Title

irt constraints — Specifying constraints

Description Quick start Syntax Remarks and examples Also see

Description

Constraints are imposed on the estimated parameters of a model. `irt` allows you to constrain a parameter to a fixed value or to constrain two or more parameters to be equal.

Quick start

2PL model for binary items `b1` to `b5`, with the discrimination parameters for items `b1` and `b2` constrained to be equal using symbolic constraints

```
irt ///
    (2pl b1, cns(a@k)) ///
    (2pl b2, cns(a@k)) ///
    (2pl b3-b5)
```

Same as above, but with the discrimination and difficulty parameters for items `b1` and `b2` constrained to fixed values

```
irt ///
    (2pl b1, cns(a@0.9 b@-1))  ///
    (2pl b2, cns(a@1.2 b@1.5)) ///
    (2pl b3-b5)
```

2PL model for binary items `b1` to `b5` and GRM for ordinal items `o1` to `o5` with discrimination parameters constrained to be equal for all items

```
irt ///
    (2pl b1-b5, cns(a@k)) ///
    (grm o1-o5, cns(a@k))
```

GRM for ordinal items `o1` to `o5` with the parameters of item `o1` constrained to fixed values

```
irt ///
    (grm o1, cns(a@1 b1@-1 b2@0 b3@1)) ///
    (grm o2-05)
```

1PL model for two groups with group-common items `b3-b7` and group-specific items `b1`, `b2`, `b8`, `b9` with the discrimination parameter constrained to be the same for all items

```
irt ///
    (0: 1pl b1 b2, cns(a@k)) ///
    (   1pl b3-b7, cns(a@k)) ///
    (1: 1pl b8 b9, cns(a@k)) ///
    , group(female)
```

Syntax

irt ... [, cns(*spec* [*spec* ...]) ...]

where *spec* is *parm*@*#* or *parm*@*symbol*.

In 1PL and 2PL models, *parm* is one of a or b, which corresponds to the discrimination or difficulty parameter in the IRT parameterization, or *parm* is one of alpha or beta, which corresponds to the slope or intercept in the slope-intercept parameterization.

In 3PL models, *parm* is one of a, b, or c, which corresponds to the discrimination, difficulty, or guessing parameter in the IRT parameterization, or *parm* is one of alpha or beta, which corresponds to the slope or intercept in the slope-intercept parameterization.

In nominal response models, *parm* is one of a1, a2, ... for the multiple discrimination parameters per item, or *parm* is one of b1, b2, ... for the multiple difficulty parameters per item.

In graded response, partial credit, and rating scale models, *parm* is a for the discrimination parameter, or *parm* is one of b1, b2, ... for the multiple difficulty parameters per item.

a is a synonym for a1, and b is a synonym for b1.

Remarks and examples

stata.com

Remarks are presented under the following headings:

Overview
Constraints in 1PL, 2PL, and 3PL models
Constraints in graded response models
Constraints in nominal response models
Constraints in partial credit models
Constraints in rating scale models

Overview

Two types of constraints can be specified in IRT models:

1. Fixed-value constraints set a parameter to a specific value. These constraints are specified by using @ and the value of the constraint, for example, a@1.5.
2. Symbolic constraints set parameters to be equal to each other. Symbolic constraints are specified by using @ and a name, for example, a@k1. Symbolic names are just names from 1 to 32 characters in length.

Constraints in 1PL, 2PL, and 3PL models

IRT models can be written using the IRT or the slope-intercept parameterization. Constraints in 1PL, 2PL, and 3PL models are applied to a, b, and c of the IRT parameterization. For instance, in *Methods and formulas* in [IRT] **irt 2pl**, we show that the 2PL model can be written in the IRT parameterization as

$$\Pr(Y_{ij}=1|a_i,b_i,\theta_j) = \frac{\exp\{a_i(\theta_j-b_i)\}}{1+\exp\{a_i(\theta_j-b_i)\}}$$

where a_i represents discrimination of item i and b_i represents the difficulty of item i. For 2PL models, we can specify constraints on these a_i and b_i parameters. Similarly, constraints on 1PL models can be applied to a and b_i parameters. The 3PL model also allows constraints on the c parameter; see *Methods and formulas* in [IRT] **irt 3pl** for information on the IRT parameterization of this model.

