5), 2)$ returns observations 1,2 , and 5 on the second variable.
d. st_data $((1,5), 2)$ returns observations 1 through 5 on the second variable.
e. st_data $((1,5 \backslash 7,9), 2)$ returns observations 1 through 5 and observations 7 through 9 on the second variable.

When a range is specified, any element of the range ( $i_{1}, i_{2}$ ) may be specified to contribute zero observations if $i_{2}=i_{1}-1$.
2. $j$ may be specified as a real row vector or as a string scalar or string row vector.
a. st_data(., .) returns the values of all variables, all observations of the Stata dataset.
b. st_data(. , 1) returns the value of the first variable, all observations.
c. st_data(., $(3,1,9)$ ) returns the values of the third, first, and ninth variables of all observations.
d. st_data(., ("mpg", "weight")) returns the values of variables mpg and weight, all observations.
e. st_data(., ("mpg weight")) does the same as d above.
f. st_data(., ("gnp", "l.gnp")) returns the values of gnp and the lag of gnp, all observations.
g. st_data(., ("gnp l.gnp")) does the same as fabove.
h. st_data(., ("mpg i.rep78")) returns the value of mpg and the 5 pseudovariables associated with i.rep78. There are 5 pseudovariables because we are imagining that auto.dta is in memory; the actual number is a function of the values taken on by the variable in the sample specified. Factor variables can be specified only with string scalars; specifying ("mpg", "i.rep78") will not work.
3. selectvar may be specified as real or as a string. Observations for which selectvar $\neq 0$ will be selected. If selectvar is real, it is interpreted as a variable number. If string, selectvar should contain the name of a Stata variable.

Specifying selectvar as "" or as missing (.) has the same result as not specifying selectvar; no observations are excluded.

Specifying selectvar as 0 means that observations with missing values of the variables specified by $j$ are to be excluded.

## Remarks and examples

Remarks are presented under the following headings:

```
Description of _st_data() and _st_sdata()
Description of st_data() and st_sdata()
Details of observation subscripting using st_data() and st_sdata()
```


## Description of _st_data() and _st_sdata()

_st_data() returns one variable's value in one observation. You refer to variables and observations by their numbers. The first variable in the Stata dataset is 1 ; the first observation is 1 .

```
_st_data(1, 1) value of 1st obs., 1st variable
_st_data(1, 2) value of 1st obs., 2nd variable
_st_data(2, 1) value of 2nd obs., 1st variable
```

_st_sdata() works the same way. _st_data() is for use with numeric variables, and _st_sdata() is for use with string variables.
_st_data() and _st_sdata() are the fastest way to obtain the value of a variable in one observation.

## Description of st_data() and st_sdata()

st_data() can be used just like _st_data(), and used that way, it produces the same result. Variables, however, can be referred to by their names or their numbers:

```
st_data(1, 1)
st_data(1, 2)
st_data(2, 1)
st_data(1, "mpg")
st_data(2, "mpg")
```

value of 1st obs., 1st variable
value of 1st obs., 2nd variable
value of 2 nd obs., 1 st variable
value of 1 st obs, variable mpg
value of 2 nd obs, variable mpg

Also, you may specify more than one variable:

```
st_data(2, (1,2,3)) value of 2nd obs., variables 1, 2, and 3
st_data(2, ("mpg","weight","displ")) value of 2nd obs., variables
    mpg, weight, and displ
st_data(2, "mpg weight displ")
(same as previous)
```

Used this way, st_data() returns a row vector.
Similarly, you may obtain multiple observations:

```
st_data((1\2\3), 10)
st_data((1,5), 10)
st_data((1,5)\(7,9), 10)
values of obs. 1, 2, and 3, variable 10
values of obs. 1 through 5, variable 10
values of obs. 1 through 5 and 7 through 9 , variable 10
```

st_sdata() works the same way as st_data().

## Details of observation subscripting using st_data() and st_sdata()

1. $i$ may be specified as a scalar: the specified, single observation is returned. $i$ must be between 1 and st_nobs () ; see [M-5] st_nvar().
2. $i$ may be specified as a scalar containing missing value: all observations are returned.
3. $i$ may be specified as a column vector: the specified observations are returned. Each element of $i$ must be between 1 and st_nobs() or may be missing. Missing is interpreted as st_nobs().
4. $i$ may be specified as a $1 \times 2$ row vector: the specified range of observations is returned; $\left(c_{1}, c_{2}\right)$ returns the $c_{2}-c_{1}+1$ observations $c_{1}$ through $c_{2}$.
$c_{2}-c_{1}+1$ must evaluate to a number greater than or equal to 0 . In general, $c_{1}$ and $c_{2}$ must be between 1 and st_nobs(), but if $c_{2}-c_{1}+1=0$, then $c_{1}$ may be between 1 and st_nobs() +1 and $c_{2}$ may be between 0 and st_nobs (). Regardless, $c_{1}==$. or $c_{2}==$. is interpreted as st_nobs().
5. $i$ may be specified as a $k \times 2$ matrix: $((1,5) \backslash(7,7) \backslash(20,30))$ specifies observations 1 through 5,7 , and 20 through 30.

## Conformability

```
_st_data \((i, j)\), _st_sdata \((i, j)\) :
    \(i: \quad 1 \times 1\)
    \(j: \quad 1 \times 1\)
    result: \(\quad 1 \times 1\)
st_data \((i, j)\), st_sdata \((i, j)\) :
        \(\begin{array}{llll}i: & n \times 1 & \text { or } & n_{2} \times 2 \\ j: & 1 \times k & \text { or } & 1 \times 1 \text { containing } k \text { elements when expanded }\end{array}\)
    result: \(\quad n \times k\)
st_data( \(i, j\), selectvar), st_sdata( \(i, j\), selectvar \():\)
    \(i\) : \(n \times 1\) or \(n_{2} \times 2\)
    \(j\) : \(1 \times k \quad\) or \(\quad 1 \times 1\) containing \(k\) elements when expanded
    selectvar: \(\quad 1 \times 1\)
        result: \(\quad(n-e) \times k, \quad\) where \(e\) is number of observations excluded by selectvar
```


## Diagnostics

_st_data $(i, j)$ returns missing (.) if $i$ or $j$ is out of range; it does not abort with error. _st_sdata $(i, j)$ returns " " if $i$ or $j$ is out of range; it does not abort with error.
st_data $(i, j)$ and st_sdata $(i, j)$ abort with error if any element of $i$ or $j$ is out of range. $j$ may be specified as variable names or variable indices. If names are specified, abbreviations are allowed. If you do not want this and no factor variables nor time-series-operated variables are specified, use st_varindex() (see [M-5] st_varindex()) to translate variable names into variable indices.

## Also see

[M-5] st_view( ) - Make matrix that is a view onto current Stata dataset
[M-5] st_store( ) - Modify values stored in current Stata dataset
[M-4] Stata - Stata interface functions
[D] putmata - Put Stata variables into Mata and vice versa

## Title

[M-5] st_dir( ) — Obtain list of Stata objects
Description Syntax Conformability Diagnostics Also see

## Description

st_dir (cat, subcat, pattern) and st_dir (cat, subcat, pattern, adorn) return a column vector containing the names matching pattern of the Stata objects described by cat-subcat.

Argument adorn is optional; not specifying it is equivalent to specifying adorn $=0$. By default, simple names are returned. If adorn $\neq 0$ is specified, the name is adorned in the standard Stata way used to describe the object. Say that one is listing the macros in e() and one of the elements is $e(\mathrm{cmd})$. By default, the returned vector will contain an element equal to "cmd". With adorn $\neq 0$, the element will be "e(cmd)".

For many objects, the adorned and unadorned forms of the names are the same.

## Syntax

```
string colvector st_dir(cat, subcat, pattern)
string colvector st_dir(cat, subcat, pattern, adorn)
```

where

$$
\begin{array}{cl}
\text { cat: } & \begin{array}{l}
\text { string scalar containing "local", "global", "r()", "e()", "s()", } \\
\text { or "char" }
\end{array} \\
\text { subcat: } & \text { string scalar containing "macro", "numscalar", "strscalar", } \\
& \text { "matrix", or, if cat==" char", "_dta" or a name. } \\
\text { pattern: } & \text { string scalar containing a pattern as defined in [M-5] strmatch() } \\
\text { adorn: } & \text { string scalar containing 0 or non-0 }
\end{array}
$$

The valid cat-subcat combinations and their meanings are

| cat | subcat | Meaning |
| :--- | :--- | :--- |
| "local" | "macro" | Stata’s local macros |
| "global" | "macro" | Stata’s global macros |
| "global" | "numscalar" | Stata's numeric scalars |
| "global" | "strscalar" | Stata’s string scalars |
| "global" | "matrix" | Stata’s matrices |
| "r()" | "macro" | macros in r() |
| "r()" | "numscalar" | numeric scalars in r() |
| "r()" | "matrix" | matrices in r() |
| "e()" | "macro" | macros in e() |
| "e()" | "numscalar" | numeric scalars in e() |
| "e()" | "matrix" | matrices in e() |
| "s()" | "macro" | macros in s() |
| "char" | "_dta" | characteristics in _dta[] |
| "char" | "name" | characteristics in variable name[] |

st_dir() returns an empty list if an invalid cat-subcat combination is specified.

## Conformability

st_dir (cat, subcat, pattern, adorn):

| cat: | $1 \times 1$ |  |
| ---: | :--- | :--- |
| subcat: | $1 \times 1$ |  |
| pattern: | $1 \times 1$ |  |
| adorn: | $1 \times 1$ | (optional) |
| result: | $k \times 1$ |  |

## Diagnostics

st_dir(cat, subcat, pattern) and st_dir (cat, subcat, pattern, adorn) abort with error if cat or subcat is invalid. If the combination is invalid, however, $J(0,1, " ")$ is returned. subcat==name is considered invalid unless cat=="char".
st_dir() aborts with error if any of its arguments are views.

## Also see

[M-4] Stata - Stata interface functions

## Title

[M-5] st_dropvar( ) - Drop variables or observations

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

st_dropvar (vars) drops the variables specified. vars is a row vector that may contain either variable names or variable indices. st_dropvar(.) drops all variables and observations.
st_dropobsin() and st_dropobsif() have to do with dropping observations.
st_dropobsin(range) specifies the observations to be dropped:
st_dropobsin(5) drops observation 5.
st_dropobsin $((5,9))$ drops observations 5 through 9.
st_dropobsin((5\8\12)) drops observations 5 and 8 and 12.
st_dropobsin $((5,7 \backslash 8,11 \backslash 13,13))$ drops observations 5 through 7,8 through 11 , and 13.
st_dropobsin(.) drops all observations (but not the variables).
st_dropobsin(J ( $0,1,$.$) ) drops no observations (or variables).$
st_dropobsif (select) specifies a st_nobs() $\times 1$ vector. Observations $i$ for which select $_{i} \neq 0$ are dropped.
st_keepvar(), st_keepobsin(), and st_keepobsif() do the same thing, except that the variables and observations to be kept are specified.

## Syntax

```
void st_dropvar(transmorphic rowvector vars)
void st_dropobsin(real matrix range)
void st_dropobsif(real colvector select)
void st_keepvar(transmorphic rowvector vars)
void st_keepobsin(real matrix range)
void st_keepobsif(real colvector select)
```


## Remarks and examples

To drop all variables and observations, code any of the following:

```
st_dropvar(.)
st_keepvar(J(1,0,.))
st_keepvar(J(1,0,""))
```

All do the same thing. Dropping all the variables clears the dataset.
Dropping all the observations, however, leaves the variables in place.

## Conformability

```
st_dropvar(vars), st_keepvar(vars):
    vars: }1\times
    result: void
st_dropobsin(range), st_keepobsin(range):
    range: }k\times1\mathrm{ or k
    result: void
st_dropobsif(select), st_keepobsif(select):
    select: st_nobs() < 1
    result: void
```


## Diagnostics

st_dropvar (vars) and st_keepvar (vars) abort with error if any element of vars is missing unless vars is $1 \times 1$, in which case they drop or keep all the variables.
st_dropvar(vars) and st_keepvar(vars) abort with error if any element of vars is not a valid variable index or name, or if vars is a view. If vars is specified as names, abbreviations are not allowed.
st_dropvar() and st_keepvar() set st_updata() (see [M-5] st_updata()) unless all variables dropped are temporary; see [M-5] st_tempname().
st_dropobsin(range) and st_keepobsin(range) abort with error if any element of range is missing unless range is $1 \times 1$, in which case they drop or keep all the observations.
st_dropobsin(range) and st_keepobsin(range) abort with error if any element of range is not a valid observation number (is not between 1 and st_nobs() [see [M-5] st_nvar()] inclusive) or if range is a view.
st_dropobsif (select) and st_keepobsif(select) abort with error if select is a view.
st_dropobsin(), st_dropobsif(), st_keepobsin(), and st_keepobsif() set st_updata() if any observations are removed from the data.

Be aware that, after dropping any variables or observations, any previously constructed views (see [M-5] st_view()) are probably invalid because views are internally stored in terms of variable and observation numbers. Subsequent use of an invalid view may lead to unexpected results or an abort with error.

## Also see

[M-4] Stata - Stata interface functions

## Title

[M-5] st_frame*() — Data frame manipulation

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

st_framecurrent () returns the name of current (working) frame.
st_framedir () returns the names of all existing frames.
st_framecreate() makes a new frame without making it the current frame.
st_framecurrent() changes the identity of current (working) frame.
st_framerename() renames the existing frame, which can be the current frame.
st_framedrop() drops or eliminates the frame that is not the current frame.
st_framedropabc() drops or eliminates all frames except the current frame. abc stands for "all but current".
st_framereset () resets Stata or Mata to contain one empty frame named default.
st_framecopy() copies or duplicates complete contents from one frame to another, clearing the previous contents of the target frame if necessary.
st_frameexists() determines whether the frame named name exists.
_st_framecreate(), _st_framecurrent(), _st_framerename(), _st_framedrop(), and _st_framecopy() perform the same action as st_framecreate(), st_framecurrent(), st_framerename(), st_framedrop(), and st_framecopy(), respectively, except that they handle errors differently. The functions without a leading underscore issue an error message, display a traceback log, and abort execution when used incorrectly. The functions with a leading underscore do not abort. They return a nonzero value and execution continues.

For an overview of frames, see [D] frames intro.

## Syntax

| string scalar | st_framecurrent() |
| :---: | :---: |
| string colvector | st_framedir() |
| void | st_framecreate (fname) |
| real scalar | _st_framecreate(fname, noisy) |
| void | st_framecurrent (fname) |
| real scalar | _st_framecurrent (fname, noisy) |
| void | st_framerename(fname, newfname) |
| real scalar | _st_framerename(fname, newfname, noisy) |
| void | st_framedrop(fname) |
| real scalar | -st_framedrop(fname, noisy) |
| void | st_framedropabc() |
| void | st_framereset() |
| void | st_framecopy (fname_to, fname_from) |
| real scalar | _st_framecopy(fname_to, fname_from, noisy) |
| real scalar | st_frameexists (name) |

where
fname is a string scalar containing a name of an existing frame.
newfname is a string scalar containing a name that is not the name of an existing frame.
name is a string scalar containing a name, whether or not of an existing frame.
noisy is a real scalar containing 0 (error messages suppressed) or any nonzero value (error messages shown).

## Remarks and examples

Stata allows more than one dataset to be stored in memory. Each is stored in a separate frame, which you name. For an overview of frames, see [D] frames intro. The st_frame*() functions let you create new frames, delete existing ones, and switch the identity of the current or working frame from one frame to another. Stata commands and Mata functions work on the current (working) frame. Data from more than one frame may be accessed simultaneously by creating Mata views onto those frames and using them in expressions.

Notice that some of the st_frame*() commands are paired:

```
    void st_framecreate(...)
real scalar _st_framecreate(..., noisy)
    void st_framecurrent (...)
real scalar _st_framecurrent (..., noisy)
    void st_framerename(...)
real scalar _st_framerename(..., noisy)
    void st_framedrop(...)
real scalar _st_framedrop(..., noisy)
    void st_framecopy(...)
real scalar _st_framecopy(..., noisy)
```

The paired functions do the same thing but handle errors differently. The functions without a leading underscore issue an error message, display a traceback log, and abort execution when used incorrectly. For example,

```
: st_framecreate("default")
frame name default already exists
    st_framecreate(): 3598 Stata returned error
        <istm>: - function returned error
r(3598);
```

The functions with a leading underscore do not abort. They return a nonzero value and execution continues. Consider the following function:

```
void example()
{
    rc = _st_framecreate("default", 1)
    printf("execution continues, rc = %f\n", rc)
}
```

Execution of it results in

```
: example()
frame name default already exists
execution continues, rc = 110
```

The error message appeared but execution continued, and the error message appeared only because we coded 1 for noisy in the call to _st_framecreate():

```
rc = _st_framecreate("default", 1)
```

Had we coded 0, the error message would not have appeared, but execution would still have continued, and we would still see the execution-continues message, and rc would have still contained 110.

The 110 is an example of a Stata return code. Stata return codes are 0 when the function runs without error. The number 110 is the particular code for already exists. Something already existed, in this case, the frame name. If we had illustrated return codes using _st_framedrop() and specified a frame name that did not exist, the return code would have been 111, meaning something does not exist, that something being the frame name.

The underscore variants exist to allow you to write more elegant code in which the output does not suggest to the user that your code has a bug when it was in fact used incorrectly by the user. We could have written example() as

```
void example()
{
        rc = _st_framecreate("default", 1)
        if (rc!=0) exit(rc)
        printf("execution continues, rc = %f\n", rc)
}
```

and then the output would have been

```
: example()
```

frame name default already exists
r(110);

## Conformability

All arguments to the st_frame*() and _st_frame*() functions are $1 \times 1$.

## Diagnostics

st_framecurrent(), st_framedir(), st_framedropabc(), and st_frameexists() always run successfully. The other st_frame*() commands abort execution when errors occur.

The _st_frame*() commands never abort. They return 0 or, when errors occur, the relevant nonzero Stata return code.

## Also see

[D] frames intro - Introduction to frames
[M-5] st_store() - Modify values stored in current Stata dataset
[M-5] st_view( ) - Make matrix that is a view onto current Stata dataset
[M-4] Stata - Stata interface functions
[D] putmata - Put Stata variables into Mata and vice versa

## Title

[M-5] st_global( ) — Obtain strings from and put strings into global macros

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Reference | Also see |  |

## Description

st_global (name) returns the contents of the specified Stata global.
st_global (name, contents) sets or resets the contents of the specified Stata global. If the Stata global did not previously exist, a new global is created. If the global did exist, the new contents replace the old.
st_global (name, contents, hcat) and st_global_hcat (name) are used to set and query the hcat corresponding to an () or r() value. They are also rarely used. See [R] Stored results and [P] return for more information.

## Syntax

| string scalar | st_global (string scalar name) |
| :--- | :--- |
| void | st_global (string scalar name, string scalar contents) |
| void | st_global (string scalar name, string scalar contents, |
|  | string scalar hcat) |

string scalar st_global_hcat(string scalar name)
where

1. name is to contain
a. global macro such as "myname"
b. $r()$ macro such as $" r$ (names) "
c. e() macro such as "e(cmd)"
d. $s()$ macro such as "s(vars)"
e. c() macro such as "c(current_date)"
f. dataset characteristic such as "_dta[date]"
g. variable characteristic such as "mpg [note]"
2. st_global (name) returns the contents of the specified Stata global. It returns "" when the global does not exist.
3. st_global(name, contents) sets or resets the contents of the specified Stata global.
4. st_global (name, " ") deletes the specified Stata global. It does this even if name is not a macro. st_global ("r(N)", "") would delete $r(N)$ whether it were a macro, scalar, or matrix.
5. st_global (name, contents, hcat) sets or resets the contents of the specified Stata global, and it sets or resets the hidden or historical status when name is an e() or r() value. Allowed hcat values are "visible", "hidden", "historical", and a string scalar release number such as such as "10", "10.1", or any string release number matching "\#[\#][.[\#[\#]]]". See $[P]$ return for a description of hidden and historical $r()$ and $e()$ values.

When st_global (name, contents) is used to set an e() or r() value, its hcat is set to "visible".
6. st_global_hcat (name) returns the hcat associated with an $e()$ or $r()$ value.

## Remarks and examples

Mata provides a suite of functions for obtaining and setting the contents of global macros, local macros, stored results, etc. It can sometimes be confusing to know which you should use. The table on the following page will help.

Stata component/action Function call

Local macro
obtain contents contents $=$ st_local("name")
create/set/replace st_local("name", contents)
delete st_local("name", "")
Global macro
obtain contents contents $=$ st_global ("name")
create/set/replace st_global("name", contents)
delete
st_global("name", "")
Global numeric scalar
obtain contents value $=$ st_numscalar("name")
create/set/replace st_numscalar("name", value)
delete
st_numscalar("name", J (0,0,.))

Global string scalar
obtain contents contents $=$ st_strscalar("name")
create/set/replace st_strscalar("name", contents)
delete st_strscalar("name", J(0,0,""))
Global matrix
obtain contents matrix = st_matrix ("name")
rowlabel = st_matrixrowstripe("name")
collabel $=$ st_matrixcolstripe("name")
create/set/replace st_matrix("name", matrix)
st_matrixrowstripe("name", rowlabel)
st_matrixcolstripe("name", collabel)
replace st_replacematrix("name", matrix)
delete st_matrix("name", J(0,0,.))
Characteristic
obtain contents contents $=$ st_global ("name[name]")
create/set/replace st_global("name[name]", contents)
delete st_global("name[name]", "")

| Stata component/action | Function call |
| :---: | :---: |
| $r$ ( ) results |  |
| macro <br> obtain contents create/set/replace | ```contents = st_global("r(name)") st_global("r(name)", contents)``` |
| numeric scalar obtain contents create/set/replace | ```value = st_numscalar("r(name)") st_numscalar("r(name)", value)``` |
| matrix <br> obtain contents <br> create/set/replace <br> replace | ```matrix = st_matrix("r(name)") rowlabel = st_matrixrowstripe("r(name)") collabel = st_matrixcolstripe("r(name)") st_matrix("r(name)", matrix) st_matrixrowstripe("r(name)", rowlabel) st_matrixcolstripe("r(name)", collabel) st_replacematrix("r(name)", matrix)``` |
| IN ALL CASES <br> delete to delete all of $r()$ | ```st_global("r(name)", "") st_rclear()``` |
| e() results | same as r() results, but code e(name) and st_eclear() |
| S() results |  |
| macro <br> obtain contents <br> create/set/replace <br> delete <br> to delete all of s() | ```contents = st_global("s(name)") st_global("s(name)", contents) st_global("s(name)", "") st_sclear()``` |
| c ( ) results |  |
| macro obtain contents | contents = st_global ("c(name) ") |
| numeric scalar obtain contents | value = st_numscalar("c(name) ") |

See [M-5] st_local(), [M-5] st_numscalar(), [M-5] st_matrix(), and [M-5] st_rclear().

## Conformability

```
st_global(name):
    name: 
    result: }1\times
st_global(name, contents):
    name: }1\times
        contents: }1\times
        result: void
st_global(name, contents, hcat):
    name: }1\times
        contents: }1\times
            hcat: 
            result: void
st_global_hcat(name):
    name: }1\times
    result: }1\times
```


## Diagnostics

st_global(name) returns "" if the name contained in name is not defined. st_global (name) aborts with error if the name is malformed, such as st_global("invalid name").
st_global (name, contents) aborts with error if the name contained in name is malformed. The maximum length of strings in Mata is significantly longer than in Stata. st_global() truncates what is stored at the appropriate maximum length if that is necessary.
st_global_hcat (name) returns "visible" when name is not an e() or r() value and returns " " when name is an e() or r() value that does not exist.

## Reference

Gould, W. W. 2008. Mata Matters: Macros. Stata Journal 8: 401-412.

## Also see

[M-5] st_rclear( ) - Clear r( ), e( ), or s()
[M-4] Stata - Stata interface functions

## Title

[M-5] st_isalias( ) - Properties of alias variable
Description Syntax Conformability Diagnostics
Also see

## Description

In all the functions, if var is specified as a name, abbreviations are not allowed.
st_isalias (var) returns 1 if var is an alias variable and 0 otherwise.
st_aliasframe (var) returns the name of the frame that var is linked to. If var is not an alias variable, or the linking variable cannot be found, then st_aliasframe() returns an empty string.
st_aliaslinkname (var) returns the name of the linking variable that was used to create var. If var is not an alias variable, then st_aliaslinkname() returns an empty string.
st_aliasvarname (var) returns the name of the variable that var is linked to. If var is not an alias variable, then st_aliasvarname() returns an empty string.

## Syntax

```
real scalar st_isalias(scalar var)
string scalar st_aliasframe(scalar var)
string scalar st_aliaslinkname(scalar var)
string scalar st_aliasvarname(scalar var)
```

where var contains a Stata variable name or a Stata variable index.

## Conformability

```
st_isalias(var):
        var: }1\times
        result: }1\times
st_aliasframe(var):
            var: }1\times
    result: }\quad1\times
st_aliaslinkname(var):
        var: }1\times
    result: }1\times
st_aliasvarname(var):
    var: }1\times
    result: }1\times
```


## Diagnostics

All functions abort with error if var is not a valid Stata variable.

## Also see

[M-5] st_addalias( ) - Add alias variable to current Stata dataset
[M-4] Stata - Stata interface functions
[D] fralias - Alias variables from linked frames
[D] frunalias - Change storage type of alias variables

## Title

[M-5] st_isfmt() - Whether valid \%fmt
Description Syntax Conformability Diagnostics Also see

## Description

st_isfmt ( $s$ ) returns 1 if $s$ contains a valid Stata \% frmt and 0 otherwise.
st_isnumfmt ( $s$ ) returns 1 if $s$ contains a valid Stata numeric $\%$ fmt and 0 otherwise.
st_isstrfmt ( $s$ ) returns 1 if $s$ contains a valid Stata string \% fmt and 0 otherwise.

## Syntax

```
real scalar st_isfmt (string scalar s)
real scalar st_isnumfmt (string scalar s)
real scalar st_isstrfmt(string scalar \(s\) )
```


## Conformability

```
st_isfmt(s), st_isnumfmt(s), st_isstrfmt(s):
            s: 
    result: }1\times
```


## Diagnostics

st_isfmt ( $s$ ), st_isnumfmt ( $s$ ), and st_isstrfmt ( $s$ ) abort with error if $s$ is a view.

## Also see

[M-4] Stata - Stata interface functions

## Title

[M-5] st_isname() - Whether valid Stata name
Description Syntax Conformability Diagnostics Also see

## Description

st_isname ( $s$ ) returns 1 if $s$ contains a valid Stata name and 0 otherwise.
st_islmname $(s)$ returns 1 if $s$ contains a valid Stata local-macro name and 0 otherwise.

## Syntax

real scalar st_isname (string scalar s)
real scalar st_islmname (string scalar s)

## Conformability

```
st_isname(s), st_islmname(s):
            s: 
    result: }1\times
```


## Diagnostics

st_isname ( $s$ ) aborts with error if $s$ is a view (but st_islmname() does not).

## Also see

[M-4] Stata - Stata interface functions

## Title

[M-5] st_local() - Obtain strings from and put strings into Stata macros

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Reference | Also see |  |

## Description

st_local (name) returns the contents of the specified local macro.
st_local (name, contents) sets or resets the contents of the specified local macro. If the macro did not previously exist, a new macro is created. If it did previously exist, the new contents replace the old. To remove a local macro from Mata, type st_local(name, "").

## Syntax

```
string scalar st_local(string scalar name)
void st_local(string scalar name, string scalar contents)
```


## Remarks and examples

See [M-5] st_global() and [M-5] st_rclear().

## Conformability

```
st_local(name):
name: }1\times
    result: }1\times
```

```
st_local(name, contents):
    name: 
    contents: }1\times
    result: void
```


## Diagnostics

st_local (name) returns "" if the name contained in name is not defined. st_local (name) aborts with error if the name is malformed.
st_local (name, contents) aborts with error if the name contained in name is malformed.

## Reference

Gould, W. W. 2008. Mata Matters: Macros. Stata Journal 8: 401-412.

## Also see

[M-4] Stata - Stata interface functions

## Title

[M-5] st_macroexpand() — Expand Stata macros in string

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

st_macroexpand ( $s$ ) returns $s$ with any quoted or dollar sign-prefixed macros expanded.
_st_macroexpand ( $S, s$ ) places in $S$ the contents of $s$ with any quoted or dollar sign-prefixed macros expanded and returns a Stata return code (it returns 0 if all went well).

## Syntax

```
string scalar st_macroexpand(string scalar s)
real scalar _st_macroexpand(S, string scalar s)
```

Note: The type of $S$ does not matter; it is replaced and becomes a string scalar.

## Remarks and examples

Be careful coding string literals containing quoted or prefixed macros because macros are also expanded at compile time. For instance, consider

```
s = st_macroexpand("regress 'varlist'")
```

'varlist' will be substituted with its value at compile time. What you probably want is
s = st_macroexpand("regress " + "‘" + "varlist" + ")")

## Conformability

st_macroexpand ( $s$ ):

| $s:$ |  |
| :--- | :--- |
| result: |  |
| $1 \times 1 \times 1$ |  |

_st_macroexpand $(S, s)$ :
input:

$$
s: \quad 1 \times 1
$$

output:
$S: \quad 1 \times 1$
result: $1 \times 1$

## Diagnostics

st_macroexpand ( $s$ ) aborts with error if $s$ is too long (exceedingly unlikely) or if macro expansion fails (also unlikely).
_st_macroexpand $(S, s)$ aborts with error if $s$ is too long.

## Also see

[M-4] Stata - Stata interface functions

## Title

[M-5] st_matrix( ) — Obtain and put Stata matrices

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

st_matrix (name) returns the contents of Stata's matrix name, or it returns $\mathrm{J}(0,0,$.$) if the matrix$ does not exist.
st_matrixrowstripe (name) returns the row stripe associated with the matrix name, or it returns $\mathrm{J}(0,2, " ")$ if the matrix does not exist.
st_matrixcolstripe (name) returns the column stripe associated with the matrix name, or it returns $\mathrm{J}(0,2, " ")$ if the matrix does not exist.
st_matrix (name, $X$ ) sets or resets the contents of the Stata matrix name to be $X$. If the matrix did not previously exist, a new matrix is created. If the matrix did exist, the new contents replace the old. Either way, the row and column stripes are also reset to contain "r1", "r2", ..., and "c1", "c2", ....
st_matrix (name, $X$ ) deletes the Stata matrix name when $X$ is $0 \times 0$ : st_matrix (name, $\mathrm{J}(0,0,)$.$) deletes Stata matrix name or does nothing if name does not exist.$
st_matrixrowstripe (name, $s$ ) and st_matrixcolstripe (name, $s$ ) change the contents to be $s$ of the row and column stripe associated with the already existing Stata matrix name. In either case, $s$ must be $n \times 2$, where $n=$ the number of rows (columns) of the underlying matrix.
st_matrixrowstripe (name, $s$ ) and st_matrixcolstripe (name, $s$ ) reset the row and column stripe to be "r1", "r2", ..., and "c1", "c2", ..., when $s$ is $0 \times 2$ (that is, J ( $0,2, "$ ") ).
st_replacematrix (name, $X$ ) resets the contents of the Stata matrix name to be $X$. The existing Stata matrix must have the same number of rows and columns as $X$. The row stripes and column stripes remain unchanged.
st_matrix (name, $X$, hcat) and st_matrix_hcat (name) are used to set and query the hcat corresponding to a Stata e() or r() matrix. They are also rarely used. See [R] Stored results and $[\mathrm{P}]$ return for more information.

## Syntax

| real matrix | st_matrix(string scalar name) |
| :--- | :--- |
| string matrix | st_matrixrowstripe (string scalar name) |
| string matrix | st_matrixcolstripe(string scalar name) |
| void | st_matrix(string scalar name, real matrix $X$ ) |
| void | st_matrix(string scalar name, real matrix $X$, string scalar hcat) |
| void | st_matrixrowstripe (string scalar name, string matrix s) |
| void | st_matrixcolstripe (string scalar name, string matrix s) |
| void | st_replacematrix(string scalar name, real matrix $X$ ) |
| string scalar | st_matrix_hcat(name) |

where

1. All functions allow name to be
a. global matrix name such as "mymatrix",
b. $r()$ matrix such as $" r(Z)$ ", or
c. e() matrix such as "e(V)".
2. st_matrix (name) returns the contents of the specified Stata matrix. It returns $\mathrm{J}(0,0,$. if the matrix does not exist.
3. st_matrix (name, $X$ ) sets or resets the contents of the specified Stata matrix. Row and column stripes are set to the default $r 1, r 2, \ldots$, and $c 1, c 2, \ldots$.
4. st_replacematrix (name, $X$ ) is an alternative way to replace existing Stata matrices. The number of rows and columns of $X$ must match the Stata matrix being replaced, and in return, the row and column stripes are not replaced.
5. st_matrix (name, $X$ ) deletes the specified Stata matrix if value $==\mathrm{J}(0,0,$.$) (if value is$ $0 \times 0$ ).
6. Neither st_matrix() nor st_replacematrix() can be used to set, replace, or delete special Stata e() matrices e(b), e(V), or e(Cns). Only Stata commands ereturn post and ereturn repost can be used to set these special matrices; see [P] ereturn. Also see [M-5] stata() for executing Stata commands from Mata.
7. st_matrix (name, $X$, hcat) sets or resets the specified Stata matrix and sets the hidden or historical status when setting a Stata e() or r() matrix. Allowed hcat values are "visible", "hidden", "historical", and a string scalar release number such as "10", "10.1", or any string release number matching "\#[\#][.[\#[\#]]]". See $[P]$ return for a description of hidden and historical stored results.
8. st_matrix_hcat(name) returns the hcat associated with a Stata e() or r() matrix.
9. st_matrixrowstripe() and st_matrixcolstripe() allow querying and resetting the row and column stripes of existing or previously created Stata matrices.

## Remarks and examples

## Processing Stata's row and column stripes

Both row stripes and column stripes are presented in the same way: each row of $s$ represents the $e q: o p . n a m e ~ a s s o c i a t e d ~ w i t h ~ a ~ r o w ~ o r ~ c o l u m n ~ o f ~ t h e ~ u n d e r l y i n g ~ m a t r i x . ~ T h e ~ f i r s t ~ c o l u m n ~ r e c o r d s ~ e q, ~$ and the second column records op.name. For instance, given the following Stata matrix

|  |  |  | eq2: | eq2: |
| ---: | ---: | ---: | ---: | ---: |
|  |  | L. | L. |  |
| mpg | turn | turn | turn | turn |
| L.mpg | 1 | 2 | 3 | 4 |
| eq2:mpg | 9 | 6 | 7 | 8 |
| eq2:L.mpg | 13 | 10 | 11 | 12 |
|  |  |  | 15 | 16 |

st_matrixrowstripe (name) returns the $4 \times 2$ string matrix

$$
\begin{aligned}
\text { "" } & \text { "mpg" } \\
\text { "" } & \text { "L.mpg" } \\
\text { "eq2" } & \text { "mpg" } \\
\text { "eq2" } & \text { "L.mpg" }
\end{aligned}
$$

and st_matrixcolstripe(name) returns

| "" | "turn" |
| ---: | :--- |
| "" | "L.turn" |
| "eq2" | "turn" |
| "eq2" | "L.turn" |

## Conformability

```
st_matrix(name):
    name: }1\times
    result: }\quadm\timesn\quad(0\times0\mathrm{ if not found)
st_matrixrowstripe(name):
    name: 
    result: }\quadm\times2 (0\times2 if not found
st_matrixcolstripe(name):
    name: }1\times
    result: }\quadn\times2 (0\times2 if not found
st_matrix(name, X):
    name: }1\times
        X: }\quadr\timesc\quad(0\times0\mathrm{ means delete )
    result: void
```

```
st_matrix(name, \(X\), hcat):
    name: \(\quad 1 \times 1\)
        \(X: \quad r \times c\)
        hcat: \(\quad 1 \times 1\)
    result: void
st_matrixrowstripe (name, \(s\) ):
    name: \(\quad 1 \times 1\)
        \(s: \quad r \times 2 \quad(0 \times 2\) means default "r1", "r2", ...)
    result: void
st_matrixcolstripe (name, \(s\) ):
    name: \(\quad 1 \times 1\)
        \(s: \quad c \times 2(0 \times 2\) means default "c1", "c2", ...)
    result: void
st_replacematrix(name, \(X\) ):
    name: \(1 \times 1\)
        \(X: \quad m \times n \quad(0 \times 0\) means delete \()\)
    result: void
st_matrix_hcat(name):
    name: \(\quad 1 \times 1\)
    result: \(\quad 1 \times 1\)
```


## Diagnostics

st_matrix(name), st_matrixrowstripe(name), and st_matrixcolstripe(name) abort with error if name is malformed. Also,

1. st_matrix (name) returns $\mathrm{J}(0,0,$.$) if Stata matrix name does not exist.$
2. st_matrixrowstripe (name) and st_matrixcolstripe (name) return $J(0,2, " ")$ if Stata matrix name does not exist. There is no possibility that matrix name might exist and not have row and column stripes.
st_matrix(name, $X$ ), st_matrixrowstripe(name, $s$ ), and st_matrixcolstripe (name, s) abort with error if name is malformed. Also,
3. st_matrixrowstripe (name, $s$ ) aborts with error if rows $(s)$ is not equal to the number of rows of Stata matrix name and rows $(s)!=0$, or if $\operatorname{cols}(s)!=2$.
4. st_matrixcolstripe (name, $s$ ) aborts with error if $\operatorname{cols}(s)$ is not equal to the number of columns of Stata matrix name and cols $(s)!=0$, or if $\operatorname{cols}(s)!=2$.
st_replacematrix (name, $X$ ) aborts with error if Stata matrix name does not have the same number of rows and columns as $X$. st_replacematrix() also aborts with error if Stata matrix name does not exist and $X!=\mathrm{J}(0,0,$.$) ; st_replacematrix () does nothing if the matrix does not$ exist and $X==\mathrm{J}(0,0,$.$) . st_replacematrix() aborts with error if name is malformed.$
st_matrix (name, $X$, hcat) aborts with error if hcat is not an allowed value.
st_matrix_hcat (name) returns "visible" when name is not a Stata e() or r() matrix and returns "" when name is an e() or r() value that does not exist.

## Also see

[M-5] st_global( ) - Obtain strings from and put strings into global macros
[M-5] st_rclear( ) - Clear r( ), e( ), or s()
[M-4] Stata - Stata interface functions

## Title

[M-5] st_numscalar( ) - Obtain values from and put values into Stata scalars

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

st_numscalar (name) returns the value of the specified Stata numeric scalar, or it returns $\mathrm{J}(0,0,$. if the scalar does not exist.
st_numscalar (name, value) sets or resets the value of the specified numeric scalar, assuming value $!=\mathrm{J}(0,0,$.$) . st_numscalar (name, value) deletes the specified scalar if value ==\mathrm{J}(0,0,$.$) .$ st_numscalar ("x", J ( $0,0,$.$) ) erases the scalar \mathrm{x}$, or it does nothing if scalar x did not exist. st_strscalar (name) returns the value of the specified Stata string scalar, or it returns $\mathrm{J}(0,0, " ")$ if the scalar does not exist.
st_strscalar(name, value) sets or resets the value of the specified scalar, assuming value != $\mathrm{J}(0,0, " \mathrm{l})$. st_strscalar(name, value) deletes the specified scalar if value $==\mathrm{J}(0,0, " ")$. st_strscalar ("x", J(0,0,"")) erases the scalar x , or it does nothing if scalar x did not exist.

Concerning deletion of a scalar, it does not matter whether you code st_numscalar (name, $\mathrm{J}(0,0,)$.$) or st_strscalar(name, \mathrm{J}(0,0, " "))$; both yield the same result.
st_numscalar (name, value, hcat) and st_numscalar_hcat(name) are used to set and query the hcat corresponding to an $e()$ or $r()$ value. They are also rarely used. See [R] Stored results and $[\mathrm{P}]$ return for more information.

## Syntax

```
real st_numscalar(string scalar name)
void st_numscalar(string scalar name, real value)
void st_numscalar(string scalar name, real value, string scalar hcat)
string st_numscalar_hcat(string scalar name)
string st_strscalar(string scalar name)
void st_strscalar(string scalar name, string value)
```

where

1. Functions allow name to be
a. global scalar such as "myname",
b. $r()$ scalar such as $" r($ mean $)$ ",
c. e() scalar such as "e(N)", or
d. c() scalar such as "c(namelenchar)".

Note that string scalars never appear in $r$ () and $e()$; thus (b) and (c) do not apply to st_strscalar().
2. st_numscalar(name) and st_strscalar (name) return the value of the specified Stata scalar. They return a $1 \times 1$ result if the specified Stata scalar exists and return a $0 \times 0$ result otherwise.
3. st_numscalar (name, value) and st_strscalar (name, value) set or reset the contents of the specified Stata scalar.
4. st_numscalar(name, value) and st_strscalar(name, value) delete the specified Stata scalar if value $=\mathrm{J}(0,0,$.$) (if value is 0 \times 0$ ).
5. st_numscalar (name, value, hcat) sets or resets the specified Stata scalar and sets or resets the hidden or historical status when name is an e() or r() value. Allowed hcat values are "visible", "hidden", "historical", and a string scalar release number such as such as "10", "10.1", or any string release number matching "\#[\#][.[\#[\#]]]". See $[\mathrm{P}]$ return for a description of hidden and historical stored results.

When st_numscalar (name, value) is used to set an e() or r() value, its hcat is set to "visible".

There is no three-argument form of st_strscalar() because there are no r() or e() string scalar values.

## Remarks and examples

See [M-5] st_global() and [M-5] st_rclear( ).

## Conformability

```
st_numscalar(name), st_strscalar(name):
```

name: $1 \times 1$
result: $\quad 1 \times 1 \quad$ or $\quad 0 \times 0$
st_numscalar(name, value), st_strscalar(name, value):
name: $1 \times 1$
value: $\quad 1 \times 1 \quad$ or $\quad 0 \times 0$
result: void
st_numscalar(name, value, hcat):
name: $1 \times 1$
value: $\quad 1 \times 1$
hcat: $\quad 1 \times 1$
result: void
st_numscalar (name):
$\begin{array}{ll}\text { name: } & 1 \times 1 \\ \text { result: } & 1 \times 1\end{array}$
result: $\quad 1 \times 1$

## Diagnostics

All functions abort with error if name is malformed.
st_numscalar (name) and st_strscalar (name) return $\mathrm{J}(0,0,$.$) or \mathrm{J}(0,0, " \mathrm{l})$ if Stata scalar name does not exist. They abort with error, however, if the name is malformed.
st_numscalar (name, value, hcat) aborts with error if hcat is not an allowed value.
st_numscalar_hcat (name) returns "visible" when name is not an e() or r() value and returns "" when name is an e() or r() value that does not exist.

## Also see

[M-5] st_rclear( ) - Clear r( ), e( ), or s()
[M-4] Stata - Stata interface functions

## Title

[M-5] st_nvar( ) - Numbers of variables and observations

Description Syntax Conformability Diagnostics Also see

## Description

st_nvar() returns the number of variables defined in the dataset currently loaded in Stata.
st_nobs() returns the number of observations defined in the dataset currently loaded in Stata.

## Syntax

```
real scalar st_nvar()
real scalar st_nobs()
```


## Conformability

```
st_nvar(), st_nobs():
    result: }1\times
```


## Diagnostics

None.

## Also see

[M-4] Stata - Stata interface functions

## Title

[M-5] st_rclear() - Clear $r(), \mathrm{e}()$, or s()

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

st_rclear() clears Stata's r() stored results.
st_eclear() clears Stata's e() stored results.
st_sclear() clears Stata's s() stored results.

## Syntax

```
void st_rclear()
void st_eclear()
void st_sclear()
```


## Remarks and examples

Returning results in $r(), \mathrm{e}()$, or s() is one way of communicating results calculated in Mata back to Stata; see [M-1] Ado. See [R] Stored results for a description of $e(), r()$, and $s()$.

Use st_rclear(), st_eclear(), or st_sclear() to clear results, and then use st_global() to define macros, st_numscalar() to define scalars, and st_matrix() to define Stata matrices in $r()$, e(), or $s()$. For example,

```
st_rclear()
st_global("r(name)", "tab") see [M-5] st_global()
st_numscalar("r(N)", n1+n2) see [M-5] st_numscalar()
st_matrix("r(table)", X+Y) see [M-5] st_matrix()
```

It is not necessary to clear before saving, but it is considered good style unless it is your intention to add to previously stored results.

If a stored result already exists, st_global(), st_numscalar(), and st_matrix() may be used to redefine it and even to redefine it to a different type. For instance, continuing with our example, later in the same code might appear

```
if (...) {
    st_matrix("r(name)", X)
}
```

Stored result r (name) was previously defined as a macro containing "tab", and, even so, can now be redefined to become a matrix.

If you want to eliminate a particular stored result, use st_global() to change its contents to " ":

```
st_global("r(name)", "")
```

Do this regardless of the type of the stored result. Here we use st_global () to clear stored result $r$ (name), which might be a macro and might be a matrix.

## Conformability

st_rclear(), st_eclear(), and st_sclear() take no arguments and return void.

## Diagnostics

st_rclear(), st_eclear(), and st_sclear() cannot fail.

## Also see

[M-5] st_global( ) - Obtain strings from and put strings into global macros
[M-5] st_matrix( ) - Obtain and put Stata matrices
[M-5] st_numscalar( ) - Obtain values from and put values into Stata scalars
[M-4] Stata - Stata interface functions

## Title

[M-5] st_store( ) - Modify values stored in current Stata dataset

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

These functions mirror _st_data(), st_data(), and st_sdata(). Rather than returning the contents from the Stata dataset, these commands change those contents to be as given by the last argument.

## Syntax

```
void _st_store(real scalar i, real scalar j, real scalar x)
void st_store(real matrix i, rowvector j, real matrix X)
void st_store(real matrix i, rowvector j, scalar selectvar, real matrix X)
void _st_sstore(real scalar i, real scalar j, string scalar s)
void st_sstore(real matrix i, rowvector j, string matrix X)
void st_sstore(real matrix i, rowvector j, scalar selectvar, string matrix X) (1,2,3)
```

where

1. $i$ may be specified in the same way as with st_data().
2. $j$ may be specified in the same way as with st_data(), except that time-series operators may not be specified.
3. selectvar may be specified in the same way as with st_data().

See [M-5] st_data().

## Remarks and examples

See [M-5] st_data( ).

## Conformability

```
_st_store(i,j, x), _st_sstore(i, j, x):
    i: }\quad1\times
    j: 
    x: }\quad1\times
    result: void
```

st_store ( $i, j, X$ ), st_sstore ( $i, j, X)$ :
$i: \quad n \times 1 \quad$ or $\quad n_{2} \times 2$
$j: \quad 1 \times k$
X: $\quad n \times k$
result: void
st_store ( $i, j$, selectvar, $X$ ), st_sstore ( $i, j$, selectvar, $X)$ :
$i: \quad n \times 1 \quad$ or $\quad n_{2} \times 2$
$j: \quad 1 \times k$
selectvar: $\quad 1 \times 1$
$X: \quad(n-e) \times k, \quad$ where $e$ is number of observations excluded by selectvar result: void

## Diagnostics

_st_store $(i, j, x)$ and _st_sstore $(i, j, s)$ do nothing if $i$ or $j$ is out of range; they do not abort with error.
st_store $(i, j, X)$ and st_sstore $(i, j, s)$ abort with error if any element of $i$ or $j$ is out of range. $j$ may be specified as a vector of variable names or as a vector of variable indices. If names are specified, abbreviations are allowed. If you do not want this, use st_varindex() (see [M-5] st_varindex( )) to translate variable names into variable indices.
st_store() and st_sstore() abort with error if $X$ is not p-conformable with the matrix that st_data() (st_sdata()) would return.

## Also see

[M-5] st_addvar( ) - Add variable to current Stata dataset
[M-5] st_data( ) - Load copy of current Stata dataset
[M-4] Stata - Stata interface functions
[D] putmata - Put Stata variables into Mata and vice versa

## Title

[M-5] st_subview() — Make view from view

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

st_subview $(X, V, i, j)$ creates new view matrix $X$ from existing view matrix $V . V$ is to have been created from a previous call to st_view() (see [M-5] st_view()) or st_subview ().

Although st_subview () is intended for use with view matrices, it may also be used when $V$ is a regular matrix. Thus code may be written in such a way that it will work without regard to whether a matrix is or is not a view.
$i$ may be specified as a $1 \times 1$ scalar, a $1 \times 1$ scalar containing missing, as a column vector of row numbers, as a row vector specifying a row-number range, or as a $k \times 2$ matrix specifying both:
a. st_subview $(X, V, 1,2)$ makes $X$ equal to the first row of the second column of $V$.
b. st_subview $(X, V, ., 2)$ makes $X$ equal to all rows of the second column of $V$.
c. st_subview $(X, V,(1 \backslash 2 \backslash 5), 2)$ makes $X$ equal to rows 1,2 , and 5 of the second column of $V$.
d. st_subview $(X, V,(1,5), 2)$ makes $X$ equal to rows 1 through 5 of the second column of $V$.
e. st_subview $(X, V,(1,5 \backslash 7,9), 2)$ makes $X$ equal to rows 1 through 5 and 7 through 9 of the second column of $V$.
f. When a range is specified, any element of the range ( $i_{1}, i_{2}$ ) may be set to contribute zero observations if $i_{2}=i_{1}-1$. For example, $(1,0)$ is not an error and neither is $(1,0 \backslash 5,7)$.
$j$ may be specified in the same way as $i$, except transposed, to specify the selected columns:
a. st_subview ( $X, V, 2,$. ) makes $X$ equal to all columns of the second row of $V$.
b. st_subview $(X, V, 2,(1,2,5))$ makes $X$ equal to columns 1,2 , and 5 of the second row of $V$.
c. st_subview $(X, V, 2,(1 \backslash 5))$ makes $X$ equal to columns 1 through 5 of the second row of $V$.
d. st_subview $(X, V, 2,((1 \backslash 5),(7 \backslash 9)))$ makes $X$ equal to columns 1 through 5 and 7 through 9 of the second row of $V$.
e. When a range is specified, any element of the range $\left(j_{1} \backslash j_{2}\right)$ may be set to contribute zero columns if $j_{2}=j_{1}-1$. For example, ( $1 \backslash 0$ ) is not an error and neither is $((1 \backslash 0),(5 \backslash 7))$.

Obviously, notations for $i$ and $j$ can be specified simultaneously:
a. st_subview $(X, V, .,$.$) makes X$ a duplicate of $V$.
b. st_subview $(X, V, .,(1 \backslash 5))$ makes $X$ equal to columns 1 through 5 of all rows of $X$.
c. st_subview $(X, V,(10,25),(1 \backslash 5))$ makes $X$ equal to columns 1 through 5 of rows 10 through 25 of $X$.

Also, st_subview() may be used to create views with duplicate variables or observations from $V$.

## Syntax

void st_subview ( $X$, transmorphic matrix $V$, real matrix $i$, real matrix $j$ )
where

1. The type of $X$ does not matter; it is replaced.
2. $V$ is typically a view, but that is not required. $V$, however, must be real or string.

## Remarks and examples

Say that you need to make a calculation on matrices $X$ and $Y$, which might be views. Perhaps the calculation is invsym $\left(X^{\prime} X\right) * X^{\prime} Y$. Regardless, you start as follows:

```
st_view(X, ., "v2 v3 v4", 0)
st_view(Y, ., "v1 v7" , 0)
```

You are already in trouble. You smartly coded fourth argument as 0 , meaning exclude the missing values, but you do not know that the same observations were excluded in the manufacturing of X as in the manufacturing of Y .

If you had previously created a touse variable in your dataset marking the observations to be used in the calculation, one solution would be

```
st_view(X, ., "v2 v3 v4", "touse")
st_view(Y, ., "v1 v7" , "touse")
```

That solution is recommended, but let's assume you did not do that. The other solution is

```
st_view(M, ., "v2 v3 v4 v1 v7", 0)
st_subview(X, M, ., (1,2,3))
st_subview(Y, M, ., (4,5))
```

The first call to st_view() will eliminate observations with missing values on any of the variables, and the second two st_subview () calls will create the matrices you wanted, obtaining them from the correctly formed M. Basically, the two st_subview() calls amount to the same thing as

$$
\begin{aligned}
& X=M[.,(1,2,3)] \\
& Y=M[.,(4,5)]
\end{aligned}
$$

but you do not want to code that because then matrices $X$ and $Y$ would contain copies of the data, and you are worried that the dataset might be large.

For a second example, let's pretend that you are processing a panel dataset and making calculations from matrix $X$ within panel. Your code looks something like

```
st_view(id, ., "panelid", 0)
for (i=1; i<=rows(id); i=j+1) {
    j = endobs(id, i)
    st_view(X, (i,j), "v1 v2 ...", 0)
}
```

where you have previously written function endobs () to be

```
scalar endobs(vector id, scalar i)
{
    scalar j
    for (j=i+1; j<=rows(id); j++) {
        if (id[j]!=id[i]) return(j-1)
        }
    return(rows(id))
}
```

In any case, there could be a problem. Missing values of variable panelid might not align with missing values of variables v 1 , v2, etc. The result could be that observation and row numbers are not in accordance or that there appears to be a group that, in fact, has all missing data. The right way to handle the problem is

```
st_view(M, ., "panelid v1 v2 ...", 0)
st_subview(id, M, ., 1)
for (i=1; i<=rows(id); i=j+1) {
    j = endobs(id, i)
    st_subview(X, M, (i,j), (2\cols(M)))
}
```


## Conformability

st_subview $(X, V, i, j)$ :
input:

$$
\begin{array}{rllll}
V: & & r \times c & & \\
i: & & 1 \times 1, n \times 1, & \text { or } & n_{2} \times 2 \\
j: & & 1 \times 1,1 \times k, & \text { or } & 2 \times k_{2}
\end{array}
$$

output:

$$
X: \quad n \times k
$$

## Diagnostics

st_subview ( $X, V, i, j$ ) aborts with error if $i$ or $j$ are out of range. $i$ and $j$ refer to row and column numbers of $V$, not observation and variable numbers of the underlying Stata dataset.

## Also see

[M-5] select() - Select rows, columns, or indices
[M-5] st_view( ) - Make matrix that is a view onto current Stata dataset
[M-4] Stata - Stata interface functions

## Title

[M-5] st_tempname() — Temporary Stata names

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

st_tempname() returns a Stata temporary name, the same as would be returned by Stata's tempvar and tempname commands; see [P] macro.
st_tempname ( $n$ ) returns $n$ temporary Stata names, $n \geq 0$.
st_tempfilename() returns a Stata temporary filename, the same as would be returned by Stata's tempfile command; see [P] macro.
st_tempfilename ( $n$ ) returns $n$ temporary filenames, $n \geq 0$.

## Syntax

```
string scalar st_tempname()
string rowvector st_tempname(real scalar n)
string scalar st_tempfilename()
string rowvector st_tempfilename(real scalar n)
```


## Remarks and examples

Remarks are presented under the following headings:
Creating temporary objects
When temporary objects will be eliminated

## Creating temporary objects

st_tempname()s can be used to name Stata's variables, matrices, and scalars. Although in Stata a distinction is drawn between tempvars and tempnames, there is no real distinction, and so st_tempname () handles both in Mata. For instance, one can create a temporary variable by coding

```
idx = st_addvar("double", st_tempname())
```

See [M-5] st_addvar().
One creates a temporary file by coding
fh = fopen(st_tempfilename(), "w")

See [M-5] fopen().

## When temporary objects will be eliminated

Temporary objects do not vanish when the Mata function ends, nor when Mata itself ends. They are removed when the ado-file (or do-file) calling Mata terminates.

Forget Mata for a minute. Stata eliminates temporary variables and files when the program that created them ends. That same rule applies to Mata: Stata eliminates them, not Mata, and that means that the ado-file or do-file that called Mata will eliminate them when that ado-file or do-file ends. Temporary variables and files are not eliminated by Mata when the Mata function ends. Thus Mata functions can create temporary objects for use by their ado-file callers, should that prove useful.

## Conformability

```
st_tempname(), st_tempfilename():
    result: 1 < 1
st_tempname(n), st_tempfilename(n):
    n: 1 < 1
    result: 1 }\times
```


## Diagnostics

st_tempname( $n$ ) and st_tempfilename( $n$ ) abort with error if $n<0$ and return $\mathrm{J}(1,0, " \mathrm{l})$ if $n=0$.

## Also see

[M-5] st_addvar( ) - Add variable to current Stata dataset
[M-4] Stata - Stata interface functions

## Title

[M-5] st_tsrevar() — Create time-series op.varname variables

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

st_tsrevar ( $s$ ) is the equivalent of Stata's [TS] tsrevar programming command: it generates temporary variables containing the evaluation of any op.varname combinations appearing in $s$.
_st_tsrevar ( $s$ ) does the same thing as st_tsrevar(). The two functions differ in how they respond to invalid elements of $s$. st_tsrevar() aborts with error, and _st_tsrevar() places missing in the appropriate element of the returned result.

## Syntax

```
real rowvector st_tsrevar(string rowvector s)
real rowvector _st_tsrevar(string rowvector s)
```


## Remarks and examples

Both of these functions help achieve efficiency when using views and time-series variables. Assume that in vars, you have a list of Stata variable names, some of which might contain time-series op.varname combinations such as l.gnp. For example, vars might contain
vars = "gnp r l.gnp"

If you wanted to create in $V$ a view on the data, you would usually code

```
st_view(V, ., vars)
```

We are not going to do that, however, because we plan to do many calculations with $V$ and, to speed execution, we want any op.varname combinations evaluated just once, as $V$ is created. Of course, if efficiency were our only concern, we would code

$$
V=\text { st_data }(., \text { vars })
$$

Assume, however, that we have lots of data, so memory is an issue, and yet we still want as much efficiency as possible given the constraint of not copying the data. The solution is to code
st_view(V, ., st_tsrevar(tokens(vars)))
st_tsrevar() will create temporary variables for each op.varname combination (l.gnp in our example), and then return the Stata variable indices of each of the variables, whether newly created or already existing. If gnp was the second variable in the dataset, $r$ was the 23 rd , and in total there were 54 variables, then returned by st_tsrevar () would be ( $2,23,55$ ). Variable 55 is new, created by st_tsrevar(), and it contains the values of l.gnp. The new variable is temporary and will be dropped automatically at the appropriate time.

## Conformability

```
st_tsrevar(s), _st_tsrevar(s):
    s: }1\times
    result: }1\times
```


## Diagnostics

st_tsrevar() aborts with error if any variable name is not found or any op.varname combination is invalid.
_st_tsrevar () puts missing in the appropriate element of the returned result for any variable name that is not found or any op.varname combination that is invalid.

## Also see

[M-5] st_varindex( ) - Obtain variable indices from variable names
[M-5] st_varname( ) - Obtain variable names from variable indices
[M-4] Stata - Stata interface functions

## Title

[M-5] st_updata( ) — Determine or set data-have-changed flag

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

st_updata() returns 0 if the data in memory have not changed since they were last saved and returns a nonzero otherwise.
st_updata(value) sets the data-have-changed flag to 0 if value $=0$ and a nonzero otherwise.

## Syntax

```
real scalar st_updata()
void st_updata(real scalar value)
```


## Remarks and examples

Stata's describe command (see [D] describe) reports whether the data have changed since they were last saved. Stata's use command (see [D] use) refuses to load a new dataset if the data currently in memory have not been saved since they were last changed. Other components of Stata also react to the data-have-changed flag.
st_updata() allows you to respect that same flag.
Also, as a Mata programmer, you must set the flag if your function changes the data in memory. Mata attempts to set the flag for you (for instance, when you add a new variable using st_addvar() [see [M-5] st_addvar( )]), but there are other places where the flag ought to be set, and you must do so. For instance, Mata does not set the flag every time you change a value in the dataset. Setting the flag what may be many thousands of times would reduce performance too much.

Moreover, even when Mata does set the flag, it might do so inappropriately, because the logic of your program fooled Mata. For instance, perhaps you added a variable and later dropped it. In such cases, the appropriate code is

```
priorupdatavalue = st_updata()
st_updata(priorupdatavalue)
```


## Conformability

st_updata():
result: $\quad 1 \times 1$
st_updata(value):
value: $\quad 1 \times 1$
result: void

## Diagnostics

None.

## Also see

[M-4] Stata - Stata interface functions

## Title

[M-5] st_varformat( ) - Obtain/set format, etc., of Stata variable
Description Syntax Conformability Diagnostics Also see

## Description

st_varformat(var) returns the display format associated with var, such as "\% 9.0 gc ". st_varformat (var, fmt) changes var's display format.
st_varlabel (var) returns the variable label associated with var, such as "Sex of Patient", or it returns " " if var has no variable label. st_varformat (var, label) changes var's variable label.
st_varvaluelabel (var) returns the value-label name associated with var, such as "origin", or it returns "" if var has no value label. st_varvaluelabel (var, labelname) changes the value-label name associated with var.

## Syntax

```
string scalar st_varformat(scalar var)
void st_varformat(scalar var, string scalar fmt)
string scalar st_varlabel(scalar var)
void st_varlabel(scalar var, string scalar label)
string scalar st_varvaluelabel(scalar var)
void st_varvaluelabel(scalar var, string scalar labelname)
```

where var contains a Stata variable name or a Stata variable index.

## Conformability

```
st_varformat(var), st_varlabel(var), st_varvaluelabel(var):
            var: }1\times
    result: }1\times
st_varformat(var, fmt), st_varlabel(var, label), st_varvaluelabel(var, labelname):
    var: }1\times
    value: }1\times
    result: void
```


## Diagnostics

In all functions, if $v a r$ is specified as a name, abbreviations are not allowed.
All functions abort with error if var is not a valid Stata variable.
st_varformat (var, fmt) aborts with error if fmt does not contain a valid display format for var. st_varlabel (var, label) will truncate label if it is too long.
st_varvaluelabel (var, labelname) aborts with error if var is a Stata string variable or if labelname does not contain a valid name (assuming labelname is not ""). It is not required, however, that the label name exist.

## Also see

[M-4] Stata - Stata interface functions

## Title

[M-5] st_varindex() - Obtain variable indices from variable names

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

st_varindex $(s)$ returns the variable index associated with each variable name recorded in $s$. st_varindex ( $s$ ) does not allow variable-name abbreviations such as "pr" for "price".
st_varindex ( $s$, abbrev) does the same thing but allows you to specify whether variable-name abbreviations are to be allowed. Abbreviations are allowed if abbrev $\neq 0$. st_varindex $(s)$ is equivalent to st_varindex ( $s, 0$ ).
_st_varindex () does the same thing as st_varindex(). The two functions differ in how they respond when a name is not found. st_varindex() aborts with error, and _st_varindex() places missing in the appropriate element of the returned result.

## Syntax

```
real rowvector st_varindex(string rowvector s)
real rowvector st_varindex(string rowvector s, real scalar abbrev)
real rowvector _st_varindex(string rowvector s)
real rowvector _st_varindex(string rowvector s, real scalar abbrev)
```


## Remarks and examples

Remarks are presented under the following headings:
Overview
Cautions when using variable indices: A variable index can change

## Overview

These functions require that each element of $s$ contain a variable name, such as
s = ("price", "mpg", "weight")

If you have one string containing multiple names

$$
s=(\text { "price mpg weight") }
$$

then use tokens () to split it into the desired form, as in

$$
k=\text { st_varindex }(\text { tokens }(s))
$$

See [M-5] tokens( ).

## Cautions when using variable indices: A variable index can change

Accessing variable data using a variable index is faster than referencing the variable by name. However, variable indices can change when variables are created or removed. If a variable is created or removed while your code is using a variable index, there is a chance the index will switch to another variable.

## Conformability

```
st_varindex(s, abbrev), _st_varindex(s, abbrev):
            s: }1\times
    abbrev: }1\times1\quad\mathrm{ (optional)
    result: }1\times
```


## Diagnostics

st_varindex() aborts with error if any name is not found.
_st_varindex () puts missing in the appropriate element of the returned result for any name that is not found.

## Also see

[M-5] st_varname( ) - Obtain variable names from variable indices
[M-5] tokens() - Obtain tokens from string
[M-4] Stata - Stata interface functions

## Title

[M-5] st_varname() - Obtain variable names from variable indices

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

st_varname ( $k$ ) returns the Stata variable names associated with the variable indices stored in $k$. For instance, with the automobile data in memory

```
names = st_varname((1..3))
```

results in names being ("make", "price", "mpg").
st_varname ( $k$, tsmap) does the same thing but allows you to specify whether you want the actual or logical variable names of any time-series-operated variables created by the Mata function st_tsrevar () (see [M-5] st_tsrevar()) or by the Stata command tsrevar (see [TS] tsrevar).
st_varname ( $k$ ) is equivalent to st_varname ( $k, 0$ ); actual variable names are returned.
st_varname $(k, 1)$ returns logical variable names.

## Syntax

```
string rowvector st_varname(real rowvector k)
string rowvector st_varname(real rowvector k, real scalar tsmap)
```


## Remarks and examples

To understand the actions of st_varname ( $k, 1$ ), pretend that variable 58 was created by st_tsrevar():
k = st_tsrevar(("gnp", "r", "l.gnp"))

Pretend that k now contains $(12,5,58)$. Variable 58 is a new, temporary variable, containing l.gnp values. Were you to ask for the actual names of the variables

```
actualnames = st_varname(k)
```

actualnames would contain ("gnp", "r", "__00004a"), although the name of the last variable will vary because it is a temporary variable. Were you to ask for the logical names,
logicalnames = st_varname(k, 1)
you would get back ("gnp", "r", "L.gnp").

## Conformability

st_varname ( $k$, tsmap)
$k: \quad 1 \times c$
tsmap: $\quad 1 \times 1 \quad$ (optional)
result: $\quad 1 \times c$

## Diagnostics

st_varname ( $k$ ) and st_varname ( $k$, tsmap) abort with error if any element of $k$ is less than 1 or greater than st_nvar(); see [M-5] st_nvar().

## Also see

[M-5] st_tsrevar( ) - Create time-series op.varname variables
[M-5] st_varindex( ) - Obtain variable indices from variable names
[M-4] Stata - Stata interface functions

## Title

[M-5] st_varrename() — Rename Stata variable
Description Syntax Conformability Diagnostics Also see

## Description

st_varrename (var, newname) changes the name of var to newname.
If var is specified as a name, abbreviations are not allowed.

## Syntax

> void st_varrename (scalar var, string scalar newname)
where var contains a Stata variable name or a Stata variable index.

## Conformability

```
st_varrename(var, newname):
            var: 
    newname: }1\times
        result: void
```


## Diagnostics

st_varrename (var, newname) aborts with error if var is not a valid Stata variable or if newname is not a valid name or if a variable named newname already exists.

## Also see

[M-4] Stata - Stata interface functions

## Title

[M-5] st_vartype( ) - Storage type of Stata variable

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

In all the functions, if var is specified as a name, abbreviations are not allowed.
st_vartype (var) returns the storage type of the var, such as float, double, or str18.
st_isnumvar (var) returns 1 if var is a numeric variable and 0 otherwise.
st_isstrvar (var) returns 1 if var is a string variable and 0 otherwise.
If $v a r$ is an alias variable, then the storage type of the linked variable is used. If the linked variable cannot be found, then st_vartype() returns unknown, and st_isnumvar() and st_isstrvar() abort with error.

## Syntax

```
string scalar st_vartype(scalar var)
real scalar st_isnumvar(scalar var)
real scalar st_isstrvar(scalar var)
```

where var contains a Stata variable name or a Stata variable index.

## Remarks and examples

st_isstrvar(var) and st_isnumvar (var) are antonyms. Both functions are provided merely for convenience; they tell you nothing that you cannot discover from st_vartype (var).

## Conformability

```
st_vartype(var):
    var: }1\times
    result: }1\times
st_isnumvar(var), st_isstrvar(var):
    var: }1\times
    result: }1\times
```


## Diagnostics

All functions abort with error if var is not a valid Stata variable.

## Also see

[M-4] Stata - Stata interface functions

## Title

[M-5] st_view( ) - Make matrix that is a view onto current Stata dataset

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Reference | Also see |  |

## Description

st_view() and st_sview() create a matrix that is a view onto the current Stata dataset.

## Syntax

```
void st_view(V, real matrix i, rowvector j)
void st_view(V, real matrix i, rowvector j, scalar selectvar)
void st_sview(V, real matrix i, rowvector j)
void st_sview(V, real matrix i, rowvector j, scalar selectvar)
```

where

1. The type of $V$ does not matter; it is replaced.
2. $i$ may be specified in the same way as with st_data().
3. $j$ may be specified in the same way as with st_data( ). Factor variables and time-seriesoperated variables may be specified.
4. selectvar may be specified in the same way as with st_data().

See [M-5] st_data().

## Remarks and examples

Remarks are presented under the following headings:

```
Overview
Advantages and disadvantages of views
When not to use views
Cautions when using views 1: Conserving memory
Cautions when using views 2: Assignment
Cautions when using views 3: View connections are ephemeral Efficiency
```


## Overview

st_view() serves the same purpose as st_data()—and st_sview() serves the same purpose as st_sdata()-except that, rather than returning a matrix that is a copy of the underlying values, st_view() and st_sview() create a matrix that is a view onto the Stata dataset itself.

To understand the distinction, consider
X = st_data(., "mpg displ weight")
and
st_view(X, ., "mpg displ weight")

Both commands fill in matrix X with the same data. However, were you to code

$$
X[2,1]=123
$$

after the st_data() setup, you would change the value in the matrix X, but the Stata dataset would remain unchanged. After the st_view() setup, changing the value in the matrix would cause the value of mpg in the second observation to change to 123 .

## Advantages and disadvantages of views

Views make it easy to change the dataset, and that can be an advantage or a disadvantage, depending on your goals.

Putting that aside, views are in general better than copies because 1) they take less time to set up and 2) they consume less memory. The memory savings can be considerable. Consider a 100,000-observation dataset on 30 variables. Coding
X = st_data(., .)
creates a new matrix that is 24 MB in size. Meanwhile, the total storage requirement for
st_view(X, ., .)
is roughly 128 bytes!
There is a cost; when you use the matrix X , it takes longer to access the individual elements. You would have to do a lot of calculation with $X$, however, before that sum of the longer access times would equal the initial savings in setup time, and even then, the longer access time is probably worth the savings in memory.

## When not to use views

Do not use views as a substitute for scalars. If you are going to loop through the data an observation at a time, and if every usage you will make of $X$ is in scalar calculations, use _st_data(). There is nothing faster for that problem.

Putting aside that extreme, views become more efficient relative to copies the larger they are; that is, it is more efficient to use st_data() for small amounts of data, especially if you are going to make computationally intensive calculations with it.

## Cautions when using views 1: Conserving memory

If you are using views, it is probably because you are concerned about memory, and if you are, you want to be careful to avoid making copies of views. Copies of views are not views; they are copies. For instance,

```
st_view(V, ., .)
Y = V
```

That innocuous looking $\mathrm{Y}=\mathrm{V}$ just made a copy of the entire dataset, meaning that if the dataset had 100,000 observations on 30 variables, Y now consumes 24 MB . Coding $\mathrm{Y}=\mathrm{V}$ may be desirable in certain circumstances, but in general, it is better to set up another view.

Similarly, watch out for subscripts. Consider the following code fragment

```
st_view(V, ., .)
for (i=1; i<=cols(V); i++) {
    sum = colsum(V[,i])
}
```

The problem in the above code is the $\mathrm{V}[, \mathrm{i}]$. That creates a new column vector containing the values from the ith column of V. Given 100,000 observations, that new column vector needs 800 k of memory. Better to code would be

```
for (i=1; i<=cols(V); i++) {
    st_view(v, ., i)
    sum = colsum(v)
}
```

If you need $V$ and $v$, that is okay. You can have many views of the data setup simultaneously.
Similarly, be careful using views with operators. X ' X makes a copy of X in the process of creating the transpose. Use functions such as cross () (see [M-5] cross()) that are designed to minimize the use of memory.

Do not be overly concerned about this issue. Making a copy of a column of a view amounts to the same thing as introducing a temporary variable in a Stata program-something that is done all the time.

## Cautions when using views 2: Assignment

The ability to assign to a view and so change the underlying data can be either convenient or dangerous, depending on your goals. When making such assignments, there are two things you need be aware of.

The first is more of a Stata issue than it is a Mata issue. Assignment does not cause promotion. Coding

$$
\mathrm{V}[1,2]=4059.125
$$

might store 4059.125 in the first observation of the second variable of the view. Or, if that second variable is an int, what will be stored is 4059 , or if it is a byte, what will be stored is missing.

The second caution is a Mata issue. To reassign all the values of the view, code

$$
\mathrm{V}[., .]=\text { matrix_expression }
$$

Do not code
V = matrix_expression

The second expression does not assign to the underlying dataset, it redefines V to be a regular matrix.
Mata does not allow the use of views as the destination of assignment when the view contains factor variables or time-series-operated variables such as i.rep78 or l.gnp.