The rules for specifying constraints in the IRT parameterization are the following:

1. Both fixed-value and symbolic constraints are allowed on the discrimination parameter, a.

   For instance, to constrain the discrimination parameter to 0.8 in a 1PL model, we could type

   ```
   . irt 1pl q1-q10, cns(a@0.8)
   ```

   To constrain all discrimination parameters in a 2PL model to be equal, reducing it to a 1PL model, we type

   ```
   . irt 2pl q1-q10, cns(a@k1)
   ```

2. Fixed-value constraints are allowed on the difficulty parameter b when a fixed-value constraint is also set on the corresponding a.

   For instance, in a 2PL model, to constrain the discrimination parameter to 0.8 and the difficulty parameter to -2 for item q1, we could type

   ```
   . irt                         ///
       (2pl q1, cns(a@0.8 b@-2)) ///
       (2pl q2-q10)
   ```

   Notice that we use the hybrid syntax to separate the item on which we are placing constraints from items that have no parameter constraints; see [IRT] **irt hybrid**.

3. Both fixed-value and symbolic constraints are allowed on the guessing parameter, c, in the 3PL model.

   To constrain the guessing parameter for all items to 0.1, we could type

   ```
   . irt 3pl q1-q10, cns(c@0.1)
   ```

   To constrain the guessing parameter for q1 and q2 to be equal (and different from the common guessing parameter for q3-q10), we could type

   ```
   . irt                  ///
       (3pl q1, cns(c@k1)) ///
       (3pl q2, cns(c@k1)) ///
       (3pl q3-q10)
   ```

We can also set constraints on the α and β parameters in the slope-intercept parameterization. For instance, in *Methods and formulas* in [IRT] **irt 2pl**, we show that the slope-intercept parameterization of the 2PL model is

$$\Pr(Y_{ij} = 1|\alpha_i, \beta_i, \theta_j) = \frac{\exp(\alpha_i\theta_j + \beta_i)}{1 + \exp(\alpha_i\theta_j + \beta_i)}$$

where α_i is the slope for item i and β_i is the intercept for item i. 1PL and 3PL models have corresponding slope-intercept parameterizations.

Constraints on α and β are most often used in group IRT models to test for differential item functioning.

The rules for specifying constraints in the slope-intercept parameterization are the following:

1. Symbolic constraints are allowed on the slope parameter, α.

   To constrain the slope of item q1 to be equal across groups and let the intercepts vary across groups, we could type

```
. irt                            ///
    (0: 2pl q1, cns(alpha@k1)) ///
    (1: 2pl q1, cns(alpha@k1)) ///
    (2pl q2-q10)
```

2. Symbolic constraints are allowed on the intercept parameter, β.

   To constrain the intercept of item q1 to be equal across groups and let the slopes vary across groups, we could type

```
. irt                           ///
    (0: 2pl q1, cns(beta@k1)) ///
    (1: 2pl q1, cns(beta@k1)) ///
    (2pl q2-q10)
```

▷ Example 1: Fixed-value constraints in a 2PL model

Using masc1.dta, we could fit a 2PL model by typing

```
. use https://www.stata-press.com/data/r18/masc1
(Data from De Boeck & Wilson (2004))
. irt (2pl q1) (2pl q2 q3 q4)
```

Notice that we used a hybrid specification of our 2PL model with q1 separated from the other items. They hybrid syntax is commonly used when fitting models with constraints because it allows us to apply constraints to a chosen item or set of items; see [IRT] **irt hybrid** for information on this syntax.

We now constrain the discrimination to 1.5 and the difficulty to −0.5 for `q1` by adding the `cns(a@1.5 b@-0.5)` option.

```
. irt (2pl q1, cns(a@1.5 b@-.5)) (2pl q2 q3 q4)
Fitting fixed-effects model:
Iteration 0:   Log likelihood = -2042.1777
Iteration 1:   Log likelihood =  -2041.492
Iteration 2:   Log likelihood = -2041.4917
Iteration 3:   Log likelihood = -2041.4917
Fitting full model:
Iteration 0:   Log likelihood = -2000.6715
Iteration 1:   Log likelihood = -1995.1431
Iteration 2:   Log likelihood = -1995.0644
Iteration 3:   Log likelihood = -1995.0643
Hybrid IRT model                                     Number of obs = 800
Log likelihood = -1995.0643
```