## Cautions when using views 3: View connections are ephemeral

For faster data access, an st_view() connection accesses data using variable indices, not variable names. However, variable indices can change when variables are created or removed. If a variable is created or removed while your code is using a view connection, there is a chance the view will switch to another variable.

## Efficiency

Whenever possible, specify argument $i$ of st_view $(V, i, j)$ and st_sview ( $V, i, j$ ) as . (missing value) or as a row vector range (for example, $\left(i_{1}, i_{2}\right)$ ) rather than as a column vector list.

Specify argument $j$ as a real row vector rather than as a string whenever st_view() and st_sview() are used inside loops with the same variables (and the view does not contain factor variables nor time-series-operated variables). This prevents Mata from having to look up the same names over and over again.

## Conformability

st_view $(V, i, j)$, st_sview $(V, i, j)$ :
input:

| $i:$ | $n \times 1$ | or | $n_{2} \times 2$ |
| :--- | :--- | :--- | :--- |
| $j:$ | $1 \times k$ | or | $1 \times 2$ containing $k$ elements when expanded |

output:
$V: \quad n \times k$
st_view ( $V, i, j$, selectvar), st_sview ( $V, i, j$, selectvar):
input:

$$
\begin{array}{rrlr}
i: & n \times 1 & \text { or } & n_{2} \times 2 \\
j: & 1 \times k & \text { or } & 1 \times 2 \text { containing } k \text { elements when expanded } \\
\text { selectvar: } & 1 \times 1 & &
\end{array}
$$

output:
$V: \quad(n-e) \times k, \quad$ where $e$ is number of observations excluded by selectvar

## Diagnostics

st_view ( $i, j[$, selectvar $]$ ) and st_sview ( $i, j[$, selectvar $]$ ) abort with error if any element of $i$ is outside the range of observations or if a variable name or index recorded in $j$ is not found. Variable-name abbreviations are allowed. If you do not want this and no factor variables nor time-series-operated variables are specified, use st_varindex() (see [M-5] st_varindex()) to translate variable names into variable indices.
st_view() and st_sview() abort with error if any element of $i$ is out of range as described under the heading Details of observation subscripting using st_data() and st_sdata() in [M-5] st_data().

Some functions do not allow views as arguments. If example $(X)$ does not allow views, you can still use it by coding

```
... example(X=V) ...
```

because that will make a copy of view V in X . Most functions that do not allow views mention that in their Diagnostics section, but some do not because it was unexpected that anyone would want to use a view in that case. If a function does not allow a view, you will see in the traceback log:

```
: myfunction(...)
    example(): }3103\mathrm{ view found where array required
        mysub(): - function returned error
    myfunction(): - function returned error
        <istmt>: - function returned error
r(3103);
```

The above means that function example() does not allow views.

## Reference

Gould, W. W. 2005. Mata Matters: Using views onto the data. Stata Journal 5: 567-573.

## Also see

[M-5] select() - Select rows, columns, or indices
[M-5] st_data( ) - Load copy of current Stata dataset
[M-5] st_subview() - Make view from view
[M-5] st_viewvars( ) - Variables and observations of view
[M-4] Stata - Stata interface functions
[D] putmata - Put Stata variables into Mata and vice versa

## Title

[M-5] st_viewvars( ) - Variables and observations of view

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

st_viewvars ( $V$ ) returns the indices of the Stata variables corresponding to the columns of $V$.
st_viewobs ( $V$ ) returns the Stata observation numbers corresponding to the rows of $V$. Returned is either a $1 \times 2$ row vector recording the observation range or an $N \times 1$ column vector recording the individual observation numbers.

## Syntax

```
real rowvector st_viewvars(matrix V)
real vector st_viewobs(matrix V)
```

where $V$ is required to be a view.

## Remarks and examples

The results returned by these two functions are suitable for inclusion as arguments in subsequent calls to st_view() and st_sview(); see [M-5] st_view().

## Conformability

```
st_viewvars ( \(V\) ):
    \(V: \quad N \times k\)
    result: \(1 \times k\)
st_viewobs( \(V\) ):
    \(V: \quad N \times k\)
    result: \(\quad 1 \times 2\) or \(N \times 1\)
```


## Diagnostics

st_viewvars ( $V$ ) and st_viewobs ( $V$ ) abort with error if $V$ is not a view.

## Also see <br> [M-5] st_view( ) - Make matrix that is a view onto current Stata dataset <br> [M-4] Stata - Stata interface functions

## Title

[M-5] st_vlexists( ) - Use and manipulate value labels

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

st_vlexists (name) returns 1 if value label name exists and returns 0 otherwise.
st_vldrop(name) drops value label name if it exists.
st_vlmap (name, values) maps values through value label name and returns the result.
st_vlsearch (name, text) does the reverse; it returns the value corresponding to the text.
st_vlload(name, values, text) places value label name into values and text.
st_vlmodify(name, values, text) creates a new value label or modifies an existing one.

## Syntax

```
real scalar st_vlexists(name)
void st_vldrop(name)
string matrix st_vlmap(name, real matrix values)
real matrix st_vlsearch(name, string matrix text)
void st_vlload(name, values, text)
void st_vlmodify(name, real colvector values, string colvector text)
```

where name is string scalar and where the types of values and text in st_vlload() are irrelevant because they are replaced.

## Remarks and examples

Value labels are named and record a mapping from numeric values to text. For instance, a value label named sexlbl might record that 1 corresponds to male and 2 to female. Values labels are attached to Stata numeric variables. If a Stata numeric variable had the value label sexlbl attached to it, then the 1 s and 2 s in the variable would display as male and female. How other values would appear-if there were other values-would not be affected.

Remarks are presented under the following headings:

[^14]
## Value-label mapping

Let us consider value label sexlbl mapping 1 to male and 2 to female.
st_vlmap("sexlbl", values) would map the $r \times c$ matrix values through sexlbl and return an $r \times c$ string matrix containing the result. Any values for which there was no mapping would result in "". Thus

```
: res = st_vlmap("sexlbl", 1)
: res
male
res = st_vlmap("sexlbl", (2,3,1))
res
1
```


st_vlsearch (name, text) performs the reverse mapping:

```
txt = st_vlsearch("sexlbl", ("female","","male"))
txt
    1 2 3
    1
        2 . 1
```


## Value-label creation and editing

st_vlmodify (name, values, text) creates new value labels and modifies existing ones.
If value label sexlbl did not exist, coding

```
: st_vlmodify("sexlbl", (1\2), ("male"\"female"))
```

would create it. If the value label did previously exist, the above would modify it so that 1 now corresponds to male and 2 to female, regardless of what 1 or 2 previously corresponded to, if they corresponded to anything. Other mappings that might have been included in the value label remain unchanged. Thus
: st_vlmodify("sexlbl", 3, "unknown")
would add another mapping to the label. Values are deleted by specifying the text as " ", so

```
: st_vlmodify("sexlbl", 3, "")
```

would remove the mapping for 3 (if there was a mapping). If you remove all the mappings, the value label itself is automatically dropped:

```
: st_vlmodify("sexlbl", (1\2), (""\""))
```

results in value label sexlbl being dropped if 1 and 2 were the final mappings in it.

## Loading value labels

st_vlload(name, values, text) returns the value label in values and text, where you can do with it as you please. Thus you could code

```
st_vlload("sexlbl", values, text)
    ...
st_vldrop("sexlbl")
st_vlmodify("sexlbl", values, text)
```


## Conformability

```
st_vlexists(name):
    name: }1\times
    result: }1\times
st_vldrop(name):
    name: 
    result: void
st_vlmap(name, values):
    name: }1\times
    values: }\quadr\times
    result: }\quadr\times
st_vlsearch(name, text):
    name: 
        text: }r\times
    result: }\quadr\times
```

st_vlload(name, values, text):
input:
name: $\quad 1 \times 1$
output:
values: $\quad k \times 1$
text: $\quad k \times 1$
st_vlmodify(name, values, text):
name: $\quad 1 \times 1$
values: $\quad m \times 1$
text: $\quad m \times 1$
result: void

## Diagnostics

The only conditions under which the above functions abort with error is when name is malformed or Mata is out of memory. Functions tolerate all other problems.
st_vldrop (name) does nothing if value label name does not exist.
st_vlmap (name, values) returns J(rows(values), cols(values), "") if value label name does not exist. When the value label does exist, individual values for which there is no recorded mapping are returned as "".
st_vlsearch(name, text): returns J (rows (values), cols(values), .) if value label name does not exist. When the value label does exist, individual text values for which there is no corresponding value are returned as . (missing).
st_vlload(name, values, text): sets values and text to be $0 \times 1$ when value label name does not exist.
st_vlmodify(name, values, text): creates the value label if it does not already exist. Value labels may map only integers and .a, . b, .., . z. Attempts to insert a mapping for . are ignored. Noninteger values are truncated to integer values. If an element of text is " ", then the corresponding mapping is removed.

## Also see

[M-4] Stata - Stata interface functions

## Title

[M-5] stata() - Execute Stata command

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

stata (cmd) executes the Stata command contained in the string scalar cmd. Output from the command appears at the terminal, and any macros contained in cmd are expanded.
stata (cmd, nooutput) does the same thing, but if nooutput $\neq 0$, output produced by the execution is not displayed. stata ( $\mathrm{cmd}, 0$ ) is equivalent to stata ( cmd ).
stata (cmd, nooutput, nomacroexpand) does the same thing but, before execution, suppresses expansion of any macros contained in $c m d$ if nomacroexpand $\neq 0$. stata ( $c m d, 0,0$ ) is equivalent to stata ( cmd ).
_stata() repeats the syntaxes of stata(). The difference is that, whereas stata() aborts with error if the execution results in a nonzero return code, _stata() returns the resulting return code.

## Syntax

```
void stata(cmd)
void stata(cmd, nooutput)
void stata(cmd, nooutput, nomacroexpand)
real scalar _stata(cmd)
real scalar _stata(cmd, nooutput)
real scalar _stata(cmd, nooutput, nomacroexpand)
```

where
cmd: string scalar
nooutput: real scalar
nomacroexpand: real scalar

## Remarks and examples

The command you execute may invoke a process that causes another instance of Mata to be invoked. For instance, Stata program $A$ calls Mata function $m l()$, which executes stata() to invoke Stata program $B$, which in turn calls Mata function $m 2()$, which ....
stata ( $c m d$ ) and _stata ( $c m d$ ) execute $c m d$ at the current run level. This means that any local macros refer to local macros in the caller's space. Consider the following:

```
program example
    local x = "value from A"
    mata: myfunc()
    display "'x'"
end
mata void myfunc()
{
    stata('"local x = "new value""))
}
```

After example executes mata: myfunc(), 'x' will be "new value".
That stata() and _stata() work that way was intentional: Mata functions can modify the caller's environment so that they may create temporary variables for the caller's use, etc., and you only have to exercise a little caution. Executing stata() functions to run other ado-files and programs will cause no problems because other ado-files and programs create their own new environment in which temporary variables, local macros, etc., are private.

Also, do not use stata() or _stata() to execute a multiline command or to execute the first line of what could be considered a multiline command. Once the first line is executed, Stata will fetch the remaining lines from the caller's environment. For instance, consider


In the example above, myfunc() will consume the display "hello" and \} lines.

## Conformability

stata(cmd, nooutput, nomacroexpand):
cmd: $\quad 1 \times 1$
nooutput: $1 \times 1$ (optional)
nomacroexpand: $\quad 1 \times 1$ (optional)
result: void
_stata(cmd, nooutput, nomacroexpand):

| cmd: | $1 \times 1$ |  |
| ---: | :--- | :--- |
| nooutput: | $1 \times 1$ | (optional) |
| nomacroexpand: | $1 \times 1$ | (optional) |
| result: | $1 \times 1$ |  |

## Diagnostics

stata() aborts with error if $c m d$ is too long (exceedingly unlikely), if macro expansion fails, or if execution results in a nonzero return code.
_stata() aborts with error if cmd is too long.

## Also see

[M-3] mata stata - Execute Stata command
[M-4] Stata - Stata interface functions

## Title

[M-5] stataversion( ) - Version of Stata being used

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

stataversion() returns the version of Stata/Mata that is running, multiplied by 100. For instance, if you have Stata 18 installed on your computer, stataversion() returns 1800.
statasetversion() returns the version of Stata that has been set by the user-the version of Stata that Stata is currently emulating-multiplied by 100. Usually stataversion() == statasetversion(). If the user has set a previous version-say, version 8 by typing version 8 in Statastatasetversion() will return a number less than stataversion().
statasetversion(version) allows you to reset the version being emulated. Results are the same as using Stata's version command; see [P] version. version, however, is specified as an integer equal to 100 times the version you want.

## Syntax

```
real scalar stataversion()
real scalar statasetversion()
void statasetversion(real scalar version)
```

Note: The version number is multiplied by 100: Stata 2.0 is 200 , Stata 5.1 is 510 , Stata 18.0 is 1800 , and StataNow 18.5 is 1850.

## Remarks and examples

It is usually not necessary to reset statasetversion(). If you do reset statasetversion(), good form is to set it back when you are finished:

```
current_version = statasetversion()
statasetversion(desired_version)
statasetversion(current_version)
```


## Conformability

stataversion():
result: $\quad 1 \times 1$
statasetversion():
result: $\quad 1 \times 1$
statasetversion(version):
version: $1 \times 1$
result: void

## Diagnostics

statasetversion(version) aborts with error if version is less than 100 or greater than stataversion().

## Also see

[M-5] bufio( ) - Buffered (binary) I/O
[M-5] byteorder( ) - Byte order used by computer
[M-4] Programming - Programming functions

## Title

[M-5] strdup( ) - String duplication
Description Syntax Conformability Diagnostics Also see

## Description

There is no strdup() function. Instead, the multiplication operator is used:

```
3*"example" = "exampleexampleexample"
"café"*2 = "cafécafé"
0*"this" = ""
```


## Syntax

$$
\begin{aligned}
& n * s \\
& s * n \\
& n: * s \\
& s: * n
\end{aligned}
$$

where $n$ is real and $s$ is string.

## Conformability

$n * s, s * n:$

| $n:$ |  | $1 \times 1$ |
| ---: | :--- | ---: |
| $s:$ | $r \times c$ |  |
| result $:$ |  | $r \times c$ |

$n: * s, s: * n$ :
$n: \quad r_{1} \times c_{1}$
$s: \quad r_{2} \times c_{2}, \quad n$ and $s$ c-conformable
result: $\quad \max \left(r_{1}, r_{2}\right) \times \max \left(c_{1}, c_{2}\right)$

## Diagnostics

If $n<0$, the result is as if $n=0: " "$ is returned.
If $n$ is not an integer, the result is as if $\operatorname{trunc}(n)$ were specified.

## Also see

[M-4] String - String manipulation functions

## Title

[M-5] strlen() — Length of string in bytes

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

strlen $(s)$ returns the length of-the number of bytes contained in-the string $s$.
When $s$ is not a scalar, strlen() returns element-by-element results.

## Syntax

```
real matrix strlen(string matrix s)
```


## Remarks and examples

Stata understands length() as a synonym for its strlen() function. Do not, however, use length() in Mata when you mean strlen(). Mata's length() function returns the length (number of elements) of a vector.

Use ustrlen() to obtain the length of a string in Unicode characters. Use udstrlen() to obtain the length of a string in display columns.

## Conformability

```
strlen(s):
    s: }\quadr\times
    result: }\quadr\times
```


## Diagnostics

strlen $(s)$, when $s$ is a binary string (a string containing binary 0 ), returns the overall length of the string, not the location of the binary 0 . Use $\operatorname{strpos}(s, \operatorname{char}(0))$ if you want the location of the binary 0 ; see [M-5] strpos().

## Also see

[M-5] fmtwidth() — Width of \%fmt
[M-5] strpos() - Find substring in string
[M-5] udstrlen() - Length of Unicode string in display columns
[M-5] ustrlen( ) - Length of Unicode string in Unicode characters
[M-4] String - String manipulation functions

## Title

[M-5] ustrlen() — Length of Unicode string in Unicode characters
Description Syntax Remarks and examples Conformability Diagnostics
Also see

## Description

ustrlen ( $s$ ) returns the number of Unicode characters in the Unicode string $s$. An invalid UTF-8 sequence is counted as one Unicode character. Note that any Unicode character besides ASCII characters ( $0-127$ ) takes more than 1 byte in UTF- 8 encoding, for example, "é" takes 2 bytes.
ustrinvalident ( $s$ ) returns the number of invalid UTF-8 sequences in $s$. An invalid UTF-8 sequence can contain one byte or multiple bytes.

When $s$ is not a scalar, functions return element-by-element results.

## Syntax

```
real matrix ustrlen(string matrix s)
real matrix ustrinvalidcnt(string matrix s)
```


## Remarks and examples

ustrlen $(s)$, when $s$ is a binary string (a string containing null terminator char(0)), returns the overall length of the Unicode string. Note that null terminator char (0) is a valid Unicode code point.

Use udstrlen() to obtain the length of a string in display columns. Use strlen() to obtain the length of a string in bytes. See [U] 12.4.2.2 Displaying Unicode characters.

## Conformability

```
ustrlen(s), ustrinvalidcnt(s):
            s: }\quadr\times
    result: }\quadr\times
```


## Diagnostics

ustrlen ( $s$ ) and ustrinvalident ( $s$ ) return negative error codes if an error occurs.

## Also see

[M-5] strlen( ) - Length of string in bytes
[M-5] udstrlen() - Length of Unicode string in display columns
[M-4] String - String manipulation functions
[U] 12.4.2.2 Displaying Unicode characters

## Title

[M-5] udstrlen( ) — Length of Unicode string in display columns

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

udstrlen ( $s$ ) returns the number of columns needed to display the Unicode string $s$ in Stata's Results window.

When $s$ is not a scalar, udstrlen() returns element-by-element results.

## Syntax

real matrix udstrlen(string matrix $s$ )

## Remarks and examples

Unicode characters from the Chinese, Japanese, and Korean languages usually require two display columns. A Latin character usually requires one column. Any invalid UTF-8 sequence requires one column. See [U] 12.4.2.2 Displaying Unicode characters for details.

Use ustrlen() to obtain the length of a string in Unicode characters. Use strlen() to obtain the length of a string in bytes.

## Conformability

```
udstrlen(s):
    s: r}\quadr\times
    result: }\quadr\times
```


## Diagnostics

udstrlen $(s)$ returns a negative error code if an error occurs.

## Also see

[M-5] strlen() - Length of string in bytes
[M-5] ustrlen() - Length of Unicode string in Unicode characters
[M-4] String - String manipulation functions
[U] 12.4.2.2 Displaying Unicode characters

## Title

[M-5] strmatch( ) - Determine whether string matches pattern

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

strmatch ( $s$, pattern) returns 1 if $s$ matches pattern and 0 otherwise.
When arguments are not scalar, strmatch() returns element-by-element results.

## Syntax

real matrix strmatch (string matrix $s$, string matrix pattern)

## Remarks and examples

In pattern, * means that 0 or more characters go here and ? means that exactly one Unicode character goes here. Thus pattern="*" matches anything and pattern="? $\mathrm{p} * \mathrm{x}$ " matches all strings whose second character is $p$ and whose last character is $x$.

## Conformability

strmatch (s, pattern):

| $s:$ | $r_{1} \times c_{1}$ |
| ---: | :--- |
| pattern: | $r_{2} \times c_{2}, \quad s$ and pattern r-conformable |
| result: | $\max \left(r_{1}, r_{2}\right) \times \max \left(c_{1}, c_{2}\right)$ |

## Diagnostics

In strmatch ( $s$, pattern), if $s$ or pattern contain a binary 0 (they usually would not), the strings are considered to end at that point.

## Also see

[M-4] String - String manipulation functions

## Title

[M-5] strofreal( ) — Convert real to string
Description Syntax Remarks and examples Conformability Diagnostics
Also see

## Description

strofreal ( $R$ ) returns $R$ as a string using Stata's $\% 9.0 \mathrm{~g}$ format. strofreal ( $R$ ) is equivalent to strofreal( $R, \quad " \% 9.0 \mathrm{~g} ")$.
strofreal ( $R$, format) returns $R$ as a string formatted using format.
Leading blanks are trimmed from the result.
When arguments are not scalar, strofreal() returns element-by-element results.

## Syntax

string matrix strofreal (real matrix $R$ )
string matrix strofreal (real matrix $R$, string matrix format)

## Remarks and examples

strofreal() will ignore the format specified with set dp . To display numeric values as strings while respecting the format specified with set $d p$, use sprintf().

## Conformability

```
strofreal(R,format):
            R: }\quad\mp@subsup{r}{1}{}\times\mp@subsup{c}{1}{
    format: }\quad\mp@subsup{r}{2}{}\times\mp@subsup{c}{2}{},\quadR\mathrm{ and format r-conformable (optional)
    result: max (r},\mp@subsup{r}{2}{})\times\operatorname{max}(\mp@subsup{c}{1}{},\mp@subsup{c}{2}{}
```


## Diagnostics

strofreal ( $R$, format) returns "." if format is invalid.

## Also see

[M-5] strtoreal() - Convert string to real
[M-4] String - String manipulation functions

## Title

[M-5] strpos() — Find substring in string

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

strpos(haystack, needle) returns the location of the first occurrence of needle in haystack, 0 if needle does not occur, or 1 if needle is empty.
strrpos(haystack, needle) returns the location of the last occurrence of needle in haystack, 0 if needle does not occur, or 1 if needle is empty.

When arguments are not scalar, strpos() returns element-by-element results.

## Syntax

real matrix strpos (string matrix haystack, string matrix needle)
real matrix strrpos (string matrix haystack, string matrix needle)

## Remarks and examples

When working with binary strings, one can find the first or last location of the binary 0 using $\operatorname{strpos}(s, \operatorname{char}(0))$ or strrpos( $s$, char(0)).

Use ustrpso() or ustrrpos() to search based on characters rather than on bytes.

## Conformability

```
strpos(haystack, needle), strrpos(haystack, needle):
    haystack: }\quad\mp@subsup{r}{1}{}\times\mp@subsup{c}{1}{
    needle: }\quad\mp@subsup{r}{2}{}\times\mp@subsup{c}{2}{},\quad\mathrm{ haystack and needle r-conformable
    result: }\quad\operatorname{max}(\mp@subsup{r}{1}{},\mp@subsup{r}{2}{})\times\operatorname{max}(\mp@subsup{c}{1}{},\mp@subsup{c}{2}{}
```


## Diagnostics

strpos(haystack, needle) and strrpos(haystack, needle) return 0 if needle is not found in haystack.

## Also see

[M-5] ustrpos() - Find substring in Unicode string
[M-4] String - String manipulation functions

## Title

[M-5] ustrpos( ) — Find substring in Unicode string

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

ustrpos ( $s, s f[, n]$ ) returns the character position in $s$ at which $s f$ is first found; otherwise, it returns 0 . If $n$ is specified and is larger than zero, the search starts at the $n$th Unicode character of $s$. ustrrpos ( $s, s f[, n]$ ) returns the position in $s$ at which $s f$ is last found; otherwise, it returns 0 . If $n$ is specified and is larger than zero, the part between the first Unicode character and the $n$th Unicode character of $s$ is searched.

When $s$ is not a scalar, these functions return element-by-element results.

## Syntax

real matrix ustrpos(string matrix $s$, string scalar sf [, real scalar $n]$ )
real matrix ustrrpos(string matrix $s$, string scalar sf [, real scalar $n]$ )

## Remarks and examples

An invalid UTF-8 sequence in $s$ or $s f$ is replaced with a Unicode replacement character \ufffd before the search is performed.

Use strpos() or strrpos() to find the byte-based location of a substring in a string.

## Conformability

```
\(\operatorname{ustrpos}(s, s f[, n])\), ustrrpos (s,sf[,n]):
            \(s: \quad r \times c\)
            \(s f: \quad 1 \times 1\)
            \(n: \quad 1 \times 1\)
    result: \(\quad r \times c\)
```


## Diagnostics

None.

## Also see

[M-5] strpos() - Find substring in string
[M-4] String - String manipulation functions
[U] 12.4.2 Handling Unicode strings

## Title

[M-5] strreverse( ) - Reverse string

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

strreverse ( $s$ ) reverses the ASCII string $s$.
When $s$ is not a scalar, strreverse() returns element-by-element results.

## Syntax

string matrix strreverse(string matrix s)

## Remarks and examples

strreverse() is intended for use with ASCII characters. For Unicode characters beyond the ASCII range, the encoded bytes are reversed.

Use ustrreverse() to return a string with its Unicode characters in reverse order.

## Conformability

```
strreverse(s)
    s: }\quadr\times
    result: }\quadr\times
```


## Diagnostics

None.

## Also see

[M-5] ustrreverse() - Reverse Unicode string
[M-4] String - String manipulation functions

## Title

[M-5] ustrreverse( ) - Reverse Unicode string

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

ustrreverse (string matrix $s$ ) reverses the Unicode string $s$.
When arguments are not scalar, this function returns element-by-element results.

## Syntax

string matrix ustrreverse (string matrix s)

## Remarks and examples

The function does not take Unicode character equivalence into consideration. Hence, a Unicode character in a decomposed form will not be reversed as one unit. An invalid UTF-8 sequence is replaced with a Unicode replacement character \ufffd.

Use strreverse() to return a string with its bytes in reverse order.

## Conformability

```
ustrreverse(s):
\begin{tabular}{rl}
\(s:\) & \(r \times c\) \\
result \(:\) & \\
\(r \times c\)
\end{tabular}
```


## Diagnostics

ustrreverse ( $s$ ) returns an empty string if an error occurs.

## Also see

[M-5] strreverse() - Reverse string
[M-4] String - String manipulation functions
[U] 12.4.2 Handling Unicode strings

## Title

[M-5] strtoname( ) - Convert a string to a Stata 13 compatible name

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

strtoname ( $s, p$ ) returns $s$ translated into a Stata name. Each character in $s$ that is not allowed in a Stata name is converted to an underscore character, _. If the first character in $s$ is a numeric character and $p$ is not 0 , then the result is prefixed with an underscore. The result is truncated to 32 bytes. strtoname $(s)$ is equivalent to strtoname ( $s, 1$ ).

When arguments are not scalar, strtoname() returns element-by-element results.

## Syntax

```
string matrix strtoname(string matrix s, real scalar p)
string matrix strtoname(string matrix s)
```


## Remarks and examples

strtoname() handles strings with only ASCII characters. Use ustrtoname() to produce Stata names with Unicode characters.

```
strtoname("StataName") returns "StataName".
strtoname("not a Stata name") returns "not_a_Stata_name".
strtoname("0 is off") returns "_0_is_off".
strtoname("0 is off", 0) returns "0_is_off".
```


## Conformability

```
strtoname(s, p):
    s: 
    result: }r\times
strtoname(s):
    s: }\quadr\times
    result: }\quadr\times
```


## Diagnostics

None.

## Also see

[M-5] ustrtoname() - Convert a Unicode string to a Stata name
[M-4] String - String manipulation functions

## Title

[M-5] ustrtoname() - Convert a Unicode string to a Stata name

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

ustrtoname ( $s[, p]$ ) converts a Unicode string to a Stata name. Each character in $s$ that is not allowed in a Stata name is converted to an underscore character, _. If the first character in $s$ is a numeric character and $p$ is specified and not zero, then the result is prefixed with an underscore. The result is truncated to 32 Unicode characters.

When arguments are not scalar, ustrtoname() returns element-by-element results.

## Syntax

string matrix ustrtoname (string matrix $s[$, real scalar $p]$ )

## Remarks and examples

An invalid UTF-8 sequence is converted to an underscore character, -.
Use strtoname() if you need to produce a Stata 13 compatible name.

## Conformability

```
ustrtoname(s [, p]):
\begin{tabular}{rr}
\(s:\) & \(r \times c\) \\
\(p:\) & \(1 \times 1\) \\
\(r e s u l t:\) & \(r \times c\)
\end{tabular}
```


## Diagnostics

ustrtoname ( $s[, p]$ ) returns an empty string if an error occurs.

## Also see

[M-5] strtoname () - Convert a string to a Stata 13 compatible name
[M-4] String - String manipulation functions
[U] 12.4.2 Handling Unicode strings

## Title

[M-5] strtoreal() — Convert string to real

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

strtoreal ( $S$ ) returns $S$ converted to real. Elements of $S$ that cannot be converted are returned as . (missing value).
_strtoreal $(S, R)$ does the same as above-it returns the converted values in $R$-and it returns the number of elements that could not be converted. In such cases, the corresponding value of $R$ contains . (missing).

## Syntax

real matrix strtoreal (string matrix $S$ )
real scalar _strtoreal (string matrix $S, R$ )

## Remarks and examples

strtoreal("1.5") returns (numeric) 1.5.
strtoreal(" $-2.5 e+1$ ") returns (numeric) -25 .
strtoreal("not a number") returns (numeric) . (missing).
Typically, strtoreal ( $S$ ) and _strtoreal $(S, R$ ) are used with scalars, but if applied to a vector or matrix $S$, element-by-element results are returned.

In performing the conversion, leading and trailing blanks are ignored: "1.5" and " 1.5 " both convert to (numeric) 1.5, but " 1.5 kilometers" converts to . (missing). Use strtoreal (tokens (S) [1]) to convert just the first space-delimited part.

All Stata numeric formats are understood, such as $0,1,-2,1.5,1.5 \mathrm{e}+2$, and $-1.0 \mathrm{x}+8$, as well as the missing-value codes ., . a, .b, ..., . z.

Thus using strtoreal ( $S$ ), if an element of $S$ converts to . (missing), you cannot tell whether the element was valid and equal to "." or the element was invalid and so defaulted to . (missing), such as if $S$ contained "cat" or "dog" or "1.5 kilometers".

When it is important to distinguish between these cases, use _strtoreal ( $S, R$ ). The conversion is returned in $R$ and the function returns the number of elements that were invalid. If _strtoreal() returns 0 , then all values were valid.

## Conformability

strtoreal (S):
input:

$$
S: \quad r \times c
$$

output:

$$
\text { result: } \quad r \times c
$$

_strtoreal (S, R):
input:
output:

$$
\begin{aligned}
S: & r \times c \\
R: & r \times c \\
\text { result: } & 1 \times 1
\end{aligned}
$$

## Diagnostics

strtoreal ( $S$ ) returns a missing value wherever an element of $S$ cannot be converted to a number.
_strtoreal ( $S, R$ ) does the same, but the result is returned in $R$.

## Also see

[M-5] strofreal( ) - Convert real to string
[M-4] String - String manipulation functions

## Title

[M-5] strtrim( ) — Remove blanks

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

stritrim (s) returns $s$ with all consecutive, internal blanks collapsed to one blank.
strltrim ( $s$ ) returns $s$ with leading blanks removed.
strrtrim ( $s$ ) returns $s$ with trailing blanks removed.
$\operatorname{strtrim}(s)$ returns $s$ with leading and trailing blanks removed.
When $s$ is not a scalar, these functions return element-by-element results.

## Syntax

```
string matrix stritrim(string matrix s)
string matrix strltrim(string matrix s)
string matrix strrtrim(string matrix s)
string matrix strtrim(string matrix s)
```


## Remarks and examples

Use ustrtrim(), ustrrtrim(), and ustrltrim() to remove Unicode whitespace and blank characters.

## Conformability

```
stritrim(s), strltrim(s), strrtrim(s), strtrim(s):
    s: r}\quadr\times
    result: }\quadr\times
```


## Diagnostics

None.

## Also see

[M-5] ustrtrim( ) - Remove Unicode whitespace characters
[M-4] String - String manipulation functions

## Title

[M-5] ustrtrim( ) — Remove Unicode whitespace characters

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

ustrltrim ( $s$ ) returns $s$ with leading Unicode whitespaces removed.
ustrrtrim ( $s$ ) returns $s$ with trailing Unicode whitespaces removed.
ustrtrim ( $s$ ) returns $s$ with leading and trailing Unicode whitespaces removed.
When $s$ is not a scalar, these functions return element-by-element results.

## Syntax

```
string matrix ustrltrim(string matrix s)
string matrix ustrrtrim(string matrix s)
string matrix ustrtrim(string matrix s)
```


## Remarks and examples

The functions remove all Unicode whitespace characters. Unicode considers an additional set of whitespace characters besides the ASCII space character char (32). For example, ASCII character 9 (horizontal tab) is a Unicode whitespace character. Hence, ustrtrim(char(9))="" but str$\operatorname{trim}(\operatorname{char}(9))=\operatorname{char}(9)$. ASCII codes char (10), char (11), char (12), and char (13) are also Unicode whitespace characters. See http://unicode.org/charts/collation/chart_Whitespace.html for the list of all Unicode whitespace characters.

Use functions strtrim(), strltrim(), strrtrim(), and stritrim() to trim only the ASCII space character char (32) in a string.

## Conformability

```
ustrltrim(s), ustrrtrim(s), ustrtrim(s):
            s: }\quadr\times
        result: }\quadr\times
```


## Diagnostics

ustrltrim $(s)$, ustrrtrim ( $s$ ), and ustrtrim ( $s$ ) return an empty string if an error occurs.

## Also see

[M-5] strtrim( ) - Remove blanks
[M-4] String - String manipulation functions
[U] 12.4.2 Handling Unicode strings

## Title

[M-5] strupper( ) - Convert ASCII letter to uppercase (lowercase)

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

strupper ( $s$ ) returns $s$, converted to uppercase.
strlower ( $s$ ) returns $s$, converted to lowercase.
strproper ( $s$ ) returns a string with the first ASCII letter capitalized and any other ASCII letters capitalized that immediately follow characters that are not letters; all other letters are converted to lowercase.

When $s$ is not a scalar, these functions return element-by-element results.

## Syntax

```
string matrix strupper(string matrix s)
string matrix strlower(string matrix s)
string matrix strproper(string matrix s)
```


## Remarks and examples

strproper("mR. joHn a. sMitH") returns Mr. John A. Smith.
strproper("jack o'reilly") returns Jack O'Reilly.
strproper ("2-cent's worth") returns 2-Cent'S Worth.
Use ustrupper() and ustrlower() to convert Unicode characters in a string to uppercase and lowercase.

## Conformability

```
strupper(s), strlower(s), strproper(s):
            s: r
    result: }\quadr\times
```


## Diagnostics

None.

## Also see

[M-5] ustrupper( ) - Convert Unicode string to uppercase, lowercase, or titlecase
[M-4] String - String manipulation functions

## Title

[M-5] ustrupper( ) - Convert Unicode string to uppercase, lowercase, or titlecase

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

ustrupper ( $s[, l o c]$ ) converts the characters in Unicode string $s$ to uppercase under the given locale loc. If loc is not specified, the locale_functions setting is used. The result can be longer or shorter than the input string; for example, the uppercase form of the German letter $\beta$ (code point $\backslash u 00 \mathrm{df}$ ) is two capital letters " SS ". The same $s$ but different loc can produce different results; for example, the uppercase letter " $i$ " is " $I$ " in English, but it is " I " with a dot in Turkish.
ustrlower ( $s[, l o c]$ ) converts the characters in Unicode string $s$ to lowercase under the given locale loc. If loc is not specified, the locale_functions setting is used. The result can be longer or shorter than the input Unicode string in bytes. The same $s$ but different loc can produce different results; for example, the lowercase letter of " $I$ " is " $i$ " in English, but it is " $i$ " without a dot in Turkish. The same Unicode character can be mapped to different Unicode characters based on its surrounding characters; for example, Greek capital letter sigma, $\Sigma$, has two lowercase alternatives: $\sigma$ or, if it is the final character of a word, $\varsigma$.
ustrtitle ( $s[, l o c]$ ) converts the Unicode words in string $s$ to titlecase. Note that a Unicode word is different from the space-delimited words produced by function word(). A Unicode word is a language unit based on either a set of word-boundary rules or dictionaries for some languages (Chinese, Japanese, and Thai). The titlecase is also locale dependent and context sensitive; for example, lowercase " ij " in titlecase form is " IJ " in Dutch, but it is " Ij " in English. If loc is not specified, the locale_functions setting is used.

When $s$ is not a scalar, these functions return element-by-element results.

## Syntax

string matrix $\quad$ ustrupper (string matrix $s[$, string scalar loc $]$ )
string matrix
ustrlower (string matrix $s[$, string scalar loc $])$
string matrix
ustrtitle (string matrix $s[$, string scalar loc $]$ )

## Remarks and examples

Use functions strupper () and strlower () to convert only ASCII letters to uppercase and lowercase.

## Conformability

```
ustrupper \((s[, l o c])\), ustrlower \((s[, l o c])\), ustrtitle \((s[, l o c])\) :
    \(s: \quad r \times c\)
    result: \(\quad r \times c\)
```


## Diagnostics

ustrupper $(s[, l o c])$, ustrlower $(s[, l o c])$, and ustrtitle $(s[, l o c])$ return an empty string if an error occurs.

## Also see

[M-5] strupper() - Convert ASCII letter to uppercase (lowercase)
[M-4] String - String manipulation functions
[U] 12.4.2.2 Displaying Unicode characters

## Title

[M-5] subinstr() - Substitute text

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

subinstr ( $s$, old, new) returns $s$ with all occurrences of old changed to new.
subinstr ( $s$, old, new, cnt) returns $s$ with the first cnt occurrences of old changed to new. All occurrences are changed if cnt contains missing.
subinword ( $s$, old, new) returns $s$ with all occurrences of old on word boundaries changed to new. subinword ( $s$, old, new, cnt) returns $s$ with the first cnt occurrences of old on word boundaries changed to new. All occurrences are changed if cnt contains missing.

When arguments are not scalar, these functions return element-by-element results.

## Syntax

string matrix subinstr (string matrix s, string matrix old, string matrix new) string matrix subinstr (string matrix $s$, string matrix old, string matrix new, real matrix cnt)
string matrix subinword(string matrix s, string matrix old, string matrix new) string matrix subinword (string matrix $s$, string matrix old, string matrix new, real matrix cnt)

## Remarks and examples

```
subinstr("th thin man", "th", "the") returns "the thein man".
subinword("th thin man", "th", "the") returns "the thin man".
```

See [M-5] usubinstr() if your string contains Unicode characters.

## Conformability

```
subinstr(s, old, new, cnt), subinword(s, old, new, cnt):
            s: }\quad\mp@subsup{r}{1}{}\times\mp@subsup{c}{1}{
        old:}\quad\mp@subsup{r}{2}{}\times\mp@subsup{c}{2}{
        new: }\quad\mp@subsup{r}{3}{}\times\mp@subsup{c}{3}{
        cnt: }\quad\mp@subsup{r}{4}{}\times\mp@subsup{c}{4}{}\mathrm{ (optional); s, old, new, cnt r-conformable
    result: }\quad\operatorname{max}(\mp@subsup{r}{1}{},\mp@subsup{r}{2}{},\mp@subsup{r}{3}{},\mp@subsup{r}{4}{})\times\operatorname{max}(\mp@subsup{c}{1}{},\mp@subsup{c}{2}{},\mp@subsup{c}{3}{},\mp@subsup{c}{4}{}
```


## Diagnostics

subinstr ( $s$, old, new, cnt) and subinword(s, old, new, cnt) treat cnt $<0$ as if $c n t=0$ was specified; the original string $s$ is returned.

## Also see

[M-5] substr() - Extract substring
[M-5] _substr( ) - Substitute into string
[M-5] usubinstr( ) - Replace Unicode substring
[M-5] usubstr() — Extract Unicode substring
[M-5] _usubstr( ) — Substitute into Unicode string
[M-4] String - String manipulation functions
[U] 12.4.2 Handling Unicode strings

## Title

[M-5] usubinstr( ) — Replace Unicode substring

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

usubinstr ( $s$, old, new, cnt) replaces the first cnt occurrences of the Unicode string old with the Unicode string new in $s$. If cnt is missing, all occurrences are replaced.

When arguments are not scalar, this function returns element-by-element results.

## Syntax

string matrix usubinstr (string matrix s, string matrix old, string matrix new, real matrix cnt)

## Remarks and examples

An invalid UTF-8 sequence in $s$, old, or new is replaced with the Unicode replacement character $\backslash u f f f d$ before replacement is performed.

Use subinstr () if your string does not contain Unicode characters or if you want to replace the substring based on bytes.

## Conformability

```
usubinstr(s, old, new, cnt):
            s: r
    old: }\quadr\timesc\mathrm{ or }1\times
    new: }\quadr\timesc\mathrm{ or }1\times
    cnt: }\quadr\timesc\mathrm{ or }1\times
    result: }\quadr\times
```


## Diagnostics

usubinstr ( $s$, old, new, cnt) returns an empty string if an error occurs.

## Also see

[M-5] subinstr( ) - Substitute text
[M-5] substr() — Extract substring
[M-5] _substr() — Substitute into string
[M-5] usubstr() — Extract Unicode substring
[M-5] _usubstr( ) — Substitute into Unicode string
[M-4] String - String manipulation functions
[U] 12.4.2 Handling Unicode strings

## Title

[M-5] sublowertriangle( ) - Return a matrix with zeros above a diagonal

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

sublowertriangle $(A, p)$ returns $A$ with the elements above a diagonal set to zero. In the returned matrix, $A[i, j]=0$ for all $i-j<p$. If it is not specified, $p$ is set to zero.
_sublowertriangle() mirrors sublowertriangle() but modifies $A$.
_sublowertriangle $(A, p)$ sets $A[i, j]=0$ for all $i-j<p$. If it is not specified, $p$ is set to zero.

## Syntax

numeric matrix sublowertriangle(numeric matrix $A$ [, numeric scalar $p]$ )
void _sublowertriangle(numeric matrix A [, numeric scalar p])
where argument $p$ is optional.

## Remarks and examples

Remarks are presented under the following headings:
Get lower triangle of a matrix
Nonsquare matrices

## Get lower triangle of a matrix

If $A$ is a square matrix, then sublowertriangle $(A, 0)=$ lowertriangle $(A)$.
sublowertriangle() is a generalization of lowertriangle().
We begin by defining A

$$
: A=(1,2,3 \backslash 4,5,6 \backslash 7,8,9)
$$

sublowertriangle (A, 0) returns A with zeros above the main diagonal as does lowertriangle():
: sublowertriangle(A, 0)
123

| 1 | 1 | 0 | 0 |
| :--- | :--- | :--- | :--- |
| 2 | 5 | 0 |  |
| 7 | 8 | 9 |  |

sublowertriangle(A, 1) returns $A$ with zeros in the main diagonal and above.

```
: sublowertriangle(A, 1)
1 2 3
```

1

| 1 | 0 | 0 | 0 |
| :--- | :--- | :--- | :--- |
| 2 | 4 | 0 | 0 |
| 3 | 7 | 8 | 0 |

sublowertriangle( $\mathrm{A}, \mathrm{p}$ ) can take negative $p$. For example, setting $\mathrm{p}=-1$ yields
: sublowertriangle(A, -1)

|  | 1 | 2 | 3 |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
| 2 | 1 2 0 <br> 4 5 6 <br> 7 8 9 <br>    |  |  |

## Nonsquare matrices

sublowertriangle() and _sublowertriangle() may be used with nonsquare matrices.
For instance, we define a nonsquare matrix A

$$
: A=(1,2,3,4 \backslash 5,6,7,8 \backslash 9,10,11,12)
$$

We use sublowertriangle() to obtain the lower triangle of A:

```
: sublowertriangle(A, 0)
```

| 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- |

1

| 1 | 0 | 0 | 0 |
| ---: | ---: | ---: | ---: |
| 5 | 6 | 0 | 0 |
| 9 | 10 | 11 | 0 |

## Conformability

## sublowertriangle (A, $p$ ):

input:

$$
\begin{array}{ll}
A: & r \times c \\
p: & 1 \times 1 \text { (optional) }
\end{array}
$$

output:

$$
\text { result: } \quad r \times c
$$

_sublowertriangle ( $A, p$ ):
input:

$$
\begin{array}{ll}
A: & r \times c \\
p: & 1 \times 1 \text { (optional) }
\end{array}
$$

output:
A: $\quad r \times c$

## Diagnostics

None.

## Also see

[M-4] Manipulation - Matrix manipulation

## Title

[M-5] _substr() — Substitute into string

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

_substr ( $s$, tosub, pos) substitutes tosub into $s$ at byte position pos. The first byte position of $s$ is pos = 1. _substr() may be used with text or binary strings.

Do not confuse _substr() with substr(), which extracts substrings; see [M-5] substr().

## Syntax

void _substr (string scalar s, string scalar tosub, real scalar pos)

## Remarks and examples

If $s$ contains "abcdef", then _substr ( $s$, "XY", 2) changes $s$ to contain "aXYdef".

## Conformability

_substr (s, tosub, pos):
input:

$$
\begin{aligned}
& s: 1 \times 1 \\
& \text { tosub: } \\
& 1 \times 1 \\
& \text { pos: } 1 \times 1
\end{aligned}
$$

output:

$$
s: \quad 1 \times 1
$$

## Diagnostics

_substr ( $s$, tosub, pos) does nothing if tosub=="".
_substr ( $s$, tosub, pos) may not be used to extend $s$ : _substr () aborts with error if substituting tosub into $s$ would result in a string longer than the original $s$. _substr () also aborts with error if pos $\leq 0$ or pos $\geq$. unless tosub is "".
_substr ( $s$, tosub, pos) aborts with error if $s$ or tosub are views.

## Also see

[M-5] subinstr( ) - Substitute text
[M-5] substr() — Extract substring
[M-5] usubinstr( ) - Replace Unicode substring
[M-5] usubstr() — Extract Unicode substring
[M-5] _usubstr( ) - Substitute into Unicode string
[M-4] String - String manipulation functions
[U] 12.4.2 Handling Unicode strings

## Title

[M-5] _usubstr() — Substitute into Unicode string

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

_usubstr (s, tosub, pos) substitutes tosub into $s$ at Unicode character position pos. The first Unicode character position of $s$ is pos $=1$. _usubstr () may be used with text or binary strings.

Do not confuse _usubstr () with usubstr (), which extracts Unicode substrings; see [M-5] usubstr().

## Syntax

void _usubstr(string scalar s, string scalar tosub, real scalar pos)

## Remarks and examples

If $s$ contains "café", then _usubstr ( $s$, "fe", 3) changes $s$ to contain "cafe".
Invalid UTF-8 sequences in both $s$ and tosub are replaced with the Unicode replacement character \ufffd before substitution.

## Conformability

_usubstr (s, tosub, pos):
input:

| s: | $1 \times 1$ |
| ---: | :--- |
| tosub: | $1 \times 1$ |
| pos: | $1 \times 1$ |

output:

$$
s: \quad 1 \times 1
$$

## Diagnostics

_usubstr ( $s$, tosub, pos) does nothing if tosub=="".
_usubstr () aborts with an error message if substituting tosub into $s$ would result in a string longer than the original $s$ in Unicode characters. _usubstr() also aborts with an error message if pos $\leq 0$ or pos $\geq$. unless tosub=="".
_usubstr ( $s$, tosub, pos) aborts with an error if $s$ or tosub are views.

## Also see

[M-5] subinstr( ) - Substitute text
[M-5] substr() — Extract substring
[M-5] _substr( ) - Substitute into string
[M-5] usubinstr( ) - Replace Unicode substring
[M-5] usubstr() — Extract Unicode substring
[M-4] String - String manipulation functions
[U] 12.4.2 Handling Unicode strings

## Title

[M-5] substr() — Extract substring
Description Syntax Remarks and examples Conformability Diagnostics
Also see

## Description

substr ( $s, b, l$ ) returns the substring of ASCII string $s$ starting at position $b$ and continuing for a length of $l$ characters.

For non-ASCII strings, $b$ and $l$ are interpreted as byte positions. To obtain character-based substrings of Unicode strings, see [M-5] usubstr( ).
$\operatorname{substr}(s, b)$ is equivalent to $\operatorname{substr}(s, b,$.$) for strings that do not contain binary 0$. If there is a binary 0 to the right of $b$, the substring from $b$ up to but not including the binary 0 is returned.

When arguments are not scalar, substr() returns element-by-element results.

## Syntax

```
string matrix substr(string matrix s, real matrix b, real matrix l)
string matrix substr(string matrix s, real matrix b)
```


## Remarks and examples

substr $(s, b, l)$ returns the substring of ASCII string $s$ starting at position $b$ and continuing for a length of $l$, where

1. $b$ specifies the starting position; the first character of the string is $b=1$.
2. $b>0$ is interpreted as distance from the start of the string; $b=2$ means starting at the second character.
3. $b<0$ is interpreted as distance from the end of string; $b=-1$ means starting at the last character; $b=-2$ means starting at the second from the last character.
4. $l$ specifies the length; $l=2$ means for two characters.
5. $l<0$ is treated the same as $l=0$ : no characters are copied.
6. $l \geq$. is interpreted to mean to the end of the string.
$\operatorname{substr}(s, b)$ is equivalent to $\operatorname{substr}(s, b,$.$) for strings that do not contain binary 0$. If there is a binary 0 to the right of $b$, the substring from $b$ up to but not including the binary 0 is returned.

If your string contains Unicode characters, see [M-5] usubstr() and [M-5] udsubstr().

## Conformability

substr $(s, b, l)$ :

| $s:$ | $r_{1} \times c_{1}$ |
| ---: | :--- |
| $b:$ | $r_{2} \times c_{2}$ |
| $l:$ | $r_{3} \times c_{3} ; \quad s, b$, and $l$ r-conformable |
| result: | $\max \left(r_{1}, r_{2}, r_{3}\right) \times \max \left(c_{1}, c_{2}, c_{3}\right)$ |

substr $(s, b)$ :
s: $\quad r_{1} \times c_{1}$
$b: \quad r_{2} \times c_{2} ; \quad s$ and $b$ r-conformable
result: $\quad \max \left(r_{1}, r_{2}\right) \times \max \left(c_{1}, c_{2}\right)$

## Diagnostics

In substr $(s, b, l)$ and $\operatorname{substr}(s, b)$, if $b$ describes a position before the beginning of the string or after the end, "" is returned. If $b+l$ describes a position to the right of the end of the string, results are as if a smaller value for $l$ were specified.

## Also see

[M-5] subinstr( ) - Substitute text
[M-5] _substr() — Substitute into string
[M-5] usubinstr( ) — Replace Unicode substring
[M-5] usubstr() — Extract Unicode substring
[M-5] _usubstr( ) - Substitute into Unicode string
[M-4] String - String manipulation functions
[U] 12.4.2 Handling Unicode strings

## Title

[M-5] usubstr( ) — Extract Unicode substring

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

usubstr ( $s, n 1, n 2$ ) returns the Unicode substring of $s$, starting at Unicode character $n 1$, for a length of $n 2$. If $n l<0, n l$ is interpreted as the distance from the last Unicode character of $s$; if $n 2=.($ missing $)$, the remaining portion of the Unicode string is returned.
ustrleft $(s, n)$ returns the first $n$ Unicode characters of the Unicode string $s$.
ustrright $(s, n)$ returns the last $n$ Unicode characters of the Unicode string $s$.
When arguments are not scalar, the functions return element-by-element results.

## Syntax

```
string matrix usubstr(string matrix s, real matrix n1, real matrix n2)
string matrix ustrleft(string matrix s, real matrix n)
string matrix ustrright(string matrix }s\mathrm{ , real matrix n)
```


## Remarks and examples

$n \leq 0$ is interpreted as the distance from the end of the Unicode string; $n=-1$ means the distance starting at the last Unicode character.

An invalid UTF-8 sequence is replaced with a Unicode replacement character \ufffd. Null terminator char ( 0 ) in a binary string is a valid UTF-8 character and will be counted and treated as such.

Use udsubstr() to extract a substring based on display columns. Use substr() to extract a substring based on bytes.

## Conformability

```
usubstr (s, b, l):
    s: r
    b: }\quadr\timesc\mathrm{ or }1\times
    l: }\quadr\timesc\mathrm{ or }1\times
    result: }\quadr\times
```


## Diagnostics

usubstr $(s, b, l)$, ustrleft $(s, b, l)$, and ustrright $(s, b, l)$ return an empty string if an error occurs.

## Also see

[M-5] subinstr( ) - Substitute text
[M-5] substr() - Extract substring
[M-5] _substr() — Substitute into string
[M-5] usubinstr( ) — Replace Unicode substring
[M-5] _usubstr( ) - Substitute into Unicode string
[M-4] String - String manipulation functions
[U] 12.4.2 Handling Unicode strings

## Title

[M-5] udsubstr() — Extract Unicode substring based on display columns

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

udsubstr $(s, n 1, n 2)$ returns the Unicode substring of $s$, starting at Unicode character $n 1$, for $n 2$ display columns. If $n 2=$. (missing), the remaining portion of the Unicode string is returned. If $n 2$ display columns from Unicode character $n l$ is in the middle of a Unicode character, the substring stops at the previous Unicode character.

When arguments are not scalar, udsubstr() returns element-by-element results.

## Syntax

string matrix udsubstr (string matrix $s$, real matrix $n 1$, real matrix $n 2$ )

## Remarks and examples

$n 1<0$ is interpreted as distance from the end of the Unicode string; $n l=-1$ means starting at the last Unicode character.

An invalid UTF-8 sequence is replaced with a Unicode replacement character \ufffd. Null terminator char ( 0 ) in a binary string is a valid UTF-8 character and will be counted and treated as such.

Use usubstr() to extract a substring based on Unicode characters. Use substr() to extract a substring based on bytes.

## Conformability

```
udsubstr(s, n1, n2):
            s: r
        nl: }\quadr\timesc\mathrm{ or 1 }\times
        n2: }\quadr\timesc\mathrm{ or }1\times
    result: }\quadr\times
```


## Diagnostics

udsubstr ( $s, n 1, n 2$ ) returns an empty string if an error occurs.

## Also see

[M-5] subinstr( ) - Substitute text
[M-5] substr() — Extract substring
[M-5] _substr() — Substitute into string
[M-5] usubinstr( ) - Replace Unicode substring
[M-5] usubstr() — Extract Unicode substring
[M-5] _usubstr() - Substitute into Unicode string
[M-4] String - String manipulation functions
[U] 12.4.2 Handling Unicode strings

## Title

[M-5] sum() — Sums

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

rowsum ( $Z$ ) and rowsum ( $Z$, missing) return a column vector containing the sum over the rows of Z.
colsum ( $Z$ ) and colsum ( $Z$, missing) return a row vector containing the sum over the columns of Z.
sum $(Z)$ and $\operatorname{sum}(Z, m i s s i n g)$ return a scalar containing the sum over the rows and columns of $Z$. quadrowsum (), quadcolsum (), and quadsum () are quad-precision variants of the above functions. The sum is accumulated in quad precision and then rounded to double precision and returned.

Argument missing determines how missing values are treated. If missing is not specified, results are the same as if missing $=0$ were specified: missing values are treated as zero. If missing $=1$ is specified, missing values are treated as missing values.

These functions may be used with real or complex matrix $Z$.

## Syntax

$$
\begin{array}{lr}
\text { numeric colvector } & \text { rowsum(numeric matrix } Z[\text {, missing }] \text { ) } \\
\text { numeric rowvector } & \text { colsum(numeric matrix } Z[\text {, missing }] \text { ) } \\
\text { numeric scalar } & \text { sum(numeric matrix } Z[\text {, missing }] \text { ) } \\
\text { numeric colvector } & \text { quadrowsum(numeric matrix } Z[\text {, missing }] \text { ) } \\
\text { numeric rowvector } & \text { quadcolsum(numeric matrix } Z[\text {, missing }] \text { ) } \\
\text { numeric scalar } & \text { quadsum(numeric matrix } Z[\text {, missing }] \text { ) }
\end{array}
$$

where optional argument missing is a real scalar that determines how missing values in $Z$ are treated:

1. Specifying missing as 0 is equivalent to not specifying the argument; missing values in $Z$ are treated as contributing 0 to the sum.
2. Specifying missing as 1 (or nonzero) specifies that missing values in $Z$ are to be treated as missing values and to turn the sum to missing.

## Remarks and examples

All functions return the same type as the argument, real if argument is real, complex if complex.

## Conformability

```
rowsum ( \(Z\), missing), quadrowsum ( \(Z\), missing):
\begin{tabular}{rll}
\(Z:\) & \(r \times c\) & \\
missing: & \(1 \times 1\) & (optional) \\
result: & \(r \times 1\) &
\end{tabular}
colsum ( \(Z\), missing), quadcolsum ( \(Z\), missing \()\) :
            \(Z: \quad r \times c\)
    missing: \(\quad 1 \times 1 \quad\) (optional)
        result: \(1 \times c\)
```

$\operatorname{sum}(Z$, missing $)$, quadsum ( $Z$, missing $)$ :

| $Z:$ | $r \times c$ |  |
| ---: | :--- | :--- |
| missing: | $1 \times 1$ | (optional) |
| result: | $1 \times 1$ |  |

## Diagnostics

If missing $=0$, missing values are treated as contributing zero to the sum; they do not turn the sum to missing. Otherwise, missing values turn the sum to missing.

## Also see

[M-5] cross() - Cross products
[M-5] mean() - Means, variances, and correlations
[M-5] runningsum ( ) - Running sum of vector
[M-4] Mathematical - Important mathematical functions
[M-4] Utility - Matrix utility functions

## Title

[M-5] svd() - Singular value decomposition

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | References | Also see |  |

## Description

$\operatorname{svd}(A, U, s, V t)$ calculates the singular value decomposition of $A: m \times n, m \geq n$, returning the result in $U, s$, and Vt. Singular values returned in $s$ are sorted from largest to smallest.
$\operatorname{svdsv}(A)$ returns the singular values of $A: m \times n, m \geq n$ or $m<n$ (that is, no restriction), in a column vector of length $\min (m, n) . U$ and $V t$ are not calculated.
$\quad \operatorname{svd}(A, s, V t)$ does the same as $\operatorname{svd}()$, except that it returns $U$ in $A$. Use of _svd() conserves memory.
_svdsv(A) does the same as $\operatorname{svdsv}()$, except that, in the process, it destroys $A$. Use of _svdsv() conserves memory.
_svd_la() is the interface to the LAPACK SVD routines and is used in the implementation of the previous functions. There is no reason you should want to use it.

## Syntax

| void | $\operatorname{svd}($ numeric matrix $A, U, s, V t)$ |
| :--- | :--- |
| real colvector | $\operatorname{svdsv}($ numeric matrix $A)$ |
| void | $\quad$ _svd $($ numeric matrix $A, s, V t)$ |
| real colvector | $\quad$ _svdsv(numeric matrix $A)$ |
| real scalar | _svd_la(numeric matrix $A, s, V t)$ |

## Remarks and examples

Remarks are presented under the following headings:

```
Introduction
Possibility of convergence problems
```

Documented here is the thin SVD, appropriate for use with $A: m \times n, m \geq n$. See [M-5] fullsvd() for the full SVD, appropriate for use in all cases. The relationship between the two is discussed in Relationship between the full and thin SVDs in [M-5] fullsvd().

Use of the thin SVD-the functions documented here-is preferred when $m \geq n$.

## Introduction

The SVD is used to compute accurate solutions to linear systems and least-squares problems, to compute the 2 -norm, and to determine the numerical rank of a matrix.

The singular value decomposition (SVD) of $A: m \times n, m \geq n$, is given by

$$
A=U * \operatorname{diag}(s) * V^{\prime}
$$

where

$$
\begin{aligned}
U: & m \times n \text { and } U^{\prime} U=\mathrm{I}(n) \\
s: & n \times 1 \\
V: & n \times n \text { and orthogonal (unitary) }
\end{aligned}
$$

When $A$ is complex, the transpose operator ${ }^{\prime}$ is understood to mean the conjugate transpose operator.
Vector $s$ contains the singular values, and those values are real even when $A$ is complex. $s$ is ordered so that the largest singular value appears first, then the next largest, and so on.

Function $\operatorname{svd}(A, U, s, V t)$ returns $U, s$, and $V t=V^{\prime}$.
Function $\operatorname{svdsv}(A)$ returns $s$, omitting the calculation of $U$ and $V t$. Also, whereas svd() is suitable for use only in the case $m \geq n$, $\operatorname{svdsv}()$ may be used in all cases.

## Possibility of convergence problems

It is possible, although exceedingly unlikely, that the SVD routines could fail to converge. svd(), $\operatorname{svdsv}(), \quad \_\operatorname{svd}()$, and _svdsv() then return singular values in $s$ equal to missing.

In coding, it is perfectly acceptable to ignore this possibility because (1) it is so unlikely and (2) even if the unlikely event occurs, the missing values will properly reflect the situation. If you do wish to check, in addition to checking missing $(s)>0$ (see [M-5] missing()), you must also check missing $(A)==0$ because that is the other reason $s$ could contain missing values. Convergence was not achieved if missing $(s)>0$ \& missing $(A)==0$. If you are calling one of the destructive-of- $A$ versions of SVD, remember to check missing $(A)==0$ before extracting singular values.

## Conformability

$\operatorname{svd}(A, U, s, V t):$
input:

$$
A: \quad m \times n, \quad m \geq n
$$

output:

$$
\begin{aligned}
U: & m \times n \\
s: & n \times 1 \\
V t: & n \times n
\end{aligned}
$$

svdsv (A):
A: $\quad m \times n, \quad m \geq n \quad$ or $\quad m<n$
result: $\quad \min (m, n) \times 1$
$\quad \_\operatorname{svd}(A, s, V t):$
input:
A: $\quad m \times n, \quad m \geq n$
output:
A: $\quad m \times n, \quad$ contains $U$
$s: \quad n \times 1$
Vt: $\quad n \times n$
_svdsv (A):
input:
A: $\quad m \times n, \quad m \geq n \quad$ or $\quad m<n$
output:
A: $\quad 0 \times 0$
result: $\quad \min (m, n) \times 1$
_svd_la $(A, s, V t)$ :
input:
A: $\quad m \times n, \quad m \geq n$
output:
A: $\quad m \times n, \quad$ contains $U$
$s: \quad n \times 1$
Vt: $\quad n \times n$
result: $\quad 1 \times 1$

## Diagnostics

$\operatorname{svd}(A, U, s, V t)$ and $\_\operatorname{svd}(A, s, V t)$ return missing results if $A$ contains missing. In all other cases, the routines should work, but there is the unlikely possibility of convergence problems, in which case missing results will also be returned; see Possibility of convergence problems above.
$\operatorname{svdsv}(A)$ and $\quad \operatorname{svdsv}(A)$ return missing results if $A$ contains missing values or if there are convergence problems.
_svd() and _svdsv() abort with error if $A$ is a view.
Direct use of _svd_la() is not recommended.

Singular value decompositions have multiple roots, as is engagingly explained by Stewart (1993). Eugenio Beltrami (Italy, 1835-1900), Camille Jordan (France, 1838-1922), and James Joseph Sylvester (Britain, 1814-1897) came to them through what we now call linear algebra, while Erhard Schmidt (Germany, 1876-1959) and Hermann Klaus Hugo Weyl (Germany, 1885-1955) approached them from integral equations. Although none of them used the matrix terminology or notation that is familiar to modern workers, seeing the structure within sets of equations was a more familiar task to them than it was to us. The terminology "singular values" appears to come from the literature on integral equations. The use of SVDs as workhorses in modern numerical analysis owes most to Gene Howard Golub (United States, 1932-2007).

## References

Stewart, G. W. 1993. On the early history of the singular value decomposition. SIAM Review 35: 551-566. https://doi.org/10.1137/1035134.

Trefethen, L. N. 2007. Obituary: Gene H. Golub (1932-2007). Nature 450: 962. https://doi.org/10.1038/450962a.

## Also see

[M-5] fullsvd( ) - Full singular value decomposition
[M-5] norm( ) - Matrix and vector norms
[M-5] $\operatorname{pinv}()$ - Moore-Penrose pseudoinverse
[M-5] rank( ) - Rank of matrix
[M-5] svsolve( ) - Solve AX=B for X using singular value decomposition
[M-4] Matrix - Matrix functions

## Title

[M-5] svsolve( ) - Solve AX=B for X using singular value decomposition

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

svsolve ( $A, B, \ldots$ ), uses singular value decomposition to solve $A X=B$ and return $X$. When $A$ is singular, svsolve() computes the minimum-norm least-squares generalized solution. When rank is specified, in it is placed the rank of $A$.
_svsolve $(A, B, \ldots)$ does the same thing, except that it destroys the contents of $A$ and it overwrites $B$ with the solution. Returned is the rank of $A$.

In both cases, tol specifies the tolerance for determining whether $A$ is of full rank. tol is interpreted in the standard way-as a multiplier for the default if $t o l>0$ is specified and as an absolute quantity to use in place of the default if $t o l \leq 0$ is specified.

## Syntax

| numeric matrix | svsolve $(A, B)$ |
| :--- | :--- |
| numeric matrix | svsolve $(A, B$, rank $)$ |
| numeric matrix | svsolve $(A, B$, rank, tol $)$ |
| real scalar | -svsolve $(A, B)$ |
| real scalar | _svsolve $(A, B$, tol $)$ |

where
A: numeric matrix
B: $\quad$ numeric matrix
rank: irrelevant; real scalar returned
tol: real scalar

## Remarks and examples

svsolve $(A, B, \ldots)$ is suitable for use with square or nonsquare, full-rank or rank-deficient matrix $A$. When $A$ is of full rank, svsolve() returns the same solution as lusolve() (see [M-5] lusolve()), ignoring roundoff error. When $A$ is singular, svsolve() returns the minimum-norm least-squares generalized solution. qrsolve() (see [M-5] qrsolve()), an alternative, returns a generalized leastsquares solution that amounts to dropping rows of $A$.

Remarks are presented under the following headings:

[^15]
## Derivation

We wish to solve for $X$

$$
\begin{equation*}
A X=B \tag{1}
\end{equation*}
$$

Perform singular value decomposition on $A$ so that we have $A=U S V^{\prime}$. Then (1) can be rewritten as

$$
U S V^{\prime} X=B
$$

Premultiplying by $U^{\prime}$ and remembering that $U^{\prime} U=I$, we have

$$
S V^{\prime} X=U^{\prime} B
$$

Matrix $S$ is diagonal and thus its inverse is easily calculated, and we have

$$
V^{\prime} X=S^{-1} U^{\prime} B
$$

When we premultiply by $V$, remembering that $V V^{\prime}=I$, the solution is

$$
\begin{equation*}
X=V S^{-1} U^{\prime} B \tag{2}
\end{equation*}
$$

See [M-5] svd() for more information on the SVD.

## Relationship to inversion

For a general discussion, see Relationship to inversion in [M-5] lusolve( ).
For an inverse based on the SVD, see $[\mathrm{M}-5] \operatorname{pinv}() \cdot \operatorname{pinv}(A)$ amounts to svsolve ( $A, \mathrm{I}$ (rows $(A)$ )), although pinv() has separate code that uses less memory.

## Tolerance

In (2) above, we are required to calculate the inverse of diagonal matrix $S$. The generalized solution is obtained by substituting zero for the $i$ th diagonal element of $S^{-1}$, where the $i$ th diagonal element of $S$ is less than or equal to eta in absolute value. The default value of eta is

$$
\text { eta }=\operatorname{epsilon}(1) * \operatorname{rows}(A) * \max (S)
$$

If you specify tol $>0$, the value you specify is used to multiply eta. You may instead specify tol $\leq$ 0 and then the negative of the value you specify is used in place of eta; see [M-1] Tolerance.

## Conformability

svsolve ( $A, B$, rank, tol):
input:

| $A:$ | $m \times n$ |  |
| ---: | :--- | :--- |
| $B:$ | $m \times k$ |  |
| tol: | $1 \times 1$ | (optional) |

output:
$\begin{array}{rlr}\text { rank: } & & 1 \times 1 \\ \text { result: } & n \times k & \text { (optional) }\end{array}$
_svsolve ( $A, B$, tol):
input:

$$
\begin{array}{rll}
A: & m \times n & \\
B: & m \times k & \\
\text { tol: } & 1 \times 1 & \text { (optional) }
\end{array}
$$

output:

$$
\begin{aligned}
A: & 0 \times 0 \\
B: & m \times k \\
\text { result: } & 1 \times 1
\end{aligned}
$$

## Diagnostics

svsolve $(A, B, \ldots)$ and _svsolve $(A, B, \ldots)$ return missing results if $A$ or $B$ contain missing. _svsolve $(A, B, \ldots)$ aborts with error if $A$ (but not $B$ ) is a view.

## Also see

[M-5] cholsolve( ) - Solve AX=B for X using Cholesky decomposition
[M-5] lusolve( ) - Solve AX=B for X using LU decomposition
[M-5] qrsolve( ) - Solve AX=B for X using QR decomposition
[M-5] solvelower() - Solve $\mathrm{AX}=\mathrm{B}$ for X , A triangular
[M-4] Matrix - Matrix functions
[M-4] Solvers - Functions to solve $A X=B$ and to obtain A inverse

## Title

[M-5] swap() — Interchange contents of variables

Description Syntax Remarks and examples Conformability<br>Diagnostics Also see

## Description

$\operatorname{swap}(A, B)$ interchanges the contents of $A$ and $B . A$ and $B$ are not required to be of the same type or dimension.

## Syntax

void $\operatorname{swap}($ transmorphic matrix $A$, transmorphic matrix $B$ )

## Remarks and examples

There is no faster way than $\operatorname{swap}(A, B)$ to assign $A=B$ when you do not care about the contents of $B$ after the assignment. For instance, you have the code

$$
\begin{aligned}
& A=B \\
& B=\ldots \text { (matrix expression)... }
\end{aligned}
$$

Faster is

```
swap(A,B)
B = ...(matrix expression)...
```

The execution time of $\operatorname{swap}()$ is independent of the size of $A$ and $B$, and $\operatorname{swap}()$ conserves memory to boot. Pretend that $B$ is a $900 \times 900$ matrix. After $A=B$ is executed, but before $B$ is reassigned, two copies of the $900 \times 900$ matrix exist. That does not happen with $\operatorname{swap}()$.

## Conformability

$\operatorname{swap}(A, B)$ :
input:

$$
\begin{array}{ll}
A: & r_{1} \times c_{1} \\
B: & \\
r_{2} \times c_{2}
\end{array}
$$

output:

$$
\begin{array}{ll}
A: & \\
r_{2} \times c_{2} \\
B: & \\
r_{1} \times c_{1}
\end{array}
$$

## Diagnostics

$\operatorname{swap}(A, B)$ works only with variables. Do not code, for instance, $\operatorname{swap}(A[i, j], A[j, i])$. It is not an error, but it will have no effect.

## Also see

[M-4] Programming - Programming functions

## Title

[M-5] Toeplitz( ) - Toeplitz matrices

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Reference | Also see |  |

## Description

Toeplitz $(c l, r l)$ returns the Toeplitz matrix defined by $c l$ being its first column and $r l$ being its first row. A Toeplitz matrix $T$ is characterized by $T[i, j]=T[i-1, j-1], i, j>1$. In a Toeplitz matrix, each diagonal is constant.

Vectors $c l$ and $r l$ specify the first column and first row of $T$.

## Syntax

numeric matrix Toeplitz(numeric colvector cl, numeric rowvector rl)

## Remarks and examples

$c l[1]$ is used to fill $T[1,1]$, and $r l[1]$ is not used.
To obtain the symmetric (Hermitian) Toeplitz matrix, code Toeplitz ( $v, v^{\prime}$ ) (if $v$ is a column vector), or Toeplitz $\left(v^{\prime}, v\right)$ if $v$ is a row vector.

## Conformability

Toeplitz (cl, rl):

| $c l:$ |  | $r \times 1$ |
| ---: | :--- | ---: | :--- |
| $r l:$ |  | $1 \times c$ |
| result: |  | $r \times c$ |

## Diagnostics

None.

Otto Toeplitz (1881-1940) was born in Breslau, Germany (now Wrocław, Poland), and educated there in mathematics. He researched and taught at universities in Göttingen, Kiel, and Bonn, making many contributions to algebra and analysis, but he was dismissed in 1935 for being a Jew. Toeplitz emigrated to Palestine in 1939 but died a few months later in Jerusalem. He was fascinated by the history of mathematics and wrote a popular work with Hans Rademacher, The Enjoyment of Mathematics.

## Reference

Robinson, A. 1976. Toeplitz, Otto. In Vol. 13 of Dictionary of Scientific Biography, ed. C. C. Gillispie. New York: Scribner's.

## Also see

[M-4] Standard - Functions to create standard matrices

## Title

[M-5] tokenget( ) - Advanced parsing

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

These functions provide advanced parsing. If you simply wish to convert strings into row vectors by separating on blanks, converting "mpg weight displ" into ("mpg", "weight", "displ"), see [M-5] tokens().