	Coefficient	Std. err.	z	P>|z|	[95% conf.	interval]
2pl						
q1						
Discrim	1.5	(constrained)				
Diff	-.5	(constrained)				
2pl						
q2						
Discrim	.70149	.1469275	4.77	0.000	.4135175	.9894626
Diff	-.1455087	.1131457	-1.29	0.198	-.3672702	.0762528
q3						
Discrim	.9136072	.1960573	4.66	0.000	.5293419	1.297873
Diff	-1.726684	.2977765	-5.80	0.000	-2.310316	-1.143053
q4						
Discrim	.742854	.1556356	4.77	0.000	.4378138	1.047894
Diff	.3541431	.1220373	2.90	0.004	.1149543	.5933319

We see that the fixed-value constraints on `q1` appear in the first table in the output.

◁

▷ Example 2: Constraining slopes in a group 2PL model

IRT models are reported using the IRT parameterization where $b_i = -\beta_i/\alpha_i$ and $a_i = \alpha_i$. Typically, constraints are set on the a's and b's as in the previous example. Note, however, that to set a constraint on one of the b's, we need to constrain both the underlying α and β parameters. Therefore, we are required to set a constraint on a any time we set a constraint on b.

Sometimes, we instead set constraints directly on the β parameters of the slope-intercept metric. Constraints on β do not require constraints on corresponding α parameters. Constraints specified in this slope-intercept metric are most often used in the context of multiple-group models.

For example, in a group 2PL model, we may want to constrain the intercepts to be the same across groups while allowing the slopes to differ across groups. Using masc2.dta, we can fit a group 2PL model by typing

```
. use https://www.stata-press.com/data/r18/masc2
(Data from De Boeck & Wilson (2004))
. irt (0: 2pl q1) (1: 2pl q1) (2pl q2 q3 q4), group(female)
```

This model allows the slope and intercept for q1 to differ across groups and constrains the slopes and intercepts for all other items across groups; see [IRT] **irt, group()** for information on group IRT models.

Now we can constrain the intercept for item q1 to be equal across the two groups and allow only the slope of q1 to vary across groups by typing

```
. irt (0: 2pl q1, cns(beta@k1)) (1: 2pl q1, cns(beta@k1)) (2pl q2 q3 q4),
> group(female)
Fitting fixed-effects model:
Iteration 0:  Log likelihood = -3848.9104
Iteration 1:  Log likelihood = -3845.4263
Iteration 2:  Log likelihood = -3845.4252
Iteration 3:  Log likelihood = -3845.4252
Group: Male
Group: Female
Fitting full model:
Iteration 0:  Log likelihood = -3786.3332
Iteration 1:  Log likelihood = -3779.6101
Iteration 2:  Log likelihood = -3778.4145
Iteration 3:  Log likelihood = -3778.3648
Iteration 4:  Log likelihood = -3778.3646
Hybrid IRT model                                   Number of obs = 1,500
Log likelihood = -3778.3646
Group: Male
```

	Coefficient	Std. err.	z	P>|z|	[95% conf.	interval]
2pl						
q1						
Discrim	1.756291	.6093519	2.88	0.004	.5619834	2.950599
Diff	-.3976132	.0983202	-4.04	0.000	-.5903172	-.2049092
2pl						
q2						
Discrim	.7065962	.160211	4.41	0.000	.3925883	1.020604
Diff	-.0184657	.1007856	-0.18	0.855	-.2160018	.1790704
q3						
Discrim	.7360524	.1741027	4.23	0.000	.3948173	1.077287
Diff	-2.006872	.4360328	-4.60	0.000	-2.861481	-1.152264
q4						
Discrim	.602399	.1225595	4.92	0.000	.3621868	.8426112
Diff	.5431222	.1520137	3.57	0.000	.2451809	.8410635
mean(Theta)	0	(omitted)				
var(Theta)	1	(constrained)				

```
Group: Female
```