## Syntax

```
t= tokeninit([wchars [, pchars[, qchars[,allownum [, allowhex ]]]]])
t= tokeninitstata()
void tokenset (t, string scalar s)
string rowvector tokengetall(t)
string scalar tokenget (t)
string scalar tokenpeek(t)
string scalar tokenrest(t)
real scalar tokenoffset (t)
void tokenoffset(t, real scalar offset)
string scalar tokenwchars(t)
void tokenwchars(t, string scalar wchars)
string rowvector tokenpchars(t)
void tokenpchars(t, string rowvector pchars)
string rowvector tokenqchars(t)
void tokenqchars(t, string rowvector qchars)
real scalar tokenallownum(t)
void tokenallownum(t, real scalar allownum)
real scalar tokenallowhex (t)
void tokenallowhex(t, real scalar allowhex)
```

where
$t$ is transmorphic and contains the parsing environment information. You obtain a $t$ from tokeninit() or tokeninitstata() and then pass $t$ to the other functions.
wchars is a string scalar containing the characters to be treated as whitespace, such as " ", (" "+char (9)) , or "".
pchars is a string rowvector containing the strings to be treated as parsing characters, such as "" and (">", "<", ">=", "<="). "" and $J(1,0, " ")$ are given the same interpretation: there are no parsing characters.
qchars is a string rowvector containing the character pairs to be treated as quote characters. "" (that is, empty string) is given the same interpretation as $J(1,0, " ")$; there are no quote characters. qchars ('"""") (that is, the two-character string quote indicates that " is to be treated as open quote and " is to be treated as close quote. qchars = ('""""), ("("")"') indicates that, in addition, '" is to be treated as open quote and "' as close quote. In a syntax that did not use < and > as parsing characters, qchars $=("<>")$ would indicate that < is to be treated as open quote and > as close quote.
allownum is a string scalar containing 0 or 1 . allownum $=1$ indicates that numbers such as 12.23 and $1.52 \mathrm{e}+02$ are to be returned as single tokens even in violation of other parsing rules.
allowhex is a string scalar containing 0 or 1 . allowhex $=1$ indicates that numbers such as $1.921 \mathrm{fb} 54442 \mathrm{~d} 18 \mathrm{X}+001$ and $1.0 \mathrm{x}+\mathrm{a}$ are to be returned as single tokens even in violation of other parsing rules.

## Remarks and examples

Remarks are presented under the following headings:

```
Concepts
    White-space characters
    Parsing characters
    Quote characters
    Overrides
    Setting the environment to parse on blanks with quote binding
    Setting the environment to parse full Stata syntax
    Setting the environment to parse tab-delimited files
Function overview
    tokeninit() and tokeninitstata()
    tokenset()
    tokengetall()
    tokenget(), tokenpeek(), and tokenrest()
    tokenoffset()
    tokenwchars(), tokenpchars(), and tokenqchars()
    tokenallownum and tokenallowhex()
```


## Concepts

Parsing refers to splitting a string into pieces, which we will call tokens. Parsing as implemented by the token* () functions is defined by (1) the whitespace characters wchars, (2) the parsing characters pchars, and (3) the quote characters qchars.

## White-space characters

Consider the string "this that what". If there are no whitespace characters, no parsing characters, and no quote characters, that is, if wchars $=$ pchars $=$ qchars $=" "$, then the result of parsing "this that what " would be one token that would be the string just as it is: "this that what".

If wchars were instead " ", then parsing "this that what" results in ("this", "that", "what"). Parsing "this that what" (note the multiple blanks) would result in the same thing. White-space characters separate one token from the next but are not otherwise significant.

## Parsing characters

If we instead left wchars $="$ " and set pchars $="$ ", "this that what" parses into ("this", " ", "that", " ", "what") and parsing "this that what" results in ("this", " ", "that", " ", " ", " ", "what").
pchars are like wchars except that they are themselves significant.
pchars do not usually contain space. A more reasonable definition of pchars is ("+", "-"). Then parsing "x+y" results in ("x", "+", "y"). Also, the parsing characters can be character combinations. If pchars $=("+", "-", "++", ~ "--")$, then parsing "x+y++" results in ("x", "+", "y", "++") and parsing "x+++y" results in ("x", "++", "+", "y"). Longer pchars are matched before shorter ones regardless of the order in which they appear in the pchars vector.

## Quote characters

qchars specifies the quote characters. Pieces of the string being parsed that are surrounded by quotes are returned as one token, ignoring the separation that would usually occur because of the wchars and pchars definitions. Consider the string
mystr= "x = y"

Let wchars $=$ " " and pchars include "=". That by itself would result in the above string parsing into the five tokens

| mystr | $=$ | "x | $=$ | $\mathrm{y} "$ |
| :--- | :--- | :--- | :--- | :--- |

Now let qchars $=(6 " " \| ")$; that is, qchars is the two-character string "". Parsing then results in the three tokens

| mystr | $=$ | $\mathrm{x}=\mathrm{y} "$ |
| :--- | :--- | :--- |

Each element of qchars contains a character pair: the open character followed by the close character. We defined those two characters as " and " above, that is, as being the same. The two characters can differ. We might define the first as ' and the second as '. When the characters are different, quotations can nest. The quotation "he said "hello"" makes no sense because that parses into ("he said ", hello, ""). The quotation 'he said 'hello'', however, makes perfect sense and results in the single token 'he said 'hello''.

The quote characters can themselves be multiple characters. You can define open quote as '" and close as "': qchars = ('"‘"")"'). Or you can define multiple sets of quotation characters, such as qchars = (‘""""), ("‘"")"').

The quote characters do not even have to be quotes at all. In some context you might find it convenient to specify them as (" ()"). With that definition, " $(2 \times(3+2))$ " would parse into one token. Specifying them like this can be useful, but in general we recommend against it. It is usually better to write your code so that quote characters really are quote characters and to push the work of handling other kinds of nested expressions back onto the caller.

## Overrides

The token*() functions provide two overrides: allownum and allowhex. These have to do with parsing numbers. First, consider life without overrides. You have set wchars $="$ " and pchars $=("=", "+", "-", " * ", " / ")$. You attempt to parse

$$
y=x+1 e+13
$$

The result is

| $y$ | $=$ | $x$ | + | $1 e$ | + | 13 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |

when what you wanted was

| $y$ | $=$ | $x$ | + | $1 e+13$ |
| :--- | :--- | :--- | :--- | :--- |

Setting allownum $=1$ will achieve the desired result. allownum specifies that, when a token could be interpreted as a number, the number interpretation is to be taken even in violation of the other parsing rules.

Setting allownum $=1$ will not find numbers buried in the middle of strings, such as the $1 \mathrm{e}+3$ in "xis $1 \mathrm{e}+3$ ", but if the number occurs at the beginning of the token according to the parsing rules set by wchars and pchars, allownum $=1$ will continue the token in violation of those rules if that results in a valid number.

The override allowhex is similar and Stata specific. Stata (and Mata) provide a unique and useful way of writing hexadecimal floating-point numbers in a printable, short, and precise way: $\pi$ can be written $1.921 \mathrm{fb} 54442 \mathrm{~d} 18 \mathrm{X}+001$. Setting allowhex $=1$ allows such numbers.

## Setting the environment to parse on blanks with quote binding

Stata's default rule for parsing do-file arguments is "parse on blanks and bind on quotes". The settings for duplicating that behavior are

$$
\begin{aligned}
\text { wchars } & =" " \\
\text { pchars } & =(" ") \\
\text { qchars } & =(‘ " " " "), \quad("(" ") " \prime) \\
\text { allownum } & =0 \\
\text { allowhex } & =0
\end{aligned}
$$

This behavior can be obtained by coding

$$
t=\text { tokeninit(" ", "", ('""""), ("("")"'), 0, 0) }
$$

or by coding

$$
t=\text { tokeninit }()
$$

because in tokeninit () the arguments are optional and "parse on blank with quote binding" is the default.

With those settings, parsing '"first second "third fourth" fifth"' results in ("first", "second", '""third fourth""', "fifth").

This result is a little different from that of Stata because the third token includes the quote binding characters. Assume that the parsed string was obtained by coding

```
res = tokengetall ( }t
```

The following code will remove the open and close quotes, should that be desirable.

```
for (i=1; i<=cols(res); i++) {
    if (substr(res[i], 1, 1)==`"""') {
        res[i] = substr(res[i], 2, strlen(res[i])-2)
    }
        else if (substr(res[i], 1, 2)=="‘" + '"""') {
        res[i] = substr(res[i], 3, strlen(res[i])-4)
    }
}
```


## Setting the environment to parse full Stata syntax

To parse full Stata syntax, the settings are

```
    wchars \(=\) " "
    pchars = ( "\", "~", "!", "=", ":", ";", ",",
        "?", "!", "@", "\#", "==", "!=", ">=",
        "<=", "<", ">", "\&", "|", "\&\&", "||",
        "+", "-", "++", "--", "*", "/", "~",
        " (", ")", "[", "]", "\{", "\}")
    qchars = ('""""), ("'"")"', char(96)+char(39))
allownum \(=1\)
allowhex \(=1\)
```

The above is a slight oversimplification. Stata is an interpretive language and Stata does not require users to type filenames in quotes, although Stata does allow it. Thus "\" is sometimes a parsing character and sometimes not, and the same is true of "/". As Stata parses a line from left to right, it will change pchars between two tokenget () calls when the next token could be or is known to be a filename. Sometimes Stata peeks ahead to decide which way to parse. You can do the same by using the tokenpchars() and tokenpeek() functions.

To obtain the above environment, code

```
t= tokeninitstata()
```


## Setting the environment to parse tab-delimited files

The token*() functions can be used to parse lines from tab-delimited files. A tab-delimited file contains lines of the form

$$
\langle\text { field } 1\rangle\langle\text { tab }\rangle\langle\text { field } 2\rangle\langle\text { tab }\rangle\langle\text { field } 3\rangle
$$

The parsing environment variables are

$$
\begin{aligned}
\text { wchars } & =" " \\
\text { pchars } & =(\operatorname{char}(9)) \quad \text { (i.e., tab) } \\
\text { qchars } & =(" ") \\
\text { allownum } & =0 \\
\text { allowhex } & =0
\end{aligned}
$$

To set this environment, code

$$
t=\text { tokeninit ("", char (9), "", 0, 0) }
$$

Say that you then parse the line
Farber, William $\langle$ tab $\quad 2201.00\langle t a b\rangle 12$
The results will be
("Farber, William", char(9), " 2201.00", char(9), "12")
If the line were
Farber, William $\langle t a b\rangle\langle t a b\rangle 12$
the result would be
("Farber, William", char(9), char (9), "12")
The tab-delimited format is not well defined when the missing fields occur at the end of the line. A line with the last field missing might be recorded

Farber, William $\langle t a b\rangle \quad 2201.00\langle t a b\rangle$
or
Farber, William $\langle t a b\rangle \quad 2201.00$
A line with the last two fields missing might be recorded
Farber, William $\langle t a b\rangle\langle t a b\rangle$
or
Farber, William $\langle$ tab $\rangle$
or
Farber, William

The following program would correctly parse lines with missing fields regardless of how they are recorded:

```
real rowvector readtabbed(transmorphic t, real scalar n)
{
    real scalar i
    string rowvector res
    string scalar token
    res = J(1, n, "")
    i = 1
    while ((token = tokenget(t))!="") {
            if (token==char(9)) i++
            else res[i] = token
    }
    return(res)
}
```


## Function overview

The basic way to proceed is to initialize the parsing environment and store it in a variable,

$$
\mathrm{t}=\text { tokeninit }(\ldots)
$$

and then set the string s to be parsed,

```
tokenset(t, s)
```

and finally use tokenget () to obtain the tokens one at a time (tokenget () returns "" when the end of the line is reached), or obtain all the tokens at once using tokengetall( $t$ ). That is, either

```
while((token = tokenget(t)) != "") {
    ... process token ...
}
```

or

```
tokens = tokengetall(t)
for (i=1; i<=cols(tokens); i++) {
    ... process tokens[i] ...
}
```

After that, set the next string to be parsed,

```
tokenset(t, nextstring)
```

and repeat.

## tokeninit() and tokeninitstata()

tokeninit() and tokeninitstata() are alternatives. tokeninitstata() is generally unnecessary unless you are writing a fairly complicated function.

Whichever function you use, code

$$
t=\text { tokeninit }(\ldots)
$$

or

$$
t=\text { tokeninitstata() }
$$

If you declare $t$, declare it transmorphic. $t$ is in fact a structure containing all the details of your parsing environment, but that is purposely hidden from you so that you cannot accidentally modify the environment.
tokeninit() allows up to five arguments:

```
    t= tokeninit(wchars, pchars, qchars, allownum, allowhex)
```

You may omit arguments from the end. If omitted, the default values of the arguments are

$$
\begin{aligned}
\text { allowhex } & =0 \\
\text { allownum } & =0 \\
\text { qchars } & =(\text { '""""', ("'"")"') } \\
\text { pchars } & =(" ") \\
\text { wchars } & =" "
\end{aligned}
$$

Notes:

1. Concerning wchars:
a. wchars is a string scalar. The whitespace characters appear one after the other in the string. The order in which the characters appear is irrelevant.
b. Specify wchars as " " to treat blank as whitespace.
c. Specify wchars as " "+char(9) to treat blank and tab as whitespace. Including $t a b$ is necessary only when strings to be parsed are obtained from a file; strings obtained from Stata already have the $t a b$ characters removed.
d. Any character can be treated as a whitespace character, including letters.
e. Specify wchars as "" to specify that there are no whitespace characters.
2. Concerning pchars:
a. pchars is a string rowvector. Each element of the vector is a separate parse character. The order in which the parse characters are specified is irrelevant.
b. Specify pchars as ("+", "-") to make + and - parse characters.
c. Parse characters may be character combinations such as ++ or $>=$. Character combinations may be up to four characters long.
d. Specify pchars as "" or $\mathrm{J}(1,0, " \mathrm{l})$ to specify that there are no parse characters. It makes no difference which you specify, but you will realize that $J(1,0, " ")$ is more logically consistent if you think about it.
3. Concerning qchars:
a. qchars is a string rowvector. Each element of the vector contains the open followed by the close characters. The order in which sets of quote characters are specified is irrelevant.
b. Specify qchars as ('"""")) to make " an open and close character.
c. Specify qchars as ('""""), ("'""")") to make "" and '"") quote characters.
d. Individual quote characters can be up to two characters long.
e. Specify qchars as "" or $J(1,0, " ")$ to specify that there are no quote characters.

## tokenset()

After tokeninit() or tokeninitstata(), you are not yet through with initialization. You must tokenset ( $s$ ) to specify the string scalar you wish to parse. You tokenset() one line, parse it, and if you have more lines, you tokenset () again and repeat the process. Often you will need to parse only one line. Perhaps you wish to write a program to parse the argument of a complicated option in a Stata ado-file. The structure is

```
program ...
    syntax ... [, ... MYoption(string) ...]
    mata: parseoption('"'myoption'"')
end
mata:
void parseoption(string scalar option)
{
        transmorphic t
        t = tokeninit(...)
        tokenset(t, option)
}
end
```


## Notes:

1. When you tokenset $(s)$, the contents of $s$ are not stored. Instead, a pointer to $s$ is stored. This approach saves memory and time, but it means that if you change $s$ after setting it, you will change the subsequent behavior of the token*() functions.
2. Simply changing $s$ is not sufficient to restart parsing. If you change $s$, you must tokenset ( $s$ ) again.

## tokengetall()

You have two alternatives in how to process the tokens. You can parse the entire line into a row vector containing all the individual tokens by using tokengetall(),

```
tokens = tokengetall ( }t\mathrm{ )
```

or you can use tokenget () to process the tokens one at a time, which is discussed in the next section.

Using tokengetall (), tokens [1] will be the first token, tokens [2] the second, and so on. There are, in total, cols (tokens) tokens. If the line was empty or contained only whitespace characters, cols(tokens) will be 0 .

## tokenget(), tokenpeek(), and tokenrest()

tokenget () returns the tokens one at a time and returns " " when the end of the line is reached. The basic loop for processing all the tokens in a line is

```
while ( (token = tokenget(t)) != "") {
}
```

tokenpeek () allows you to peek ahead at the next token without actually getting it, so whatever is returned will be returned again by the next call to tokenget (). tokenpeek() is suitable only for obtaining the next token after tokenget (). Calling tokenpeek() twice in a row will not return the next two tokens; it will return the next token twice. To obtain the next two tokens, code

```
current = tokenget (t) // get the current token
t2 = t // copy parse environment
next_1 = tokenget(t2) // peek at next token
next_1 = tokenget(t2) // peek at token after that
current = tokenget (t) // get next token
```

If you declare $t 2$, declare it transmorphic.
tokenrest() returns the unparsed portion of the tokenset() string. Assume that you have just gotten the first token by using tokenget(). tokenrest() would return the rest of the original string, following the first token, unparsed. tokenrest ( $t$ ) returns substr (original_string, tokenoffset ( $t$ ), .).

## tokenoffset()

tokenoffset() is useful only when you are using the tokenget() rather than tokengetall() style of programming. Let the original string you tokenset () be "this is an example". Right after you have tokenset () this string, tokenoffset() is 1 :

```
        this is an example
        |
tokenoffset() = 1
```

After getting the first token (say it is "this"), tokenoffset () is 5:

```
this is an example
tokenoffset() = 5
```

tokenoffset () is always located on the first character following the last character parsed.

The syntax of tokenoffset () is

```
tokenoffset(t)
```

and

```
tokenoffset(t, newoffset)
```

The first returns the current offset value. The second resets the parser's location within the string.

## tokenwchars(), tokenpchars(), and tokenqchars()

tokenwchars(), tokenpchars(), and tokenqchars() allow resetting the current wchars, pchars, and qchars. As with tokenoffset(), they come in two syntaxes.

With one argument, $t$, they return the current value of the setting. With two arguments, $t$ and newvalue, they reset the value.

Resetting in the midst of parsing is an advanced issue. The most useful of these functions is tokenpchars (), since for interactive grammars, it is sometimes necessary to switch on and off a certain parsing character such as /, which in one context means division and in another is a file separator.

## tokenallownum and tokenallowhex()

These two functions allow obtaining the current values of allownum and allowhex and resetting them.

## Conformability

```
tokeninit(wchars, pchars, qchars, allownum, allowhex):
    wchars: 1 1 1 (optional)
        pchars: }1\times\mp@subsup{c}{p}{}\quad\mathrm{ (optional)
        qchars: 1 1 < cq
        allownum: 1 < 1 (optional)
        allowhex: 1 1 1 (optional)
        result: transmorphic
tokeninitstata():
    result: transmorphic
tokenset(t, s):
    t: transmorphic
    s: }\quad1\times
    result: void
tokengetall(t):
    t: transmorphic
        result: }1\times
tokenget(t), tokenpeek(t), tokenrest (t):
            t: transmorphic
        result: }1\times
```

```
tokenoffset(t), tokenwchars(t), tokenallownum(t), tokenallowhex(t):
    t: transmorphic
    result: }1\times
tokenoffset(t, newvalue), tokenwchars(t, newvalue),
    tokenallownum(t, newvalue), tokenallowhex(t, newvalue):
                    t: transmorphic
        newvalue: }1\times
            result: void
tokenpchars(t), tokenqchars(t):
            t: transmorphic
        result: }1\times
tokenpchars(t, newvalue), tokenqchars(t, newvalue):
            t: transmorphic
        newvalue: }1\times
        result: void
```


## Diagnostics

None.

## Also see

[M-5] invtokens() - Concatenate string rowvector into string scalar
[M-5] tokens() - Obtain tokens from string
[M-5] ustrword() - Obtain Unicode word from Unicode string
[M-4] Programming - Programming functions
[M-4] String - String manipulation functions
[ P$]$ gettoken - Low-level parsing
[P] tokenize - Divide strings into tokens

## Title

[M-5] tokens() — Obtain tokens from string

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

tokens ( $s$ ) returns the contents of $s$, split into words.
tokens ( $s$, parsechars) returns the contents of $s$ split into tokens based on parsechars.
tokens ( $s$ ) is equivalent to tokens ( $s, "$ ").
If you need more advanced parsing, see [M-5] tokenget().

## Syntax

```
string rowvector tokens(string scalar s)
string rowvector tokens(string scalar s, string scalar parsechars)
```


## Remarks and examples

tokens () is commonly used to split a string containing a sequence of variable names into a row vector, each element of which contains one variable name:

```
tokens("mpg weight displacement") = ("mpg", "weight", "displacement")
```

Some Stata interface functions require that variable names be specified in this form. This is required, for instance, by st_varindex () ; see [M-5] st_varindex( ). If you had a string scalar vars containing one or more variable names, you could obtain their variable indices by coding

```
indices = st_varindex(tokens(vars))
```


## Conformability

tokens (s, parsechars)

| $s:$ | $1 \times 1$ |  |
| ---: | :--- | :--- |
| parsechars: | $1 \times 1 \quad$ (optional) |  |
| result: | $1 \times w, \quad w=$ number of words (tokens) in $s$ |  |

## Diagnostics

If $s$ contains " ", tokens() returns $\mathrm{J}(1,0, "$ ").
If $s$ contains double-quoted or compound-double-quoted material, the quotes are stripped and that material is returned as one token. For example,

```
tokens('"this "is an" example"') = ("this", "is an", "example")
```

If $s$ contains quoted material and the quotes do not match, results are as if the appropriate number of close quotes were added to the end of $s$. For example,

```
tokens('"this "is an example"') = ("this", "is an example")
```


## Also see

[M-5] invtokens() - Concatenate string rowvector into string scalar
[M-5] tokenget() — Advanced parsing
[M-5] ustrword() - Obtain Unicode word from Unicode string
[M-4] String - String manipulation functions
[P] gettoken - Low-level parsing
[P] tokenize - Divide strings into tokens

## Title

[M-5] trace() - Trace of square matrix

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

trace ( $A$ ) returns the sum of the diagonal elements of $A$. Returned result is real if $A$ is real, complex if $A$ is complex.
trace $(A, B)$ returns trace $(A B)$, the calculation being made without calculating or storing the off-diagonal elements of $A B$. Returned result is real if $A$ and $B$ are real and is complex otherwise.
$\operatorname{trace}(A, B, t)$ returns trace $(A B)$ if $t=0$ and returns trace $\left(A^{\prime} B\right)$ otherwise, where, if either $A$ or $B$ is complex, transpose is understood to mean conjugate transpose. Returned result is real if $A$ and $B$ are real and is complex otherwise.

## Syntax

```
numeric scalar trace(numeric matrix A)
numeric scalar trace(numeric matrix A, numeric matrix B)
numeric scalar trace(numeric matrix A, numeric matrix B, real scalar t)
```


## Remarks and examples

$\operatorname{trace}(A, B)$ returns the same result as $\operatorname{trace}(A * B)$ but is more efficient if you do not otherwise need to calculate $A * B$.
$\operatorname{trace}(A, B, 1)$ returns the same result as trace $\left(A^{\prime} B\right)$ but is more efficient.
For real matrices $A$ and $B$,

$$
\begin{aligned}
\operatorname{trace}\left(A^{\prime}\right) & =\operatorname{trace}(A) \\
\operatorname{trace}(A B) & =\operatorname{trace}(B A)
\end{aligned}
$$

and for complex matrices,

$$
\begin{aligned}
\operatorname{trace}\left(A^{\prime}\right) & =\operatorname{conj}(\operatorname{trace}(A)) \\
\operatorname{trace}(A B) & =\operatorname{trace}(B A)
\end{aligned}
$$

where, for complex matrices, transpose is understood to mean conjugate transpose.

Thus for real matrices,

| To calculate | Code |
| :--- | :--- |
| $\operatorname{trace}(A B)$ | $\operatorname{trace}(A, B)$ |
| $\operatorname{trace}\left(A^{\prime} B\right)$ | $\operatorname{trace}(A, B, 1)$ |
| $\operatorname{trace}\left(A B^{\prime}\right)$ | $\operatorname{trace}(A, B, 1)$ |
| $\operatorname{trace}\left(A^{\prime} B^{\prime}\right)$ | $\operatorname{trace}(A, B)$ |

and for complex matrices,

| To calculate | Code |
| :--- | :--- |
| $\operatorname{trace}(A B)$ | $\operatorname{trace}(A, B)$ |
| $\operatorname{trace}\left(A^{\prime} B\right)$ | $\operatorname{trace}(A, B, 1)$ |
| $\operatorname{trace}\left(A B^{\prime}\right)$ | $\operatorname{conj}(\operatorname{trace}(A, B, 1))$ |
| $\operatorname{trace}\left(A^{\prime} B^{\prime}\right)$ | $\operatorname{conj}(\operatorname{trace}(A, B))$ |

Transpose in the first column means conjugate transpose.

## Conformability

trace ( $A$ ):
A: $\quad n \times n$
result: $\quad 1 \times 1$
trace $(A, B)$ :

| $A:$ | $n \times m$ |
| ---: | :--- |
| $B:$ | $m \times n$ |
| result: | $1 \times 1$ |

trace $(A, B, t)$

| $A:$ | $n \times m$ if $t=0, m \times n$ otherwise |
| ---: | :--- |
| $B:$ | $m \times n$ |
| $t:$ | $1 \times 1$ |
| result $:$ | $1 \times 1$ |

## Diagnostics

trace ( $A$ ) aborts with error if $A$ is not square.
$\operatorname{trace}(A, B)$ and $\operatorname{trace}(A, B, t)$ abort with error if the matrices are not conformable or their product is not square.

The trace of a $0 \times 0$ matrix is 0 .

## Also see

[M-4] Matrix - Matrix functions

## Title

[M-5] _transpose() — Transposition in place

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

_transpose ( $A$ ) replaces $A$ with $A^{\prime}$. Coding _transpose ( $A$ ) is equivalent to coding $A=A^{\prime}$, except that execution can take a little longer and less memory is used. When $A$ is complex, $A$ is replaced with its conjugate transpose; see [M-5] transposeonly() if transposition without conjugation is desired.

## Syntax

void _transpose (numeric matrix $A$ )

## Remarks and examples

In some calculation, you need $A^{\prime}$
$X=\ldots$ calculation using $A^{\prime} \ldots$
If $A$ is large, you can save considerable memory by coding
_transpose (A)
$X=\ldots$ calculation $u$ sing $A \ldots$
_transpose ( $A$ )

## Conformability

_transpose ( $A$ ):
input:

```
                A: }\quadr\times
```

output:

$$
A: \quad c \times r
$$

## Diagnostics

_transpose ( $A$ ) aborts with error if $A$ is a view.

[^16]
## Title

[M-5] transposeonly() - Transposition without conjugation

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

transposeonly $(A)$ returns $A$ with its rows and columns interchanged. When $A$ is real, the actions of transposeonly ( $A$ ) are indistinguishable from coding $A^{\prime}$; see [M-2] op_transpose. The returned result is the same, and the execution time is the same, too. When $A$ is complex, however, transposeonly $(A)$ is equivalent to coding conj $\left(A^{\prime}\right)$, but transposeonly () obtains the result more quickly.
_transposeonly ( $A$ ) interchanges the rows and columns of $A$ in place-without use of additional memory-and returns the transposed (but not conjugated) result in $A$.

## Syntax

numeric matrix transposeonly (numeric matrix $A$ )
void $\quad$ _transposeonly (numeric matrix $A$ )

## Remarks and examples

transposeonly() is useful when you are coding in the programming, rather than the mathematical, sense. Say that you have two row vectors, a and b, and you want to place the two vectors together in a matrix $R$, and you want to turn them into column vectors. If $a$ and $b$ were certain to be real, you could just code

$$
R=\left(a^{\prime}, b^{\prime}\right)
$$

The above line, however, would result in not just the organization but also the values recorded in $R$ changing if a or b were complex. The solution is to code

$$
R=\text { (transposeonly(a), transposeonly(b)) }
$$

The above line will work for real or complex a and b. If you were concerned about memory consumption, you could instead code

```
R = (a \ b)
_transposeonly(R)
```


## Conformability

transposeonly ( $A$ ):
A: $\quad r \times c$
result: $\quad c \times r$
_transposeonly (A):
input:
A: $\quad r \times c$
output:
A: $\quad c \times r$

## Diagnostics

_transposeonly ( $A$ ) aborts with error if $A$ is a view.

## Also see

[M-2] op_transpose - Conjugate transpose operator
[M-5] _transpose( ) - Transposition in place
[M-4] Manipulation - Matrix manipulation

## Title

[M-5] trunc() — Round to integer

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

These functions convert noninteger values to integers by moving toward 0 , moving down, moving up, or rounding. These functions are typically used with scalar arguments, and they return a scalar in that case. When used with vectors or matrices, the operation is performed element by element.
trunc $(R)$ returns the integer part of $R$.
floor $(R)$ returns the largest integer $i$ such that $i \leq R$.
ceil $(R)$ returns the smallest integer $i$ such that $i \geq R$.
round ( $R$ ) returns the integer closest to $R$.
round $(R, U)$ returns the values of $R$ rounded in units of $U$ and is equivalent to round ( $(R: / U)): * U$. For instance, round ( $R, 2$ ) returns $R$ rounded to the closest even number. round ( $R, .5$ ) returns $R$ rounded to the closest multiple of one half. round ( $R, 1$ ) returns $R$ rounded to the closest integer and so is equivalent to round ( $R$ ).

## Syntax

```
real matrix trunc(real matrix R)
real matrix floor(real matrix R)
real matrix ceil(real matrix R)
real matrix round(real matrix R)
real matrix round(real matrix R, real matrix }U\mathrm{ )
```


## Remarks and examples

Remarks are presented under the following headings:
Relationship to Stata's functions
Examples of rounding

## Relationship to Stata's functions

trunc() is equivalent to Stata's int () function.
floor(), ceil(), and round() are equivalent to Stata's functions of the same name.

## Examples of rounding

| $x$ | $\operatorname{trunc}(x)$ | floor $(x)$ | ceil $(x)$ | round $(x)$ |
| :--- | :---: | :---: | :---: | :---: |
| 1 | 1 | 1 | 1 | 1 |
| 1.3 | 1 | 1 | 2 | 1 |
| 1.6 | 1 | 1 | 2 | 2 |
| -1 | -1 | -1 | -1 | -1 |
| -1.3 | -1 | -2 | -1 | -1 |
| -1.6 | -1 | -2 | -1 | -2 |

## Conformability

trunc $(R)$, floor $(R)$, ceil $(R)$ :

| $R:$ | $r \times c$ |
| ---: | :--- |
| result: | $r \times c$ |

round ( $R$ ):

| $R:$ |  | $r \times c$ |
| ---: | :--- | ---: |
| result $:$ |  | $r \times c$ |

round ( $R, U$ ):
$R: \quad r_{1} \times c_{1}$
$U: \quad r_{2} \times c_{2}, \quad R$ and $U$ r-conformable
result: $\quad \max \left(r_{1}, r_{2}\right) \times \max \left(c_{1}, c_{2}\right)$

## Diagnostics

Most Stata and Mata functions return missing when arguments contain missing, and in particular, return . whether the argument is ., .a, .b, ..., .z. The logic is that performing the operation on a missing value always results in the same missing-value result. For example, sqrt (.a)==.

These functions, however, when passed a missing value, return the particular missing value. Thus $\operatorname{trunc}(. a)==. a$, floor $(. b)==. b, \operatorname{ceil}(. c)==. c$, and round (.d) $==. d$.

For round() with two arguments, this applies to the first argument and only when the second argument is not missing. If the second argument is missing (whether ., .a, ..., or . $z$ ), then . is returned.

For round () with one or two arguments, values of $R$ exactly at midpoints (where it may not be clear whether to round up or down) are always rounded up to the larger value. For example, round (4.5) is 5 and round $(-4.5)$ is -4 . Note that rounding a number is based on the floating-point number representation of the number instead of the number itself. So round () is sensitive to representation errors and precision limits. For example, 0.15 has no exact floating-point number representation. Therefore, round $(0.15,0.1)$ is 0.1 instead of 0.2 .

## Also see

[M-4] Scalar - Scalar mathematical functions

## Title

[M-5] uniqrows() — Obtain sorted, unique values

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

uniqrows $(P)$ returns a sorted matrix containing the unique rows of $P$.
uniqrows ( $P$, freq) does the same but lets you specify whether the frequencies with which each combination occurs should be calculated. Using uniqrows $(P, 0)$ is the same as using uniqrows $(P)$. uniqrows $(P, 1)$ specifies that the frequencies with which each combination occurs should be calculated.

## Syntax

> transmorphic matrix uniqrows (transmorphic matrix $P$ )
> transmorphic matrix uniqrows (transmorphic matrix $P$, freq)
where

$$
\begin{gathered}
\text { freq }=0 \text { (frequencies are not calculated) or } \\
1 \text { (frequencies are calculated) }
\end{gathered}
$$

## Remarks and examples

```
: x
            1 2 3
\begin{tabular}{|lll|}
\hline 4 & 5 & 7 \\
4 & 5 & 6 \\
1 & 2 & 3 \\
4 & 5 & 6 \\
\hline
\end{tabular}
: uniqrows(x)
    1 2 3
    1 1 2 3
    2 
    3 
: uniqrows(x, 1)
            1 2 3
\begin{tabular}{l|llll}
1 & 1 & 2 & 3 & 1 \\
2 & 4 & 5 & 6 & 2 \\
3 & 4 & 5 & 7 & 1
\end{tabular}
```


## Conformability

uniqrows $(P, 0)$ :
$P: \quad r_{1} \times c_{1}$
result: $\quad r_{2} \times c_{1}, \quad r_{2} \leq r_{1}$
uniqrows $(P, 1)$ :

$$
\begin{aligned}
P: & r_{1} \times c_{1} \\
\text { result: } & r_{2} \times c_{1}+1, \quad r_{2} \leq r_{1}
\end{aligned}
$$

## Diagnostics

In uniqrows $(P)$, if rows $(P)==0, \mathrm{~J}(0, \operatorname{cols}(P)$, missingof $(P))$ is returned.
If rows $(P)>0$ and $\operatorname{cols}(P)==0, \mathrm{~J}(1,0$, missingof $(P))$ is returned.

## Also see

[M-5] sort() - Reorder rows of matrix
[M-4] Manipulation - Matrix manipulation

## Title

[M-5] unitcircle( ) - Complex vector containing unit circle

Description Syntax Conformability Diagnostics Also see

## Description

unitcircle ( $n$ ) returns a column vector containing $C(\cos (\theta), \sin (\theta)$ ) for $0 \leq \theta \leq 2 \pi$ in $n$ points.

## Syntax

complex colvector unitcircle(real scalar $n$ )

## Conformability

```
unitcircle(n):
    n: }\quad1\times
    result: }\quadn\times
```


## Diagnostics

None.

## Also see

[M-4] Standard - Functions to create standard matrices

## Title

[M-5] unlink() — Erase file

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

unlink(filename) erases filename if it exists, does nothing if filename does not exist, and aborts with error if filename exists but cannot be erased.
_unlink(filename) does the same, except that, if filename cannot be erased, rather than aborting with error, _unlink() returns a negative error code. _unlink() returns 0 if filename was erased or filename did not exist.

## Syntax

| void | unlink (string scalar filename) |
| :--- | :--- |
| real scalar | _unlink $($ string scalar filename $)$ |

## Remarks and examples

To remove directories, see rmdir() in [M-5] chdir().

## Conformability

```
unlink(filename)
    filename: }\quad1\times
        result: void
_unlink(filename)
    filename: }1\times
        result: }1\times
```


## Diagnostics

unlink(filename) aborts with error when _unlink() would give a negative result.
_unlink(filename) returns a negative result if the file cannot be erased and returns 0 otherwise. If the file did not exist, 0 is returned. When there is an error, most commonly returned are -3602 (filename invalid) or -3621 (file is read-only).

## Also see

[M-4] IO - I/O functions

## Title

[M-5] urlencode( ) - Convert URL into percent-encoded ASCII format
Description Syntax Conformability Diagnostics Also see

## Description

urlencode ( $s[$, useplus $]$ ) converts a string into percent-encoded ASCII format for web transmission. It replaces reserved ASCII characters with a \% followed by two hexadecimal digits. The function replaces a space with $\% 20$ if useplus is not specified or is 0 ; it replaces a space with a + if useplus is 1 .
urldecode ( $s$ ) decodes the string obtained from urlencode() and returns the original string.

## Syntax

string scalar urlencode (string scalar $s$ [, real scalar useplus])
string scalar urldecode (string scalar $s$ )

## Conformability

```
urlencode ( \(s[\), useplus \(]\) ), urldecode \((s)\) :
            \(s: \quad 1 \times 1\)
    useplus: \(\quad 1 \times 1\)
    result: \(\quad 1 \times 1\)
```


## Diagnostics

None.

## Also see

[M-4] String - String manipulation functions

## Title

[M-5] ustrcompare() - Compare or sort Unicode strings

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

ustrcompare ( $s 1, s 2[, l o c]$ ) compares two Unicode strings. The function returns -1 , 1 , or 0 if $s 1$ is less than, greater than, or equal to $s 2$, respectively. If loc is not specified, the locale_functions setting is used.
ustrsortkey ( $s$, loc $[, l o c]$ ) generates a null-terminated byte array. The sort command on the sort keys of two Unicode strings $s 1$ and $s 2$ produces the same order from ustrcompare ( $s 1, s 2, l o c$ ). If loc is not specified, the locale_functions setting is used. The result is also diacritic and case sensitive. If you need different behavior, you should use the extended function ustrsortkeyex ().
ustrcompareex (sl, s2, loc, st, case, cslv, norm, num, alt, fr) is an extended version of ustrcompare(). It provides more options for the comparison behaviors.
ustrsortkeyex (s, loc, st, case, cslv, norm, num, alt, fr) is an extended version of ustrsortkey (). It provides more options for the comparison behaviors.

The additional options are as follows:
st controls the strength of the comparison:
-1 default value for the locale
1 primary; base-letter differences, such as "a" and "b"
2 secondary; diacritical differences of the same base letter, such as "a" and "ä"
3 tertiary; case differences of the same base letter, such as "a" and "A"
4 quaternary; used to distinguish between Katakana and Hiragana for JIS 4061 collation standard
5 identical; code-point order of the string; rarely useful
Numbers other than those listed above are treated as tertiary.
case controls the uppercase and lowercase letter order. Possible values are 1 (uppercase first), 2 (lowercase first), or 0 (use tertiary strength; advanced option). -1 means the default value for the locale should be used. Any other values are treated as 0 .
$c s l v$ controls whether an extra case level between the secondary level and the tertiary level is generated.
Possible values are 0 (off) or 1 (on). -1 means the default value for the locale should be used. Any other values are treated as 0 . Combining this setting to be "on" and the strength setting to be primary can achieve the effect of ignoring the diacritical differences but preserving the case differences. If the setting is "on", the result is also affected by the case setting.
norm controls whether the normalization check and normalizations are performed. Possible values are 0 (off) or 1 (on). -1 means the default value for the locale should be used. Any other values are treated as 0 . Most languages do not require normalization for comparison. Normalization is needed in languages that use multiple combining characters, such as Arabic, ancient Greek, or Hebrew. For more information about normalization, see [M-5] ustrnormalize( ),
num controls how contiguous digit substrings are sorted. Possible values are 0 (off) or 1 (on). -1 means the default value for the locale should be used. Any other values are treated as 0 . If the setting is "on", substrings consisting of digits are sorted based on the numeric value. For example, " 100 " is after " 20 " instead of before it. Note that digit substrings are limited to 254 digits and that plus or minus signs, decimals, and exponents are not supported.
alt controls how spaces and punctuation characters are handled. Possible values are 0 (use primary weights) or 1 (alternative handling). Any other values are treated as 0 . If the setting is 1 (alternative handling), "onsite", "on-site", and "on site" are considered the same.
$f r$ controls the direction of secondary strength. Possible values are 0 (off) or 1 (on). -1 means the default value for the locale should be used. All other values are treated as "off". If the setting is "on", the diacritical letters are sorted backward. Note that the setting is "on" by default only for Canadian French locale (fr_CA).

When $s 1$ and $s 2$ are not scalar, these functions return element-by-element results.

## Syntax

real matrix ustrcompare(string matrix s1, string matrix $s 2[$, string scalar loc $]$ )
string matrix ustrsortkey(string matrix $s$ [, string scalar loc $]$ )
real matrix ustrcompareex (string matrix s1, s2, string scalar loc, real scalar st, case, cslv, norm, num, alt, fr)
string matrix ustrsortkeyex (string matrix $s$, string scalar loc, real scalar st, case, cslv, norm, num, alt, fr)

## Remarks and examples

Unicode string comparison is locale dependent. For example, z $<0 ̈$ in Swedish but $0 \quad<\mathrm{z}$ in German. The comparison is diacritic and case sensitive. If you need different behavior, such as case-insensitive comparison, you should use the extended comparison function ustrcompareex (). Unicode string comparison is language sensitive, which is different from the byte value comparison used by sort. For example, capital letter "Z" (byte value 90) comes before lowercase "a" (byte value 97) in terms of byte value but comes after "a" in any English dictionary.

An invalid UTF-8 sequence is replaced with the Unicode replacement character \ufffd.

## Conformability

```
ustrcompare(s1, s2 [,loc]):
    sl: }\quadr\times
        s2: }\quadr\times
        loc: 
    result: }\quadr\times
ustrsortkey(s [,loc]):
            s: r
        loc: 
    result: }\quadr\times
ustrcompareex(s1, s2, loc, st, case, cslv, norm, num, alt, fr):
        sl: r
        s2: }\quadr\times
        loc: 
            st: 
            case: }1\times
            cslv: }1\times
            norm: }\quad1\times
            num: }1\times
            alt: 
            fr: 
    result: }\quadr\times
ustrsortkeyex(s,loc, st, case, cslv, norm, num, alt, fr):
            s: r}\times
        loc: 
            st: 
            case: }\quad1\times
            cslv: }1\times
            norm: }\quad1\times
            num: }1\times
            alt: 
            fr: 
    result: }\quadr\times
```


## Diagnostics

ustrcompare() and ustrcompareex () return a negative number other than -1 if an error occurs. ustrsortkey() and ustrsortkeyex() return an empty string if an error occurs.

## Also see

[M-5] sort() - Reorder rows of matrix
[M-4] String - String manipulation functions
[U] 12.4.2 Handling Unicode strings

## Title

[M-5] ustrfix() - Replace invalid UTF-8 sequences in Unicode string

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

ustrfix (s [, rep]) replaces each invalid UTF-8 sequence with a Unicode character. If rep is specified and it starts with a Unicode character, the Unicode character is used. Otherwise, the Unicode replacement character \ufffd is used.

When arguments are not scalar, the function returns element-by-element results.

## Syntax

string matrix ustrfix (string matrix $s[$, string scalar rep $]$ )

## Remarks and examples

An invalid UTF-8 sequence may contain one byte or multiple bytes.

## Conformability

ustrfix (matrix s $[, r]$ )
$s: \quad r \times c$
rep: $\quad 1 \times 1$
result: $\quad r \times c$

## Diagnostics

ustrfix (matrix $s[, r]$ ) returns an empty string if an error occurs.

## Also see

[M-5] ustrto() - Convert a Unicode string to or from a string in a specified encoding
[M-5] ustrunescape() - Convert escaped hex sequences to Unicode strings
[M-4] String - String manipulation functions
[U] 12.4.2 Handling Unicode strings

## Title

[M-5] ustrnormalize( ) - Normalize Unicode string

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

ustrnormalize ( $s$, norm) normalizes Unicode string $s$ to one of the five normalization forms specified by norm.

When $s$ is not a scalar, the function returns element-by-element results.

## Syntax

string matrix ustrnormalize(string matrix s, string matrix norm)

## Remarks and examples

Unicode normalization removes the Unicode string differences caused by Unicode character equivalence. For example, the character " $i$ " with two dots as in naïve can be represented either by a single Unicode code point, $\backslash u 00$ ef, or by two code points, $\backslash u 0069$, which is the regular "i", and $\backslash u 0308$, which is the diaeresis character. The code point $\backslash u 00 e f$ and the code-point sequence $\backslash u 0069 \backslash u 0308$ are considered Unicode equivalent. According to the Unicode standard, they should be treated as the same single character in Unicode string operations, such as display, comparison, and selection. But Stata does not support multiple code-point characters; each code point is considered a single Unicode character. Hence, $\backslash u 0069 \backslash u 0308$ is displayed as two characters in the Results window. ustrnormalize () can be used to deal with this issue by normalizing \u0069\u0308 to its canonical equivalent composited \NFC form \u00ef.
norm must be one of $\mathrm{nfc}, \mathrm{nfd}, \mathrm{nfkc}, \mathrm{nfkd}$, or nfkcc . The function returns an empty string for any other value of norm.
nfc specifies Normalization Form C, which normalizes decomposed Unicode code points to canonical composited form. nfd specifies Normalization Form D, which normalizes composited Unicode code points to canonical decomposed form. nfc and nfd produce canonical equivalent form. nfkc and nfkd are similar to nfc and nfd but produce compatibility equivalent form. nfkcc is similar to nfkc but also handles case folding. For details, see http://unicode.org/reports/tr15/.

## Conformability

ustrnormalize(s, norm):

```
            s: r
norm: }\quadr\timesc\mathrm{ or 1 }\times
result: }\quadr\times
```


## Diagnostics

None.

## Also see

[M-4] String - String manipulation functions
[U] 12.4.2 Handling Unicode strings

## Title

[M-5] ustrsplit( ) — Split string into parts based on a Unicode regular expression

```
Description Syntax Remarks and examples Conformability
Also see
```


## Description

ustrsplit ( $s$, ustrregexp) returns the contents of $s$ split into parts based on ustrregexp.

## Syntax

string rowvector ustrsplit(string scalar s, string scalar ustrregexp)

## Remarks and examples

ustrsplit() splits a string into parts according to a Unicode regular expression.
For example,

```
ustrsplit("$12.31 €6.75", "[$€]") = ("", "12.31 ", "6.75")
```

Note that the first element of the result is an empty string. This is because ustrsplit() encountered the first " $\$$ " in the string being split and an empty string (that is, nothing) was to the left of that "\$". Assuming you put the above result into a string scalar named "result", you can type the following to remove that leading empty string and any other all-whitespace parts:

```
select(result, ustrlen(ustrtrim(result)):!=0)
```

The example above splits on any dollar sign or Euro symbol. To split on any Unicode character that is considered a currency symbol, we can use the regular expression [ $\backslash \mathrm{p}\{\mathrm{Sc}\}$ ],

```
ustrsplit("$12.31 €6.75 ¥100.50 #25.45", "[\p{Sc}]")
```

which returns

```
("", "12.31 ", "6.75 ","100.50 ","25.45")
```


## Conformability

```
ustrsplit(s, ustrregexp):
    s: }\quad1\times
    result: }\quad1\times1,w=\mathrm{ number of parts in s
```


## Also see

[M-5] ustrword() - Obtain Unicode word from Unicode string
[M-4] String - String manipulation functions
[P] tokenize - Divide strings into tokens
[U] 12.4.2 Handling Unicode strings

## Title

[M-5] ustrto( ) — Convert a Unicode string to or from a string in a specified encoding

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

ustrto ( $s$, enc, mode) converts the Unicode string $s$ to a string encoded in enc. Any invalid UTF-8 sequence in $s$ is replaced with a Unicode replacement character \ufffd. mode controls how unsupported Unicode characters in the encoding enc are handled. The possible values for mode are 1, which substitutes any unsupported characters with the enc's substitution string; 2, which skips any unsupported characters; 3, which stops at the first unsupported character and returns an empty string; or 4, which replaces any unsupported character with an escaped hex digit sequence \uhhhh or \Uhhhhhhhh. The hex digit sequence contains either four or eight hex digits depending on the Unicode character's code-point value. Any other values are treated as 1.
ustrfrom ( $s$, enc, mode) converts a string $s$ in encoding enc to a UTF-8 encoded Unicode string. mode controls how invalid byte sequences in $s$ are handled. The possible values for mode are 1, which substitutes an invalid byte sequence with a Unicode replacement character \ufffd; 2, which skips any invalid byte sequences; 3, which stops at the first invalid byte sequence and returns an empty string; or 4 , which replaces any byte in an invalid sequence with an escaped hex digit sequence $\% \mathrm{Xhh}$. Any other values are treated as 1 .

When arguments are not scalar, ustrto() returns element-by-element results.

## Syntax

string matrix ustrto (string matrix $s$, string scalar enc, real scalar mode)
string matrix ustrfrom(string matrix $s$, string scalar enc, real scalar mode)

## Remarks and examples

Type unicode encoding list to list available encodings. See [U] 12.4.2.3 Encodings and see the unicode encoding command in [D] unicode for details.

The substitution character for both ASCII and Latin-1 encoding is char (26)
A good use of mode $=4$ (escape) is to check what invalid bytes a Unicode string ust contains by examining the result of ustrfrom(ust, "utf-8", 4).

## Conformability

```
ustrto(s, enc, mode), ustrfrom(s, enc, mode):
            s: r}\times
    enc: }\quad1\times
    mode: }1\times
    result: }\quadr\times
```


## Diagnostics

ustrto(s, enc, mode) and ustrfrom(s, enc, mode) return an empty string if an error occurs.

## Also see

[M-5] ustrfix( ) - Replace invalid UTF-8 sequences in Unicode string
[M-5] ustrunescape( ) - Convert escaped hex sequences to Unicode strings
[M-4] String - String manipulation functions
[U] 12.4.2 Handling Unicode strings
[U] 12.4.2.3 Encodings

## Title

[M-5] ustrunescape() - Convert escaped hex sequences to Unicode strings

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Diagnostics | Also see |  |

## Description

ustrunescape ( $s$ ) returns a string of Unicode characters corresponding to the escaped sequences in $s$.
ustrtohex $(s[, n])$ returns a string of escaped hex digit Unicode characters, up to 200, specified in $s$. If $n$ is specified and is larger than one, the result starts at the $n$th Unicode character of $s$; otherwise, it starts at the first Unicode character.

When $s$ is not a scalar, these functions return element-by-element results.

## Syntax

> string matrix ustrunescape (string matrix s)
> string matrix ustrtohex(string matrix $s$ [, real scalar $n]$ )

## Remarks and examples

The following escape sequences are recognized by ustrunescape():

| 4 hex digit form | \uhhhh |
| :--- | :--- |
| 8 hex digit form | \Uhhhhhhhh |
| $1-2$ hex digit form | \xhh |
| $1-3$ octal digit form | \ooo |

where $h$ is in [0-9A-Fa-f] and $o$ is in [0-7]. The standard ANSI $C$ escape sequences $\backslash \mathrm{a}, \backslash \mathrm{b}, \backslash \mathrm{t}$, $\backslash \mathrm{n}, \backslash \mathrm{v}, \backslash \mathrm{f}, \backslash \mathrm{r}, \backslash \mathrm{e}, \backslash ", \backslash ', \backslash ?$, and $\backslash \backslash$ are recognized as well. The function returns an empty string if an escape sequence is badly formed. Note that the 8 hex digit form \Uhhhhhhhh starts with a capital "U".
ustrtohex () converts each Unicode character in string $s$ to an escaped hex string. Unicode code points in the range $\backslash u 0000-\backslash u f f f f$ are converted to 4 hex digit form \uhhhh. Unicode code points greater than \uffff are converted to 8 hex digit form \Uhhhhhhhh.
ustrtohex () converts an invalid UTF-8 sequence to the Unicode replacement character \ufffd.
The null terminator char (0) is a valid Unicode character and its escaped form is $\backslash u 0000$.

## Conformability

ustrunescape ( $s$ ), ustrtohex ( $s[, n]$ ):

| $s:$ | $r \times c$ |
| ---: | :--- |
| $n:$ |  |
| result $:$ |  |
|  | $r \times c$ |

## Diagnostics

None.

## Also see

[M-5] ustrfix ( ) - Replace invalid UTF-8 sequences in Unicode string
[M-5] ustrto( ) - Convert a Unicode string to or from a string in a specified encoding [M-4] String - String manipulation functions
[U] 12.4.2 Handling Unicode strings

## Title

[M-5] ustrword() — Obtain Unicode word from Unicode string

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

ustrword ( $s, n$ ) returns the $n$th Unicode word in the Unicode string $s$. Positive numbers count Unicode words from the beginning of $s$, and negative numbers count Unicode words from the end of $s .1$ is the first word in $s$, and -1 is the last Unicode word in $s$. The function uses the locale_functions setting.
ustrword ( $s, n, l o c$ ) returns the $n$th Unicode word in the Unicode string $s$. Positive numbers count Unicode words from the beginning of $s$, and negative numbers count Unicode words from the end of $s .1$ is the first word in $s$, and -1 is the last Unicode word in $s$. The function uses the locale specified in loc.
ustrwordcount ( $s$ ) returns the number of nonempty Unicode words in the Unicode string $s$. An empty Unicode word is a Unicode word consisting of only Unicode whitespace characters. The function uses the locale_functions setting.
ustrwordcount ( $s, l o c$ ) returns the number of nonempty Unicode words in the Unicode string $s$. An empty Unicode word is a Unicode word consisting of only Unicode whitespace characters. The function uses the locale specified in loc.

When $s$ and $n$ are not scalar, these functions return element-by-element results.

## Syntax

```
string matrix ustrword(string matrix s, real matrix n)
string matrix ustrword(string matrix s, real matrix n, string scalar loc)
real matrix ustrwordcount(string matrix s)
real matrix ustrwordcount(string matrix s, string scalar loc)
```


## Remarks and examples

A Unicode word is different from a word produced by the function word(). The word in word() is a space-separated token. A Unicode word is a language unit based on either a set of word boundary rules or dictionaries for some language such as Chinese, Japanese, and Thai.

An invalid UTF-8 sequence is replaced with a Unicode replacement character \ufffd.
The null terminator char (0) is a nonempty Unicode word.

## Conformability

$\operatorname{ustrword}(s, n)$, ustrword $(s, n, l o c)$ :

| $s:$ | $r \times c$ |
| ---: | :--- |
| $n:$ | $r \times c$ or $1 \times 1$ |
| loc: | $1 \times 1$ |
| result: | $r \times c$ |

ustrwordcount ( $s$ ), ustrwordcount ( $s$, loc):
$s: \quad r \times c$
loc: $\quad 1 \times 1$
result: $\quad r \times c$

## Diagnostics

ustrword () returns an empty string if an error occurs. ustrwordcount () returns a negative number if an error occurs.

## Also see

[M-5] invtokens( ) - Concatenate string rowvector into string scalar
[M-5] tokenget() - Advanced parsing
[M-5] tokens() - Obtain tokens from string
[M-5] ustrsplit() - Split string into parts based on a Unicode regular expression
[M-4] String - String manipulation functions
[FN] String functions
[U] 12.4.2 Handling Unicode strings

## Title

[M-5] valofexternal() — Obtain value of external global

| Description | Syntax | Remarks and examples | Conformability |
| :--- | :--- | :--- | :--- |
| Diagnostics | Also see |  |  |

## Description

valofexternal (name) returns the contents of the external global matrix, vector, or scalar whose name is specified by name; it returns $\mathrm{J}(0,0,$.$) if the external global is not found.$

Also see Linking to external globals in [M-2] Declarations.

## Syntax

transmorphic matrix valofexternal(string scalar name)

## Remarks and examples

Also see [M-5] findexternal(). Rather than returning a pointer to the external global, as does findexternal(), valofexternal() returns the contents of the external global. This is useful when the external global contains a scalar:

```
tol = valofexternal("tolerance")
if (tol==J(0,0,.)) tol = 1e-6
```

Using findexternal(), one alternative would be

```
if \(((p=\) findexternal("tolerance")) \(==\) NULL) tol \(=1 \mathrm{e}-6\)
else tol \(=* p\)
```

For efficiency reasons, use of valofexternal() should be avoided with nonscalar objects; see [M-5] findexternal().

## Conformability

valofexternal (name):

```
name: }1\times
result: }\quadr\timesc\mathrm{ or 0 0 0 if not found
```


## Diagnostics

valofexternal() aborts with error if name contains an invalid name.
valofexternal (name) returns $\mathrm{J}(0,0,$.$) if name does not exist.$

## Also see

[M-5] findexternal() - Find, create, and remove external globals
[M-4] Programming - Programming functions

## Title

[M-5] Vandermonde() — Vandermonde matrices
Description Syntax Remarks and examples Conformability

Diagnostics
Reference
Also see

## Description

Vandermonde ( $x$ ) returns the Vandermonde matrix containing the geometric progression of $x$ in each row

$$
\left[\begin{array}{cccccc}
1 & x_{1} & x_{1}^{2} & x_{1}^{3} & \ldots & x_{1}^{n-1} \\
1 & x_{2} & x_{2}^{2} & x_{2}^{3} & \ldots & x_{2}^{n-1} \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
1 & x_{n} & x_{n}^{2} & x_{n}^{3} & \ldots & x_{n}^{n-1}
\end{array}\right]
$$

where $n=\operatorname{rows}(x)$. Some authors use the transpose of the above matrix.

## Syntax

numeric matrix Vandermonde(numeric colvector $x$ )

## Remarks and examples

Vandermonde matrices are useful in polynomial interpolation.

## Conformability

Vandermonde ( $x$ ):

| $x:$ | $n \times 1$ |
| ---: | :--- |
| result: |  |
|  | $n \times n$ |

## Diagnostics

None.

Alexandre-Théophile Vandermonde (1735-1796) was born in Paris. His first passion was music (particularly the violin) and he turned to mathematics only at the age of 35 . Four papers dated 1771 and 1772 are his entire mathematical output, although all contain good work. He also worked in experimental science and the theory of music, arguing that musicians should ignore all theory and trust their trained ears, and was busy with various committees and other administration. Vandermonde was a strong supporter of the French Revolution. He is now best known for the Vandermonde determinant, even though it does not appear in any of his papers, and for the associated matrix. Lebesgue later conjectured that the attribution arises from a misreading of Vandermonde's notation.

## Reference

Jones, P. S. 1976. Vandermonde, Alexandre-Théophile. In Vol. 13 of Dictionary of Scientific Biography, ed. C. C. Gillispie, 571-572. New York: Scribner's.

## Also see

[M-4] Standard - Functions to create standard matrices

## Title

[M-5] vec() - Stack matrix columns

| Description | Syntax | Remarks and examples |
| :--- | :--- | :--- |
| Conformability | Diagnostics | Also see |

## Description

$\operatorname{vec}(T)$ returns $T$ transformed into a column vector with one column stacked onto the next.
$\operatorname{vech}(T)$ returns square and typically symmetric matrix $T$ transformed into a column vector; only the lower half of the matrix is recorded.
invvech ( $v$ ) returns vech()-style column vector $v$ transformed into a symmetric (Hermitian) matrix.

## Syntax

transmorphic colvector vec (transmorphic matrix $T$ )
transmorphic colvector vech (transmorphic matrix $T$ )
transmorphic matrix invvech (transmorphic colvector $v$ )

## Remarks and examples

Remarks are presented under the following headings:
Example of vec()
Example of vech() and invvech()

## Example of vec()

```
: x
            1 2 3
                1 2 3
2 4
: vec(x)
                1
1
2 4
3 2
4 5
5 3
6 6
```


## Example of vech() and invvech()

```
: x
[symmetric]
            1 2 3
    l}\begin{array}{llll}{1}&{1}&{}&{}\\{2}&{2}&{4}&{}\\{3}&{3}&{6}&{9}
: v = vech(x)
: v
        1
\begin{tabular}{l|l|}
\cline { 2 - 2 } & 1 \\
2 & 1 \\
3 & 2 \\
4 & 4 \\
5 & 6 \\
6 & 9 \\
\hline
\end{tabular}
: invvech(v)
[symmetric]
        1 2 3
    1
    2
    3
        2 4
        3 6 9
```


## Conformability

$\operatorname{vec}(T)$ :

$$
\begin{aligned}
T: & r \times c \\
\text { result: } & r * c \times 1
\end{aligned}
$$

$\operatorname{vech}(T)$ :
T: $\quad n \times n$
result: $\quad(n(n+1) / 2 \times 1)$
invvech ( $v$ ):

$$
\begin{aligned}
v: & (n(n+1) / 2 \times 1) \\
\text { result: } & n \times n
\end{aligned}
$$

## Diagnostics

$\operatorname{vec}(T)$ cannot fail.
$\operatorname{vech}(T)$ aborts with error if $T$ is not square. vech() records only the lower triangle of $T$; it does not require $T$ be symmetric.
invvech $(v)$ aborts with error if $v$ does not have $0,1,3,6,10, \ldots$ rows.

## Also see

[M-4] Manipulation - Matrix manipulation

## Title

[M-5] xI() — Excel file I/O class
Description Syntax Remarks and examples Appendix Also see

## Description

The xl() class allows you to create Excel 1997/2003 (.xls) files and Excel 2007/2013 (.xlsx) files and load them from and to Mata matrices. The two Excel file types have different data size limits that you can read about in the technical note Excel data size limits of [D] import excel. The xl () class is supported on Windows, Mac, and Linux.

## Syntax

If you are trying to import or export an Excel file to or from Stata, see [D] import excel. If you are trying to export a table created by Stata to Excel, see [RPT] putexcel.

The syntax diagrams below describe a Mata class. For help with class programming in Mata, see [M-2] class.

Syntax is presented under the following headings:

## Step 1: Initialization

$$
B=\mathrm{xl}()
$$

## Step 2: Creating and opening an Excel workbook

| (void) | B.create_book(string scalar filename, string scalar sheetname |
| :--- | :--- |
|  | $[, " \mathrm{xls}\| \| \mathrm{xlsx}[$, string scalar locale $]])$ |
| (void) | B.load_book(string scalar filename $[$, string scalar locale $]$ ) |
| (void) | B.clear_book(string scalar filename) |
| (void) | B.set_mode("open" $\mid$ "closed") |
| (void) | B.close_book() |

## Step 3: Working with Excel worksheets

| (void) | B.add_sheet (string scalar sheetname) |
| :--- | :--- |
| (void) | B.set_sheet (string scalar sheetname) |
| (void) | B.move_sheet (string scalar sheetname, real scalar idx) |
| (void) | B.clear_sheet (string scalar sheetname) |
| (void) | B.delete_sheet (string scalar sheetname) |
| string colvector | B.get_sheets () |

## Step 4: Excel active worksheet settings

| (void) | B.set_gridlines("on" $\mid$ "off") |
| :--- | :--- |
| (void) | B.set_merge(real vector row, real vector col) |
| (void) | B.delete_merge(real scalar row, real scalar col) |
| (void) | B.set_split(real scalar row, real scalar col) |
| (void) | B.set_named_range(real vector row, real vector col, string scalar name) |
| (void) | B.delete_named_range(string scalar name) |
| (void) | B.insert_horizontal_page_break(real scalar row) |
| (void) | B.delete_horizontal_page_break(real scalar row) |
| (void) | B.insert_vertical_page_break(real scalar col) |
| (void) | B.delete_vertical_page_break(real scalar col) |
| (void) | B.set_header (string scalar header $[$, real scalar margin $]$ ) |
| (void) | B.set_footer (string scalar footer $[$, real scalar margin]) |

## Step 5: Reading and writing data from and to an Excel worksheet

| string matrix | $B$. get_string (real vector row, real vector col) |
| :---: | :---: |
| real matrix | $B$.get_number (real vector row, real vector col [, "asdate" \| "asdatetime"]) |
| string matrix | $B . \mathrm{get}$-cell_type (real vector row, real vector col) |
| (void) | $B$. put_string (real scalar row, real scalar col, string matrix s) |
| (void) | ```B.put_number(real scalar row, real scalar col, real matrix r [, "asdate" \| "asdatetime" | "asdatenum" | "asdatetimenum" ])``` |
| (void) |  |
| (void) | $B$. put_picture (real scalar row, real scalar col, string scalar filename) |
| (void) | B.put_link(real scalar row, real scalar col, string scalar link, string scalar link_text) |
| (void) | $B$.set_missing ([real scalar num $\mid$ string scalar val $]$ ) |

## Step 6: Formatting cells in an Excel worksheet

| (void) | $B$.set_number_format (real vector row, real vector col, string scalar format) |
| :---: | :---: |
| (void) | B.set_vertical_align (real vector row, real vector col, string scalar align) |
| (void) | B.set_horizontal_align(real vector row, real vector col, string scalar align) |
| (void) | B.set_border (real vector row, real vector col, string scalar style [, string scalar color $]$ ) |
| (void) | B.set_left_border (real vector row, real vector col string scalar style [, string scalar color]) |
| (void) | B.set_right_border (real vector row, real vector col, string scalar style [, string scalar color]) |
| (void) | $B$.set_top_border (real vector row, real vector col, string scalar style [, string scalar color]) |
| (void) | B.set_bottom_border (real vector row, real vector col, string scalar style [, string scalar color]) |
| (void) | B.set_diagonal_border (real vector row, real vector col, string scalar direction, string scalar style $[$, string scalar color $]$ ) |
| (void) | $B$.set_fill_pattern (real vector row, real vector col, string scalar pattern, string scalar fgcolor [, string scalar bgcolor $]$ ) |
| (void) | $B$. set_column_width (real scalar coll, real scalar col2, real scalar width) |
| (void) | $B$. set_row_height (real scalar row1, real scalar row2, real scalar height) |

## Step 7: Formatting text in an Excel worksheet

| (void) | B.set_font (real vector row, real vector col, string scalar fontname, real scalar size [, string scalar color $]$ ) |
| :---: | :---: |
| (void) | $B$. set_font_bold(real vector row, real vector col, "on"\|"off") |
| (void) | $B$. set_font_italic (real vector row, real vector col, "on"\|"off") |
| (void) | $B$. set_font_strikeout (real vector row, real vector col, "on"\| "off") |
| (void) | $B$. set_font_underline (real vector row, real vector col, "on"\| "off") |
| (void) | $B$.set_font_script (real vector row, real vector col, "sub" \| "super" | "normal") |
| (void) | $B$. set_text_wrap(real vector row, real vector col, "on"\|"off") |
| (void) | $B$. set_shrink_to_fit(real vector row, real vector col, "on"\|"off") |
| (void) | $B$. set_text_rotate (real vector row, real vector col, real scalar rotation) |
| (void) | $B$. set_text_indent (real vector row, real vector col, real scalar indent) |
| (void) | $B$. set_format_lock(real vector row, real vector col, "on"\|"off") |
| (void) | $B$. set_format_hidden(real vector row, real vector col, "on"\|"off") |

## Step 8: Formatting cell ranges in an Excel worksheet

| real scalar | $B$. add_fmtid() |
| :---: | :---: |
| (void) | $B$. set_fmtid(real vector row, real vector col, real scalar fmitid) |
| (void) | B.set_column_fmtid(real vector cols, real scalar width, real scalar hidden, real scalar fmitid) |
| (void) | B.fmtid_set_number_format (real scalar fmitid, string scalar format) |
| (void) | B.fmtid_set_vertical_align (real scalar fmtid, string scalar align) |
| (void) | B.fmtid_set_horizontal_align(real scalar fmid, string scalar align) |
| (void) | B.fmtid_set_border (real scalar fmide, string scalar style [, string scalar color $]$ ) |
| (void) | B.fmtid_set_left_border (real scalar fintid, string scalar style [, string scalar color $]$ ) |
| (void) | B.fmtid_set_right_border (real scalar fmitid, string scalar style [, string scalar color $]$ ) |
| (void) | B.fmtid_set_top_border (real scalar fmide, string scalar style [, string scalar color]) |
| (void) | B.fmtid_set_bottom_border (real scalar fintid, string scalar style [, string scalar color $]$ ) |
| (void) | B.fmtid_set_diagonal_border (real scalar fintid, string scalar direction, string scalar style $[$, string scalar color $]$ ) |
| (void) | B.fmtid_set_fill_pattern(real scalar fintid, string scalar pattern, string scalar fgcolor [, string scalar bgcolor ]) |
| (void) | B.fmtid_set_column_width (real scalar fmtid, real scalar coll, real scalar col2, real scalar width) |
| (void) | B.fmtid_set_row_height (real scalar fmtid, real scalar rowl, real scalar row2, real scalar height) |
| (void) | B.fmtid_set_text_wrap (real scalar fmidid, "on"\|"off") |
| (void) | B.fmtid_set_shrink_to_fit(real scalar fmide "on"\|"off") |
| (void) |  |
| (void) |  |
| (void) |  |
| (void) |  |


| real scalar | $B$. add_fontid() |
| :---: | :---: |
| (void) | B.fmtid_set_fontid(real scalar fmid, real scalar fontid) |
| (void) | B.fontid_set_font (real scalar fontid, string scalar fontname, real scalar size $[$, string scalar color $]$ ) |
| (void) | B.fontid_set_font_bold(real scalar fontid, "on"\|"off") |
| (void) |  |
| (void) | $B . f$ ontid_set_font_strikeout (real scalar fontid, "on"\|"off") |
| (void) | $B . f$ ontid_set_font_underline(real scalar fontid, "on"\|"off") |
| (void) | B.fontid_set_font_script (real scalar fontid, "sub" \| "super" | "normal") |

## Utility functions for use in all steps

(varies) $\quad B$. query $([$ string scalar item $])$
real vector $\quad B . g e t \_c o l n u m$ (string vector)
string vector $B$. get_colletter (real vector)
(void) B.set_keep_cell_format("on"|"off")
(void) B.set_error_mode("on"|"off")
real scalar B.get_last_error()
string scalar B.get_last_error_message()
where item can be
filename
mode
filetype
sheetname
missing

## Remarks and examples

Remarks are presented under the following headings:
Definition of $B$
Specifying the Excel workbook
Working with Excel worksheets
Excel active worksheet settings
Reading data from Excel
Writing data to Excel
Dealing with missing values
Dealing with dates
Formatting functions
Numeric formatting
Text alignment
Cell borders
Fonts
Other
Formatting examples
Range formatting functions
Adding format IDs
Setting formats by ID
Cell formatting functions
Adding font IDs
Setting font IDs for format IDs
Font formatting functions
Range formatting examples
Utility functions
Handling errors
Error codes

## Definition of $B$

A variable of type xl is called an instance of the xl() class. $B$ is an instance of xl() . You can use the class interactively:

```
b = xl()
b.create_book("results", "Sheet1")
```

In a function, you would declare one instance of the xl() class $B$ as a scalar.

```
void myfunc()
{
        class xl scalar b
        b = xl()
        b.create_book("results", "Sheet1")
}
```

When using the class inside other functions, you do not need to create the instance explicitly as long as you declare the member-instance variable to be a scalar:

```
void myfunc()
{
    class xl scalar b
    b.create_book("results", "Sheet1")
}
```


## Specifying the Excel workbook

To read from or write to an existing Excel workbook, you need to tell the xl() class about that workbook. To create a new workbook to write to, you need to tell the xl () class what to name that workbook and what type of Excel file that workbook should be. Excel 1997/2003 (.xls) files and Excel 2007/2010 (.xlsx) files can be created. You must either load or create a workbook before you can use any sheet or read or write member functions of the xl () class.
B.create_book(filename, sheetname [, "xls"|"xlsx" [, locale]]) creates an Excel workbook named filename with the sheet sheetname. By default, an .xlsx file is created. If you use the optional xls argument, then an .xls file is created. locale specifies the locale used by the workbook. You might need this option when working with extended ASCII character sets. This option has no effect on Microsoft Windows. The default locale is UTF-8.

## B.load_book(filename [, locale])

loads an existing Excel workbook. Once it is loaded, you can read from or write to the workbook. locale specifies the locale used by the workbook. You might need this option when working with extended ASCII character sets. This option has no effect on Microsoft Windows. The default locale is UTF-8.

## B.clear_book(filename)

removes all worksheets from an existing Excel workbook.
To create an .xlsx workbook, code

```
b = xl()
b.create_book("results", "Sheet1", "xlsx")
```

To load an .xls workbook, code

```
\(\mathrm{b}=\mathrm{xl}\) ()
b.load_book("Budgets.xls")
```

The xl () class will open and close the workbook for each member function you use that reads from or writes to the workbook. This is done by default, so you do not have to worry about opening and closing a file handle. This can be slow if you are reading or writing data at the cell level. In these cases, you should leave the workbook open, deal with your data, and then close the workbook. The following member functions allow you to control how the class handles file I/O.
B.set_mode("open" | "closed")
sets whether the workbook file is left open for reading or writing data. set_mode("closed"), the default, means that the workbook is opened and closed after every sheet or read or write member function.
B.close_book()
closes a workbook file if the file has been left open using set_mode("open").
Below is an example of how to speed up file I/O when writing data.

```
b = xl()
b.create_book("results", "year1")
b.set_mode("open")
for(i=1;i<10000;i++) {
    b.put_number(i,1,i)
}
b.close_book()
```


## Working with Excel worksheets

The following member functions are used to modify the settings of a worksheet named sheetname. By default, if you do not specify a worksheet, the xl () class will use the first worksheet in the workbook to read data from and write data to.
$B$.add_sheet (sheetname)
adds a new worksheet named sheetname to the workbook and sets the active worksheet to that sheet.
B.set_sheet (sheetname)
sets the active worksheet to sheetname.
B.move_sheet (sheetname, idx)
moves worksheet sheetname to the $i d x$ position in a workbook.
B.clear_sheet (sheetname)
clears all cell values for sheetname.
B.delete_sheet (sheetname)
deletes sheetname from the workbook.
B.get_sheets() returns a string colvector of all the sheetnames in the current workbook.