	Coefficient	Std. err.	z	P>|z|	[95% conf.	interval]
2pl						
q1						
Discrim	1.000017	.2965882	3.37	0.001	.4187151	1.581319
Diff	-.6983125	.2424141	-2.88	0.004	-1.173435	-.2231896
2pl						
q2						
Discrim	.7065962	.160211	4.41	0.000	.3925883	1.020604
Diff	-.0184657	.1007856	-0.18	0.855	-.2160018	.1790704
q3						
Discrim	.7360524	.1741027	4.23	0.000	.3948173	1.077287
Diff	-2.006872	.4360328	-4.60	0.000	-2.861481	-1.152264
q4						
Discrim	.602399	.1225595	4.92	0.000	.3621868	.8426112
Diff	.5431222	.1520137	3.57	0.000	.2451809	.8410635
mean(Theta)	-.0903098	.1215655	-0.74	0.458	-.3285738	.1479542
var(Theta)	1.295406	.4927367			.6146562	2.730106

We do not see the estimates of β in this output, but we can use the `estmetric` option to display the results in the slope-intercept parameterization.

```
. irt, estmetric

Hybrid IRT model                                    Number of obs = 1,500
Log likelihood = -3778.3646

 ( 1)  [q1]0bn.female - [q1]1.female = 0
 ( 2)  [q2]0bn.female - [q2]1.female = 0
 ( 3)  [q2]0bn.female#c.Theta - [q2]1.female#c.Theta = 0
 ( 4)  [q3]0bn.female - [q3]1.female = 0
 ( 5)  [q3]0bn.female#c.Theta - [q3]1.female#c.Theta = 0
 ( 6)  [q4]0bn.female - [q4]1.female = 0
 ( 7)  [q4]0bn.female#c.Theta - [q4]1.female#c.Theta = 0
 ( 8)  [/]mean(Theta)#0bn.female = 0
 ( 9)  [/]var(Theta)#0bn.female = 1
Group: Male                                           Number of obs = 761
```

	Coefficient	Std. err.	z	P>|z|	[95% conf. interval]	
q1						
Theta	1.756291	.6093519	2.88	0.004	.5619834	2.950599
_cons	.6983246	.1370853	5.09	0.000	.4296424	.9670067
q2						
Theta	.7065962	.160211	4.41	0.000	.3925883	1.020604
_cons	.0130478	.0709493	0.18	0.854	-.1260102	.1521058
q3						
Theta	.7360524	.1741027	4.23	0.000	.3948173	1.077287
_cons	1.477163	.0933292	15.83	0.000	1.294241	1.660085
q4						
Theta	.602399	.1225595	4.92	0.000	.3621868	.8426112
_cons	-.3271763	.0668258	-4.90	0.000	-.4581524	-.1962002
mean(Theta)	0	(omitted)				
var(Theta)	1	(constrained)				

```
Group: Female                                         Number of obs = 739
```

	Coefficient	Std. err.	z	P>|z|	[95% conf. interval]	
q1						
Theta	1.000017	.2965882	3.37	0.001	.4187151	1.581319
_cons	.6983246	.1370853	5.09	0.000	.4296424	.9670067
q2						
Theta	.7065962	.160211	4.41	0.000	.3925883	1.020604
_cons	.0130478	.0709493	0.18	0.854	-.1260102	.1521058
q3						
Theta	.7360524	.1741027	4.23	0.000	.3948173	1.077287
_cons	1.477163	.0933292	15.83	0.000	1.294241	1.660085
q4						
Theta	.602399	.1225595	4.92	0.000	.3621868	.8426112
_cons	-.3271763	.0668258	-4.90	0.000	-.4581524	-.1962002
mean(Theta)	-.0903098	.1215655	-0.74	0.458	-.3285738	.1479542
var(Theta)	1.295406	.4927367			.6146562	2.730106

Now we see that the equality constraints on the β parameters for q1 were imposed across groups. The β's are the intercepts labeled _cons with a value of 0.698.

◁

Constraints in graded response models

Constraints for the GRM differ from the 1PL, 2PL, and 3PL models because each item in a GRM has multiple b parameters. In the cns() option, we refer to these parameters as b1, b2, b3, ...

The rules for constraints in the IRT parameterization of the GRM are the following:

1. Both fixed-value and symbolic constraints are allowed on the discrimination parameter, a.

   For instance, to constrain the discrimination parameters for all items to 0.8, we could type

   ```
   . irt grm q1-q5, cns(a@0.8)
   ```

   To constrain all discrimination parameters to be equal, we type

   ```
   . irt grm q1-q5, cns(a@k1)
   ```

2. Fixed-value constraints are allowed on the difficulty parameters, b_1, b_2, . . . , when a fixed-value constraint is also set on the corresponding a.