You may need to make a change to all the sheets in a workbook. get_sheets() can help you do this.

```
void myfunc()
{
    class xl scalar b
    string colvector sheets
    real scalar i
    b.load_book("results")
    sheets = b.get_sheets()
    for(i=1;i<=rows(sheets);i++) {
        b.set_sheet(sheets[i])
        b.clear_sheet(sheets[i])
    }
}
```

To create a new workbook with multiple new sheets, code

```
b.create_book("Budgets", "Budget 2009")
for(i=10;i<=13;i++) {
    sheet = "Budget 20" + strofreal(i)
    b.add_sheet(sheet)
}
```


## Excel active worksheet settings

The following member functions are sheet settings:
B.set_gridlines("on"|"off")
sets whether gridlines are displayed for active worksheet. The default is "on".
B.set_merge (row, col)
merges the cells in active worksheet for each cell in the Excel cell range specified in row and col. Both row and col can be a $1 \times 2$ real vector. The first value in the vectors must be the starting (upper-left) cell in the worksheet to which you want to merge. The second value must be the ending (lower-right) cell in the worksheet to which you want to merge.
B.delete_merge (row, col)
deletes the merged cells in the active worksheet for any cells merged with the cell specified by row and col.
B.set_split(row, col)
splits the active sheet at position row and col or specifies the position of frozen pane.
B.set_named_range (row, col, name)
names the cell range in active worksheet for each cell in the Excel cell range specified in row and col. Both row and col can be a $1 \times 2$ real vector. The first value in the vectors must be the starting (upper-left) cell in the worksheet to which you want to merge. The second value must be the ending (lower-right) cell in the worksheet to which you want to merge.

## $B$.delete_named_range (name)

deletes the named range name from the list of named cell ranges stored in the active worksheet.

## $B$.insert_horizontal_page_break(row)

inserts a page break at row row in the active worksheet.

## B.delete_horizontal_page_break(row)

deletes a page break at row row in the active worksheet.
$B$.insert_vertical_page_break(col)
inserts a page break at column col in the active worksheet.

## B.delete_vertical_page_break(col)

deletes a page break at column col in the active worksheet.
$B$. set_header (header [, margin])
adds a worksheet header header with margin margin to the active worksheet.
B.set_footer (footer [, margin])
adds a worksheet footer footer with margin margin to the active worksheet.

## Reading data from Excel

The following member functions of the xl () class are used to read data. Both row and col can be a real scalar or a $1 \times 2$ real vector.
B.get_string(row, col)
returns a string matrix containing values in a cell range depending on the range specified in row and col.
$B$.get_number (row, col [, "asdate" | "asdatetime" ])
returns a real matrix containing values in a cell range depending on the range specified in row and col.
B.get_cell_type(row, col)
returns a string matrix containing the string values numeric, string, date, datetime, or blank for each cell in the Excel cell range specified in row and col.

To get the value in cell A1 from Excel into a string scalar, code

```
string scalar val
val = b.get_string(1,1)
```

If A1 contained the value "Yes", then val would contain "Yes". If A1 contained the numeric value 1 , then val would contain "1". get_string() will convert numeric values to strings.

To get the value in cell A1 from Excel into a real scalar, code

```
real scalar val
val = b.get_number(1,1)
```

If A1 contained the value "Yes", then val would contain a missing value. get_number will return a missing value for a string value. If A1 contained the numeric value 1 , then val would contain the value 1 .

To read a range of data into Mata, you must specify the cell range by using a $1 \times 2$ rowvector. To read row 1, columns B through F of a worksheet, code

```
string rowvector cells
real rowvector cols
cols = (2,6)
cells = b.get_string(1,cols)
```

To read rows 1 through 3 and columns B through D of a worksheet, code

```
real matrix cells
real rowvector rows, cols
rows = (1,3)
cols = (2,4)
cells = b.get_number(rows,cols)
```


## Writing data to Excel

The following member functions of the xl() class are used to write data. row and col are real scalars. When you write a matrix or vector, row and col are the starting (upper-left) cell in the worksheet to which you want to begin saving.
B.put_string(row, col, s)
writes a string scalar, vector, or matrix to an Excel worksheet.
B.put_number (row, col, r [, "asdate" | "asdatetime" | "asdatenum" | asdatetimenum]) writes a real scalar, vector, or matrix to an Excel worksheet.
B.put_formula(row, col, s)
writes a string scalar, vector, or matrix containing valid Excel formulas to an Excel worksheet.

## B.put_picture(row, col, filename)

writes a portable network graphics (.png), JPEG (.jpg), device-independent bitmap (.dib), enhanced metafile (.emf), or tagged image file format (.tiff) file to an Excel worksheet.
B.put_link(row, col, link, link_text)
writes an HTTP link, Excel file link, or sheet link to an Excel worksheet.
To write the string "Auto Dataset" in cell A1 of a worksheet, code

```
b.put_string(1, 1, "Auto Dataset")
```

To write "mpg", "rep78", and "headroom" to cells B1 through D1 in a worksheet, code

```
names = ("mpg", "rep78", "headroom")
```

b.put_string(1, 2, names)

To write values $22,17,22,20$, and 15 to cells B2 through B6 in a worksheet, code

```
mpg_vals = (22\17\22\20\15)
b.put_number(2, 2, mpg_vals)
```

To sum the cells A1 through A4 in cell A6 in a worksheet, code

```
b.put_formula(1, 6, "SUM(A1:A4)")
```

To write the file mygraph.png to starting cell D15 in a worksheet, code

```
b.put_picture(4, 15, "mygraph.png")
```

To write a file link in cell A15 of the current worksheet to the file auto.xlsx saved in the same working directory as the current workbook, code
b.put_link(1, 15, "file://auto.xlsx", "Auto")

To write a cell link in cell A16 to cell C2 for active workbook auto.xlsx, Sheet1, code
b.put_link(1, 16, "[auto.xlsx]Sheet1:C2", "MPG1")

To write an HTTP link in cell A17 to stata.com, code
b.put_link(1, 17, "https://www.stata.com", "Stata")

## Dealing with missing values

set_missing() sets how Mata missing values are to be treated when writing data to a worksheet. Here are the three syntaxes:
B.set_missing() specifies that missing values be written as blank cells. This is the default.
B.set_missing(num) specifies that missing values be written as the real scalar num.
$B$.set_missing (val) specifies that missing values be written as the string scalar val.
Let's look at an example.

```
my_mat = J (1,3,.)
b.load_book("results")
b.set_sheet("Budget 2012")
b.set_missing(-99)
b.put_number(1, 1, my_mat)
b.set_missing("no data")
b.put_number(2, 1, my_mat)
b.set_missing()
b.put_number(3, 1, my_mat)
```

This code would write the numeric value -99 in cells A1 through C1 and "no data" in cells A2 through C2; cells A3 through C3 would be blank.

## Dealing with dates

Say that cell A1 contained the date value $1 / 1 / 1960$. If you coded

```
mydate = b.get_number(1,1)
mydate
21916
```

the value displayed, 21916, is the number of days since 31dec1899. This is how Excel stores its dates. If we used the optional get_number() argument "asdate" or "asdatetime", mydate would contain 0 because the date $1 / 1 / 1960$ is 0 for both $t d$ and tc dates. To store $1 / 1 / 1960$ in Mata, code

```
mysdate = b.get_string(1,1)
mysdate
1/1/1960
```

To write dates to Excel, you must tell the xl() class how to convert the date to Excel's date or datetime format. To write the date 1/1/1960 12:00:00 to Excel, code

```
b.put_number(1,1,0, "asdatetime")
```

To write the dates $1 / 1 / 1960,1 / 2 / 1960$, and $1 / 3 / 1960$ to Excel column A, rows 1 through 3, code

```
date_vals = (0\1\2)
b.put_number(1, 1, date_vals, "asdate")
```

"asdate" and "asdatetime" apply an Excel date format to the transformed date value when written. Use "asdatenum" or "asdatetimenum" to write the transformed Excel date number and preserve the cell's format.

Note: Excel has two different date systems; see the technical note Dates and times in [D] import excel.

## Formatting functions

The following member functions of the xl () class are used to format cells of the active worksheet. Both row and col can be a real scalar or a $1 \times 2$ real vector. The first value in the vectors must be the starting (upper-left) cell in the worksheet to which you want to format. The second value must be the ending (lower-right) cell in the worksheet to which you want to format.

## Numeric formatting

B.set_number_format (row, col, format)
sets the numeric format for each cell in the Excel cell range specified in row and col.

## Text alignment

B.set_vertical_align(row, col, align)
sets the text to vertical alignment for each cell in the Excel cell range specified in row and col. align may be "top", "center", "bottom", "justify", or "distributed".
B.set_horizontal_align(row, col, align)
sets the text to horizontal alignment for each cell in the Excel cell range specified in row and col. align may be "left", "center", "right", "fill", "justify", "merge", or "distributed".

## Cell borders

B.set_border (row, col, style [, color $]$ )
sets the top, left, right, and bottom border style and color for each cell in the Excel cell range specified in row and col.
B.set_left_border (row, col, style [, color]) sets the left border style and color for each cell in the Excel cell range specified in row and col.
B.set_right_border (row, col, style [, color])
sets the right border style and color for each cell in the Excel cell range specified in row and col.
B.set_top_border (row, col, style [, color])
sets the top border style and color for each cell in the Excel cell range specified in row and col.
B.set_bottom_border (row, col, style [, color])
sets the bottom border style and color for each cell in the Excel cell range specified in row and col.
B.set_diagonal_border (row, col, direction, style [, color] $)$
sets the diagonal border direction, style, and color for each cell in the Excel cell range specified in row and col. direction may be "none", "down", "up", or "both".
B.set_fill_pattern(row, col, pattern, fgcolor [, bgcolor])
sets the fill color for each cell in the Excel cell range specified in row and col.
B.set_column_width(coll, col2, width)
sets the column width for each cell in the Excel cell column range specified in coll through col2. Column width is measured as the number of characters ( $0-255$ ) rendered in Excel's default style's font.
B.set_row_height (row1, row 2 , height)
sets the row height for each cell in the Excel cell row range specified in row 1 through row 2 . height is measured in point size.

## Fonts

The following member functions of the xl () class are used to format text of a given cell in the active worksheet. Both row and col can be a real scalar or a $1 \times 2$ real vector. The first value in the vectors must be the starting (upper-left) cell in the worksheet that you want to format. The second value must be the ending (lower-right) cell in the worksheet that you want to format.
B.set_font (row, col, fontname, size [, color])
sets the font, font size, and font color for each cell in the Excel cell range specified in row and col.
B.set_font_bold(row, col, "on"|"off")
bolds or unbolds text for each cell in the Excel cell range specified in row and col.
B.set_font_italic (row, col, "on"|"off")
italicizes or unitalicizes text for each cell in the Excel cell range specified in row and col.
B.set_font_strikeout (row, col, "on"|"off")
strikesout or unstrikesout text for each cell in the Excel cell range specified in row and col.
B.set_font_underline(row, col, "on"|"off")
underlines or ununderlines text for each cell in the Excel cell range specified in row and col.
B.set_font_script(row, col, "sub"|"super"|"normal")
sets the script type for each cell in the Excel cell range specified in row and col.

## Other

The following member functions of the xl() class control other various cell formatting for a given cell in the active worksheet. Both row and col can be a real scalar or a $1 \times 2$ real vector. The first value in the vectors must be the starting (upper-left) cell in the worksheet to which you want to format. The second value must be the ending (lower-right) cell in the worksheet to which you want to format.
B.set_text_wrap(row, col, "on"|"off")
sets whether text is wrapped for each cell in the Excel cell range specified in row and col.
B.set_shrink_to_fit(row, col, "on"|"off")
sets whether text is shrunk-to-fit the cell width for each cell in the Excel cell range specified in row and col.
B.set_text_rotate (row, col, rotation)
sets the text rotation for each cell in the Excel cell range specified in row and col.
$B$.set_text_indent(row, col, indent)
sets the text indention for each cell in the Excel cell range specified in row and col. indent must be an integer less than or equal to 15 .
B.set_format_lock(row, col, "on"|"off")
sets the locked protection property for each cell in the Excel cell range specified in row and col.
B.set_format_hidden(row, col, "on"|"off")
sets the hidden protection property for each cell in the Excel cell range specified in row and col.

## Formatting examples

To change a cell's numeric format so that a number has commas and two decimal points and places all negative numbers in braces (number_sep_d2_negbra) for rows 2 through 7 and columns 2 through 4 for a worksheet, code

```
real rowvector rows, cols
b = xl()
rows = (2,7)
cols = (2,4)
b.set_number_format(rows, cols, "number_sep_d2_negbra")
```

To add a medium-thick border to all cell sides for the same cell range, code

```
b.set_border(rows, cols, "medium")
```

To change the font and font color for rows 1 through 7, column 1, code

```
rows = (1,7)
b.set_font(rows, 1, "Arial", 12, "white")
```

and to change the background fill color of the same cells, code

```
b.set_fill_pattern(rows, 1, "solid", "white", "lightblue")
```

To bold the text in cell B1 through C3, code

```
rows = (1,3)
cols = (2,3)
b.set_font_bold(rows, cols, "on")
```


## Range formatting functions

By default, the xl () class creates a new format ID for each font and cell format change you make in a workbook using the standard xl () class formatting functions. Depending on how many format changes you make to a workbook, the number of format IDs can cause the Excel workbook to become so large that it will open slowly in Excel. To prevent this, you can create a format ID with specific font and format settings and apply that format ID across a cell range. Once a format ID has been attached to a cell range, any changes to the format ID are automatically applied to the cells.

Font formatting also has its own ID system, but you must attach a font ID to a format ID for the font ID to apply to the cell range. You can use one font ID with multiple format IDs. There is a limit of 512 font IDs per workbook.

## Adding format IDs

B.add_fmtid()
returns a new format ID and adds it to the current workbook.

## Setting formats by ID

B.set_fmtid (row, col, fmid) sets the format ID to fintid for each cell in the Excel cell range specified in row and col.
B.set_column_fmtid(cols, width, hidden, fmtid)
sets the format ID to fintid for a column or column range in the active worksheet specified in cols, with column width and hidden specified.

## Cell formatting functions

The cell formatting functions below are used when formatting cells using a format ID.
B.fmtid_set_number_format (fmtid, format) sets the numeric format for the specified format ID.
B.fmtid_set_vertical_align(fmtid, align) sets the vertical alignment of the text for the specified format ID. align may be "top", "center", "bottom", "justify", or "distributed".
B.fmtid_set_horizontal_align(fmid, align)
sets the horizontal alignment of the text for the specified format ID. align may be "left", "center", "right", "fill", "justify", "merge", or "distributed".
B.fmtid_set_border (fintid, style [, color])
sets the top, left, right, and bottom border style and color for the specified format ID.
B.fmtid_set_left_border (fmitid, style [, color])
sets the left border style and color for the specified format ID.
B.fmtid_set_right_border (fintid, style [, color])
sets the right border style and color for the specified format ID.
B.fmtid_set_top_border (fintid, style [, color])
sets the top border style and color for the specified format ID.
B.fmtid_set_bottom_border (fmid , style [, color])
sets the bottom border style and color for the specified format ID.
B.fmtid_set_diagonal_border (fmidid, direction, style [, color])
sets the diagonal border direction, style, and color for the specified format ID. direction may be "none", "down", "up", or "both".
B.fmtid_set_fill_pattern(fmid, pattern, fgcolor [, bgcolor])
sets the fill color for the specified format ID.
B.fmtid_set_column_width(frntid, coll, col2, width)
sets the column width for the specified format ID in the Excel column range specified in coll through col2. Column width is measured as the number of characters ( $0-255$ ) rendered in Excel's default style's font.
B.fmtid_set_row_height (fintid, row1, row2, height)
sets the row height for the specified format ID in the Excel row range specified in rowl through row2. height is measured in point size.
B.fmtid_set_text_wrap(fmtid, "on"|"off")
sets whether text is wrapped for the specified format ID.
B.fmtid_set_shrink_to_fit(fmidid, "on"|"off")
sets whether text is shrunk to fit the cell width for the specified format ID.
B.fmtid_set_text_rotate (fmid, rotation)
sets the text rotation for the specified format ID.
$B . f m t i d \_s e t \_t e x t \_i n d e n t(f m i d, ~ i n d e n t) ~$
sets the text indention for the specified format ID. indent must be an integer less than or equal to 15 .
B.fmtid_set_format_lock(fmid, "on" | "off") sets the locked protection property for the specified format ID.
B.fmtid_set_format_hidden(fmidid, "on"|"off")
sets the hidden protection property for the specified format ID.

## Adding font IDs

$B$.add_fontid()
returns a new font ID and adds it to the current workbook.

## Setting font IDs for format IDs

B.fmtid_set_fontid(fmtid, fontid) sets the font ID for the specified format ID.

## Font formatting functions

The font formatting functions below are used when formatting fonts using a font ID.
$B . f o n t i d \_s e t \_f o n t(f o n t i d$, fontname, size $[$, color $]$ ) sets the font, font size, and font color for the specified font ID.
B.fontid_set_font_bold(fontid, "on"|"off")
bolds or unbolds text for the specified font ID.
B.fontid_set_font_italic(fontid, "on"|"off") italicizes or unitalicizes text for the specified font ID.
B.fontid_set_font_strikeout (fontid, "on"|"off") strikesout or unstrikesout text for the specified font ID.
B.fontid_set_font_underline(fontid, "on"|"off") underlines or ununderlines text for the specified font ID.
B.fontid_set_font_script(fontid, "sub" | "super" | "normal") sets the script type for the specified font ID.

## Range formatting examples

To create a format ID with a numeric format that places all negative numbers in braces, uses commas for thousands separators, and specifies two digits after the decimal, (number_sep_d2_negbra), code

```
b = xl()
fmt_id1 = b.add_fmtid()
b.fmtid_set_number_format(fmt_id1, "number_sep_d2_negbra")
```

To also change the format ID to have a medium-thick border for all cell sides, code

```
b.fmtid_set_border(fmt_id1, "medium")
```

To apply these format changes for rows 2 through 7 and columns 2 through 4 for a worksheet, code

```
rows = (2,7)
cols = (2,4)
b.set_fmtid(rows, cols, fmt_id1)
```

To create a font ID with an Arial font and a font color of blue, code

```
font_id1 = b.add_fontid()
b.fontid_set_font(font_id1, "Arial", 12, "blue")
```

To apply these font changes to the format ID fmt_id1

```
b.fmtid_set_fontid(fmt_id1, font_id1)
```

To create a new format ID that sets the background fill color to lightblue, code

```
fmt_id2 = b.add_fmtid()
b.fmtid_set_fill_pattern(fmt_id2, "solid", "white", "lightblue")
```

To apply these format changes to cell A1 for a worksheet, code

```
b.set_fmtid(1, 1, fmt_id2)
```

To also apply the font_id1 font changes to row 1 column 1, type

```
b.fmtid_set_fontid(fmt_id2, font_id1)
```

By adding the font settings in font_id1 to fmt_id2, the font formatting is automatically applied to row 1 column 1.

## Utility functions

The following functions can be used whenever you have an instance of the xl () class.
query () returns information about an $x l()$ class. Here are the syntaxes for query ():
void B.query()
string scalar B.query ("filename")
real scalar B.query("mode")
real scalar B.query ("filetype")
string scalar B.query("sheetname")
transmorphic scalar B.query("missing")

## $B$.query ()

lists the current values and settings of the class.

## B.query("filename")

returns the filename of the current workbook.
$B$.query("mode")
returns 0 if the workbook is always closed by member functions or returns 1 if the current workbook is open.

## B.query("filetype")

returns 0 if the workbook is of type .xls or returns 1 if the workbook is of type .xlsx.

## B.query("sheetname")

returns the active sheetname in a string scalar.
B.query("missing")
returns $\mathrm{J}(1,0,$.$) (if set to blanks), a string scalar, or a real scalar depending on$ what was set with set_missing().

When working with different Excel file types, you need to know the type of Excel file you are using because the two file types have different column and row limits. You can use xl. query ("filetype") to obtain that information.

```
if (xl.query("filetype")) {
}
else {
}
```

B.get_colnum()
returns a vector of column numbers based on the Excel column labels in the string vector argument.

To get the column number for Excel columns AA and AD, code

```
mycol = ("AA","AD")
col = b.get_colnum(mycol)
col
    1 2
    27 30
```

B.get_colletter()
returns a vector of column letters based on the column numbers in the real vector argument.
To get the column letter for Excel columns 1 and 29, code

```
: mycol = (1, 29)
: col = b.get_colletter(mycol)
col
    1 2
1
        A AC
```

The following function is used for cell formats and styles.

## B.set_keep_cell_format("on"|"off")

sets whether the put_number() class member function preserves a cell's style and format when writing a value. By default, preserving a cell's style and format is "off".

The following functions are used for error handling with an instance of class xl .
B.set_error_mode("on"|"off")
sets whether xl() class member functions issue errors. By default, errors are turned "on".
B.get_last_error ()
returns the last error code issued by the xl() ) class if set_error_mode() is set "off".
B.get_last_error_message()
returns the last error message issued by the xl() class if set_error_mode() is set "off".

## Handling errors

Turning errors off for an instance of the xl () class is useful when using the class in an ado-file. You should issue a Stata error code in the ado-file instead of a Mata error code. For example, in Mata, when trying to load a file that does not exist within an instance, you will receive the error code $r$ (16103) :

```
: b = xl()
: b.load_book("zzz")
file zzz.xls could not be loaded
r(16103);
```

The correct Stata error code for this type of error is 603 , not 16103 . To issue the correct error, code

```
b = xl()
b.set_error_mode("off")
b.load_book("zzz")
if (b.get_last_error()==16103) {
    error(603)
}
```

You should also turn off errors if you set_mode("open") because you need to close your Excel file before exiting your ado-file. You should code

```
b = xl()
b.set_mode("open")
b.set_error_mode("off")
b.load_book("zzz")
b.put_string(1,300, "test")
if (b.get_last_error()==16116) {
    b.close_book()
    error(603)
}
```

If set_mode("closed") is used, you do not have to worry about closing the Excel file because it is done automatically.

## Error codes

The error codes specific to the xl () class are the following:
Code Meaning

16101 file not found
16102 file already exists
16103 file could not be opened
16104 file could not be closed
16105 file is too big
16106 file could not be saved
16111 worksheet not found
16112 worksheet already exists
16113 could not clear worksheet
16114 could not add worksheet
16115 could not read from worksheet
16116 could not write to worksheet
16121 invalid syntax
16122 invalid range
16130 could not read cell format
16131 could not write cell format
16132 invalid column format
16133 invalid column width
16134 invalid row format
16135 invalid row height
16136 invalid color
16140 invalid number format
16141 invalid alignment format
16142 invalid border style format
16143 invalid border direction format
16144 invalid fill pattern style format
16145 invalid font format
16146 invalid font size format
16147 invalid font name format
16148 invalid cell format

## Appendix

## Codes for numeric formats

| format | Example |
| :--- | ---: |
| number | 1000 |
| number_d2 | 1000.00 |
| number_sep | 100,000 |
| number_sep_d2 | $100,000.00$ |
| number_sep_negbra | $(1,000)$ |
| number_sep_negbrared | $(1,000)$ |
| number_d2_sep_negbra | $(1,000.00)$ |
| number_d2_sep_negbrared | $(1000.00)$ |
| currency_negbra | $(\$ 4000)$ |
| currency_negbrared | $(\$ 4000)$ |
| currency_d2_negbra | $(\$ 4000.00)$ |
| currency_d2_negbrared | $(\$ 4000.00)$ |
| account | 5,000 |
| accountcur | 5,000 |
| account_d2 | $5,000.00$ |
| account_d2_cur | $5,000.00$ |
| percent | $75 \%$ |
| percent_d2 | $75.00 \%$ |
| scientific_d2 | $10.00 \mathrm{E}+1$ |
| fraction_onedig | $101 / 2$ |
| fraction_twodig | $1023 / 95$ |
| date | $3 / 18 / 2007$ |
| date_d_mon_yy | $18-M a r-07$ |
| date_d_mon | $18-M a r$ |
| date_mon_yy | Mar-07 |
| time_hmm_AM | $8: 30$ AM |
| time_HMMSS_AM | $8: 30: 00$ AM |
| time_HMM | $8: 30$ |
| time_HMMSS | $8: 30: 00$ |
| time_MMSS | $30: 55$ |
| time_H0MMSS | $20: 30: 55$ |
| time_MMSS0 | $30: 55.0$ |
| date_time | $3 / 18 / 20078: 30$ |
| text | this is text |
|  |  |

## Custom formatting

format also can be a custom code string formed by sections. Up to four sections of format codes can be specified. The format codes, separated by semicolons, define the formats for positive numbers, negative numbers, zero values, and text, in that order. If only two sections are specified, the first is used for positive numbers and zeros, and the second is used for negative numbers. If only one section is specified, it is used for all numbers. The following is a four section example:

```
#,###.00_);[Red](#,###.00);0.00;"sales "@
```

The following table describes the different symbols that are available for use in custom number formats:

| Symbol | Description | Cell <br> value | Fmt <br> code | Cell displays |
| :---: | :---: | :---: | :---: | :---: |
| 0 | Digit placeholder (add zeros) | 8.9 | \#. 00 | 8.90 |
| \# | Digit placeholder (no zeros) | 8.9 | \#.\#\# | 8.9 |
| ? | Digit placeholder (add space) | 8.9 | 0.0? | 8.9 |
|  | Decimal point |  |  |  |
| \% | Percentage | . 1 | \% | 10\% |
| , | Thousands separator |  |  |  |
| E- E+ e- e+ | Scientific format | 12200000 | $0.00 \mathrm{E}+00$ | $1.22 \mathrm{E}+07$ |
| \$-+/() : space | Display the symbol | 12 | (000) | (012) |
| $\backslash$ | Escape character | 3 | $0 \backslash!$ | 3 ! |
| * | Repeat character (fill in cell width) | 3 | 3* | 3 xxxxx |
| - | Skip width of next character | -1.2 | -0.0 | 1.2 |
| "text" | Display text in quotes | 1.23 | 0.00 "a" | 1.23 a |
| © | Text placeholder | b | "a"@"c" | abc |

The following table describes the different codes that are available for custom datetime formats:

| Fmt <br> code | Description | Cell <br> displays |
| :--- | :--- | :--- |
| m | Months | $1-12$ |
| mm | Months | $01-12$ |
| mmm | Months | Jan-Dec |
| mmmm | Months | January-December |
| mmmmm | Months | J-D |
| d | Days | $1-31$ |
| dd | Days | $01-31$ |
| ddd | Days | Sun-Sat |
| dddd | Days | Sunday-Saturday |
| yy | Years | $00-99$ |
| yyyy | Years | $1909-9999$ |
| h | Hours | $0-23$ |
| hh | Hours | $00-23$ |
| m | Minutes | $0-59$ |
| mm | Minutes | $00-59$ |
| s | Seconds | $0-59$ |
| ss | Seconds | $00-59$ |
| h AM/PM | Time | 5 AM |
| $\mathrm{h}: \mathrm{mm}$ AM/PM | Time | $5: 36$ PM |
| $\mathrm{h}: \mathrm{mm}: \mathrm{ss}$ A/P | Time | $5: 36: 03 \mathrm{P}$ |
| $\mathrm{h}: \mathrm{mm}: \mathrm{ss} .00$ | Time | $5: 34: 03.75$ |
| $[\mathrm{~h}]: \mathrm{mm}$ | Elapsed time | $1: 22$ |
| $[\mathrm{~mm}]:$ ss | Elapsed time | $64: 16$ |
| $[\mathrm{ss}] .00$ | Elapsed time | 3733.71 |

## Custom formatting: Text color

To set the text color for a section of the format, type the name of one of the colors listed in the table under Format colors in square brackets in the section. The color must be the first item in the section.

## Custom formatting: Conditional formatting

To set number formats that will be applied only if a number meets a specified condition, enclose the condition in square brackets. The condition consists of a comparison operator and a value. Comparison operators include the following:

| Code | Description |
| :--- | :--- |
| $=$ | Equal to |
| $>$ | Greater than |
| $<$ | Less than |
| $>=$ | Greater than or equal to |
| $<=$ | Less than or equal to |
| $<>$ | Not equal to |

For example, the following format displays numbers that are less than or equal to 100 in a red font and numbers that are greater than 100 in a blue font:

$$
\text { [Red] }[<=100] \text {; [Blue] [>100] }
$$

If the cell value does not meet any of the criteria, then pound signs (\#) are displayed across the width of the cell.

## Codes for border styles

```
style
none
thin
medium
dashed
dotted
thick
double
hair
medium_dashed
dash_dot
medium_dash_dot
dash_dot_dot
medium_dash_dot_dot
slant_dash_dot
```


## Codes for fill pattern styles

```
pattern
none
solid
gray50
gray75
gray25
horstripe
verstripe
revdiagstripe
diagstripe
diagcrosshatch
thickdiagcrosshatch
thinhorstripe
thinverstripe
thinrevdiagstripe
thindiagstripe
thinhorcrosshatch
thindiagcrosshatch
gray12p5
gray6p25
```


## Codes for text rotation

| rotation | Meaning |
| :--- | :--- |
| $0-90$ | text rotated counterclockwise 0 to 90 degrees |
| $91-180$ | text rotated clockwise 1 to 90 degrees |
| 255 | vertical text |

## Format colors

color may be any of the color names listed below or an RGB (red, green, blue) value specified in double quotes ("255 255 255").

| aliceblue antiquewhite | deeppink deepskyblue |
| :---: | :---: |
| aqua | dimgray |
| aquamarine | dodgerblue |
| azure | firebrick |
| beige | floralwhite |
| bisque | forestgreen |
| black | fuchsia |
| blanchedalmond | gainsboro |
| blue | ghostwhite |
| blueviolet | gold |
| brown | goldenrod |
| burlywood | gray |
| cadetblue | green |
| chartreuse | greenyellow |
| chocolate | honeydew |
| coral | hotpink |
| cornflowerblue | indianred |
| cornsilk | indigo |
| crimson | ivory |
| cyan | khaki |
| darkblue | lavender |
| darkcyan | lavenderblush |
| darkgoldenrod | lawngreen |
| darkgray | lemonchiffon |
| darkgreen | lightblue |
| darkkhaki | lightcoral |
| darkmagenta | lightcyan |
| darkolivegreen | lightgoldenrodyellow |
| darkorange | lightgray |
| darkorchid | lightgreen |
| darkred | lightpink |
| darksalmon | lightsalmon |
| darkseagreen | lightseagreen |
| darkslateblue | lightskyblue |
| darkslategray | lightslategray |
| darkturquoise | lightsteelblue |
| darkviolet | lightyellow |


| lime | peru |
| :--- | :--- |
| limegreen | pink |
| linen | plum |
| magenta | powerblue |
| maroon | purple |
| mediumaquamarine | red |
| mediumblue | rosybrown |
| mediumorchid | royalblue |
| mediumpurple | saddlebrown |
| mediumseagreen | salmon |
| mediumslateblue | sandybrown |
| mediumspringgreen | seagreen |
| mediumturquoise | seashell |
| mediumvioletred | sienna |
| midnightblue | silver |
| mintcream | skyblue |
| mistyrose | slateblue |
| moccasin | snow |
| navajowhite | springgreen |
| navy | steelblue |
| oldlace | tan |
| olive | teal |
| olivedrab | thistle |
| orange | tomato |
| orangered | turquoise |
| orchid | violet |
| palegoldenrod | wheat |
| palegreen | white |
| paleturquoise | whitesmoke |
| palevioletred | yellow |
| papayawhip | yellowgreen |
| peachpuff |  |

Note: .xls files can only contain 56 unique colors.
fgcolor may be any color name specified in color or an RGB (red, green, blue) value specified in double quotes ("255 255 255").
bgcolor may be any color name specified in color or an RGB (red, green, blue) value specified in double quotes ("255 255 255").

## Also see

[M-5] _docx*() - Generate Office Open XML (.docx) file
[M-5] Pdf*() - Create a PDF file
[M-4] IO - I/O functions
[D] import excel - Import and export Excel files
[RPT] putdocx intro - Introduction to generating Office Open XML (.docx) files
[RPT] putexcel - Export results to an Excel file
[RPT] putpdf intro - Introduction to generating PDF files

## [M-6] Mata glossary of common terms

## Title

[M-6] Glossary

## Description

Commonly used terms are defined here.

## Mata glossary

## arguments

The values a function receives are called the function's arguments. For instance, in lud $(A, L, U)$, $A, L$, and $U$ are the arguments.

## array

An array is any indexed object that holds other objects as elements. Vectors are examples of 1dimensional arrays. Vector $\mathbf{v}$ is an array, and $\mathbf{v}[1]$ is its first element. Matrices are 2-dimensional arrays. Matrix $\mathbf{X}$ is an array, and $\mathbf{X}[1,1]$ is its first element. In theory, one can have 3-dimensional, 4-dimensional, and higher arrays, although Mata does not directly provide them. See [M-2] Subscripts for more information on arrays in Mata.

Arrays are usually indexed by sequential integers, but in associative arrays, the indices are strings that have no natural ordering. Associative arrays can be 1-dimensional, 2-dimensional, or higher. If $A$ were an associative array, then $A$ ["first"] might be one of its elements. See [M-5] asarray() for associative arrays in Mata.

## binary operator

A binary operator is an operator applied to two arguments. In $2-3$, the minus sign is a binary operator, as opposed to the minus sign in -9 , which is a unary operator.

## broad type

Two matrices are said to be of the same broad type if the elements in each are numeric, are string, or are pointers. Mata provides two numeric types, real and complex. The term broad type is used to mask the distinction within numeric and is often used when discussing operators or functions. One might say, "The comma operator can be used to join the rows of two matrices of the same broad type," and the implication of that is that one could join a real to a complex. The result would be complex. Also see type, eltype, and orgtype.

## c-conformability

Matrix, vector, or scalar $A$ is said to be c-conformable with matrix, vector, or scalar $B$ if they have the same number of rows and columns (they are $p$-conformable), or if they have the same number of rows and one is a vector, or if they have the same number of columns and one is a vector, or if one or the other is a scalar. c stands for colon; c-conformable matrices are suitable for being used with Mata's :op operators. $A$ and $B$ are c-conformable if and only if

| $A$ | $B$ |
| :---: | :---: |
| $r \times c$ | $r \times c$ |
| $r \times 1$ | $r \times c$ |
| $1 \times c$ | $r \times c$ |
| $1 \times 1$ | $r \times c$ |
| $r \times c$ | $r \times 1$ |
| $r \times c$ | $1 \times c$ |
| $r \times c$ | $1 \times 1$ |

The idea behind c-conformability is generalized elementwise operation. Consider $C=A: * B$. If $A$ and $B$ have the same number of rows and have the same number of columns, then $\left\|C_{i j}\right\|=\left\|A_{i j}{ }^{*} B_{i j}\right\|$. Now say that $A$ is a column vector and $B$ is a matrix. Then $\left\|C_{i j}\right\|=\left\|A_{i} * B_{i j}\right\|$ : each element of $A$ is applied to the entire row of $B$. If $A$ is a row vector, each column of $A$ is applied to the entire column of $B$. If $A$ is a scalar, $A$ is applied to every element of $B$. And then all the rules repeat, with the roles of $A$ and $B$ interchanged. See [M-2] op_colon for a complete definition.

## class programming

See object-oriented programming.

## colon operators

Colon operators are operators preceded by a colon, and the colon indicates that the operator is to be performed elementwise. $A: * B$ indicates element-by-element multiplication, whereas $A * B$ indicates matrix multiplication. Colons may be placed in front of any operator. Usually one thinks of elementwise as meaning $c_{i j}=a_{i j}<o p>b_{i j}$, but in Mata, elementwise is also generalized to include c-conformability. See [M-2] op_colon.

## column stripes

See row and column stripes.

## column-major order

Matrices are stored as vectors. Column-major order specifies that the vector form of a matrix is created by stacking the columns. For instance,

| : A |  |  |
| :---: | :---: | :---: |
|  | 1 | 2 |
|  | 1 4 <br> 1 2 <br> 2 5 <br> 3 3 |  |

is stored as

1 | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | 1 | 2 | 3 | 4 | 5 |

in column-major order. The LAPACK functions use column-major order. Mata uses row-major order. See row-major order.

## colvector

See vector, colvector, and rowvector.

## complex

A matrix is said to be complex if its elements are complex numbers. Complex is one of two numeric types in Stata, the other being real. Complex is generally used to describe how a matrix is stored and not the kind of numbers that happen to be in it: complex matrix $Z$ might happen to contain real numbers. Also see type, eltype, and orgtype.

## condition number

The condition number associated with a numerical problem is a measure of that quantity's amenability to digital computation. A problem with a low condition number is said to be well conditioned, whereas a problem with a high condition number is said to be ill conditioned.

Sometimes reciprocals of condition numbers are reported and yet authors will still refer to them sloppily as condition numbers. Reciprocal condition numbers are often scaled between 0 and 1 , with values near epsilon(1) indicating problems.

## conformability

Conformability refers to row-and-column matching between two or more matrices. For instance, to multiply $A * B, A$ must have the same number of columns as $B$ has rows. If that is not true, then the matrices are said to be nonconformable (for multiplication).

Three kinds of conformability are often mentioned in the Mata documentation: p-conformability, $c$-conformability, and $r$-conformability.

## conjugate

If $z=a+b \mathrm{i}$, the conjugate of $z$ is $\operatorname{conj}(z)=a-b \mathrm{i}$. The conjugate is obtained by reversing the sign of the imaginary part. The conjugate of a real number is the number itself.

## conjugate transpose

See transpose.

## data matrix

A dataset containing $n$ observations on $k$ variables in often stored in an $n \times k$ matrix. An observation refers to a row of that matrix; a variable refers to a column. When the rows are observations and the columns are variables, the matrix is called a data matrix.

## declarations

Declarations state the eltype and orgtype of functions, arguments, and variables. In

```
real matrix myfunc(real vector A, complex scalar B)
{
        real scalar i
}
```

the real matrix is a function declaration, the real vector and complex scalar are argument declarations, and real scalar i is a variable declaration. The real matrix states the function returns a real matrix. The real vector and complex scalar state the kind of arguments myfunc() expects and requires. The real scalar i helps Mata to produce more efficient compiled code.

Declarations are optional, so the above could just as well have read

```
function myfunc(A, B)
{
}
```

When you omit the function declaration, you must substitute the word function.
When you omit the other declarations, transmorphic matrix is assumed, which is fancy jargon for a matrix that can hold anything. The advantages of explicit declarations are that they reduce the chances you make a mistake either in coding or in using the function, and they assist Mata in producing more efficient code. Working interactively, most people omit the declarations.

See [M-2] Declarations for more information.

## defective matrix

An $n \times n$ matrix is defective if it does not have $n$ linearly independent eigenvectors.

## dereference

Dereferencing is an action performed on pointers. Pointers contain memory addresses, such as $0 \times 2 \mathrm{a} 1228$. One assumes something of interest is stored at $0 \times 2 \mathrm{a} 1228$, say, a real scalar equal to 2 . When one accesses that 2 via the pointer by coding $* p$, one is said to be dereferencing the pointer. Unary $*$ is the dereferencing operator.

## diagonal matrix

A matrix is diagonal if its off-diagonal elements are zero; $A$ is diagonal if $A[i, j]==0$ for $i!=j$. Usually, diagonal matrices are also square. Some definitions require that a diagonal matrix also be a square matrix.

## diagonal of a matrix

The diagonal of a matrix is the set of elements $A[i, j]$.

## dyadic operator

Synonym for binary operator.

## eigenvalues and eigenvectors

A scalar, $\lambda$, is said to be an eigenvalue of square matrix $\mathbf{A}: n \times n$ if there is a nonzero column vector $\mathbf{x}: n \times 1$ (called an eigenvector) such that

$$
\begin{equation*}
\mathbf{A} \mathbf{x}=\lambda \mathbf{x} \tag{1}
\end{equation*}
$$

Equation (1) can also be written as

$$
(\mathbf{A}-\lambda \mathbf{I}) \mathbf{x}=0
$$

where $\mathbf{I}$ is the $n \times n$ identity matrix. A nontrivial solution to this system of $n$ linear homogeneous equations exists if and only if

$$
\begin{equation*}
\operatorname{det}(\mathbf{A}-\lambda \mathbf{I})=0 \tag{2}
\end{equation*}
$$

This $n$ th-degree polynomial in $\lambda$ is called the characteristic polynomial or characteristic equation of $\mathbf{A}$, and the eigenvalues $\lambda$ are its roots, also known as the characteristic roots.

The eigenvector defined by (1) is also known as the right eigenvector, because matrix $\mathbf{A}$ is postmultiplied by eigenvector $\mathbf{x}$. See [M-5] eigensystem() and left eigenvectors.

## eltype

See type, eltype, and orgtype.

## epsilon(1), etc.

epsilon(1) refers to the unit roundoff error associated with a computer, also informally called machine precision. It is the smallest amount by which a number may differ from 1. For IEEE double-precision variables, epsilon(1) is approximately $2.22045 \mathrm{e}-16$.
epsilon $(x)$ is the smallest amount by which a real number can differ from $x$, or an approximation thereof; see [M-5] epsilon().

## exp

exp is used in syntax diagrams to mean "any valid expression may appear here"; see [M-2] exp.

## external variable

See global variable.

## frames

Frames, also known as data frames, are in-memory areas where datasets are analyzed. Stata can hold multiple datasets in memory, and each dataset is held in a memory area called a frame. A variety of commands exist to manage frames and manipulate the data in them. See [D] frames.

## function

The words program and function are used interchangeably. The programs that you write in Mata are in fact functions. Functions receive arguments and optionally return results.

Examples of functions that are included with Mata are sqrt(), ttail(), and substr(). Such functions are often referred to as the built-in functions or the library functions. Built-in functions refer to functions implemented in the C code that implements Mata, and library functions refer to functions written in the Mata programming language, but many users use the words interchangeably because how functions are implemented is of little importance. If you have a choice between using a built-in function and a library function, however, the built-in function will usually execute more quickly and the library function will be easier to use. Mostly, however, features are implemented one way or the other and you have no choice.

Also see underscore functions.
For a list of the functions that Mata provides, see [M-4] Intro.

## generalized eigenvalues

A scalar, $\lambda$, is said to be a generalized eigenvalue of a pair of $n \times n$ square numeric matrices $\mathbf{A}, \mathbf{B}$ if there is a nonzero column vector $\mathbf{x}: n \times 1$ (called a generalized eigenvector) such that

$$
\begin{equation*}
\mathbf{A} \mathbf{x}=\lambda \mathbf{B} \mathbf{x} \tag{1}
\end{equation*}
$$

Equation (1) can also be written as

$$
(\mathbf{A}-\lambda \mathbf{B}) \mathbf{x}=0
$$

A nontrivial solution to this system of $n$ linear homogeneous equations exists if and only if

$$
\begin{equation*}
\operatorname{det}(\mathbf{A}-\lambda \mathbf{B})=0 \tag{2}
\end{equation*}
$$

In practice, the generalized eigenvalue problem for the matrix pair $(\mathbf{A}, \mathbf{B})$ is usually formulated as finding a pair of scalars $(w, b)$ and a nonzero column vector $\mathbf{x}$ such that

$$
w \mathbf{A} \mathbf{x}=b \mathbf{B} \mathbf{x}
$$

The scalar $w / b$ is a generalized eigenvalue if $b$ is not zero.
Infinity is a generalized eigenvalue if $b$ is zero or numerically close to zero. This situation may arise if $\mathbf{B}$ is singular.

The Mata functions that compute generalized eigenvalues return them in two complex vectors, $\mathbf{w}$ and $\mathbf{b}$ of length $n$. If $\mathbf{b}[i]=0$, the $i$ th generalized eigenvalue is infinite, otherwise the $i$ th generalized eigenvalue is $\mathbf{w}[i] / \mathbf{b}[i]$.

## global variable

Global variables, also known as external variables and as global external variables, refer to variables that are common across programs and which programs may access without the variable being passed as an argument.

The variables you create interactively are global variables. Even so, programs cannot access those variables without engaging in another step, and global variables can be created without your creating them interactively.

To access (and create if necessary) global external variables, you declare the variable in the body of your program:

```
function myfunction(...)
{
        external real scalar globalvar
}
```

See Linking to external globals in [M-2] Declarations.
There are other ways of creating and accessing global variables, but the declaration method is recommended. The alternatives are crexternal(), findexternal(), and rmexternal() documented in [M-5] findexternal() and valofexternal() documented in [M-5] valofexternal().

## hashing, hash functions, and hash tables

Hashing refers to a technique for quickly finding information corresponding to an identifier. The identifier might be a name, a Social Security number, fingerprints, or anything else on which the information is said to be indexed. The hash function returns a many-to-one mapping of identifiers onto a dense subrange of the integers. Those integers, called hashes, are then used to index a hash table. The selected element of the hash table specifies a list containing identifiers and information. The list is then searched for the particular identifier desired. The advantage is that rather than searching a single
large list, one need only search one of $K$ smaller lists. For this to be fast, the hash function must be quick to compute and produce roughly equal frequencies of hashes over the range of identifiers likely to be observed.

## Hermitian matrix

Matrix $A$ is Hermitian if it is equal to its conjugate transpose; $A=A^{\prime}$; see transpose. This means that each off-diagonal element $a_{i j}$ must equal the conjugate of $a_{j i}$, and that the diagonal elements must be real. The following matrix is Hermitian:

$$
\left[\begin{array}{cc}
2 & 4+5 i \\
4-5 i & 6
\end{array}\right]
$$

The definition $A=A^{\prime}$ is the same as the definition for a symmetric matrix, although usually the word symmetric is reserved for real matrices and Hermitian, for complex matrices. In this manual, we use the word symmetric for both; see symmetric matrices.

## Hessenberg decomposition

The Hessenberg decomposition of a matrix, $\mathbf{A}$, can be written as

$$
\mathbf{Q}^{\prime} \mathbf{A Q}=\mathbf{H}
$$

where $\mathbf{H}$ is in upper Hessenberg form and $\mathbf{Q}$ is orthogonal if $\mathbf{A}$ is real or unitary if $\mathbf{A}$ is complex. See [M-5] hessenbergd( ).

## Hessenberg form

A matrix, $\mathbf{A}$, is in upper Hessenberg form if all entries below the first subdiagonal are zero: $A_{i j}=0$ for all $i>j+1$.

A matrix, $\mathbf{A}$, is in lower Hessenberg form if all entries above the first superdiagonal are zero: $A_{i j}=0$ for all $j>i+1$.

## instance and realization

Instance and realization are synonyms for variable, as in Mata variable. For instance, consider a real scalar variable $X$. One can equally well say that $X$ is an instance of a real scalar or a realization of a real scalar. Authors represent a variable this way when they wish to emphasize that $X$ is not representative of all real scalars but is just one of many real scalars. Instance is often used with structures and classes when the writer wishes to emphasize the difference between the values contained in the variable and the definition of the structure or the class. It is confusing to say that $V$ is a class $C$, even though it is commonly said, because the reader might confuse the definition of $C$ with the specific values contained in $V$. Thus careful authors say that $V$ is an instance of class $C$.

## istmt

An istmt is an interactive statement, a statement typed at Mata's colon prompt.

## $\mathrm{J}(\mathrm{r}, \mathrm{c}$, value)

J() is the function that returns an $r \times c$ matrix with all elements set to value; see [M-5] J(). Also, $J()$ is often used in the documentation to describe the various types of void matrices; see void matrix. Thus the documentation might say that such-and-such returns $\mathrm{J}(0,0,$.$) under certain conditions.$ That is another way of saying that such-and-such returns a $0 \times 0$ real matrix.

When $r$ or $c$ is 0 , there are no elements to be filled in with value, but even so, value is used to determine the type of the matrix. Thus $\mathrm{J}(0,0,1 i)$ refers to a $0 \times 0$ complex matrix, $\mathrm{J}(0,0$, $" ")$ refers to a $0 \times 0$ string matrix, and $J(0,0$, NULL) refers to a $0 \times 0$ pointer matrix.

In the documentation, $J()$ is used for more than describing $0 \times 0$ matrices. Sometimes, the matrices being described are $r \times 0$ or are $0 \times c$. Say that a function example $(X)$ is supposed to return a column vector; perhaps it returns the last column of $X$. Now say that $X$ is $0 \times 0$. Function example () still should return a column vector, and so it returns a $0 \times 1$ matrix. This would be documented by noting that example () returns $\mathrm{J}(0,1,$.$) when X$ is $0 \times 0$.

## LAPACK

LAPACK stands for Linear Algebra PACKage and forms the basis for many of Mata's linear algebra capabilities; see [M-1] LAPACK.

## left eigenvectors

A vector $\mathbf{x}: n \times 1$ is said to be a left eigenvector of square matrix $\mathbf{A}: n \times n$ if there is a nonzero scalar, $\lambda$, such that

$$
\mathbf{x} \mathbf{A}=\lambda \mathbf{x}
$$

## Ival

lval stands for left-hand-side value and is defined as the property of being able to appear on the left-hand side of an equal-assignment operator. Matrices are lvals in Mata, and thus

$$
x=\ldots
$$

is valid. Functions are not lvals; thus, you cannot code

```
substr(mystr,1,3) = "abc"
```

lvals would be easy to describe except that pointers can also be lvals. Few people ever use pointers. See [M-2] op_assignment for a complete definition.

## machine precision

See epsilon(1), etc.

## .mata source code file

By convention, we store the Mata source code for function function() in file function.mata; see [M-1] Source.

## matrix

The most general organization of data, containing $r$ rows and $c$ columns. Vectors, column vectors, row vectors, and scalars are special cases of matrices.

## .mlib library

The object code of functions can be collected and stored in a library. Most Mata functions, in fact, are located in the official libraries provided with Stata. You can create your own libraries. See [M-3] mata mlib.

## .mo file

The object code of a function can be stored in a .mo file, where it can be later reused. See [M-1] How and $[\mathrm{M}-3]$ mata mosave.

## monadic operator

Synonym for unary operator.

## NaN

NaN stands for Not a Number and is a special computer floating-point code used for results that cannot be calculated. Mata (and Stata) do not use NaNs. When NaNs arise, they are converted into . (missing value).

## norm

A norm is a real-valued function $f(x)$ satisfying

$$
\begin{array}{ll}
f(0) & =0 \\
f(x) & >0 \\
f(c x) & =|c| f(x) \\
f(x+y) & \leq f(x)+f(y)
\end{array} \quad \text { for all } x \neq 0
$$

The word norm applied to a vector $x$ usually refers to its Euclidean norm, $p=2$ norm, or length: the square root of the sum of its squared elements. The are other norms, the popular ones being $p=$ 1 (the sum of the absolute values of its elements) and $p=$ infinity (the maximum element). Norms can also be generalized to deal with matrices. See [M-5] norm().

## NULL

A special value for a pointer that means "points to nothing". If you list the contents of a pointer variable that contains NULL, the address will show as $0 x 0$. See pointer.

## numeric

A matrix is said to be numeric if its elements are real or complex; see type, eltype, and orgtype.

## object code

Object code refers to the binary code that Mata produces from the source code you type as input. See [M-1] How.

## object-oriented programming

Object-oriented programming is a programming concept that treats programming elements as objects and concentrates on actions affecting those objects rather than merely on lists of instructions. Objectoriented programming uses classes to describe objects. Classes are much like structures with a primary difference being that classes can contain functions (known as methods) as well as variables. Unlike structures, however, classes may inherit variables and functions from other classes, which in theory makes object-oriented programs easier to extend and modify than non-object-oriented programs.

## observations and variables

A dataset containing $n$ observations on $k$ variables in often stored in an $n \times k$ matrix. An observation refers to a row of that matrix; a variable refers to a column.

## operator

An operator is,+- , and the like. Most operators are binary (or dyadic), such as + in $A+B$ and $*$ in $C * D$. Binary operators also include logical operators such as \& and I ("and" and "or") in $E \& F$ and $G \mid H$. Other operators are unary (or monadic), such as ! (not) in ! J, or both unary and binary, such as - in $-K$ and in $L-M$. When we say "operator" without specifying which, we mean binary operator. Thus colon operators are in fact colon binary operators. See [M-2] exp.

## optimization

Mata compiles the code that you write. After compilation, Mata performs an optimization step, the purpose of which is to make the compiled code execute more quickly. You can turn off the optimization step-see [M-3] mata set-but doing so is not recommended.

## orgtype

See type, eltype, and orgtype.

## orthogonal matrix and unitary matrix

$A$ is orthogonal if $A$ is square and $A^{\prime} A==I$. The word orthogonal is usually reserved for real matrices; if the matrix is complex, it is said to be unitary (and then transpose means conjugate-transpose). We use the word orthogonal for both real and complex matrices.

If $A$ is orthogonal, then $\operatorname{det}(A)= \pm 1$.

## p-conformability

Matrix, vector, or scalar $A$ is said to be p-conformable with matrix, vector, or scalar $B$ if $\operatorname{rows}(A)==\operatorname{rows}(B)$ and $\operatorname{cols}(A)==\operatorname{cols}(B) . p$ stands for plus; p-conformability is one of the properties necessary to be able to add matrices together. p-conformability, however, does not imply that the matrices are of the same type. Thus ( $1,2,3$ ) is p-conformable with $(4,5,6)$ and with ("this", "that", "what") but not with (4\5\6).

## permutation matrix and permutation vector

A permutation matrix is an $n \times n$ matrix that is a row (or column) permutation of the identity matrix. If $P$ is a permutation matrix, then $P * A$ permutes the rows of $A$ and $A * P$ permutes the columns of $A$. Permutation matrices also have the property that $P^{-1}=P^{\prime}$.

A permutation vector is a $1 \times n$ or $n \times 1$ vector that contains a permutation of the integers 1 , $2, \ldots, n$. Permutation vectors can be used with subscripting to reorder the rows or columns of a matrix. Permutation vectors are a memory-conserving way of recording permutation matrices; see [M-1] Permutation.

## pointer

A matrix is said to be a pointer matrix if its elements are pointers.
A pointer is the address of a variable. Say that variable $X$ contains a matrix. Another variable $p$ might contain $137,799,016$ and, if $137,799,016$ were the address at which $X$ were stored, then $p$ would be said to point to $X$. Addresses are seldom written in base 10 , and so rather than saying $p$ contains $137,799,016$, we would be more likely to say that $p$ contains $0 x 836 a 568$, which is the way we write numbers in base 16. Regardless of how we write addresses, however, $p$ contains a number and that number corresponds to the address of another variable.

In our program, if we refer to $p$, we are referring to $p$ 's contents, the number 0x836a568. The monadic operator * is defined as "refer to the address" or "dereference": $* p$ means $X$. We could code $\mathrm{Y}=* \mathrm{p}$ or $\mathrm{Y}=\mathrm{X}$, and either way, we would obtain the same result. In our program, we could refer to $X[i, j]$ or ( $* p$ ) $[i, j]$, and either way, we would obtain the $i, j$ element of $X$.

The monadic operator \& is how we put addresses into $p$. To load $p$ with the address of $X$, we code $p=\& X$.

The special address 0 (zero, written in hexadecimal as $0 x 0$ ), also known as NULL, is how we record that a pointer variable points to nothing. A pointer variable contains NULL or it contains a valid address of another variable.

See [M-2] pointers for a complete description of pointers and their use.

## pragma

"(Pragmatic information) A standardised form of comment which has meaning to a compiler. It may use a special syntax or a specific form within the normal comment syntax. A pragma usually conveys non-essential information, often intended to help the compiler to optimise the program." See The Free On-line Dictionary of Computing, http://foldoc.org/, Editor Denis Howe. For Mata, see [M-2] pragma.

## rank

Terms in common use are rank, row rank, and column rank. The row rank of a matrix $A$ : $m \times n$ is the number of rows of $A$ that are linearly independent. The column rank is defined similarly, as the number of columns that are linearly independent. The terms row rank and column rank, however, are used merely for emphasis; the ranks are equal and the result is simply called the rank of A.

For a square matrix $A$ (where $m==n$ ), the matrix is invertible if and only if $\operatorname{rank}(A)==n$. One often hears that A is of full rank in this case and rank deficient in the other. See $[\mathrm{M}-5] \operatorname{rank}()$.

## r-conformability

A set of two or more matrices, vectors, or scalars $A, B, \ldots$, are said to be r-conformable if each is $c$ conformable with a matrix of max (rows $(A)$, rows $(B), \ldots$ ) rows and max (cols $(A)$, cols $(B)$, ...) columns.
r -conformability is a more relaxed form of c-conformability in that, if two matrices are c-conformable, they are r-conformable, but not vice versa. For instance, $A: 1 \times 3$ and $B: 3 \times 1$ are r-conformable but not c-conformable. Also, c-conformability is defined with respect to a pair of matrices only; r-conformability can be applied to a set of matrices.
r -conformability is often required of the arguments for functions that would otherwise naturally be expected to require scalars. See $R$-conformability in [M-5] normal() for an example.

## real

A matrix is said to be a real matrix if its elements are all reals and it is stored in a real matrix. Real is one of the two numeric types in Mata, the other being complex. Also see type, eltype, and orgtype.

## row and column stripes

Stripes refer to the labels associated with the rows and columns of a Stata matrix; see Stata matrix.

## row-major order

Matrices are stored as vectors. Row-major order specifies that the vector form of a matrix is created by stacking the rows. For instance,

is stored as

| 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 2 | 3 | 4 | 5 | 6 |

in row-major order. Mata uses row-major order. The LAPACK functions use column-major order. See column-major order.

## rowvector

See vector, colvector, and rowvector.

## scalar

A special case of a matrix with one row and one column. A scalar may be substituted anywhere a matrix, vector, column vector, or row vector is required, but not vice versa.

## Schur decomposition

The Schur decomposition of a matrix, $\mathbf{A}$, can be written as

$$
\mathbf{Q}^{\prime} \mathbf{A Q}=\mathbf{T}
$$

where $\mathbf{T}$ is in Schur form and $\mathbf{Q}$, the matrix of Schur vectors, is orthogonal if $\mathbf{A}$ is real or unitary if A is complex. See [M-5] schurd().

## Schur form

There are two Schur forms: real Schur form and complex Schur form.
A real matrix is in Schur form if it is block upper triangular with $1 \times 1$ and $2 \times 2$ diagonal blocks. Each $2 \times 2$ diagonal block has equal diagonal elements and opposite sign off-diagonal elements. The real eigenvalues are on the diagonal and complex eigenvalues can be obtained from the $2 \times 24$ diagonal blocks.

A complex square matrix is in Schur form if it is upper triangular with the eigenvalues on the diagonal.

## source code

Source code refers to the human-readable code that you type into Mata to define a function. Source code is compiled into object code, which is binary. See [M-1] How.

## square matrix

A matrix is square if it has the same number of rows and columns. A $3 \times 3$ matrix is square; a $3 \times 4$ matrix is not.

## Stata matrix

Stata itself, separate from Mata, has matrix capabilities. Stata matrices are separate from those of Mata, although Stata matrices can be gotten from and put into Mata matrices; see [M-5] st_matrix( ). Stata matrices are described in [P] matrix and [U] 14 Matrix expressions.

Stata matrices are exclusively numeric and contain real elements only. Stata matrices also differ from Mata matrices in that, in addition to the matrix itself, a Stata matrix has text labels on the rows and columns. These labels are called row stripes and column stripes. One can think of rows and columns as having names. The purpose of these names is discussed in [U] 14.2 Row and column names. Mata matrices have no such labels. Thus three steps are required to get or to put all the information recorded in a Stata matrix: 1) getting or putting the matrix itself; 2) getting or putting the row stripe from or into a string matrix; and 3) getting or putting the column stripe from or into a string matrix. These steps are discussed in [M-5] st_matrix().

## string

A matrix is said to be a string matrix if its elements are strings (text); see type, eltype, and orgtype. In Mata, a string may be text or binary and may be up to $2,147,483,647$ characters (bytes) long.

## structure

A structure is an eltype, indicating a set of variables tied together under one name. struct mystruct might be

```
struct mystruct {
    real scalar n1, n2
    real matrix X
}
```

If variable a was declared a struct mystruct scalar, then the scalar a would contain three pieces: two real scalars and one real matrix. The pieces would be referred to as a.n1, a.n2, and a.X. If variable $b$ were also declared a struct mystruct scalar, it too would contain three pieces, b.n1, b.n2, and b. X. The advantage of structures is that they can be referred to as a whole. You can code $\mathrm{a} . \mathrm{n} 1=\mathrm{b} . \mathrm{n} 1$ to copy one piece, or you can code $\mathrm{a}=\mathrm{b}$ if you wanted to copy all three pieces. In all ways, $a$ and $b$ are variables. You may pass a to a subroutine, for instance, which amounts to passing all three values.

Structures variables are usually scalar, but they are not limited to being so. If A were a struct mystruct matrix, then each element of A would contain three pieces, and one could refer, for instance, to $A[2,3] . \mathrm{n} 1, \mathrm{~A}[2,3] . \mathrm{n} 2$, and $\mathrm{A}[2,3] . \mathrm{X}$, and even to $\mathrm{A}[2,3] . \mathrm{X}[3,2]$.

See [M-2] struct.

## subscripts

Subscripts are how you refer to an element or even a submatrix of a matrix.
Mata provides two kinds of subscripts, known as list subscripts and range subscripts.
In list subscripts, $A[2,3]$ refers to the $(2,3)$ element of $A$. $A[(2 \backslash 3),(4,6)]$ refers to the submatrix made up of the second and third rows, fourth and sixth columns, of $A$.

In range subscripts, $A[|2,3|]$ also refers to the $(2,3)$ element of $A$. $A[|2,3 \backslash 4,6|]$ refers to the submatrix beginning at the $(2,3)$ element and ending at the $(4,6)$ element.

See [M-2] Subscripts for more information.

## symmetric matrices

Matrix $A$ is symmetric if $A=A^{\prime}$. The word symmetric is usually reserved for real matrices, and in that case, a symmetric matrix is a square matrix with $a_{i j}==a_{j i}$.

Matrix $A$ is said to be Hermitian if $A=A^{\prime}$, where the transpose operator is understood to mean the conjugate-transpose operator; see Hermitian matrix. In Mata, the ' operator is the conjugate-transpose operator, and thus, in this manual, we will use the word symmetric both to refer to real, symmetric matrices and to refer to complex, Hermitian matrices.

Sometimes, you will see us follow the word symmetric with a parenthesized Hermitian, as in, "the resulting matrix is symmetric (Hermitian)". That is done only for emphasis.

The inverse of a symmetric (Hermitian) matrix is symmetric (Hermitian).

## symmetriconly

Symmetriconly is a word we have coined to refer to a square matrix whose corresponding off-diagonal elements are equal to each other, whether the matrix is real or complex. Symmetriconly matrices have no mathematical significance, but sometimes, in data-processing and memory-management routines, it is useful to be able to distinguish such matrices.

## time-series-operated variable

Time-series-operated variables are a Stata concept. The term refers to op .varname combinations such as L.gnp to mean the lagged value of variable gnp. Mata's [M-5] st_data() function works with time-series-operated variables just as it works with other variables, but many other Stata-interface functions do not allow op.varname combinations. In those cases, you must use [M-5] st_tsrevar().

## titlecase

Titlecasing is a Unicode concept implemented in Mata in the ustrtitle() function. To "titlecase" a phrase means to convert to Unicode titlecase the first letter of each Unicode word. This is almost, but not exactly, like capitalizing the first letter of each Unicode word. Like capitalization, titlecasing letters is locale-dependent, which means that the same letter might have different titlecase forms in different locales. In some locales, the titlecase form of a letter is different than the capital form of that same letter. For example, in some locales, capital letters at the beginning of words are not supposed to have accents on them, even if that capital letter by itself would have an accent.

## traceback log

When a function fails-either because of a programming error or because it was used incorrectly-it produces a traceback log:

```
: myfunction(2,3)
            solve(): 3200 conformability error
            mysub(): - function returned error
        myfunction(): - function returned error
        <istmt>: - function returned error
r(3200);
```

The log says that solve() detected the problem—arguments are not conformable—and that solve() was called by mysub() was called by myfunction() was called by what you typed at the keyboard. See [M-2] Errors for more information.

## transmorphic

Transmorphic is an eltype. A scalar, vector, or matrix can be transmorphic, which indicates that its elements may be real, complex, string, pointer, or even a structure. The elements are all the same type; you are just not saying which they are. Variables that are not declared are assumed to be transmorphic, or a variable can be explicitly declared to be transmorphic. Transmorphic is just fancy jargon for saying that the elements of the scalar, vector, or matrix can be anything and that, from one instant to the next, the scalar, vector, or matrix might change from holding elements of one type to elements of another.

See [M-2] Declarations.

## transpose

The transpose operator is written different ways in different books, including ${ }^{\prime}$, superscript *, superscript $T$, and superscript $H$. Here we use the ' notation: $A^{\prime}$ means the transpose of $A, A$ with its rows and columns interchanged.

In complex analysis, the transpose operator, however it is written, is usually defined to mean the conjugate transpose; that is, one interchanges the rows and columns of the matrix and then one takes the conjugate of each element, or one does it in the opposite order-it makes no difference. Conjugation simply means reversing the sign of the imaginary part of a complex number: the conjugate of $1+2 i$ is $1-2 i$. The conjugate of a real is the number itself; the conjugate of 2 is 2 .

In Mata, ${ }^{\prime}$ is defined to mean conjugate transpose. Since the conjugate of a real is the number itself, $A^{\prime}$ is regular transposition when $A$ is real. Similarly, we have defined ${ }^{\prime}$ so that it performs regular transposition for string and pointer matrices. For complex matrices, however, ' also performs conjugation.

If you have a complex matrix and simply want to transpose it without taking the conjugate of its elements, see $[\mathrm{M}-5]$ transposeonly( ). Or code $\operatorname{conj}\left(A^{\prime}\right)$. The extra $\operatorname{conj}()$ will undo the undesired conjugation performed by the transpose operator.

Usually, however, you want transposition and conjugation to go hand in hand. Most mathematical formulas, generalized to complex values, work that way.

## triangular matrix

A triangular matrix is a matrix with all elements equal to zero above the diagonal or all elements equal to zero below the diagonal.

A matrix $A$ is lower triangular if all elements are zero above the diagonal, that is, if $A[i, j]==0, j$ $>i$.

A matrix $A$ is upper triangular if all elements are zero below the diagonal, that is, if $A[i, j]==0, j$ $<i$.

A diagonal matrix is both lower and upper triangular. That is worth mentioning because any function suitable for use with triangular matrices is suitable for use with diagonal matrices.

A triangular matrix is usually square.
The inverse of a triangular matrix is a triangular matrix. The determinant of a triangular matrix is the product of the diagonal elements. The eigenvalues of a triangular matrix are the diagonal elements.

## type, eltype, and orgtype

The type of a matrix (or vector or scalar) is formally defined as the matrix's eltype and orgtype, listed one after the other-such as real vector-but it can also mean just one or the other-such as the eltype real or the orgtype vector.
eltype refers to the type of the elements. The eltypes are

| real | numbers such as 1, 2, 3.4 |
| ---: | :--- |
| complex | numbers such as $1+2 \mathrm{i}, 3+0 \mathrm{i}$ |
| string | strings such as "bill" |
| pointer | pointers such as \&varname |
| struct | structures |
| numeric | meaning real or complex |
| transmorphic | meaning any of the above |

orgtype refers to the organizational type orgtype specifies how the elements are organized. The orgtypes are

```
    matrix two-dimensional arrays
    vector one-dimensional arrays
colvector one-dimensional column arrays
rowvector one-dimensional row arrays
    scalar single items
```

The fully specified type is the element and organization types combined, as in real vector.

## unary operator

A unary operator is an operator applied to one argument. In -2 , the minus sign is a unary operator. In ! $(a==b \mid a==c)$, ! is a unary operator.

## underscore functions

Functions whose names start with an underscore are called underscore functions, and when an underscore function exists, usually a function without the underscore prefix also exists. In those cases, the function is usually implemented in terms of the underscore function, and the underscore function is harder to use but is faster or provides greater control. Usually, the difference is in the handling of errors.

For instance, function fopen() opens a file. If the file does not exist, execution of your program is aborted. Function _fopen() does the same thing, but if the file cannot be opened, it returns a special value indicating failure, and it is the responsibility of your program to check the indicator and to take the appropriate action. This can be useful when the file might not exist, and if it does not, you wish to take a different action. Usually, however, if the file does not exist, you will wish to abort, and use of fopen() will allow you to write less code.

## unitary matrix

See orthogonal matrix.

## UTF-8

UTF-8 is the way of encoding Unicode characters chosen by Stata for its strings. It is backward compatible with ASCII encoding in the sense that plain ASCII characters are encoded the same in UTF-8 as in ASCII and that strings are still null terminated. Characters beyond plain ASCII are encoded using two to four bytes per character. As with other Unicode encodings, all possible Unicode characters (code points) can be represented by UTF-8.

## variable

In a program, the entities that store values $(a, b, c, \ldots, x, y, z)$ are called variables. Variables are given names of 1 to 32 characters long. To be terribly formal about it: a variable is a container; it contains a matrix, vector, or scalar and is referred to by its variable name or by another variable containing a pointer to it.

Also, variable is sometimes used to refer to columns of data matrices; see data matrix.

## vector, colvector, and rowvector

A special case of a matrix with either one row or one column. A vector may be substituted anywhere a matrix is required. A matrix, however, may not be substituted for a vector.

A colvector is a vector with one column.
A rowvector is a vector with one row.
A vector is either a rowvector or colvector, without saying which.

## view

A view is a special type of matrix that appears to be an ordinary matrix, but in fact the values in the matrix are the values of certain or all variables and observations in the Stata dataset that is currently in memory. Its values are not just equal to the dataset's values; they are the dataset's values: if an element of the matrix is changed, the corresponding variable and observation in the Stata dataset also changes. Views are obtained by st_view() and are efficient; see [M-5] st_view().

## void function

A function is said to be void if it returns nothing. For instance, the function [M-5] printf() is a void function; it prints results, but it does not return anything in the sense that, say, [M-5] sqrt() does. It would not make any sense to code $\mathrm{x}=\mathrm{printf}$ ("hi there"), but coding $\mathrm{x}=$ sqrt(2) is perfectly logical.

## void matrix

A matrix is said to be void if it is $0 \times 0, r \times 0$, or $0 \times c$; see [M-2] void.

## Also see

[M-0] Intro - Introduction to the Mata manual
[M-1] Intro - Introduction and advice

## Subject and author index

See the combined subject index and the combined author index in the Stata Index.


[^0]:    Also see
    [M-5] lapack( ) - Linear algebra package (LAPACK) functions
    [R] Copyright LAPACK - LAPACK copyright notification
    [M-1] Intro - Introduction and advice

[^1]:    Also see
    [M-5] epsilon() - Unit roundoff error (machine precision)
    [M-5] solve_tol() - Tolerance used by solvers and inverters
    [M-1] Intro - Introduction and advice

[^2]:    Also see
    [M-2] Declarations - Declarations and types
    [M-2] struct - Structures
    [P] class - Class programming
    [M-2] Intro - Language definition

[^3]:    Assignment suppresses display
    The equal-assignment operator
    lvals, what appears on the left-hand side
    Row, column, and element lvals
    Pointer lvals

[^4]:    Introduction
    Structures and functions must have different names
    Structure variables must be explicitly declared
    Declare structure variables to be scalars whenever possible
    Vectors and matrices of structures
    Structures of structures
    Pointers to structures
    Operators and functions for use with structure members
    Operators and functions for use with entire structures
    Listing structures
    Use of transmorphics as passthrus
    Saving compiled structure definitions Saving structure variables

[^5]:    Also see
    [M-2] Intro - Language definition

[^6]:    Also see
    [M-3] Imbuild - Easily create function library
    [M-3] mata mosave - Save function's compiled code in object file
    [M-3] Intro - Commands for controlling Mata

[^7]:    Detailed description
    Example 1: Scalar keys and scalar contents
    Example 2: Scalar keys and matrix contents
    Example 3: Vector keys and scalar contents; sparse matrix
    Setting the efficiency parameters

[^8]:    Also see
    [M-4] Mathematical - Important mathematical functions

[^9]:    Also see
    [M-5] optimize( ) - Function optimization
    [M-5] Quadrature( ) - Numerical integration
    [M-4] Mathematical - Important mathematical functions
    [M-4] Statistical - Statistical functions
    [R] set iter - Control iteration settings

[^10]:    Also see
    [M-5] qrinv() - Generalized inverse of matrix via QR decomposition
    [M-5] qrsolve( ) - Solve AX=B for X using QR decomposition
    [M-4] Matrix - Matrix functions

[^11]:    Derivation
    Relationship to inversion
    Tolerance

[^12]:    Step 1: Problem initialization
    Step 2: Problem definition
    Quadrature(): Definition of one integration problem
    QuadratureVec(): Definition of a vector of integration problems
    Step 3: Perform integration
    Step 4: Display or obtain results
    Quadrature()
    Quadrature Vec()
    Utility function for use in all steps
    Definition of $q$
    Functions defining the integration problem q.setEvaluator() and q.getEvaluator( )
    q.setLimits() and q.getLimits()
    q.setTechnique() and q.getTechnique()
    q.setMaxiter() and q.getMaxiter()

[^13]:    Derivation
    Tolerance

[^14]:    Value-label mapping
    Value-label creation and editing
    Loading value labels

[^15]:    Derivation Relationship to inversion Tolerance

[^16]:    Also see
    [M-2] op_transpose - Conjugate transpose operator
    [M-5] conj() - Complex conjugate
    [M-5] transposeonly () - Transposition without conjugation
    [M-4] Manipulation - Matrix manipulation