   For instance, to constrain the discrimination parameter to 0.8 and the difficulty parameters to -2, -1, and 0 for item q1, we could type

   ```
   . irt                                    ///
       (grm q1, cns(a@0.8 b1@-2 b2@-1 b3@0)) ///
       (grm q2-q5)
   ```

Constraints in nominal response models

Constraints for the NRM differ from the GRM models because each item in an NRM has multiple a parameters in addition to multiple b parameters. In the cns() option, we refer to these a parameters as a1, a2, a3, ...

The rules for constraints in the IRT parameterization of the NRM are the following:

1. Both fixed-value and symbolic constraints are allowed on the discrimination parameters, a_1, a_2, ...

   For instance, to constrain the discrimination parameters for item q1 to 1 and 1.1, we could type

   ```
   . irt                         ///
       (nrm q1, cns(a1@1 a2@1.1) ///
       (nrm q2-q5)
   ```

   To constrain discrimination parameters for q1 and q2 to be equal, we type

   ```
   . irt                            ///
       (nrm q1 q2, cns(a1@k1 a2@k2) ///
       (nrm q3-q5)
   ```

2. Fixed-value constraints are allowed on the difficulty parameters, b_1, b_2, ..., when a fixed-value constraint is also set on the corresponding discrimination parameters a_1, a_2, ...

   For instance, to constrain the discrimination parameters to 1 and 1.1 and constrain the difficulty parameters to −1 and 1 for item q1, we could type

```
. irt                                       ///
    (nrm q1, cns(a1@1 a2@1.1 b1@-1 b2@1)) ///
    (nrm q2-q5)
```

Constraints in partial credit models

The PCM and GPCM have multiple difficulty parameters for each item. In the cns() option, we refer to these parameters as b1, b2, b3, ...

The rules for constraints in the IRT parameterization of the PCM and GPCM are the following:

1. Both fixed-value and symbolic constraints are allowed on the discrimination parameter, a.

   For instance, to constrain the discrimination parameter in a PCM to 0.8, we could type

```
. irt pcm q1-q5, cns(a@0.8)
```

   To constrain discrimination parameters on q1 and q2 to be equal in a GPCM, we type

```
. irt                          ///
    (gpcm q1 q2, cns(a@k1)) ///
    (gpcm q3-q5)
```

2. Fixed-value constraints are allowed on the difficulty parameters, b_1, b_2, ..., when a fixed-value constraint is also set on the corresponding a.

   For instance, to constrain the discrimination parameter to 0.8 and the difficulty parameters to −2, −1, and 0 for item q1 in a GPCM, we could type

```
. irt                                        ///
    (gpcm q1, cns(a@0.8 b1@-2 b2@-1 b3@0)) ///
    (gpcm q2-q5)
```

Constraints in rating scale models

The RSM has multiple difficulty parameters for each item. In the cns() option, we refer to these parameters as b1, b2, b3, ...

The rules for constraints in the IRT parameterization of the PCM are the following:

1. Both fixed-value and symbolic constraints are allowed on the discrimination parameter, a.

   For instance, to constrain the discrimination parameter to 0.8, we could type

```
. irt rsm q1-q5, cns(a@0.8)
```

2. Fixed-value constraints are allowed on the difficulty parameters, b_1, b_2, $\ldots$, when a fixed-value constraint is also set on the corresponding a and on all b's for the item.

   For instance, to constrain the discrimination parameter to 0.8 and the difficulty parameters to -2, -1, and 0 for item q1, we could type

```
. irt                                  ///
    (rsm q1, cns(a@0.8 b1@-2 b2@-1 b3@0)) ///
    (rsm q2-q5)
```

Also see

[IRT] **irt** — Introduction to IRT models

[IRT] **irt 1pl** — One-parameter logistic model

[IRT] **irt 2pl** — Two-parameter logistic model

[IRT] **irt 3pl** — Three-parameter logistic model

[IRT] **irt grm** — Graded response model

[IRT] **irt, group( )** — IRT models for multiple groups

[IRT] **irt hybrid** — Hybrid IRT models

[IRT] **irt nrm** — Nominal response model

[IRT] **irt pcm** — Partial credit model

[IRT] **irt rsm** — Rating scale model

For suggested citations, see the FAQ on citing Stata documentation.